CASE STUDIES OF SCHEDULING IN TRAFFIC SERVICES BY OPTIMIZATION TECHNIQUE

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Abstract
This study shows three applications of scheduling in traffic services: a vehicle assignment on a community bus system, a driver scheduling on a shuttle bus system, and taxi driver scheduling. These scheduling problems are not large-scale ones. Hence, we formulate each problem as an integer optimization problem and solve it by using an optimization software. We also discuss the practical availability of these scheduling problems in terms of modeling processes and the characteristics of the problems.

Keywords: vehicle assignment, crew scheduling, traffic service.

1. Introduction
The planning processes, and scheduling tools of traffic services with highly complex systems have been widely researched and developed. Line planning, timetabling, circulation, and crew scheduling for railway systems and for airline systems are well-known examples (Arabeyre, et al., 1969; Caprara, et al., 2007; Clausen, et al., 2010). Almost all real-world applications for these studies have dealt with large-scale systems because large reductions of cost are expected. On the other hand, a recent research on service science has discussed the need of frameworks to use scheduling techniques easily even for smaller-scale service industries (Shigeno and Ikegami, 2010). In addition, the current progress in software for optimization allows for some scheduling problems to be solved by mathematical optimization modeling. Hence, widespread case studies and observations of the characteristic for models are needed to recognize the importance of scheduling techniques in smaller industries.

We show three applications of scheduling in traffic service. We deal with a vehicle assignment on a community bus system, driver scheduling on a shuttle bus system, and taxi driver scheduling. These scheduling problems are not large-scale ones. Hence, we formulate each problem as an integer optimization problem and solve it by using an optimization software. We also discuss the practical availability of these scheduling problems in terms of modeling processes and characteristics of these problems.

2. Vehicle assignment on community bus
Many local governments provide community buses in low ridership areas of the regions. Bus routes, and timetables have often been repeatedly revised to improve passenger’s utility and reduce costs. In this study, given a timetable, we assign vehicles, which mean buses in this case, and drivers to routes such that working conditions are satisfied. Although we do not optimize routes and timetables, our results are useful to decide whether revisions of routes and timetables can be done within cost restrictions. We demonstrate the practical relevance of mathematical optimization techniques to this assignment problem.

2.1 Community bus system for our study
We first give a rough description for a community bus system we handled. This system provides bus services over five routes, which start and end at a main railway station in the city. The service time and the number of services scheduled on a day of each route are shown in Table 1.

<table>
<thead>
<tr>
<th>Route</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Service time (min.)</td>
<td>60</td>
<td>60</td>
<td>40</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td>The number of services</td>
<td>14</td>
<td>7</td>
<td>5</td>
<td>7</td>
<td>5</td>
</tr>
</tbody>
</table>

These bus services are provided by four vehicles that are the same model. Each vehicle runs a given combination of services by one driver during the whole day. Hence, in this case, an assignment of drivers to services is equivalent to an assignment to vehicles to services. Since every service finishes before 18:00, the driver’s working hours do not conflict with rules of employment. Thus, we need to consider only the following conditions for driver’s break time:

(1) A driver must be provided with a rest period of more than 65 minutes if working hours exceed five hours. An employer must prepare a place with rest facilities. In this case, more than 15 minutes are necessary in order to make a round-trip to the rest place. We refer to this
total 80 minutes break time as lunch time.

(2) A driver must take a break for more than 30 minutes if his driving hour without any breaks is more than four hours. This break can be divided into more than two terms which are longer than ten minutes.

2.2 Modeling for vehicle assignments

We formulate this vehicle (equivalent to driver) assignments as an integer optimization problem. Let D and V be sets of bus services scheduled on a day and of vehicles, respectively. Each bus service $d \in D$ has a departure time, $s_d$, and an arrival time, $f_d$, at the main railway station. Assume that $s_d$ and $f_d$ are given by minutes. For example, if $d$ departs at 7:20, then $s_d = 440$. We prepare two types of 0-1 variables: $x^B_{dv}$ (resp. $x^A_{dv}$) for whether vehicle $v$ covers service $d$ assigned before (resp. after) driver’s lunch time, and $y_v$ for whether vehicle $v$ is needed to use for this community bus system. Our vehicle assignment problem, without consideration for break time, is minimizing $\sum_{v \in V} y_v$ subject to the following constraints:

\[ \sum_{v \in V} (x^B_{dv} + x^A_{dv}) = 1 \quad (\forall d \in D), \tag{1} \]

\[ \sum_{d \in D} (x^B_{dv} + x^A_{dv}) \leq |D| \cdot y_v \quad (\forall v \in V), \tag{2} \]

\[ x^B_{dv} + x^A_{dv} \leq 1 \quad (\forall d, d' \in D \text{ with } s_d \leq s_{d'} \leq f_d, \forall v \in V). \tag{3} \]

Constraint (1) implies that each bus service $d$ is provided by exactly one vehicle. It is shown by Constraint (2) whether vehicle $v$ needs to provide some services. Constraint (3) represents that a vehicle is not assigned to two services which are provided at the same period. For lunch time, we need constraints

\[ f_d \cdot x^B_{dv} - s_{d'} \cdot x^B_{dv} - 300 < M \cdot (\sum_{v \in V} x^B_{dv} + (1 - x^B_{dv})) \quad (\forall d, d' \in D, \forall v \in V), \tag{4} \]

\[ s_d \cdot x^A_{dv} + M(1 - x^A_{dv}) \geq f_d - x^B_{dv} + (65 + 15) \quad (\forall d, d' \in D, \forall v \in V), \tag{5} \]

where $M$ is a sufficiently large number. Constraint (4) represents whether a driver assigned to vehicle $v$ needs a lunch time, and constraint (5) implies that the length of the lunch time must be more than 80 minutes. For break time constraints, we use additional five types of 0-1 variables: $z^B_v$ (resp. $z^B_v$) for whether a driver assigned to vehicle $v$ needs to take a break before (resp. after) his lunch time, and $r_{dv}^B$ (resp. $r_{dv}^A$) for whether a driver assigned to vehicle $v$ takes a break of 10, 15, 20, and 30 minutes, respectively, following service $d$ that is provided before (resp. after) his lunch time.

\[ f_d \cdot x^B_{dv} - s_{d'} \cdot x^B_{dv} - 240 < M \cdot (1 - x^B_{dv} + z^B_v) \quad (\forall d, d' \in D, \forall v \in V, \forall k \in \{B, A\}), \tag{6} \]

\[ 30z^B_v \leq \sum_{v \in V} (10c^B_{dv} + 15p^B_{dv} + 20q^B_{dv} + 30r^B_{dv}) \quad (\forall v \in V, \forall k \in \{B, A\}), \tag{7} \]

\[ o_{dv}^B + p_{dv}^B + q_{dv}^B + r_{dv}^B \leq x^B_{dv} \]

\[ M(1 - o_{dv}^B) \geq \sum_{d' \in D} x^B_{dv} \]

\[ M(1 - p_{dv}^B) \geq \sum_{d' \in D} x^B_{dv} \]

\[ M(1 - q_{dv}^B) \geq \sum_{d' \in D} x^B_{dv} \]

\[ (\forall d \in D, \forall v \in V, \forall k \in \{B, A\}), \tag{8} \]

\[ x^B_{dv} + o_{dv}^B + p_{dv}^B + q_{dv}^B + r_{dv}^B - 1 \leq \sum_{d' \in D} x^B_{dv} \]

\[ (\forall v \in V, \forall k \in \{B, A\}), \tag{9} \]

where $D(d, \ell) = \{d' \in D \mid f_d \leq s_{d'} < f_d + \ell\}$. Constraint (6) implies that whether a driver assigned to vehicle $v$ needs to take a break with more than 30 minutes, which is restricted by Constraint (7). Constraint (8) represents that $o_{dv}^B$, $p_{dv}^B$, $q_{dv}^B$, and $r_{dv}^B$ follow providing service $d$. Constraint (9) is needed to forbid taking a break after the last service before the lunch time or the whole of a day.

In addition, some services need to be provided by the same bus for transitions between routes. The formulation described in this section can be used to solve our vehicle assignments in about 20 seconds using Xpress-Optimizer. This formulation is most appropriate for our case than two other formulations we tried. One of the other formulation uses 0-1 variables for whether bus service $d$ follows bus service $d'$ by the same vehicle. The other one uses 0-1 variables for whether vehicle $v$ provides bus service $d$ at time $t$.

2.3 Practical relevance

Since community bus systems usually are not so large like the system we treated, the vehicle assignment problem may be solved in practical time by using an optimization solver. This problem is useful when we consider revisions of routes and timetables. For example, we checked whether a little revision of the given departure time of a bus schedule to make better connection with a railway could be done without any increment of vehicles.

On the other hand, modeling for this problem needs to be customized for each community bus system and each driver’s working rule. For instance, in another community bus system, a vehicle is shared by several drivers during a day. In addition, we may need to consider fairness between
drivers. Thus, our further work is to develop a model which can be easily customized.

3. Driver scheduling on shuttle bus

Some successes of traffic service industry pay attention to not only safety but also their service quality. In the case of shuttle buses, high quality services can be provided by drivers who can offer the service desired by customers. Hence, it is important to make a scheduling for drivers so that the appropriate ones are assigned to customers. Our study deals with this scheduling problem, and reports how well optimization techniques can solve it.

3.1 Shuttle bus system for our study

Shuttle buses provide charter buses to transfer employees and students from a railway station to companies, factories and schools. These bus services are held regularly at fixed times. The company we studied makes “service groups” by combinations of bus services which can be provided by a driver on a day so that the number of vehicles needed is minimized. Given these service groups, fleet managers decide driver schedules and assign drivers to service groups in compliance of government legislation and of working conditions. In addition, the company has driver classifications I, II, and III, and customer attributes A, B, and C, to keep the quality of service high. A driver belonging to classification I can provide services for customers in any attributes. Managers consider that a driver in classification II has to provide services for customers in attributes B and C, and that a driver in III has to be assigned to customers in C. Moreover, it is ideal that fixed drivers provide service for a customer. The many conditions listed above complicate the process of making driver schedules. Indeed, it takes about nine hours to prepare a driver schedule for a week.

3.2 Modeling for driver scheduling

Our study reschedules the driver scheduling and investigates the practical applicability of optimization techniques. We use a schedule executed between June 1st and July 26th in 2009. There are 88 drivers, where 31 drivers are in classification I, 56 drivers are in II, and one driver is in III. There are 86 service groups. For each service group, we give attribute A if it contains customers in A, and give attribute B if it contains customers in only B and C. The remaining service groups are attributed to C. In this case, there are 5, 61, and 20 service groups in A, B, and C, respectively.

The rescheduling problem is formulated as an integer optimization problem, where the following conditions are satisfied:

C1 working hours and rest periods, which are given by a regulation of the Ministry of Health, Labour and Welfare, hold.

C2 drivers classification II are not assigned to service groups in A.

C3 drivers classification III are assigned to only service groups in C.

C4 drivers work on days which are duty days in the original schedule.

The objective function considers the following items

O1 minimizing the difference of each driver’s working hours between those in the original schedule and those in our revised schedule.

O2 minimizing the number of service groups provided by each driver.

We solve the rescheduling problem in every one-week period. After solving the first week, we use the results of previous weeks when solving problems.

We now show our rescheduling results. As shown in Table 2, our result gives more desirable assignments than the original schedule, because consistency with driver’s classifications and customer’s attributes is treated as constraints. Although our rescheduling does not change works of almost drivers, schedules of several drivers are modified for the consistency of matchings between driver’s classifications and customer’s attributes. This change influences working hours for several drivers, and the number of service groups provided by each driver. Figure 1 shows the difference of working hours during two months between in the original schedule and the reschedule. Figure 2 shows the difference of the number of service groups provided during two months between in the original schedule and the reschedule.

Table 2 Consistency with driver’s classifications and customer’s attributes

<table>
<thead>
<tr>
<th>original</th>
<th>Service group</th>
<th>result</th>
<th>Service group</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>A  8 B  37 C  17</td>
<td>I</td>
<td>A  9 B  51 C  14</td>
</tr>
<tr>
<td>II</td>
<td>A  9 B  71 C  25</td>
<td>II</td>
<td>A  0 B  69 C  26</td>
</tr>
<tr>
<td>III</td>
<td>A  0 B  1 C  1</td>
<td>III</td>
<td>A  0 B  6 C  6</td>
</tr>
</tbody>
</table>

Fig. 1 Graph representing each numbers of drivers whose working hours during two months are increased corresponding label by rescheduling.
3.3 Practical indicator

Our result is evaluated in the consistency with driver’s classifications and customer’s attribute. However, it needs to be improved in the dispersion of the working hours, and so on. In particular, we solve the problem for one-week period in more than two hours by using Xpress-Optimizer. Thus, to construct a scheduling support system, we must divide the problem into some parts, or develop heuristic algorithms. On the other hand, even though computing takes a long time, exact optimal solutions are needed when we want to know the sufficient number of drivers belonging to each classification, and the desirable number of paid vacations for each driver to provide high quality services. Providing such management indicators by modifying our model is very valuable.

4. Taxi driver scheduling

After the deregulation of taxi industry in Japan, the number of taxis increased and that per-vehicle daily operating revenues of taxis declined. Recently, the government re-regulated the taxi industry including the decrease of vehicles. Hence, taxi companies have to make work schedules of drivers with assignments of limited vehicles in accordance with passenger demand. Our study models taxi driver scheduling as optimization problems and examines its possibilities.

4.1 ABCDE diagram for taxi drivers

Shift types for taxi drivers differ with each company. We study a shift type called “ABCDE diagram” or two-day shift, which repeats the working pattern described in Table 3. In this shift type, two vehicles, 1 and 2 are shared by five drivers, A, B, C, D, and E. In Table 3, a sign [1] (resp. [2]) stands for a duty day in which the corresponding driver uses vehicle 1 (resp. 2). At a duty day, drivers usually work about 21 hours, until next early morning. Hence, the next day of duty is given for rest. By repeating this shift pattern, all vehicles always operate efficiently, if there are enough drivers. Furthermore, it has the following merits.

(1) During a period, all drivers work the same number of days. In addition, all drivers have the same numbers of duty days on each duty day of week.

(2) Since drivers sharing a single vehicle are limited, it is easy to take care of the vehicle.

(3) Drivers can notice his working days easily because of the circularity of the shift.

On the other hand, many companies cannot operate vehicles efficiently because of a lack of drivers. The lack of drivers periodically derives vacant vehicles as a result of shifts assigned no driver, which occur independently from the variance of income by days of week. To maximize efficiency, we modify the shift pattern to make several drivers work on days with higher profits.

4.2 Modeling for improved ABCDE diagrams

To clarify driver’s working conditions, we establish the following rules.

R1 The next day of a duty day has to be for rest.

R2 There are exactly two duty days during a consecutive five-days period.

R3 There are more than 12 duty days a month.

Many managers of taxi companies regard fairness and periodicity of working schedules as important. Thus, the following rule is given.

R4 For each day of week, differences of the numbers of duty days between drivers are at most one during a given period.

To maintain periodicity, we make a schedule based on the ABCDE diagram because one that is not based on the ABCED diagram shown in Figure 3 has not been accepted by some managers. We consider the following additional rules for fairness.

R5 During a given period, differences of the numbers of changes from the ABCDE diagram between drivers are at most one.

R6 During a given period, all drivers have the same number of consecutive holidays that last more than three days.
Under the above rules, we determine a schedule for drivers by formulating an integer optimization problem. Although an assignment between drivers and vehicles is important in order to take care of vehicles, we decide its assignment in accordance with the ABCDE diagram after the determination of working days. Examples of result for the first month during six months, where there are 20 drivers (A−T) who share 10 vehicles (1−10), are shown in Figures 3 and 4. The number in the tables stands for a vehicle assigned for the corresponding driver and "−−" implies rest or off. For efficiency, we suppose that managers want to increase the number of drivers working on Friday as much as possible, and to reduce the number of drivers working on Sunday. In addition, suppose that managers consider that more than seven drivers need to work in each day.

Fig. 3 Result of driver scheduling which satisfies rules R1-R4 and is not based on the ABCDE diagram

Fig. 4 Result of driver scheduling based on the ABCDE diagram and satisfying rules R1-R6

4.3 Practical applicability

We can schedule drivers by considering the demand of each day of week. Our model can be extend to include holidays, events when many people gather, and so on. Moreover, it can be used to consider holidays desired by each driver. Hence, our model has also flexibility. However, some managers tend to consider that various requirements of fairness are needed, and this leads to lower effectiveness. This tendency is caused by the fact that the salaries of the taxi driver are by a percentage pay system. Therefore, for practical use, we conclude that we should not only make a schedule by optimization but also provide by a simulation that helps managers and drivers to understand fairness and efficiency.

5. Conclusion

This study formulated three applications of scheduling in traffic service as integer optimization problems and solved the problems by an optimization solver. The first application is a vehicle assignment in a community bus system. By using optimization technique, we could easily know whether some modifications of a timetable can be done without a supplementary vehicle. The second application is a driver assignment in a shuttle bus system. In this case, optimization technique could furnish an desirable schedule. We will need to reduce computational time to solve this assignment problem for practical use. The last application is a taxi driver scheduling. This case showed that optimization is one of tools to overcome the difficulty of balance between fairness of drivers and effectiveness of income by days.

Our problems addressed here are rather small sizes. Thus, to make a schedule manually has been possible. However, introducing of optimization technique can make more effective schedule as shown in our case studies. It will be important in future to adapt optimization technique for more cases in traffic service system even if it is small size.

References


