THE DEVELOPMENT OF MULTI-AXIAL CREEP DAMAGE CONSTITUTIVE EQUATIONS FOR 0.5Cr0.5Mo0.25V FERRETTIC STEEL AT 590°C

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ABSTRACT

Within the framework of phenomenological approach a set of multi-axial creep damage constitutive equations for 0.5Cr0.5Mo0.25V ferritic steel at 590°C is developed in which a new formulation was employed. The deficiency of previous formulation and the need for improvement became apparent after a critical review of the development of creep damage constitutive equations for 316 stainless steel [1]. The need for improvement was further underpinned by a call for modification of constitutive equations. Recently, a specific formulation was proposed and validated [2-4]. This paper reports the latest development conducted by this author in the development of multi-axial creep constitutive equations for 0.5Cr0.5Mo0.25V ferritic steel at 590°C including: 1) the fundamental requirement; 2) formulation; 3) validation; and 4) conclusion. It systematically shows the suitability of this new set of constitutive equation and incapability of previous one. Furthermore, it contributes knowledge to the methodology.

Key words: creep damage constitutive equations, formulation, multi-axial states of stress, validation

1 INTRODUCTION

Creep damage phenomena in material and structural members are one of the serious problems in our modern life and have been widely studied [5-48]. Creep continuum damage mechanics, originally proposed by Kachanov in 1958, has been developed with the objective of dealing with engineering design. It is evident that significant progress has been made in various aspects including theory (such as phenomenological approach, unified irreversible thermodynamic formulation [7, 31], anisotropic damage theory [28-29]), applications [11-26], experimental observation and verification [34-37] and experimental methodology [38], and even optimisation. Creep continuum damage mechanics, as well as the damage mechanics as a whole, is developing rapidly with large number of publications produced. However, it is consciously important, if not more, to distil information and to assess and refine both the generic methodology and a specific set of constitutive equations developed to ensure the general applicability. That is the main concern of this paper.

The phenomenological approach was originated by Kachanov (1958) and it can be broadly classified into weak coupling and strong coupling between damage and deformation. In case of a weak coupling the effect of material damage on a elastic properties is disregarded and a coupling is established by introducing the damage variables into the constitutive equation of the continuum solids with the effective state variables concept is used (cf. Kachanov [12-13]; Rabotnov [14-15]; Leckie [42-43]; Hayhurst [18-19]; Leckie and Hayhurst [17, 44]; etc.). In case a fully (strong) coupled approach, damage evolution affects both elastic properties of the material and inelastic response [28-29].

Within the weak approach, a set of creep damage constitutive equations for uni-axial tension is developed first with consideration of the different creep mechanisms and coupling between the creep damage and creep strain and then be generalised for multi-axial states of stress [16-19, 42-44]. The success of this approach relies crucially on the development of a set of appropriate constitutive equations capable of depicting the observed multi-axial behaviour. This generalisation from a uni-axial one to a multi-axial one is directed by the relevant theory developed/adopted. A set of constitutive equations developed should be validated properly to ensure its general applicability.

0.5Cr0.5Mo0.25V ferritic steel is an important material in industry, say, power generation. A class of KRH (Kachanov-Rabotnov-Hayhurst) type constitutive equations for a range of temperature was developed for this material by Perrin and Hayhurst at 1996. That knowledge was immediately used by this author to produce a specific set constitutive equations at 590°C which, together with constitutive equations for weld material, heat effect zone, and Type IV, was used to predict damage evolution and failure in welded vessels. However, further development was initially motivated by a realisation of the deficiency existed in KRH's approach after a critical review [1] which was underpinned by the recently reported discrepancy between predicted damage and experimentally observed one with a call for modification of constitutive equations made by other researchers [35]. The conceptual development was partially inspired by the general plasticity theory. This paper reports the latest development conducted by this author in the development of creep constitutive equations for multi-axial state of stress for 0.5Cr0.5Mo0.25V ferritic steel at 590°C including: 1) the two fundamental requirement; 2) formulation; 3) validation; and 4) conclusion. This paper systematically shows the inadequacy of previous formulation and the suitability of new set constitutive equations in general engineering analysis. Furthermore, it contributes knowledge to the methodology.

2 REQUIREMENT [1]

The two fundamental requirements for the development of a set of creep damage constitutive equations are creep strain rate consistency and damage evolution consistency. This means that the predicted creep strain rate and damage evolution by a set of constitutive equations under various states of stress should be consistent with the experimental observations. Thus, the development of a set of creep damage constitutive equations is to fit the creep strain surfaces and damage evolution surfaces in stress states - time space, if the loading is proportional. It could be thought as mathematically a problem of two-objective optimisation and mechanically a damage evolution and creep deformation.

Integration a specific set of creep damage constitutive equations from virgin material to failure will produce:
\[ \omega = \omega_f \]
\[ t = t_{\text{multi}} \]
\[ \bar{\epsilon} = \bar{\epsilon}_{\text{multi}} \]

where \( \omega_f \) is the critical value of damage and \( \bar{\epsilon} \) is effective creep strain.

The general applicability of a set of multi-axial constitutive equations is ensured through validation. It is addressed that both lifetime and strain at failure should be considered and a wider range of stress states and loading conditions should be included. 

3 FORMULATION

3.1 Uni-axial Form

The uni-axial form creep damage constitutive equations for 0.5Cr0.5Mo0.25V ferritic steel at 590°C are written as [22-23]:

\[ \dot{\epsilon} = \frac{h}{\sigma} (1 - H) \bar{\epsilon}_e \]  

(2)

\[ H = \frac{K_e}{3} (1 - \phi)^4 \]  

(3)

\[ \dot{\phi} = \frac{2}{3} \phi \frac{\partial \phi}{\partial 

where \( A, B, C, h, H^* \) and \( K_e \) are material parameters. \( H \) represents the strain hardening that occurs during primary creep; initially \( H \) is zero and as strain is accumulated, increases to the value of \( H^* \). \( \Phi \) describes the evolution of the spacing of the carbide precipitates which is known to lead a progressive loss in the creep resistance of particle hardened alloys as ferritic steels. \( \bar{\epsilon} = \sqrt{2/3 \epsilon_y \epsilon_{\text{ij}}} \) is the rate of effective creep strain while \( \omega \) represents intergranular cavitation damage and varies from zero, for the material in virgin state, to \( \omega_f = 1/3 \), when all the grain boundaries normal to the applied stress have completely cavitated [23], at which time the material is considered to have failed.

3.2 KRH Formulation

The multi-axial KRH form is written as [22-23]:

\[ \dot{\epsilon}_{\text{ij}} = \frac{3}{2 \sigma_e} \text{Asinh}(B_{\text{ij}}(1 - H)) \]  

(6)

\[ H = \frac{K_e}{3} (1 - \phi)^4 \]  

(7)

\[ \dot{\phi} = \frac{2}{3} \phi \frac{\partial \phi}{\partial 

\[ \dot{\omega} = C \epsilon_v (\sigma_1/\sigma_e)^{v} \]  

(9)

where \( \phi \) is heavy step function and \( \nu \) is stress state index defining the multi-axial stress rupture criterion. The adopted function of states of stress \( (\sigma_1/\sigma_e)^{v} \) was originally proposed by Cane according to Perrin [23]. The value of 2.8 for \( \nu \) is obtained through calibration of isochronous rupture loci in plane stress condition and a couple of multi-axial tests [23]. A length discussion on its determination can also be found in [23].

For completeness the material constants for this material at 590°C are given below:

\[ A = 2.1618 \times 10^4 \text{ MPa h}^{-1} \]
\[ B = 0.20524 \text{ MPa}^{-2} \]
\[ C = 1.8557 \]
\[ h = 2.4326 \times 10^8 \text{ MPa} \]
\[ H^* = 0.5929 \]
\[ \text{K_e} = 9.2273 \times 10^3 \text{ MPa} \text{ h}^{-1} \]
\[ \nu = 2.8 \]

The creep curves and damage evolutions are shown in Fig.1.

3.3 New Formulation

The new form for constitutive equations is given as [2-3]:

\[ \dot{\epsilon}_{\text{ij}} = \frac{3}{2 \sigma_e} \text{Asinh}(B_{\text{ij}}(1 - H)) \]  

(10)

\[ H = \frac{h}{\sigma_e} (1 - \frac{H}{H^*}) \bar{\epsilon}_e \]  

(11)

\[ \dot{\phi} = \frac{K_e}{3} (1 - \phi)^4 \]  

(12)

\[ \dot{\omega} = C \epsilon_v (\sigma_1/\sigma_e)^{v} \]  

(13)

\[ \dot{\omega} = \omega \cdot f_1 \]  

(14)

where \( f_1 \) and \( f_2 \) functions of stress states. The function \( f_2 \) is introduced to depict the effect of states of stress on the damage evolution, which was employed in previous formulation. The additional function \( f_1 \) is introduced to better phenomenologically depict the coupling between damage and tertiary deformation and creep rupture. The physical implication is that creep deformation and creep rupture are two different processes and two separate internal variables are needed.

3.4 Specific Form of \( f_1 \) and \( f_2 \)

The specific form for function \( f_1 \) was a Huddleston’s formulation for strength theory given as [33]:

\[ f_1 = \frac{2 \sigma_e}{3 S_f} \text{exp} \left[ \frac{3 \sigma_m}{S_f} - 1 \right] \]  

(15)

where \( S_f = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2} \) and \( S_f = \sigma_1 - \sigma_m \).

The Spindler’s relation of strain at failure is adopted for function \( f_2 \): 

\[ f_2 = \left( \text{exp} \left[ \frac{1}{2} \frac{\gamma}{\sigma_1} \right] + q \frac{1}{2} \frac{3 \sigma_m}{2 \sigma_e} \right)^{-1} \]  

(16)

which equals to 1 uni-axial condition and coincides with Rice & Tracey’s equation if \( q = 0 \) and \( q = 1 \). Details for this equation can be found in [34, 41, 47-48].

Substituting equations (15-16) into equations (13-14) gives:

\[ \dot{\epsilon}_{\text{ij}} = \frac{3}{2 \sigma_e} \text{Asinh}(B_{\text{ij}}(1 - H)) \]  

(17)

\[ H = \frac{h}{\sigma_e} (1 - \frac{H}{H^*}) \bar{\epsilon}_e \]  

(18)

\[ \dot{\phi} = \frac{K_e}{3} (1 - \phi)^4 \]  

(19)

\[ \dot{\omega} = C \epsilon_v \text{exp} \left[ \frac{1}{2} \frac{3 \sigma_m}{2 \sigma_e} \right] \]  

(20)

\[ \dot{\omega} = \omega \cdot f_2 \]  

(21)

The constants \( a \) and \( b \) should be determined by the experimentally determined isochronous rupture loci.

3.5 Discussion

The KRH approach for generalisation was originated at the development of multi-axial constitutive equations for copper and aluminium at 1972 [18] in which the creep damage rate equation
is constructed through creep damage equivalent stress identified from isochronous rupture loci in plane stress conditions.

The main concepts and/or steps used in KRH approach are summarised by this author as:
1) the effective creep strain is controlled by Von Mises stress;
2) the description of tertiary creep deformation is achieved by coupling with creep damage;
3) creep damage is assumed as quasi-isotropic;
4) the effective stress is defined based on strain equivalent hypothesis and it was not consistently used for which no theoretical justification was given [6];
5) the material is deemed failed when the damage variable reaches its critical value. This critical value was originally assumed to be 1, and then modified to be a constant less than 1. It typical value ranges from 0.5 to 0.9 according to Lemaitre [6];
6) the creep rupture strength theory or creep damage equivalent stress is introduced to describe the damage evolution to achieve lifetime consistency; and
7) the material constant \( v \) is calibrated against lifetime within bi-axial states of stress.

The fundamental deficiency in this KRH approach is that the creep strain consistency requirement is not satisfied. The new formulation introduced two functions to depict the effect of states of stress on the damage evolution and creep strain, respectively. The possible physical implication for this is that the creep deformation process and creep rupture process differ and two internal variable may be needed.

**4 VALIDATION** [4]

Validation should be adequate including the proportional loading condition and the non-proportional loading conditions. The former is a must and the latter should be desirable and it could be compromised due to high cost and experimental difficulty. However, even for the proportional loading condition, the validation in previous practice was not done adequately in either the items to be assessed or the range of states of stress [4]. Here, typical results are presented within proportional loading under plane stress and plane strain conditions. Detailed information on validation can be found in [4].

The isochronous rupture loci and ratios of strain at failure for previous formulation are presented in Fig. 2 and Fig. 3, respectively. The stress states sensitivity ranges from 1 to 4 with an interval of 1.

Typical isochronous rupture loci and ratios of strain at failure for new formulation are presented in Fig. 4 to Fig. 6. The parametric characteristics of ratios of strain at failure are shown in Fig. 4 for \( p = 2.5 \). The parameter \( q \) ranges from 0 to 1 with an interval of 0.25 and both the plane stress and plane strain conditions are considered.

Typical isochronous rupture loci are presented in Fig. 5 and Fig. 6. The former illustrates its characteristics when constants \( a \) and \( b \) are zero; the latter presents the complex coupling of creep deformation, creep damage and lifetime which are consistent with experimental observations.

The following overall conclusions can be drawn:
1. The KRH formulation is not appropriately constructed suffering in: a) the predicted lifetime near bi-axial tension region is longer than that of experimental observation [27, 41, 45-46]; b) the impossibility to obtain a value for \( v \) for both plane stress and plane strain conditions as an unrealistic increase of creep strength is predicted under plane strain conditions; c) the predicted ratio of strain at failure is conjugated with isochronous rupture loci which is not consistent with experimental observation [40];
2. The new formulation is better constructed. It introduced two separate functions to depict the complex influence of stress states on creep strain and damage evolution. The two specific forms of equations were introduced to depict the stress states on creep deformation and damage evolution separately. This results in different damage evolution equation and a different coupling between damage and deformation.
3. The validation conducted under proportional loading (plane stress condition and plane strain condition) shows that this new set of constitutive equations is capable to present the characteristics of creep deformation and rupture behaviour consistent with experimental observations.

**5 CONCLUSIONS**

The development of a new set of multi-axial creep damage constitutive equations for 0.5Cr0.5Mo0.25V ferritic steel at 590°C is presented covering the critical review of fundamental requirement, formulation, specific equations, and validation under proportional loading conditions. It also contains comparison with previous formulation whenever applicable. It shows that this new set of constitutive equations is better constructed and is capable to depict creep rupture and strain at failure consistent with experimental observations. It is planned to further investigate the creep strain prior to failure and to validate under non-proportional loading conditions, which will be reported in due course.

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a. creep curves under uni-axial tension for stress of 60 MPa and 40 MPa

b. damage evolution under uni-axial tension

Figure 1 Creep and damage evolution under uni-axial tension

a. Isochronous rupture loci under plane stress

b. Isochronous rupture loci under plane strain

Figure 2 Isochronous rupture loci for previous formulation. The stress state sensitivity ranges from 1 to 4 with interval of 1. The normalising stress $\sigma_0$ is 60 MPa.
a. Ratios of strain at failure under plane stress

b. Ratios of strain at failure under plane strain

**Figure 3** Ratios of strain at failure for previous formulation. The stress state sensitivity ranges from 0 to 4 with interval of 1. The normalising stress \( \sigma_0 \) is 60 MPa.

**Figure 4** Ratios of strain at failure for new formulation. The parameter \( q \) ranges from 0 to 1 with interval of 0.25 and \( p = 2.5 \). The normalising stress \( \sigma_0 \) is 60 MPa.
The parameter $q$ is 0, 0.5 and 1 while and $p = 2.5$, $a = 0$ and $b = 0$. The normalising stress $\sigma_0$ is 60 MPa.

Figure 5 Isochronous rupture loci for new formulation. The parameter $b$ is 1, 2 and 3 while $a = 2$ and $p = 2.5$, $q = 1$. The normalising stress $\sigma_0$ is 60 MPa.