SHAKING TABLE TESTS ON FLOW DYNAMICS IN LIQUEFIED SLOPE

HIROFUMI TOYOTA†, IKUO TOWHATA†, SHIN-ICHI IMAMURA†‡ and KEN-ICHI KUDO†‡

ABSTRACT

A series of 1-g shaking-table model tests was carried out on lateral flow of a sandy slope undergoing high excess pore water pressure. Model sandy deposit of extremely low density was prepared in order to induce lateral flow to various extents whether with continued shaking or impact shaking of short duration. By changing the test conditions, the effects of the intensity of shaking, the frequency of base excitation, the slope angle, and the density of sand were investigated. To facilitate the further development of an analytical measure that predicts the liquefaction-induced lateral flow movement, a simple model was developed with a rate-dependent energy dissipation mechanism. Example calculations showed that the experimental findings in the shaking table tests were reproduced by this model.

Key words: deformation, earthquake, liquefaction, model test, sand, vibration (IGC: D7/E8)

INTRODUCTION

The recent earthquakes in 1990’s such as those in Kobe and Taiwan have revealed significant intensities of acceleration records exceeding, for example, 600 gal. Since such a strong level of acceleration had rarely been recorded in the past, modification of seismic design principles has been discussed since then. Among those discussions is the one concerning geotechnical engineering which does not expect soil to resist such remarkably strong earthquake effects. The new principle may be the idea of allowable seismic displacement of earth structures which should not be exceeded by a real displacement upon a strong earthquake shaking. Obviously, a major role will be played therein by prediction of seismically-generated displacement. To be practical for ordinary kinds of earth structures, this prediction should not require detailed and consequently time-consuming subsurface investigations. A typical situation as above is found in the fields of slopes and embankments that are subjected to subsurface liquefaction.

The authors (Towhata et al., 1992, 1999a) have developed a practical method for prediction of liquefaction-induced ground displacement which requires only standard penetration tests and/or its equivalence which are widely employed in liquefaction site investigations. This method solves an equation of motion of a flowing soil mass with due consideration of the geometry of the soil mass as well as the soil properties. In spite of the mathematical achievement, however, there are still issues on soil properties which are not clearly understood. In this respect, Towhata et al. (1995) reported briefly several factors that affect the extent of displacement. The present text attempts to present more detailed discussion about this issue by referring to results of 1-g shaking table model tests. One of the major aims of this paper is to demonstrate that those detailed findings in 1-g model tests are reproduced by a simple predictive measure. To achieve this goal, it was necessary to introduce many experimental findings in detail, which are reproduced analytically in later parts of this paper. It is thus believed that the flow of liquefied ground is controlled by a few simple principles, as addressed in this paper, although observed facts appear complicated.

The magnitude of soil displacement as a consequence of subsurface liquefaction is affected by many factors. One of the important factors is the geometry of liquefied subsoil such as its length, slope inclination, thickness of layers, and boundary conditions. The authors’ dynamic analysis has already taken geometry into consideration. Therefore, the present text focuses on another important issue which is the material properties of sand undergoing liquefaction-induced lateral flow movement. This aim is achieved by running shaking-table model tests on flow of liquefied gentle slope which is going to be described in what follows.

The rate dependent nature of sand is not a common topic of study because sand is believed to be a frictional material. However, it may not be the case under a regime of low effective stress. The authors’ group conducted a model test in which an embedded pipe was pulled laterally in a liquefied model ground at a specified rate of displace-
ment. Since the subgrade reaction force was proportional to the rate of displacement (Towhatan et al., 1999b), an idea of viscous modeling of liquefied sand came into scope. More recently, Nishimura et al. (2002) investigated the rate dependency of sand by running torsion shear tests and showed that the viscosity coefficient is of the order of hundreds of kPa·s. Benedetto et al. (2002) developed a shear device for sand which can produce an extremely high rate of strain of 5600%/s. Since the present model tests do not precisely monitor the stress and strain states in model ground, it is difficult to discuss the rate dependency of sand in detail. However, it is possible to discuss the reasonable type of rate-dependent modeling by referring to the observed behavior of liquefied model ground.

The rate-dependent approach for deformation of sand is under investigation in other fields of geotechnical engineering. Di Benedetto et al. (2002) consolidated sand specimens under 392 kPa to show that the stress-strain behavior during triaxial compression varies with the changing rate of strain. Their study, however, is not directly applicable to the present study due to the different magnitude of effective stress. Moreover, numerical analysis on debris flow is another interesting field of dynamic analysis on soil flow. Egashira et al. (1997) together with Egashira and Miyamoto (2000), for example, made use of three energy dissipation mechanisms in the flow which were namely grain-to-grain friction, viscosity of pore fluid, and intergranular collision, among which collision is considered to be rate-dependent. The study of this type, however, concerns very rapid rate of flow and may not be directly correlated with the present problem of liquefaction-induced lateral displacement.

### PREPARATION OF MODEL SLOPE MADE OF EXTREMELY LOOSE SAND

It is not uncommon that liquefaction-induced shear strain exceeds 50 to 100% as demonstrated by model tests (Sasaki et al., 1992; Ghalandarzadeh et al., 1998a, 1998b). Since this range of strain cannot be reproduced by conventional shear testing devices, a study of large-strain behavior of liquefied sand is not an easy task. An alternative approach has been undertaken, therefore, by shaking-table model tests which monitor drag force as needed to pull an embedded object in liquefied subsoil (Towhatan et al., 1999b among others). Another method of study is monitoring flow failures of liquefied model ground as accurately as possible. The present study has taken this approach.

A series of 1-g shaking table tests was carried out on flow failure of liquefied slope made of Toyoura sand; for its physical properties, refer to Table 1. It is essentially important in 1-g model tests under reduced stress level that the potential of flow failure depends chiefly upon density of sand and the effective stress level (Verdugo and Ishihara, 1996). More precisely, the dilatancy and excess pore water pressure development in sand are governed by the combined effects of the effective stress level and the density. In the present study it is supposed that the undesired effects of reduced consolidation pressure in 1-g model tests are compensated for to a certain extent by employing reduced relative density. Consequently, the relative density of model deposits less than 30% is employed here because the real liquefiable sandy deposits in mind has the relative density of 40% or more. For details, refer to APPENDIX of this paper.

![Fig. 1. Procedure of moist tamping method for preparing loose sandy ground for model tests](image-url)

Table 1. Physical properties of Toyoura sand

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific gravity, $G_s$</td>
<td>2.648</td>
</tr>
<tr>
<td>Maximum void ratio, $e_{\text{max}}$</td>
<td>0.974</td>
</tr>
<tr>
<td>Minimum void ratio, $e_{\text{min}}$</td>
<td>0.605</td>
</tr>
<tr>
<td>Mean grain size, $D_{10}$</td>
<td>0.19 mm</td>
</tr>
</tbody>
</table>

Very loose deposits of Toyoura sand were prepared by the method of moist tamping by which the state of as low as −20% relative density was attained. The procedure of this method is illustrated in Fig. 1. Initially, Toyoura sand was mixed with 5% weight of water in order to produce the apparent cohesion in a partially saturated state. The present study, in particular, rained sand through four layers of sieves which had mesh sizes of 38,
19, 16, and 5 mm from the top downward. The use of sieves helped produce a uniform sandy deposit (Miura and Toki, 1982). Each layer of sand was lightly compacted so that the desired density would be achieved at the end of the preparation. The sandy deposits thus prepared had CO₂ gas percolated from the bottom and was submerged thereafter by slowly introducing water from the bottom. The loss of apparent cohesion upon submergence made the sand slightly denser. The following text will hence refer to the relative density after this minor collapse of grain structure. 

The uniformity of a prepared model slope was examined by draining water from the model and collecting sand samples from different parts of four model slopes with their length measuring 2 m. The size of the employed sampling tube was 10 cm in length and 7.5 cm in diameter. Figure 2 shows that the locations of sampling were mostly near the surface (○ and ●) with one exception (△) in the middle. The sampling from the middle was difficult because of insufficient drainage upon the practice; high degree of saturation in extremely loose sand caused liquefaction and made sampling unsuccessful. Hence, only one data was obtained from the middle depth. As compared with the overall mean void ratio of 1.050, 1.046, 1.052, and 1.055 which were obtained by the dry weight and volume of sand, there is some extent of variation in the local void ratio. In particular, the greater void ratio was remarkable near the top of the model slope. Hence, it was decided to lightly compress the new deposit of sand by hand in that part of the model. 

Figure 1 illustrated that the sand container was tilted after sand deposition by jacking up one of its ends. In the meantime, the surface of sandy layer was made level although a slope model was intended to be prepared in the ultimate shape after removal of a bottom jack. This jacking measure was undertaken because the water-saturated deposit of extremely low density, which is the present case, was prone to static liquefaction and flow failure (Poulos et al., 1984; Castro et al., 1992, among others) when the surface was inclined. Since such a failure during the model preparation stage was not desired, the surface was held level until the last moment prior to shaking. Undesired failure was still experienced occasionally, however, upon removal of the jacking equipment, and, if this was the case, the model ground was abandoned. 

Since the present study relies substantially on the monitoring of horizontal acceleration during lateral flow, efforts were made to avoid erroneous acceleration records. The most significant error among others was the one caused by tilting of embedded acceleration transducers upon softening of liquefied sand. Since the transducers, however, recorded forces, consisting of static gravitational and inertial components, it was possible to detect the extent of tilting, θ, from the drifting component. As illustrated in Fig. 3, consequently, the horizontal component of acceleration was calculated and presented in this study. 

**METHODS OF SHAKING**

The lateral displacement of liquefied slope models was generated by two types of driving forces which were namely the gravity-induced static force and the dynamic inertial effects that were produced by horizontal shaking. It was attempted in some of the tests to isolate the gravity-induced flow in the longitudinal direction of the container (Fig. 4) from the inertial effects due to shaking. This goal was achieved by causing the liquefaction of...
sand by means of impact shaking of a short duration of time and having liquefied slope flow only afterwards. Such an isolation was ensured further even during the short duration of impact shaking by orienting the horizontal impact perpendicular to the slope direction (Fig. 4). The remaining tests, on the other hand, employed a continuous harmonic shaking which had a specified frequency as well as amplitude of acceleration. The direction of shaking was either longitudinal (parallel to the slope) or perpendicular. In the former case, the effects of static gravity force and cyclic inertial force were superimposed in the same direction.

**CONFIGURATION OF MODEL GROUND**

The rigid soil container which was employed in the present study measured 2 m in length, 40 cm in width, and 50 cm in depth. The model ground was made of Toyoura sand. Two configurations of model slope were investigated; one with 20% slope and the other with 10% slope (Fig. 5). The latter slope was underlain by a layer of compacted sand which was densified by lightly plunging the surface prior to placing the overlying loose sand. Consequently, uniform slope angle and thickness of liquefiable sand layer were achieved. A square grid of colored Toyoura sand was installed along one of the transparent side walls so that the overall deformation during flow failure could be observed visually. In addition to acceleration and excess pore water pressure in the model, a time history of lateral displacement was recorded at a single point in the slope by using a potentiometer. This device continuously recorded the displacement of an embedded plate (Fig. 5) which was connected by means of a wire to a variable electric resistance and the displacement was converted to an electric voltage. This wire was encased in a pipe so that its friction with sand would not affect the measurement.

**FREE FLOW FAILURE CAUSED BY IMPACT SHAKING IN PERPENDICULAR DIRECTION**

*Flow of Slope Made of Extremely Loose Sand*

The response of sand to a horizontal impact excitation in the perpendicular direction is going to be described hereinafter. It was aimed to investigate the displacement of liquefied 20%-slope model (Fig. 5(a)) under the gravity force, without influence of seismic inertial forces. Therefore, an impact shaking was produced in the perpendicular direction for a short while in order to trigger liquefaction by hitting with a hammer at the bottom wooden plate of the container. This container was supported by a metal spring to allow for a short period of free decayed oscillation after hitting. The high energy dissipation of the spring made the shaking decay quickly and, therefore, the flow of liquefied soil occurred after shaking solely under the gravity action. Furthermore, note that even the excitation did not produce any inertial effects in the direction of flow because of its perpendicular orientation. Toyota (1995) confirmed that, in spite of this manual excitation in the perpendicular direction, two tests of same soil conditions under identical feature of excitation gave very similar magnitude of soil movement.

Photo 1 illustrates the development of flow failure when sand was extremely loose ($D_s = -20\%$, $e = 1.049$). The liquefied slope continued its motion until the surface...
became nearly level. Note that the driving force of flow due to gravity vanished when the surface became level. It is important that the lateral displacement was negligible at the bottom of the container, and increased towards its maximal at the surface. Figure 6 demonstrates the time history of lateral displacement of the same test. At around four seconds when shaking had ceased completely, it attained the ultimate displacement of about 30 cm at which the slope became level.

The time histories of horizontal acceleration in the model are illustrated in Fig. 7. The base shaking was produced by impact in the transverse direction and its time history (A7) shows that the maximum acceleration of 1000 gal occurred with only one cycle, followed by quick decay. Accordingly, the transverse direction near the surface (A1) decayed similarly after the first peak acceleration of about 600 gal. The longitudinal acceleration (A2) was substantially large at the beginning although no excitation was made in this particular direction, probably manifesting the interaction of soil with walls of the container. Note that the longitudinal acceleration in the later stage was negligible while flow displacement continued until 4 seconds (Fig. 6). The lateral flow was, thus, a steady phenomenon without dynamic oscillation in time history.

Figure 8 reveals the time histories of excess pore water pressure. Transducers for pore water pressure were fixed to vertical embedded rods so that their location would not change during flow movement of surrounding sand. To cancel the disturbance in records due to water waves produced by a rapid flow of sandy deposits, the presented time histories are the difference of two records; one in soil and the other in overlying water. Hence, null pressure difference in the figure stands for a hydrostatic condition. The very short shaking at the bottom in the transverse direction induced liquefaction everywhere in the soft sandy deposit. The developed excess pore water pressure near the bottom (P1, P4, and P7) did not dissipate quickly until eight seconds, while the transducer at a shallow depth (P2) had most of its overburden sand disappear during flow (see change of configuration in Photo 1) and, consequently, the pore water pressure record, P2-P3, decreased and became hydrostatic quickly. It is further interesting to state that the excess pore pressures near the top of the slope (P1 and P2) were lower than the initial effective vertical stress, \(\sigma'_{vo}\), after 2 seconds and was held constant. In contrast, the pressure under the bottom of the slope (P7) became higher than \(\sigma'_{vo}\). The reason for this is the change of slope topography; the overburden pressure decreased near the top of the slope, while it kept increasing near the bottom, changing the total stress significantly.
Flow of Slope Made of Denser Sand

It is interesting to compare the behavior of extremely loose sandy slope in the preceding section with the behavior of denser sand. Photo 2 manifests the ultimate configuration of a slope which had the same initial configuration but higher relative density of 7%. The deformation in this model was evidently smaller than what happened in the looser sandy deposit in Photo 1. This is further verified by the time history of displacement in Fig. 9. It is noteworthy in this figure that the displacement reached the ultimate value at around 4 seconds when excess pore water pressure was still high (Fig. 10). Note that ceasing of motion at 4 seconds is similar to what happened in the case of loosest sand (Fig. 6).

Time histories of excess pore water pressure are manifested in Fig. 10. Most records of pressure therein was held lower than the initial effective vertical stress. This is partly due to the higher density of sand and partly due to the small deformation which did not increase the overburden pressure near the bottom of the slope so substantially as in the previous test. Furthermore, the rate of pore pressure dissipation is faster here than in Fig. 8 of looser sand.

Effects of Sand Density on Free Flow of Slope under Gravity

Figure 11 compares the time histories of lateral displacement which were recorded in five tests with different sand densities. It is clearly seen that the looser sand generates the greater magnitude of ultimate displacement. In contrast, the denser model attains the smaller ultimate displacement at which the ground surface is still inclined. Thus, it is reasonable that the flowing denser sand maintains to some extent the shear strength which keeps the displacement at smaller magnitudes. Noteworthy is that the displacement in all the five cases occurs from 2 to 5 seconds, and that, after a short halt, it is resumed once more and finally stops at 7 seconds. As a whole, the duration time of displacement is more or less constant independent of density. The reason for this halt-and-resume behavior is found in the pore pressure change during the event; see Fig. 13 later. Another important finding is that no oscillation of motion is observed whatever the density may be; strongly indicating the existence of energy-dissipation mechanism. According to the theory of free vibration, the time needed for this over-damped transient motion is produced by the rate-dependent energy dissipation which is often expressed by a
dashpot; being independent of the rigidity, mass, or the external forces. A frictional mechanism of energy dissipation can also reproduce the facts of constant flow time until its end as well as the over-damped transient motion. It cannot, however, account for the level ground surface at the end of flow of very loose sand, because the ultimate level surface implies lack of shear resistance and the over-damped motion (Fig. 11) in contrast suggests substantial shear resistance. Therefore, it appears that the flowing sand has a rate-dependent energy dissipation mechanism as well in addition to the aforementioned shear resistance.

FREE FLOW OF LIQUEFIED SLOPE WITH 10% INCLINATION

Another series of tests were conducted on free flow of a model slope of 10% inclination; see Fig. 5(b). The loose sandy deposit of 35 cm thickness was underlain by a compacted Toyoura sand. Shaking was excited by an impact again in the perpendicular direction. Therefore, the flow and inertial effects were kinematically independent of each other.

The ultimate distortion of extremely loose sandy slope ($e=1.039$ and relative density $= -18\%$) is revealed in Photo 3. The relative density in the base layer was 59%. Large shear distortion occurred only in the top loose layer, while the base compacted layer did not deform. The fact that the ground surface became nearly horizontal infers that this loose sand had only negligible shear rigidity after liquefaction and that the final stability was attained only after becoming level.

The time histories of lateral displacement are presented in Fig. 12 where two different densities of sand are concerned. The relative density in the base layer was 72% when the void ratio of loose part of sand was 1.009. Firstly, note that the ultimate magnitude of displacement is smaller than what was observed in the 20% slope model with similar densities. This is because the displacement which was needed to attain the level configuration was smaller in a slope of reduced inclination. Secondly, the two models in Fig. 12 spent similar time on flow, irrespective of the magnitude of displacement. This is consistent with the finding in Fig. 11 for 20% slope model. Thirdly, the displacement developed in two stages with an intermittent period in the middle. This behavior was similarly observed in 20% slopes, although its extent was less. To understand the reason for this two-stage motion, an additional pore pressure transducer was installed near the displacement transducer. Being different from other ones, this transducer was not fixed to any rod and was able to move together with sand and the displacement transducer. The results in Fig. 13 indicates that pore pressure increased at the beginning of the shaking, causing the first stage of flow, dropped slightly so that the motion is paused, and finally increased again to generate the second stage of motion. It seems likely that the tentative drop of pore pressure was related to positive dilatancy of sand upon its large shear deformation, and that the pressure was raised again due to its propagation from the area of higher pore pressures.

FLOW FAILURE OF LIQUEFIED SLOPE UNDERGOING CONTINUED SHAKING

Effects of Longitudinal Shaking

Since real earthquake motions have a certain duration time of strong shaking, it is meaningful to study the nature of flow displacement in the course of continued shaking. The first series of tests were conducted on the configuration of Fig. 5(a) which had 20% slope. Shaking was given in the longitudinal direction, and the seismic inertial effects and the static gravity-induced driving force were superimposed in the same direction. The shaking frequency at the base was about 3 Hz, while the amplitude of motion was variable to some extent with time; the maximum acceleration being 250 gal.

The first example of flow tests employed very low relative density of $-16\%$ (void ratio = 1.034). As shown in Fig. 14, the base shaking started at around 2 seconds and ceased at 9 seconds. The consequent lateral displacement attained the maximal extent at 6 seconds and then went back to some extent, being stabilized at around 9 seconds when the shaking was terminated. Since the
apparent backward movement was made by a minor driving force in the displacement transducer, this part of the record should not be relied on and is removed from further discussion. The ultimate shape of the model is demonstrated in Photo 4 in which the ground surface is level. The lateral displacement in the subsoil is illustrated by grids of dyed sand. It attained the maximal near the surface, while negligible at the bottom.

The importance of continued shaking is found in the magnitude of ultimate displacement. When the relative density was increased to −3% in Photo 5, the ultimate configuration was still level. The time history of lateral displacement in Fig. 15 achieved 28 cm in the ultimate stage and this is remarkably greater than what was observed in the case of impact shaking with similar void ratio; see the displacement time history in Fig. 12 for similar void ratio of $e = 0.982$. Tohwhata et al. (1999b) revealed that continued shaking prevents the development of positive dilatancy and maintains the excess pore water pressure at a high extent, which leads to the possibility of greater displacement. This point is confirmed in Fig. 15 where excess pore water pressures at the same location in two models are compared. Having similar void ratio, one model was subjected to continued shaking, and the other to impact shaking. It is evident that the test with continued shaking achieved higher pore water pressure and liquefaction. Being consistent with Tohwhata et al. (1999b), continued shaking thus produced greater residual displacement. It should be mentioned that the displacement caused by continued shaking in Fig. 15 was ceased at around 9 seconds although shaking continued longer. This is because the level configuration was achieved at 9 seconds and no more displacement was needed.

The effects of continued shaking are indicated in Fig. 16. It is evident that the ultimate displacement was similarly as large as 30 cm or more when shaking was continued for a sufficiently long time irrespective of density of sand. In contrast, when shaking was of an impact type, the residual displacement decreased when denser sand was employed. Consequently, no flow displacement is likely when void ratio is less than 0.92.
LIQUEFACTION AND FLOW DYNAMICS

Fig. 17. Response of 20% slope to 3 Hz transverse shaking (7% relative density)

Effects of Transverse Shaking
A model slope was excited by continued shaking in the transverse direction. This direction of continued shaking made the inertia force and the gravity-induced force perpendicular to each other. Hence, high pore water pressure was generated by cyclic loading, and the lateral displacement occurred under the static gravity force, independent of kinematic effects of shaking. Photo 6 shows that the transverse shaking generated a level configuration in a model of void ratio = 1.000. This is in good contrast with the case of impact shaking and free flow (Photo 7) in which the surface did not become level in spite of looser density of sand (e = 1.011). Thus, continued horizontal shaking is able to generate large displacement whether its direction is longitudinal or transverse.

The time histories of records in this test are shown in Fig. 17. The lateral displacement achieved the maximal at 9 seconds when shaking was still going on. The excess pore water pressure at the P4 station near the bottom achieved the level of $\sigma_{\text{w}}$. Finally, Fig. 18 compares the time histories of lateral displacement. It is shown herein that the rate of displacement is not affected by the direction of continued shaking when sand is extremely loose ($e = 1.034$ and 1.000).

Significance of Duration Time of Shaking
The present section has revealed that continued shaking in both longitudinal and transverse horizontal directions is able to achieve very large displacement of sand over a range of density. In contrast, in the free-flow condition, only the extremely loose sand can develop large displacement of the similar extent. This implies that practical prediction of liquefaction-induced ground displacement needs to specify the duration time of strong shaking after which no more large displacement occurs. Certainly this duration time includes both main shock that induces liquefaction and aftershocks which occur immediately after the onset of liquefaction. APPENDIX presents some idea about the range of in-situ density which corresponds to the extremely loose condition of sand in the present model tests which was subject to large free flow.

EFFECTS OF INTENSITY AND FREQUENCY OF INPUT SHAKING ON FLOW MOVEMENT
The nature of base excitation is most reasonably characterized by the amplitude and frequency. In this section, the effects of these parameters on the residual displacement are going to be investigated. The base shaking, therefore, maintained a constant amplitude of a harmonic longitudinal shaking. Since the duration of shaking was variable, the displacement after 5 seconds of strong shaking is employed for discussion. Another reason to compare the displacement at 5 seconds in place of the ultimate one is that all the model attained similar extent of displacement finally at which the configuration became level. Thus, the effects of shaking is more clearly seen at a fixed time.

Figure 19 summarizes the results. It is noteworthy that the greater base acceleration increases the lateral displacement more effectively. Another important issue may be that the shaking frequency does not affect the displacement after 5 seconds of flow.

Figure 20 investigates the effects of shaking frequency
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Another velocity.

Fig. 19. Effects of nature of base excitation on residual displacement

Fig. 20. Effects of void ratio on mean velocity of lateral soil flow

Fig. 21. Effects of nature of base excitation on mean velocity of lateral soil flow

don the development of lateral displacement. In this figure, three tests with longitudinal shaking with similar magnitudes of acceleration were employed. It seems relevant that the frequency does not have much effects on the displacement.

Another way to study the vulnerability of flow failure is the one based on the mean flow velocity. In this light, the ultimate displacement divided by the total duration time of flow was calculated and defined as the mean velocity.

Figure 21 illustrates the variation of the mean flow velocity with the void ratio prior to flow. The concerned range of base acceleration is variable as illustrated in Fig. 22. In principle, the velocity decreases as the sand is densified. When no shaking continued in the course of flow (impact shaking), the velocity would be zero below the void ratio of 0.92; no free flow is likely. This is consistent with Fig. 16. When the base shaking was continued during flow for void ratio less than 0.95, the velocity was increased to a range of 2 to 4 cm/s. When the void ratio is greater, conversely, impact shaking and continued shaking gave similar velocity. Thus, there seems to be no influence of continued shaking when sand is extremely loose.

Figure 22 employs the same set of data as in Fig. 21 in order to show the variation of the mean flow velocity with the amplitude of base acceleration. For the cases of continued harmonic shaking at 4, 8, and 16 Hz, the void ratio was relatively smaller, ranging from 0.906 to 0.941, and the flow velocity increased to some extent with the increase of shaking acceleration. The tests with impact shaking and 3 Hz shaking were generally of greater void ratio, and therefore the flow velocity was greater. However, there is still consistency among data when void ratio is 0.945 or 0.947 which is close to the void ratio range of other data. By summarizing Figs. 21 and 22, it may be said that flow velocity is affected more significantly by density of sand than the intensity of base shaking. This implies that the flow phenomenon is induced by the static gravity force more profoundly than the dynamic inertia force.

RECONSTRUCTED STRESS-STRAIN RELATIONSHIP OF SAND UNDERGOING LATERAL FLOW

General Principles

Matsuo and Koga (1990) proposed to reproduce the stress-strain behavior of sand in 1-g shaking table models by using the recorded time history of acceleration and displacement. This was followed by Sasaki et al. (1992), while Zeghal and Elgamal (1994) applied this idea to a real earthquake motion.

The idea is illustrated in Fig. 23. The equation of motion of a vertical soil column is given by

Shear stress = - (mass density) \times \int_{t_0}^{t} (Acceleration) \, dz (1)
where the effects of gravity-induced lateral earth pressures on two sides of the column are considered to cancel each other because of the gentle slope and the narrow width of the column; they make the model similar to a part of an infinite slope. Hence, Eq. (1) concerns only with the cyclic component of stress. Moreover, “−” at the head of the right-hand side of the equation is necessary due to the definitions of positive directions of acceleration and stress. Note that Eq. (1) ignores the static component of shear stress which is induced by the gravity force and changes with the progress of flow. The shear strain, on the other hand, is approximated by the measured displacement which is relative to the soil container;

\[
\text{Shear strain} = \frac{\text{Lateral displacement}}{\text{Elevation above bottom}}. \tag{2}
\]

By using the acceleration and lateral displacement, thus, it is possible to get an idea of the stress-strain relationship of sand. In what follows the displacement at the elevation of 32 cm (depth of 9 cm) is substituted in Eq. (2) (see Fig. 24) and the corresponding shear stress in Eq. (1) was calculated by using the surface acceleration and another record at 18 cm depth. Those records were obtained in the models of 20% slope with varying relative densities, shaking magnitudes, and frequencies. Mass density of 1.84 g/cm³ was assumed.

**Effects of Density of Sand**

Figure 25 illustrates an example of stress-strain diagram of Test 12 (in Fig. 14) in which a very loose model slope (relative density = −16%) was subjected to a longitudinal shaking of 3 Hz and, at maximum, 250 gal acceleration. It is noteworthy that strain develops in the positive direction when the stress lies not only in the loading (increase) phase but also in the unloading (decrease) phase. This is particularly the case when the strain is less than 0.4 (40%) and the inclination of the model slope is still large. In this period of time, the excess pore water pressure ratio (P2−P3), which was measured near the displacement transducer and whose highest value in shaking cycles was normalized by the initial effective vertical stress, is still high, although not being 1.0. It seems that the model slope of very loose sand continues to flow under the overwhelming influence of the gravity-induced load in the down-slope direction. Since this load is far greater than the stress and strength of the extremely loose sand, the flow movement is able to continue without pause. This situation, however, changes after the strain exceeds 0.4. Strain and down-slope movement develop only when the shear stress increases. This is probably because the slope angle has decreased and the influence of the static load has become less important. Another influence may come from the dissipation of excess pore water pressure. Once the slope has become nearly level, there is no more progress of lateral deformation.

Another example of stress-strain diagram is shown in Fig. 26 where the employed sand has a lower void ratio of 0.945. Being subjected to a similar kind of base shaking as in Fig. 25, the present time history of stress-strain behavior consists of two different stages. In the early stage when the strain is less than 0.3, the strain increases mostly while the cyclic stress is negative. Moreover, as long as the strain is less than 0.7, the strain develops in the positive direction when the stress increases from the negative peak towards the positive one. This suggests the overwhelming influence of the static load which is superimposed on the cyclic stress and maintains the total shear stress (static + cyclic) in the positive direction. An opposite behavior is detected in Fig. 26 when the strain
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Fig. 26. Reproduced stress-strain history in Test 11 subjected to continued longitudinal shaking (250 gal)

Fig. 27. Reproduced stress-strain history in Test 14 subjected to 180 gal of continued longitudinal shaking

Fig. 28. Reproduced stress-strain history in Test 16 subjected to 470 gal of continued longitudinal shaking

above. When shaking is stronger in Fig. 28, conversely, the strain, which starts to increase when the stress increases as well, continues to increase after the stress comes into the decreasing phase. It may be said, therefore, that stronger shaking destroys the material strength of liquefied sand more profoundly, leading to the greater residual displacement as was seen in Fig. 19.

Effects of Shaking Frequency

This section attempts to find out possible rate-dependent nature of sand by using the reproduced stress-strain curves. It is herein borne in mind that the amplitude of stress naturally changes with the frequency because of the variation of amplification with the frequency. Hence, it was decided to use the modulus of sand under different rates of shaking. Therefore, the reproduced stress-strain relationships are compared for two sets of experiments in which the shaking frequency was varied while the density of tested sand and the intensity of base shaking were held similar. In Figs. 29 and 30, the magnitudes of the tangent shear modulus are roughly manifested by the inclination of lines and compared. The modulus under 8-Hz shaking is similar or slightly greater than that under 4-Hz excitation, while the modulus under 16 Hz appears to be greater. It seems, therefore, that the tested sand has to some extent a rate-dependent stress-strain characteristic.

Another important issue in Figs. 29 and 30 is that the increment of residual strain per cycle is greater when the shaking frequency is lower; the 16 Hz shaking achieved...
Fig. 31. Correlation of flow displacement with inertia force in Test 12 subjected to 250 gal of continued longitudinal shaking

The least increment of strain per cycle in spite of the greatest stress amplitude. This finding implies that the accumulation of strain is governed by the static load rather than the cyclic loading. Since the time rate of displacement accumulation is nearly independent of shaking frequency (Fig. 20), it can be said again that lateral displacement of liquefied sand is controlled more profoundly by the static force.

CORRELATION BETWEEN INERTIA FORCE AND FLOW DISPLACEMENT

Ghalandarzadeh et al. (1998a) conducted shaking table model tests on gravity-type quay wall subjected to backfill liquefaction. They reported therein a correlation between the seismic inertia force and the development of permanent lateral displacement. The same is found in the present study as shown in what follows.

The inertia force is calculated of a soil column of a unit cross section which includes the displacement transducer;

\[ \text{Inertia force} = -(\text{Mass density}) \times (\text{Thickness of sand}) \times (\text{Base acceleration}) \]  

(3)

in which the thickness of sand is 32 + 9 = 41 cm for a 20% slope model as shown in Fig. 24.

Figures 31 and 32 indicate the correlation of lateral displacement with the inertia force. There is a clear feature in these figures that displacement develops when the inertia force decreases irrespective of the density of the sand. This is consistent with what was reported by Ghalandarzadeh et al. (1998a). It should be noted near the end of flow in Fig. 32 that a reversed direction of displacement occurs when the inertia force approaches the positive peak. The detailed plotting of this part is shown in Fig. 32(b). As stated before, there is an overwhelming influence of static load on the development of lateral displacement. In the initial stage of shaking, this static force allows only the downslope movement. When the slope angle has decreased to a nearly level configuration, as shown in Fig. 32, this static force is small and the cyclic load can cause displacement in both positive and negative directions.

The reversed (climbing the slope) development of displacement in Fig. 32(b) was not understandable because accompanying positive inertia force is oriented still in the down-slope direction. To further investigate this point, Fig. 33 was drawn in which the cyclic shear stress was reconstructed from the acceleration records and plotted against the soil strain. In the earlier phase of flow when the strain was still less than 0.5, the strain and the displacement kept developing in the positive (downslope) direction. This is because the slope was still inclined in this stage and the static gravity force plus the illustrated cyclic component was positive throughout, allowing only the positive increment of large deformation. In contrast when the shear strain was larger than 0.94 in Fig. 33, the slope angle had vanished and the static shear stress had disappeared. Hence, the shear stress varied cyclically in both positive and negative directions as illustrated. When the inertia force was increasing (DE in Figs. 21 and 33), the shear stress occurred in the negative side and the strain changed in the negative direction, too (Fig. 33). In the phase of EF, on the
contrary, the shear stress was loaded in the positive direction and deformation developed in the positive direction as well. This matching of the orientation of stress and strain increment may not be common in solid material, but is widely seen in the behavior of viscous liquid. The strange correlation in Fig. 32 was solely due to a significant phase difference between the inertia force and the shear stress (acceleration). Note that this phase difference was caused by the softening of tested sand.

**SINGLE-DEGREE-OF-FREEDOM MODEL FOR FLOW OF LIQUEFIED SLOPE**

The lateral flow of liquefied slope is modeled by a simple model which is illustrated in Fig. 34. Although being extremely simple, the essence of the model in what follows is identical with that of a two-dimensional and three-dimensional flow analyses that have been developed by the authors (Towhata et al., 1999a; Acacio et al., 2001; Kobayashi, 2001). It is therefore aimed herein to examine the validity of the principles of the previously developed analyses in which flow movement is governed chiefly by the effects of static gravity load together with the rate-dependent nature of liquefied sand. These principles are reassembled in this paper to a simple 1-degree-of-freedom model in Fig. 34 so that roles played by respective principles are clearly demonstrated. Shortcomings of this approach may be that comparison with observation and calculation is merely qualitative and that employed parameters are determined without sufficient experimental background. In spite of these problems, it is still useful to examine the essential aspects of the method by such a simple way. For detailed comparisons, refer to Towhata and Mizutani (1999) for example.

In Fig. 34, a slope has an inclination of $\theta$ angle and is subjected to the base acceleration of $-kg \sin \omega t$ that exerts an inertial force on the unit mass resting on the slope. The relative displacement between the slope and the mass is denoted by "$u$." Note that the static force includes a component which decreases with "$u."" This accounts for the fact that slope angle decreases with the development of displacement and reduces the static force with time towards zero. Thus, the static force per unit mass is given by $g \sin \theta - \omega^2 \omega u$ where $\theta$ is the initial slope angle and $\omega^2 \omega u$ stands for a linear proportionality.

The resistance involves a frictional resistance which is equal to $F$ times the static gravity force ($g \sin \theta$). Accordingly, $F$ is equivalent with the conventional static factor of safety. The present analysis assumes $F < 1$, however, because free flow tests in this paper showed that flow displacement is possible without continued shaking (Photo 1). Finally, the present model assumes a rate-dependent nature of liquefied sand, and "$h$" designates the damping ratio and $h > 1$ is assumed because no oscillation was observed in soil flow tests. Accordingly, the equation of motion is derived; when the mass is moving downwards,

$$u + 2\omega \omega u + \omega^2 \omega u = g \sin \theta + kg \sin \omega t - F \sin \theta \quad (4)$$

and, when the mass is moving upwards,

$$u + 2\omega \omega u + \omega^2 \omega u = g \sin \theta + kg \sin \omega t + F \sin \theta \quad (5)$$

In case of the extremely loose sand, the final stationary state ($u = \dot{u} = k = 0$) was associated with a level ground surface (Photo 1). From the viewpoint of Eq. (4), the stationary state is accompanied by $u = g \sin \theta / \omega^2$ because $F = 0$ is reasonable for the extremely loose sand. Therefore, $\omega$ concerns the displacement that is needed to achieve a level ground surface and should be considered as a geometrical nature. It is not related to the rigidity of soil as might be imagined. Moreover, note that no motion is initiated when the driving force, static and dynamic, is less than the resistance.

**EXAMPLE CALCULATIONS**

A series of example analyses was made of a case in which the slope inclination $= 20\%$ ($\theta = 11$ degrees), the static resistance $= 10\%$ of the gravity-induced load due to the effects of excess pore water pressure ($F = 0.1$), and $h = 10$. Since the aim of this section is simply an example demonstration, there is not much physical meaning in the selected values of individual parameters.

**Control Case**

The first analysis on the control case concerns $\omega / \omega_s = 20$ and the seismic coefficient $k = 0.5$. The ratio of $\omega / \omega_s$ should be taken sufficiently large because $\omega$ stands for the shaking frequency which is substantially greater than the natural frequency of sloshing motion, $\omega_s$, of a liquefied slope. Figure 35 is indicative of the time history of lateral displacement in which a minor fluctuation of displacement caused by inertia force is seen. The development

**Fig. 34. Simple idealization of slope subjected to lateral displacement**

**Fig. 35. Time history of calculated displacement in control case**
rate of displacement decreases with time because the normalized displacement approaches the ultimate equilibrium of $\sin \theta$. The correlation between the inertia force and the displacement in Fig. 36 illustrates that down-slope displacement develops when the inertia force is decreasing. Moreover, the upward movement becomes more evident when the displacement has increased substantially and the static force of $g \sin \theta - \omega^2 \mu$ is less important. The discrepancy between the calculated upward displacement and the observed one is that the calculated upward displacement starts immediately after the negative peak of the inertia force (Fig. 36 right) while the observed upward motion is initiated when the inertia force has become positive (Fig. 32). The reason for this difference is not understood yet.

Figure 37 reveals the calculated relationship between the resistance of the inclined floor and the lateral displacement. Similar to the previous discussion on experimental results, the resistance herein is defined by

$$\text{resistance} = k \sin \omega t - \frac{\ddot{u}}{g}$$

which is equal to $\pm F \sin \theta + (2 \omega \dot{u} + \omega^2 \mu) / g - \sin \theta$ according to Eqs. (4) and (5). Thus, Eq. (6) includes not only the true resistance of $F \sin \theta$ but also the geometrical effects, the viscous resistance and the gravity-induced load. Increasing or decreasing with time, the calculated displacement history is consistent with the experimental observation in Figs. 27 and 28. Note moreover that monotonic increase of displacement as experimentally observed in Fig. 26 can be reproduced by analysis when the slope angle is sufficiently greater than the inertial force effects so that the combined load may be kept positive.

**Combined Effects of Magnitude of Resistance and Intensity of Shaking**

Figure 19 manifested that the stronger shaking results in slightly greater displacement after a fixed duration time of shaking. Moreover, Fig. 20 demonstrated that the velocity of flow movement increases as the tested sand is made looser. This is particularly the case of very loose sand, although the ultimate displacement of more or less 30 cm was achieved at different times of shaking (Fig. 16). With these in mind, the effects of density as expressed by resistance ($F$ parameter in Eqs. (4) and (5)) and the intensity of base shaking ($k$ factor) are examined together in Fig. 38. When the resistance is small as in the control case ($F=0.1$), the displacement was reduced only slightly by decreasing the intensity of base motion to half. This implies that the static gravity plays a chief role in flow of very loose sand. The effects of the base shaking are made more profound when the resistance is nine-times greater ($F=0.9$). These findings are at least qualitatively consistent with the experimental observations.

**Effects of Shaking Frequency**

Figure 39 illustrates the calculated displacement for three kinds of shaking frequencies; $\omega_c$ of the control case,
Analysis on Free Flow

This subsection deals with the displacement without shaking effects. This is equivalent with the free flow of liquefied sand without continued shaking. To investigate the effects of sand density on the development of displacement, Eq. (4) is solved with a family of $F$ values while other parameters are held identical as in the control case above. The calculated results in Fig. 40 indicates that the ultimate displacement changes with the $F$ parameter, while the time needed to achieve the ultimate displacement is independent of $F$. These are consistent with test results (Fig. 11).

In summary, the simple model in Fig. 34 is able to reproduce the experimental findings on the effects of intensity of shaking, frequency, material properties and so on.

Limitations of Present Study

The present study aims to demonstrate that the proposed simple analytical measure is capable of reproducing the nature of liquefaction-induced flow of sandy ground as observed in 1-g model tests. It is thus fare to point out that there are two limitations which should be addressed in future studies. Firstly, the conducted model tests concern 1-g stress field and may not precisely reproduce the behavior of in-situ sand undergoing higher stress field. Although an attempt was made to solve this problem to some extent by employing very loose sand in tests, more efforts are needed by, for example, running full-scale model tests. The other limitation lies in the determination of soil parameters which are needed for analysis. Efforts in this regard have been made by Nishimura et al. (2002) as well as Towhata and Gallage (2002).

CONCLUSION

The present study aims at facilitating the development of an analytical measure in which the authors have been engaged in predicting the liquefaction-induced lateral displacement of sandy deposit. This goal was achieved by running shaking table tests on lateral displacement and interpreting the observed behavior of sandy ground. The major findings from the tests are summarized in what follows.

1) The magnitude of free flow varies with the density of sand because the shear strength of sand changes with the density.
2) Since no oscillation occurs in sandy slope after free flow, there is an energy dissipation mechanism other than the frictional resistance. At this moment, this mechanism seems to be of a rate-dependent nature.
3) Duration time of strong shaking after liquefaction governs the magnitude of ultimate displacement. Therefore, design practice needs to somehow determine this duration time for a good prediction of liquefaction-induced ground displacement.
4) The amplitude of base shaking affects the rate of development of lateral displacement. The stronger excitation develops the displacement more quickly. In contrast, the shaking frequency does not have a remarkable influence.

Based on these observation, a simple model of lateral flow was developed with a single degree of freedom in which rate-dependent nature plays a major role. It is important that example calculations with this model were able to reproduce the observed behavior of model slopes. Since this simple model is essentially equivalent with the more elaborate model which the authors have developed (Towhata et al., 1999a), it appears possible now that the authors’ advanced model will be able to predict the liquefaction-induced flow in a more reasonable manner.

ACKNOWLEDGMENT

The idea of the displacement transducer which the present study employed was given by Prof. Iwadate of the Tokyo Metropolitan University who was then working for the Central Research Institute of Electric Power Industry. The authors express their sincere gratitude for his kind assistance.

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APPENDIX: CONSIDERATION ON STRESS LEVEL IN 1-G MODEL TESTS

Since the effects of low stress level in 1-g model tests are often disputed, the present study employed reduced density of sand in order to cancel the stress level effects to some extent. Since the flow and large deformation of sand undergoing nearly undrained loading are the chief target of study, it was necessary to make an attempt to reproduce similar flow characteristics of sand in 1-g models and the in-situ conditions. Figure A1 illustrates conceptually the stress-strain behavior of sand undergoing undrained large shear deformation. Different extent of softening after the peak strength induces large deformation to a variety of magnitudes and the extent of softening is affected by density of sand as well as the confining pressure level; more undrained softening occurs when sand is looser and the effective stress is higher (Verdugo and Ishihara, 1996). Hence, the present study aims to maintain similar extent of softening in model tests by cancelling the effects of reduced stress level by means of reduced density of sand.

The extent of softening is defined by the brittleness index (Bishop et al., 1971) which is illustrated in Fig. A1. The effects of density and stress level were experimentally studied by Vargas (1998) who ran ring shear tests on Toyoura sand under constant volume conditions. Figure A2 shows the observed variation of the brittleness index when stress and density are varied. As expected, a similar value of brittleness is maintained under low stress level if sand density is reduced as well. Although the
range of data variation is not small, mean trends were
drawn for four levels of stress. Assuming that the in-situ
effective stress level prior to flow is typically 100 kPa,
while 1-g tests have 10 kPa of initial effective stress, it
may be reasonable to reduce the relative density by 25 to
30% so that a similar value of brittleness index is
maintained. Note that this idea of employing very loose
sand focuses only on the brittleness and possibility of
large distortion, while similitudes of dynamic response
and shear rigidity are out of scope.