7. On the Complete Reflection of Water Waves

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(*From J.S.N.A. Japan, Vol. 151, July 1982*)

Summary

This is a theoretical and experimental study of the water wave reflector which is floating and moored.

Solving motions of moored floating body among waves by the two-dimensional wave theory, we may predict the amplitude of reflected and transmitted wave, so that we may easily find the wave-length at which the incident wave is reflected or transmitted completely. This wave-length depends mainly on the spring constant of its mooring system. Hence, adjusting this restoring force, we may compose a complete reflector or transmitter for an incident wave of arbitrary wave-length.

Then we describe conditions of complete reflection and transmission by making use of phases of wave exciting force and mechanical impedance of mooring system in the complex plane. This description enables us to estimate easily the necessary spring constant at an arbitrary wave-length.

Experiments on a semi-submerged vertical plate and a circular cylinder show good agreements with theoretical prediction.

According to this theory, it becomes possible to design a reflector of long wave over ten times of the breadth of floating body.

1. Introduction

Various kinds of wave breaker have been developed and have received practical application in recent years. Principles of wave breaking applied to these wave breakers can be roughly classified as follows, reflection of an incident wave, conversion of wave energy into eddy energy in fluid, and combination of these principles. Wave breaking by reflection of an incident wave can be easily predicted by using a potential theory, and so the potential theory has usually been applied for comparison with model experiments and so on. But, in the case of conventional wave breakers, ideas of form or configuration of a wave breaker usually comes first, and a theoretical investigation on wave breaking may not be carried out sufficiently. This fact leads to an inferior performance of wave breaking for waves with long wave lengths, and therefore results in the scaling up of a wave breaker for improvement of the performance.

On the other hand, many studies on wave power absorption have been carried out vigorously due to the recent fossil energy crisis. Recent studies show that the maximum efficiency of wave power absorption attained by using a single mode oscillation of a two-dimensional floating body is only 50% but it becomes 100% if a complete wave reflector exists at the lee side of the body. Therefore, it is obvious that a reflector with superior performance can be effectively utilized for the purpose of the wave power absorption by a floating body.

In this paper, conditions of complete reflec-

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tion of an incident wave are derived from a theoretical investigation based on the two-dimensional wave theory, and an attempt is made to realize complete reflection at arbitrary wave periods by tuning the mechanical impedance of mooring systems. Consequently, it is found that complete reflection of an incident wave can be attained by using a device with a simple mechanical configuration and that it can reflect not only short waves but also long waves in high performance.

2. Theory

2.1 *Problem formulation (or Equation of motion)*

The coordinate system is shown in Fig. 1. We denote an incident wave η_ω, swaying motion η_1 and heaving motion η_2 of point 0 and rolling motion η_3 around the point 0 as follows:

$$\eta_\omega=\zeta_\omega e^{i(kx+\omega t)} \tag{1}$$

$$\eta_j=\zeta_j e^{i\omega t}, \qquad j=1,2,3 \tag{2}$$

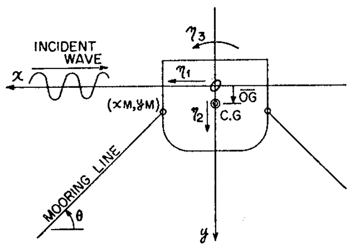

Fig. 1 Coordinate system

By using these expressions, equations of the motions in three degrees of freedom can be expressed as follows:

$$-Wk(\zeta_1-\overline{OG}\zeta_3)$$
$$=F_{11}+F_{13}+F_{1M}+F_{13M}+F_{1W} \tag{3}$$

$$-Wk\zeta_2=F_{22}+F_{2R}+F_{2M}+F_{2W} \tag{4}$$

$$Wk\overline{OG}\zeta_1-Ik\zeta_3$$
$$=F_{33}+F_{31}+F_{3R}+F_{3M}+F_{31M}+F_{3W} \tag{5}$$

Where $\overline{OG}$ denotes the distance of the center of gravity of the floating body from coordinate origin, W and I are the weight of the floating body and moment of inertia around point 0 respectively. F_{ij} is the hydrodynamic force in the i-th direction caused by j-th mode motion. By using an added mass coefficient f_{ijc} and Kochin function H_j^+, it can be expressed in the form

$$F_{ij}=\rho\omega^2(f_{ijc}-iH_i^+\overline{H_j^+})L\zeta_j \tag{6}$$

F_{2R} and F_{3R}, denote hydrodynamic restoring forces for heaving and rolling motions, and they can be expressed as:

$$F_{2R}=-\rho gLB\zeta_2 \tag{7}$$

$$F_{3R}=-\rho g\nabla\overline{GM}\zeta_3 \tag{8}$$

where L, B and ∇ are length, breadth and displacement volume of the floating body respectively.

F_{iM} and F_{ijM} are restoring forces in i-th mode motion of a mooring system caused by j-th mode motion. If the number of the mooring lines is n and the mooring lines of spring constant k_M are stretched symmetrically about the center plane, these become:

$$F_{1M}=-nk_M\cos\theta\zeta_1 \tag{9}$$

$$F_{2M}=-nk_M\sin\theta\zeta_2 \tag{10}$$

$$F_{3M}=-nk_M(x_M^2\sin\theta+y_M^2\cos\theta)\zeta_3 \tag{11}$$

$$F_{13M}=nk_M\cos\theta y_M\zeta_3 \tag{12}$$

$$F_{31M}=nk_M\cos\theta y_M\zeta_1 \tag{13}$$

Here θ shows the angle from the horizontal plane to the mooring line, and x_M and y_M are the coordinates of the points where the mooring lines are connected to the floating body.

By the use of the Kochin function, a wave exciting force in the i-th direction can be expressed as following:

$$F_{j\omega}=-\rho g\zeta_\omega H_j^+L \tag{14}$$

2.2 *Impedance*[3]

With an assumption that the amplitude of an incident wave is unity, and by substitution of Eqs. (6)～(14) into Eqs. (3), (4), (5), the equations of motions are replaced by the following:

$$Z_{11}\zeta_1+Z_{13}\zeta_3=-\rho g H_1^+/i\omega \tag{15}$$

$$Z_{22}\zeta_2=-\rho g H_2^+/i\omega \tag{16}$$

$$Z_{31}\zeta_1+Z_{33}\zeta_3=-\rho g H_3^+/i\omega \tag{17}$$

Here Z_{ij}, what is called Mechanical impedance, is expressed as follows:

$$Z_{11}=\rho\omega H_1^+\overline{H_1^+}+i\left\{\omega\left(\frac{W}{gL}+\rho f_{11C}\right)-\frac{nk_M\cos\theta}{\omega L}\right\} \tag{18}$$

$$Z_{22}=\rho\omega H_3^+\overline{H_3^+}+i\left\{\omega\left(\frac{W}{gL}+\rho f_{22C}\right)-\frac{nk_M\sin\theta}{\omega L}-\frac{\rho g B}{\omega}\right\} \tag{19}$$

$$Z_{33}=\rho\omega H_3^+\overline{H_3^+}+i\left\{\omega\left(\frac{I}{g}+\rho f_{33C}\right)-\frac{nk_M(x_M^2\sin\theta+y_M^2\cos\theta)}{\omega L}-\frac{\rho g\nabla\overline{GM}}{\omega L}\right\} \tag{20}$$

$$Z_{13}=Z_{31}=\rho\omega l_w H_1^+\overline{H_1^+}+i\left\{\omega\left(-\frac{W\overline{OG}}{gL}+\rho f_{13C}\right)+\frac{y_M nk_M\cos\theta}{\omega L}\right\} \tag{21}$$

For the convenience of later sections, we will rewrite the impedance Z_{ij} by amplitude and phase as follows:

$$Z_{11}=\rho\omega H_1^+\overline{H_1^+}\sec\beta_1 e^{i\beta_1} \tag{22}$$

$$Z_{22}=\rho\omega H_2^+\overline{H_2^+}\sec\beta_2 e^{i\beta_2} \tag{23}$$

$$Z_{33}=\rho\omega H_3^+\overline{H_3^+}\sec\beta_3 e^{i\beta_3} \tag{24}$$

$$Z_{13}=\rho\omega l_w H_1^+\overline{H_1^+}\sec\beta_{13} e^{i\beta_{13}} \tag{25}$$

2.3 *Condition of complete reflection*[3)]

Complex amplitudes of a reflection wave and a transmission wave are expressed as follows:

$$A_R=\frac{1}{2}\left(\frac{H_2^+}{\overline{H_2^+}}+\frac{H_1^+}{\overline{H_1^+}}\right)+ik(\zeta_1+l_w\zeta_3)H_1^+ +ik\zeta_2 H_2^+ \tag{26}$$

$$A_T=\frac{1}{2}\left(\frac{H_2^+}{\overline{H_2^+}}-\frac{H_1^+}{\overline{H_1^+}}\right)-ik(\zeta_1+l_w\zeta_3)H_1^+ +ik\zeta_2 H_2^+ \tag{27}$$

We will also rewrite the Kochin function by amplitude and phase as follows:

$$H_j^+=|H_j^+|e^{i\alpha_j} \tag{28}$$

By substitution of solutions of Eqs. (15)～(17) into Eqs. (26), (27) and by the use of Eqs. (52)～(25), (28), Eqs. (26), (27) are rewritten as

$$A_R=-\frac{1}{2}e^{i2(\alpha_2-\beta_2)}+\frac{1}{2}e^{i2(\alpha_1-\beta_1')} \tag{29}$$

$$A_T=-\frac{1}{2}e^{i2(\alpha_2-\beta_2)}-\frac{1}{2}e^{i2(\alpha_1-\beta_1')} \tag{30}$$

where β_1' is

$$\beta_1'=\mathrm{Arg}\{\sec\beta_1 e^{i\beta_1}\sec\beta_3 e^{i\beta_3}-\sec^2\beta_{13}e^{i2\beta_{13}}\} \tag{31}$$

The first term on the right hand of Eqs. (29), (30) shows a symmetrical wave and the second term an asymmetrical wave respectively.

Since complete reflection means

$$|A_R|=1 \tag{32}$$

a condition of complete reflection is derived from Eq. (29) and is expressed as follows:

$$\alpha_1-\alpha_2=\beta_1'-\beta_2\pm(2n+1)\frac{\pi}{2} \tag{33}$$

On the contrary, in the case of $A_T=0$, which means complete transmission, a condition of complete transmission is derived from Eq. (30) and expressed as in the following.

$$\alpha_1-\alpha_2=\beta_1'-\beta_2\pm n\pi \tag{34}$$

In the same way, a condition which makes a transmission coefficient smaller than 0.5 is obtained as follows:

$$\left|(\alpha_1-\alpha_2)-\left\{\beta_1'-\beta_2\pm 2n+1)\frac{\pi}{2}\right\}\right|<\frac{\pi}{6} \tag{35}$$

2.4 *Example of numerical calculation*

2.4.1 Vertical flat-plate

We will consider a vertical flat plate oscillating only in swaying mode. Complex response amplitude of swaying motion can be expressed by using the expression of impedance shown by Eq. (18) as follows:

$$\zeta_1=\frac{\rho g L H_1^+}{i\omega Z_{11}} \tag{36}$$

The Kochin function of a vertical flat plate is, as shown by Kotik[4],

$$H_1^{\pm}=\pm\frac{i\pi}{k}\frac{I_1(kd)+L_1(kd)}{K_1(kd)+i\pi I_1(kd)} \tag{37}$$

where $I_1(kd)$ and $K_1(kd)$ are modified Bessel functions of order 1, and $L_1(kd)$ is a Struve function of order 1.

Complex amplitudes of reflected waves and transmitted waves can be rewritten, by substituting Eq. (36) into Eqs. (26), (27) and by using the expression of Eqs. (22), (28), as in the following:

$$A_R=\frac{1}{2}-\frac{1}{2}e^{i2(\alpha_1-\beta_1)} \tag{38}$$

$$A_T=\frac{1}{2}+\frac{1}{2}e^{i2(\alpha_1-\beta_1)} \tag{39}$$

From the Eq. (38), the condition of complete wave reflection becomes

$$\alpha_1=\beta_1\pm(2n+1)\frac{\pi}{2} \tag{40}$$

Numerical values of α_1 and $\beta_1\pm(2n+1)\pi/2$ calculated by changing the spring constant of the mooring system are shown in Fig. 2. As shown by Eq. (39), the cross point of α_1 curve and $\beta_1\pm(2n+1)\pi/2$ curve is the point where complete reflection is realized. k_M' in Fig. 2 means a non-dimensional value defined by the following expression

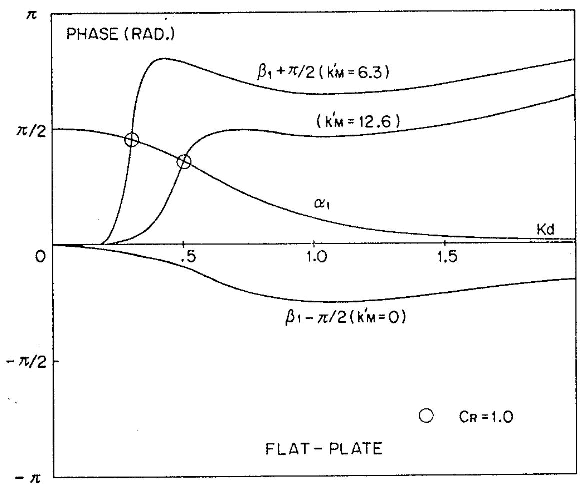

Fig. 2 Phases of Kochin function and impedance for a swaying vertical cylinder

$$k_M'=(L/W)\cdot nk_M \tag{41}$$

It is clear from Fig. 2 that there is not any frequency where complete reflection can be realized in the frequency range if the spring constant of the mooring system is zero, and that there is only one frequency where complete reflection can be realized if the spring constant is a finite value.

2.4.2 Half immersed circular cylinder

We assume that a half-immersed circular cylinder oscillates without interaction between swaying and rolling motions. Then complex amplitudes of swaying and rolling motions can be expressed in terms of the impedances of Eqs. (18) and (19) as follows:

$$\zeta_1=-\frac{\rho g H_1^+}{i\omega Z_{11}} \tag{42}$$

$$\zeta_2=-\frac{\rho g H_2^+}{i\omega Z_{22}} \tag{43}$$

Substituting Eqs. (42) and (43) into Eqs. (26) and (27), and using the expressions shown by Eqs. (22) and (23), Eqs. (26) and (27) can be replaced by the following expressions.

$$A_R=-\frac{1}{2}e^{i2(\alpha_2-\beta_2)}-\frac{1}{2}e^{i2(\alpha_1-\beta_1)} \tag{44}$$

$$A_T=-\frac{1}{2}e^{i2(\alpha_2-\beta_2)}+\frac{1}{2}e^{i2(\alpha_1-\beta_1)} \tag{45}$$

From Eq. (44), a condition of complete reflection is derived as in the following:

$$\alpha_1-\alpha_2=\beta_1-\beta_2\pm n\pi \tag{46}$$

Fig. 3 shows the result of calculation on $(\alpha_1-\alpha_2)$ and $(\beta_1-\beta_2\pm n\pi)$ in the case of a half-immersed circular cylinder moored by horizontal mooring lines. The definition of k_M' shown in Fig. 3 is the same as Eq. (41). Fig. 3 shows that the intersection of $(\alpha_1-\alpha_2)$ curve and $(\beta_1-\beta_2\pm n\pi)$ curve occurs at two point in the frequency range shown in Fig. 3 if the spring constant of the mooring lines is tuned at the appropriate values, and this fact means that complete reflection can be realized at two frequencies of these intersections.

2.4.3 Floating body of Lewis form section

As an example of a general floating body, we consider the case as shown in Fig. 4. The

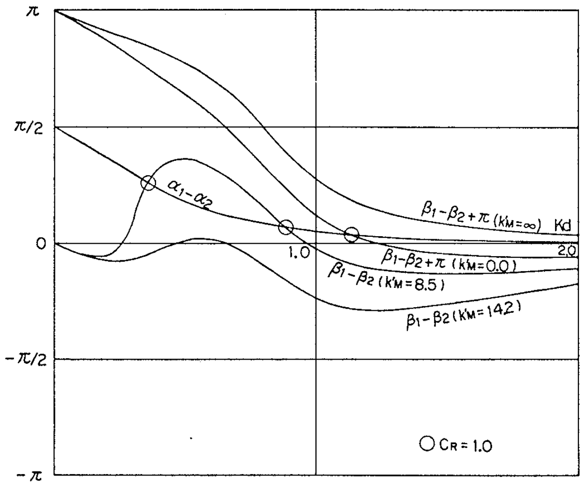

Fig. 3 Phases of Kochin functions and impedances for a swaying and heaving circular cylinder

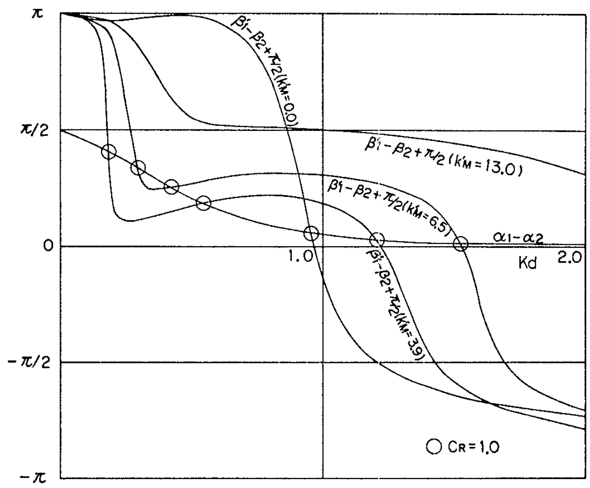

Fig. 5 Phases of Kochin functions and impedances for a Lewis form cylinder oscillating in three degree of freedom

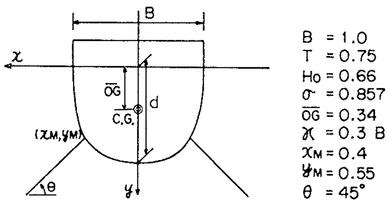

Fig. 4 Mooring of a Lewis form cylinder

condition of complete reflection which is applicable to this case has been shown by Eq. (33) already. Results of calculation on $(\alpha_1-\alpha_2)$ and $\{\beta_1-\beta_2\pm(2n+1)\pi/2\}$ are shown in Fig. 5.

In the case of $k_M'=0$, $(\alpha_1-\alpha_2)$ curve and $(\beta_1-\beta_2)$ curve cross each other at only one point, while, in case of $k_M'=3.9$, crossing of these curves occurs at three points. Therefore, complete reflection can be realized at the frequencies of these three points. In the case of $k_M'=3.9$, the difference between $(\alpha_1-\alpha t)$ curve and $(\beta_1-\beta_2)$ curve is smaller than $\pi/6$ in k_d ranging from 0.2 to 1.2, so that it is derived from Eq. (35) that the transmission coefficient takes a smaller values than 0.5. Moreover, in the case of $k_M'=13$, as the difference between $(\alpha_1-\alpha_2)$ durve and $(\beta_1-\beta_2)$ curve becomes about $\pi/2$ in a wide frequency range, it follows from Eq. (34) that complete transmission is almost realized in the frequency range.

3. Model experiments

3.1 *Vertical flat plate*

A vertical flat-plate used in the model experiment is shown in Fig. 6. This plate is attached under a slide guide which is installed on a narrow channel of 4 m length and 0.61 breadth, temporarily constructed in a wave basin of $L\times B\times H=9\times1.2\times1$ m. In the model experiment, heaving and rolling motions are restrained by the slide guide and only swaying motion is allowed. Waves are measured by three wave-height-meters, and one of them is installed at lee-side and the others at wheather side. These wave signals are analyzed by a Fourier series expansion and amplitudes of an incident wave, reflected wave and transmitted wave are obtained from the

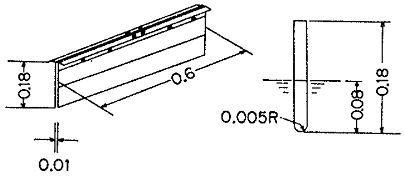

Fig. 6 Dimension and figure of the vertical plate for model test

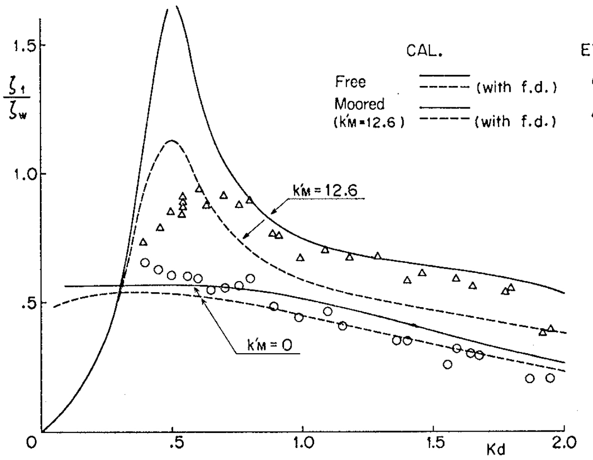

Fig. 7 Swaying motion of the vertical plate

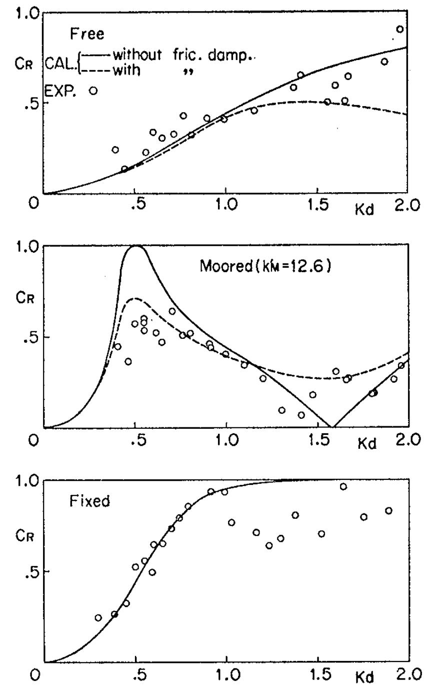

Fig. 8 Reflection coefficient of the vertical plate

Fourier series.

The amplitude response function of swaying motion is shown in Fig. 7 and the reflection coefficient in Fig. 8. In these figures, small round marks show results of the model experiment and solid and broken lines show results of theoretical calculations. The broken lines are calculated including the effect of mechanical friction, in which case impedance is expressed as follows:

$$Z_{11}=\rho\omega H_1^+\overline{H_1^+}+\mu_f+i\left\{\omega\left(\frac{W}{\rho g}+\rho f_{11C}\right)-\frac{nk_M}{\omega L}\right\} \tag{47}$$

where μ_f is the frictional resistance coefficient.

It is clear from comparing the results of the model experiment with the theoretical calculations that the results of the model experiment almost coincide with the theoretical calculation which includes the effect of mechanical friction. As shown in Fig. 8, the reflection coefficient calculated without mechanical friction becomes almost 1.0 at $K_d=0.5$, while both the results of the model experiment and the theorettcal calculation with mechanical-friction show $C_R=0.7$ at $K_d=0.5$, so that it is concluded that complete reflection can not be attained in our model experiment on a swaying vertical flat plate. Generally, if a frictional force exists, a dynamical system becomes dispersive and energy conservation laws do not hold well. As this fact inevitably leads to $C_R<1.0$, it is proper that complete reflection can not be realized in our model experiment.

3.2 *Half immersed circular cylinder*

The figure and dimensions of a circular cylinder are shown in Fig. 9. Model experiments are carried out by using the cylinder which is 2.97 m in length and 0.6 m in breadth and is half-immersed in a wave flume of L×B×H=60×3×1.5 m. Wave measurement and analysis are carried out in the same way as that used in the case of the vertical flat plate. Motions of the cylinder are measured by a motion measuring instrument equipped with

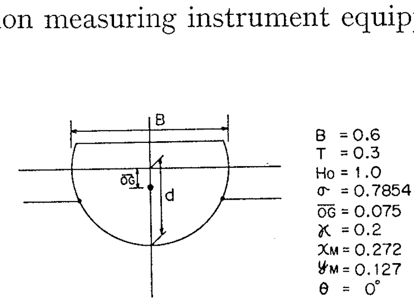

Fig. 9 Dimension and figure of the circular cylinder for model test

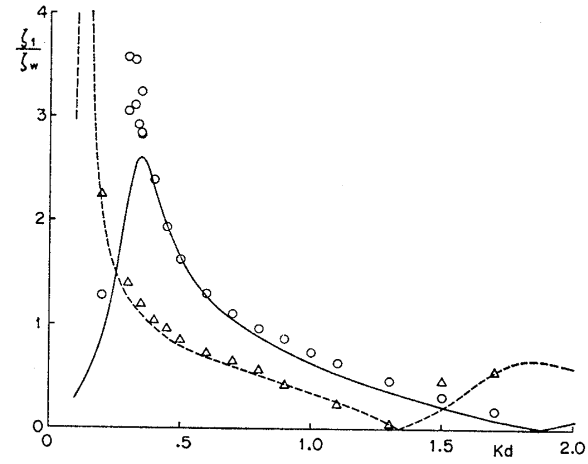

Fig. 10 Swaying motion of the circular cylinder

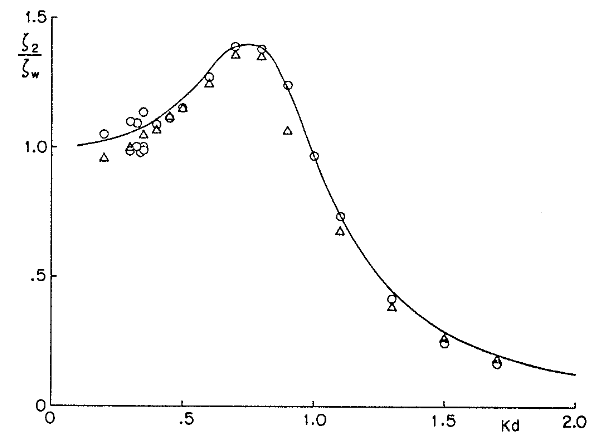

Fig. 11 Heaving motion of the circular cylinder

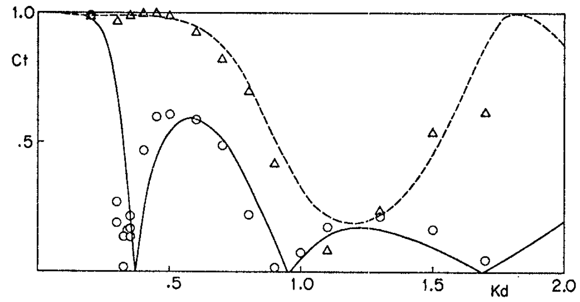

Fig. 12 Transmission coefficient of the circular cylinder

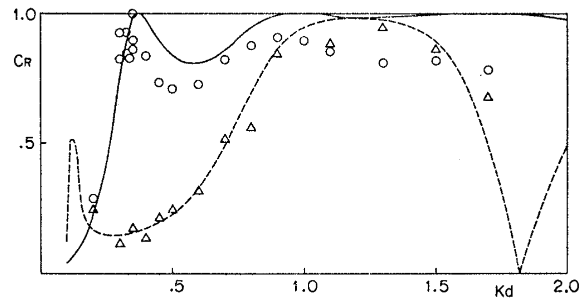

Fig. 13 Reflection coefficient of the circular cylinder

potentio-meters. Mooring lines consist of wires and coil springs, and these are connected with the cylinder at a position 0.127 m under the water surface and stretched horizontally.

Amplitude response functions of swaying and heaving motions are shown in Figs. 10 and 11, and transmission coefficients and reflection coefficients are shown in Figs. 12 and 13. Small round marks and solid lines in these figures show results of the model experiments and the theoretical calculations respectively in the case of $k_M'=9.1$, and small triangular marks and broken lines in the case of $K_M'=2.8$. These figures show that the results of the model experiments almost coincide with the results of the theoretical calculations. A slight discrepancy is observed in the reflection coefficient in Fig. 13 in the case of $K_M'=9.1$, and the experimental values are smaller than the theoretical values. Although complete reflection is not attained perfectly, it can be recognized that a result close to complete reflection is attained in the model experiments. Fig. 12 shows that the transmission coefficient becomes zero at three frequencies in the case of $K_M'=9.1$. This fact is different from the result shown in Fig. 3, but it is proper because both swaying and rolling motions interact with each other in this case. In the case of $K_M'=9.1$, the transmission coefficient becomes about 0.5 or less in the frequency range of $K_d \geqq 0.3$ and shows good performance of wave reflection.

As clarified by these results of the model experiments and the theoretical calculations, the tuning of mechanical impedance by changing the spring constant of the mooring lines leads to change of the natural frequency of swaying motion, and results in higher performance of wave reflection in the high frequency range above the natural frequency as shown in Figs. 12 and 13. Consequently, excellent performance, where the reflection coefficient is larger than 0.9 and the transmission coefficient smaller than 0.2, is attained even in the case where the wave length is longer than ten times the breadth of the floating body.

4. Conclusion

The performance of wave reflection of a floating body equipped with a mechanical mooring system was investigated by using the two-dimensional wave theory. The concept of impedance was applied to the hydrodynamic property of a floating body, then reflected and transmitted waves were described by the phases of the impedances and the Kochin functions. Moreover conditions which should be satisfied by the phases of the impedances and the Kochin functions for complete reflection were clarified. Numerical calculation on these phases were carried out with respect to a flat plate, a half-immersed circular cylinder and a Lewis form cylinder. Results of the calculation showed that complete reflection can be realized at more than two frequencies if the impedances are tuned properly by adjusting the spring constant of the mooring system.

Furthermore, model experiments were carried out concerning the flat-plate and the circular cylinder, and it was confirmed that results of the model experiments almost coincide with the results of the numerical calculations. Moreover it was verified that a long wave, more than ten times the breadth of the floating body, can be effectively reflected in the case of the circular cylinder by tuning the impedances.

For the practical application of the results of this paper a long length floating body causes problems such as structual and sea traffic. But the same property could be expected by a linear array arrangement of many short bodies in the antenna theory.

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