8. Ship Maneuvering Motion due to Tugboats and Its Mathematical Model

Junshi TAKASHINA*, Member

Summary

This paper presents results of free running model tests and simulation calculations concerning ship maneuvering motion due to tugboats. The free running model tests were carried out by the use of wind fans on the model deck which simulate the tug forces.

The mathematical model for such maneuvering motions at low speed was developed on the basis of captive model tests which consist of the static drift test with whole range of drift angle and the yaw rotating test with several combinations of towing speed and yaw angular velocity. The simulation calculations according to the mathematical model were compared with the free running test results for typical maneuvers with tugboats.

The simulation calculations show satisfactory agreement with the free running test results. This indicates that the mathematical model proposed in the present paper is available for describing a so-called low speed maneuvering motion.

1. Introduction

As increase in ship sizes and in sea traffic congestion in last decades, roles of tugboats have become more important for harbor maneuvering of ships. Although the tugs are frequently used in reality and operated with a complicated manner\(^1\), there have been still unknowns and few research works for the maneuvering characteristics of a ship assisted by tugs.

Tugboats usually assist a ship sailing at a low speed and can push or pull the ship with arbitrary forces and directions. Therefore a range of the ship maneuvering motions realized by tugs is considered much wider than that realized by a conventional rudder operation. This indicates that a mathematical model developed for the conventional maneuvering motion at a relatively high advance speed, where the surge velocity is assumed to be large relative to the sway velocity and the yaw angular velocity, may be difficult to be applied to such motions due to tugs.

A lot of efforts to develop the mathematical model applicable to motions where the surge, sway and yaw velocities are comparable, called as a low speed maneuvering motion, have been made\(^{2,3}\). Such work has been carried out by Kose\(^3\) according to the extensive captive tests.

The first objective of the present study is to investigate experimentally the ship maneuvering characteristics due to tugs by free running model tests.

Furthermore the author attempted to develop the mathematical model for the low speed maneuvering motion based on somewhat new captive model test called as yaw rotating test.

In the present paper, through comparisons between simulations according to the mathematical model and the free running test results, the basic characteristics of ship motions due to tugs are clarified and the validity of the mathematical model proposed is examined.

2. Model Tests

2.1 Ship Models

Two ship models representing a LNG carrier were used for this study and their principal particulars are given in Table 1. The larger model, Lpp=5m, was used as a free running radio controlled model, whereas the smaller model, Lpp=2.5m, was used in a captive test.
Scale ratios of these models are 1 to 54 and 1 to 108 respectively.

2.2 Free Running Tests

Free running tests were carried out to aim at investigating the basic characteristics of ship maneuvering motion due to simplified tug forces rather than simulating actual tug operations. For this purpose, a set of three wind fans was installed on the model deck as an alternative to tug boats.

Fig. 1 shows the arrangement of this wind fan system which consists of the fore and aft fans for generating lateral force and yaw moment and the midship fan for longitudinal force. The free running model tests were carried out in the large towing tank of Akishima Laboratories (Mitui Zosen) Inc., having a length of 200m, a width of 14m and a depth of 6m.

![Arrangement of wind fans of model](image)

Table 1 Principal particulars of models

<table>
<thead>
<tr>
<th></th>
<th>Free-running test</th>
<th>Captive test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length p.p.</td>
<td>5.000 m</td>
<td>2.500 m</td>
</tr>
<tr>
<td>Breadth</td>
<td>0.830 m</td>
<td>0.415 m</td>
</tr>
<tr>
<td>Draft</td>
<td>0.200 m</td>
<td>0.100 m</td>
</tr>
<tr>
<td>Displacement volume</td>
<td>0.574 m$^3$</td>
<td>0.072 m$^3$</td>
</tr>
<tr>
<td>C.G. from B (-fore)</td>
<td>0.028 m</td>
<td>0.012 m</td>
</tr>
<tr>
<td>Propeller diameter</td>
<td>0.148 m</td>
<td></td>
</tr>
<tr>
<td>Rudder area ratio</td>
<td>1/44.5</td>
<td></td>
</tr>
</tbody>
</table>

2.3 Captive Tests

As shown later, it was found that the ship motion during the free running tests had often a large drift angle and a large yaw angular velocity relative to ship speed. It is therefore necessary that the hydrodynamic forces acting on the ship are measured at a very large drift angle as well as at very slow speed motion in order to develop a mathematical model for the maneuvering motion due to tugs. There were several captive tests adequate for this purpose such as expanded planar motion tests and circular motion tests.

In the present study the following two tests, namely static drift test and yaw rotating tests, were carried out for the force measurements on the ship hull.

(A) Static drift tests

The ship was towed at a constant speed with a constant drift angle ranging from 0° to 180°.

(B) Yaw rotating tests

The ship was towed at a constant speed while rotating around a vertical axis through the center of gravity of the ship with a constant yaw angular velocity. The test were carried out for various combinations of the towing speed $U$ and the yaw angular velocity $\omega$ so as to cover a range of nondimensional yaw angular velocity $\omega'(=rL/U, L:$ ship length) from 0.26 to 2.0. In addition to the tests, tests with zero towing speed were made for several yaw angular velocities and these correspond to the yaw rotating tests for $\omega' = \infty$. On the contrary, the static drift tests correspond to that for $\omega' = 0$.

As shown in Fig. 2, the ship during a yaw rotating test experiences the forces due to coupling motion between varying drift angle and a constant yaw angular velocity and therefore a lot of information can be obtained from one test run. The yaw rotating test are
expected to save experimental cost and time comparing to the conventional tests. Another advantage of the tests may be a simple mechanism to be used.

All of the captive tests were also carried out in the large towing tank of Akishima Laboratories (Mitsui Zosen) Inc. with the small model in a bare hull condition fixing roll, pitch and heave motions.

3. Results of Captive Tests

A coordinate system presented in Fig. 3 is used to describe the ship motion as well as the forces and moment acting on ship. The motion variables, forces and moment are usually nondimensionalised according to the following scheme.

\[
\begin{align*}
\mathbf{u}', \mathbf{v}' &= \mathbf{u}, \mathbf{v}/U \\
\mathbf{r}' &= \mathbf{r}L/U \\
\mathbf{X}', \mathbf{Y}' &= \mathbf{X}, \mathbf{Y}/\frac{1}{2} \rho LdU^2 \\
\mathbf{N}' &= \mathbf{N}/\frac{1}{2} \rho L^2dU^2
\end{align*}
\]

where \(\rho\) : density of water
\(L\) : ship length between perpendiculars
\(d\) : Ship draft

3.1 Static Drift Tests

The static drift test with drift angle \(\beta\) varying from 0° to 180° were carried out at the towing speed of 0.327m/sec (Froude number \(Fn=0.066\)). The measured lateral force and yaw moment in the nondimensional forms are plotted to a base of drift angle in Fig. 4.

Among several expressions for the force and moment as a function of whole range of drift angle\(^{3,6}\), the following expression using Fourier series was adopted in the present study.

\[Y'N' = \sum_{k=1}^{K} S_k \sin k\beta\]  \hspace{1cm} (2)

Solid lines in Fig. 4 give the approximation of the measured data by eq. (2) with the coefficients of the Fourier series given in Table 2.

3.2 Yaw Rotating Tests

(1) Tests with zero ship speed

The tests were carried out for yaw angular velocity of 2, 3, 4 and 5 deg/sec. Figure 5 shows the plots of the yaw moment acting on
Table 2 Coefficients of Fourier series approximating lateral force and yaw moment on hull in static drift test

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<th>Yaw moment N'</th>
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<tr>
<td>S1</td>
<td>0.550</td>
<td>0.0098</td>
</tr>
<tr>
<td>S2</td>
<td>-0.021</td>
<td>0.0529</td>
</tr>
<tr>
<td>S3</td>
<td>-0.021</td>
<td>-0.0023</td>
</tr>
<tr>
<td>S4</td>
<td>-0.0091</td>
<td>-0.0095</td>
</tr>
<tr>
<td>S5</td>
<td>-0.0091</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 5 Yaw moment on hull in rotating test with zero forward speed

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Table 2 Coefficients of Fourier series approximating lateral force and yaw moment on hull in static drift test

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Fig. 5 Yaw moment on hull in rotating test with zero forward speed

the hull against yaw angular velocity squared \( r \) \( |r| \).

According to the ratio of yaw moment to yaw angular velocity squared, nonlinear derivative \( N'_r \) \( |r| \) can be obtained using the following scheme.

\[
N'_r \frac{1}{|r|} = N \frac{1}{2} \rho L^4 dr \frac{1}{|r|} \tag{3}
\]

The derivative \( N'_r \) \( |r| \) obtained from the tests is

\[
N'_r \frac{1}{|r|} = -0.034 \tag{4}
\]

and this value is approximately corresponding to the value, \( N'_r \frac{1}{|r|} = -1/32(-C_D/32 : C_D = 1) \), which is derived from a cross flow model with the local cross flow drag coefficient \( C_D \) constant along the ship.

(2) Tests with forward ship speed

The tests were carried out at three different towing speeds, \( U = 0.109, 0.218, 0.327 \text{ m/sec} \) with four different yaw angular velocities, \( r = 2, 3, 4, 5 \text{ deg/sec} \).

A procedure for analysis of the yaw rotating tests with forward speed are shown below.

The equations of motions of a ship during the captive tests can be given in the ship fixed coordinate system

\[
(\dot{m} + m_x) u = -(m + m_y) v + X_H - X_m
\]

\[
(\dot{m} + m_x) v = -(m + m_y) u + Y_H - Y_m
\]

\[
(I_{zz} + J_{zz}) \dot{r} = N_H - Y_m
\]

(5)

where \( m \) : mass of ship

\( m_x, m_y \) : added mass of ship

\( I_{zz} \) : moment of inertia of ship

\( J_{zz} \) : added moment of inertia of ship

and where \( X_H, Y_H \) and \( N_H \) represent the hydrodynamic damping forces and moment, and \( X_m, Y_m \) and \( N_m \) represent the forces and moment measured on force transducers.

The motions given to the ship during the yaw rotating tests are of the form:

\[
\beta = \beta (t; \beta_0 = 0 \text{ at } t = 0; \beta \text{ : time})
\]

\[
u = U \cos \beta, \quad \dot{v} = -U \sin \beta \sin \beta \]

\[
v = -U \sin \beta, \quad \dot{v} = -U \cos \beta \cos \beta \]

(6)

\( \tau = \text{constant}, \dot{i} = 0 \)

When substituting equation (6) into equation (5) and using equation (1) for nondimensionalizing, the nondimensional measured forces and moment can be obtained as follows.

\[
X'_m = (m'_x - m'_y) \tau \sin \beta + X'_H
\]

\[
Y'_m = (m'_y - m'_x) \tau \cos \beta + Y'_H
\]

\[
N'_m = N'_H
\]

(7)

where \( m'_x, m'_y = m_x, m_y - \frac{1}{2} \rho L^2 d \)

It should be noted that the measured forces
$X_m'$ and $Y_m'$ always include virtual forces due to added mass besides the damping forces.

The measured forces and moment for the drift angle between $0^\circ$ and $360^\circ$ can be expressed in terms of Fourier series with respect to the drift angle $\beta$.

$$X_m' = \sum_{k=0}^{K} (S_k X \sin k \beta + C_k X \cos k \beta)$$

$$Y_m' = \sum_{k=0}^{K} (S_k Y \sin k \beta + C_k Y \cos k \beta) \tag{8}$$

$$N_m' = \sum_{k=0}^{K} (S_k N \sin k \beta + C_k N \cos k \beta)$$

The prime Fourier coefficients resulting from harmonic analysis for $Y_m'$ and $N_m'$ are plotted against the nondimensional yaw angular velocity $r'$ in Figs. 6 and 7, where the Fourier coefficients obtained from the static drift tests are also plotted at $r'=0$. If the variations of the Fourier coefficients are given as a function of $r'$ in a definitive manner, the hydrodynamic forces and moment acting on the hull for any ship motion can be determined through equations (7) and (8).

In the present study, the variations of the Fourier coefficients are supposed to be expressed by the following equation written in general form.

$$S_k X = S_{k1} r' + S_{k2} r'^2 \mid r' \mid$$

$$S_k Y, S_k N = S_{k0} + S_{k1} \mid r' \mid + S_{k2} r'^2$$

$$C_k X = C_{k0} + C_{k1} \mid r' \mid + C_{k2} r'^2$$

$$C_k Y, C_k N = C_{k1} r' + C_{k2} r' \mid r' \mid$$

(9)

Solid lines in Figs. 6 and 7 show the approximated variation of each Fourier coefficient by equation (9), which were used as a linear approximation for the several coefficients such as $S_1 Y, S_2 N$ and $C_2 N$. The Fourier coefficients without the approximation in Figs. 6 and 7 are supposed to be constant independent of $r'$.

As a result of these approximations, the yaw moment $N_H'$ can be written as follows.

$$N_H' = S_{10} \sin \beta + (S_{20} + S_{21} \mid r' \mid)$$

$$\sin 2 \beta + S_{30} \sin 3 \beta$$

$$+ S_{10} \sin 4 \beta + C_{01} r'$$

$$+ C_{02} r' + C_{21} r' \cos 2 \beta \tag{10}$$

Furthermore the above expression in terms of Fourier series can be transformed into the conventional expression with hydrodynamic derivatives by the use of the relations between

---

*a Figure 6: Coefficients of Fourier series approximating lateral force on hull in yaw rotating test*
\[ \sin \beta = -v', \quad \cos \beta = u', \quad \sin 2\beta = -2u'v', \quad \cos 2\beta = 1 - 2v'^2, \quad \cdots \]

This leads to:
\[ N''_H = N' v' + N''_{uv} u' v' + N''_{uvu} u' v'^3 + N'_{vu} v'^2 + N'_{v'u} v' + N'_{v''u} v'^2 v' + N'_v v' + N'_v v'^2 \]

where each derivative can be obtained from the following relationships.

\[ N'_v = -S_{10} - 3S_{30}, \quad N''_{uv} = 4S_{30} \]
\[ N'_w = -2S_{20} - 4S_{40}, \quad N''_{uvu} = 8S_{40} \]
\[ N'_r = C_{01} + C_{21}, \quad N''_{vv} = -2C_{21} \]
\[ N'_v |r| = C_{02}, \quad N''_{uv} |r| = -2S_{21} \]

To determine the above derivatives, the results of the static drift tests as shown in Table 2 were used for \( S_{10}, S_{20}, S_{30}, S_{40} \) and the results of the yaw rotating with zero ship speed as given by equation (4) were used for \( C_{02} \).

The longitudinal force \( X''_H \) and lateral force \( Y''_H \) can be expressed in terms of the hydrodynamic derivatives by the same manner as \( N''_H \). However, a special attention should be paid to the virtual forces included in the measured force, as shown with a broken line in Fig. 6 for the lateral force. The final forms for \( X''_H \) and \( Y''_H \) are given in the next chapter. The determination of the mathematical model for the longitudinal force is made according to the following simplification because of some uncertainty of the measured data.

\[ X''_H = C_{10} \cos \beta + S_{11} v' \sin \beta \]

where \( C_{10} \) is given as a function of Froude number by the results of resistance tests using the larger model for the free running tests.

4. Mathematical Model
4.1 Equation of Motion

By reference to the ship fixed coordinate system, shown in Fig. 3, the Newton's equa-
Ship Maneuvering Motion due to Tugboats and Its Mathematical Model

4.2 Hull Damping Forces

The hull damping forces can be expressed as follows based on the results of the captive tests.

(1) In the case of $U=0$

\[
X_H = \left( X'_v \dot{u}' + X'_w v' r' \right) \times \frac{1}{2} \rho L d U^2
\]

\[
Y_H = \left( Y'_v \dot{v}' + Y'_w v'^3 + Y'_w v' v 
\right) \times \frac{1}{2} \rho L d U^2
\]

\[
N_H = \left( N'_v \dot{u}' + N'_w v' + N'_w v' v'^3 
\right) \times \frac{1}{2} \rho L^2 d U^2
\]

When the ship forward speed is dominant, namely $u' \neq 1$, equation (16) becomes very similar to the mathematical model for conventional ship maneuvering motion. For simulation calculation equation (16) is applicable to the case of $U \neq 0$, whatever ship speed is close to zero.

(2) In the case of $U=0$

In the case of $U=0$, ship motion is identical with the pure rotating motion around the center of gravity of the ship. The ship experiences, therefore, the yaw moment given by equation (3).

\[
X_H, Y_H = 0
\]

\[
N_H = N'_r | r' | r | \times \frac{1}{2} \rho L^4 d
\]

Table 3 gives the added mass, added moment of inertia and hydrodynamic derivatives in equations (16) and (17). The added mass $m$, and added moment of inertia $J_{zz}$ were obtained from a PMM tests and the added mass $m_x$ was estimated from Motoria's charts.

| $m_x/m$ | 0.050 |
| $m_y/m$ | 0.650 |
| $J_{zz}/mL^2$ | 0.043 |

| \(X'_v\) | \(-0.102\) |
| \(Y'_u\) | \(-0.035\) |
| \(Y'_v\) | \(-0.285\) |
| \(Y'v\) | \(-0.898\) |
| \(Y'vv\) | \(-0.050\) |
| \(Y'v'v'\) | \(-0.060\) |
| \(Y'vvr'\) | \(-0.035\) |
| \(Y'vvr'v'\) | \(-0.230\) |
| \(N'_v\) | \(-0.0028\) |
| \(N'_uv\) | \(-0.0078\) |
| \(N'_vv\) | \(-0.0093\) |
| \(N'_vvv\) | \(-0.0759\) |
| \(N'_r\) | \(-0.030\) |
| \(N'_r|r'|\) | \(-0.034\) |
| \(N'_u|v'|\) | \(-0.030\) |
| \(N'_v|v'|\) | \(-0.048\) |

The derivatives obtained from the present captive tests is considered to be reasonable by comparisons with empirical formulae such as proposed by Inoue. et. al. In such comparisons the derivatives have to be transformed into conventional derivatives on the assumption of $u' \neq 1$. 
4.3 Propeller, Rudder and Tug Forces

The longitudinal force due to propeller can be written in the following form.

\[ X_p = (1 - t_p) \frac{1}{2} \rho (|u(1 - w_p)|^2 \]

\[ + (0.7 \pi n D_p)^2 - \frac{\pi}{4} D_p^2 C_T(\theta_p) \]  \hspace{1cm} (18)

where \( t_p \) : thrust deduction coefficient
\( w_p \) : effective propeller wake fraction
\( D_p \) : propeller diameter
\( n \) : number of propeller revolution

The propeller open water characteristics \( C_T \) applicable to four quadrants are given as a function of the advance angle at 0.7 radius, \( \theta_p \), defined as

\[ \theta_p = \tan^{-1}[u(1 - w_p) / (0.7 \pi n D_p)] \]

It is assumed that both \( w_p \) and \( t_p \) are constant against the propeller operating condition but \( w_p \) varies with change in inflow angle to the propeller in a same manner as the conventional mathematical model\(^3\). The lateral force \( Y_p \) and yaw moment \( N_p \) due to propeller were ignored in this study.

The mathematical model for the rudder forces, which is basically the same as that for the conventional maneuvering motion, allows the inflow angle to the rudder to develop up to 180°.

5. Simulations and Free Running Test Results

The simulations were carried out on the basis of the mathematical model mentioned above and were compared with the results of the free running tests.

Figure 8 shows the turning motions by bow tug for various forward speeds, and Fig. 9 for the same by stern tug. Figures 10 and 11 show the turning motions and the lateral shifting motions by both bow and stern tugs. In the upper half of each figure the measured ship track are shown and the simulated ship track are compared in the lower half.

![Fig. 8 Turning motion due to tug pushing bow (T_B=-0.57kg, T_S=0.0kg)](image-url)
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Fig. 9 Turning motion due to tug pushing stern ($T_B = 0.0\text{kg}, T_S = 0.57\text{kg}$)

Fig. 10 Turning motion due to tugs pushing both bow and stern ($T_B = -0.57\text{kg}, T_S = 0.57\text{kg}$)
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The lateral force of each tug acting on the model is 0.57kg corresponding to about 90 tons full scale, which is equivalent to the thrust due to three tug boats of 3,000HP. In each figure, the ship tracks in case of initial forward speed of 0.0, 0.14, 0.28 and 0.42m/sec corresponding to 0, 2, 4 and 6 knots full scale are shown for a better understanding of the ship speed effect on the ship motions due to tug. The forward propulsion forces were given by the propeller and then the propeller kept on operating and the rudder amidship during each test. In these figures the outlines of ship are drawn at every 10 seconds.

The turning motions by the bow tug shown in Fig.8 seems not so different from the conventional maneuvering motions by rudder. It can be also found from this figure that the effect of ship forward speed on the ship motions is so significant that the turning diameter becomes large as the foward speed increases. The simulation results explain well such tendencies and give satisfactory agreements with the experimental results.

The turning motions by the stern tug shown in Fig.9 seem to be much different from that by the bow tug. The most interesting motion can be seen in a case of initial speed of 0.14m/sec, in which a drift angle varies from 0° at start to 180°. In the turning motions by the stern tug, longitudinal resistance due to turning motion, strictly speaking coupling effect of yaw and sway motion, becomes relatively large and therefore the motions vary according to whether of the longitudinal resistance and the propeller propulsion force is stronger. The both forces may be just balanced in the case of $U_0=0.28m/sec$. It can be seen that the effect of the stern tug also reduces as the ship forward speed increases. From the comparisons between the simulated and measured motions, it is concluded that the present mathematical model can describe well such complicated motions as shown in Fig. 9.

The turning motions by the bow and stern tugs shown in Fig. 10 are also affected by the ship forward speed. The simulations explain well the speed effects on the turning diameter which were observed in the model test, although the computed drift angles are somewhat different from the measured ones. These differences may be resulted from insufficient approximation of the hydrodynamic forces for large $\psi$ beyond the range of the captive tests.

The results of the lateral shifting motions shown in Fig. 11 indicate that lateral shift of the ship becomes difficult with increasing ship forward speed. The simulation results give good agreements with the test results except that the simulated heading change due to increasing forward speed seems to be exaggerated.

Figure 12 shows the comparisons of conventional turning motions by rudder of 15° and 35°[4] between the simulations and the free running tests in order to examine whether or not the present mathematical model for low speed maneuvering motions can be applied to the ordinary maneuvering motions. Good correlations can be seen between both results.

As shown in the comparisons of Figs. 8 through 12, the simulations can provide good qualitative predictions for various maneuvering motions realized by the tug operations as well as the rudder operations. There may be still room for quantitative improvement in the mathematical model proposed here, because some simplifications were made in deriving the mathematical model.
6. Conclusions

In order to study the maneuvering characteristics of a ship operated by tugs, both free running tests and captive tests were carried out and the mathematical model was derived from the captive test results. Then the simulations based on the mathematical model were compared with the results of the free running tests. The major conclusions obtained from the present study are as follows.

(1) The free running tests clarify the basic characteristics of ship maneuvering motions due to tugs involving turning motions in various uses of tugs and lateral shifting motions and indicate that such maneuvering motions largely depend on ship forward speed.

(2) The yaw rotating tests, in which a ship is...
towed with a constant speed while rotating around the vertical axis, were proposed as captive tests for a so-called low speed maneuvering motion. This test may be useful and cost effective from the view point that a lot of information is obtained from a few test runs and the instrument is quite simple.

(3) The mathematical model valid for all drift angles and for very large yaw angular velocity relative to ship speed was derived from the captive tests. This model becomes similar to the models for conventional maneuvering motions in the case of a relatively high forward speed, namely $u' \gg v'$. This fact suggests the present model is applicable to the conventional maneuvering motions.

(4) The simulations based on the mathematical model provided satisfactory predictions for various types of motion realized by tugs as well as the rudder.

(5) The comparisons made with the free running test results indicate the usefulness of the present mathematical model to predict a wide range of maneuvering motion.

7. Acknowledgement

A part of the present study was carried out as a research work in Research Panel RR742 of the Shipbuilding Research Association of Japan under the chairman of Prof. T. Koyama. The author is grateful to the useful discussion and suggestion of Prof. M. Fujino. He wishes to thank Dr. M. Hirano of Akishima Laboratories (Mitsui Zosen) Inc. for his kind encouragements and suggestions in carrying out this work. Thanks are also due to Mr. S. Moriya of Akishima Laboratories for cooperation and assistance throughout the work.

References


