# Evaluation of Formation Permeability from Borehole Stoneley Waves

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### Abstract

The propagation of borehole Stoneley waves has been shown to be sensitive to fluid mobility (the ratio of permeability to viscosity). When crossing a permeable formation Stoneley wave energy attenuates and slowness increases, a phenomenon that is well described by Biots theory and one that has been verified in laboratory experiments.

However, as mobility effects on Stoneley waves are secondary and rather small, an accurate Stoneley wave propagation model as well as an optimal inversion technique is required to perform quantitative inversions of borehole log data. Effects of the mudcake on the borehole wall, which reduces pressure communication between the borehole fluid and the formation and thereby decreases Stoneley wave mobility effects, need to be included in the Stoneley wave propagation model. As such, an elastic membrane model was devised to include these effects in the Biot model. An inversion technique, which uses both slowness and attenuation of the Stoneley wave over a range of frequencies to evaluate mobility, is proposed to optimize the sensitivity of the inversion to formation mobility.

This paper describes the implementation of an interpretation methodology based on the above technique. The Stoneley wave propagation model employed is described with 13 parameters. The error analysis shows that the accurate determination of the fluid mobility requires that critical parameters, such as the mud slowness, mud attenuation and pore-fluid modulus be precisely determined. An interpretation procedure is proposed to determine these parameters with reasonable accuracy. The fluid mobility can then be determined without the need for external calibration with other mobility measurements. Finally, with knowledge of the saturating fluid viscosity, the intrinsic permeability of the rock can be derived.

When using the proposed methodology within the applicable measurement range limited by Biot physics, the Stoneley wave can provide a continuous estimation of the formation permeability along a borehole wall. Core measurements are not required for calibration although they can be used for verification.

In addition to permeability estimation, the use of this technique has various related applications such as the optimal placement of wireline formation tests and perforation intervals. In the realm of earth science, other applications are expected to be developed as the use and interest in borehole measurements continues to grow.

Key words : borehole Stoneley wave, fluid mobility, Biot theory, mudcake effects, Stoneley frequency dispersion of slowness and attenuation

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### I. Introduction

Permeability information is essential for oil and gas production, once reserves have been identified and evaluated, to optimize well completion and field development. Permeability is needed to determine the placement of optimal perforation intervals with respect to the reservoir boundaries and the water table. More generally, permeability is needed for:

 $\cdot$  completion and production optimization to maximize production while minimizing water cut,

 $\boldsymbol{\cdot}$  production prediction and planning to maximize hydrocarbon recovery, and

· definition of drainage pattern.

As the absolute value of permeability in the reservoir is usually considered to be the most important parameter, the variations of permeability along a well are equally important.

Permeability, however, is one of the most difficult parameter to measure in an oil well. Direct measurements either provide only a few points along the well, as is the case with well testing or wireline testers, or provide measurements under different conditions in the case of core measurements. With indirect measurements, permeability is inferred from a different property (porosity, magnetic resonance or geochemical logs) using models and assumptions. As the models are not based on an exact theory and mostly based on statistical correlations, the uncertainty associated with the results is large. Another technique such as the study of invasion profiles, gives only qualitative information about permeability. The Stoneley wave is the only technique to provide a continuous and direct measurement of permeability along the well. Here, 'direct' means that Stoneley wave can detect effects of fluid movement in the formation due to a pressure communication between the borehole fluid and the formation rather than statistical correlations. However, although the principle of the measurement has been known for quite some time, obtaining a reliable and accurate permeability from the Stoneley waves has proven difficult.



Fig. 1 Effects of permeability and fractures on Stoneley waves. The Stoneley wave slowness increases and attenuates when crossing permeable zones. Crossing permeable fractures causes Stoneley wave reflections and transmission loss.

At low frequency, the mode of the Stoneley waves becomes the tube wave and propagates as a piston-like compression of the borehole fluid in the borehole. When the borehole crosses permeable zones or permeable fractures, some fluid movement occurs between the borehole and the formation. This results in some energy loss, hence attenuation, and a slowing down of the wave, hence increased Stoneley wave slowness (Fig. 1) Fractures and permeable zones have different characteristics and affect the Stoneley wave in different ways. In particular, in the case of permeable fractures, the strong, localized impedance contrasts also cause reflections and transmission loss of the Stoneley wave. Specific techniques are used to evaluate permeable fractures with the Stoneley wave (Brie et al., 1988; Hornby, 1989; Endo et al., 1997). The objective of the present work is to evaluate the permeability of nonfractured reservoirs, i.e., essentially, distributed permeability through the pore space.

In effect, the formation parameter the Stoneley wave measured is not exactly the formation permeability, but rather the fluid mobility ( $\kappa_0/\eta$ , that is, the ratio of permeabil-

ity  $\kappa_0$  to fluid viscosity  $\eta$ ) The permeability is evaluated in millidarcy (1 darcy is equal to 0.986923 × 10<sup>-12</sup> m<sup>2</sup>.) units and the viscosity in centipoises (1 centipoises is equal to 1 × 10<sup>-3</sup> Pa·s.), so that the mobility is usually given in md/cp. As the viscosity of water is about 1 cp, the mobility and the permeability take almost the same value in water-bearing reservoirs. The pore fluid viscosity is normally known with sufficient accuracy; hence a measurement of permeability can be obtained from the Stoneley wave.

There have been many attempts at evaluating permeability from the Stoneley wave. Rosenbaum (1974) proposed measuring permeability with the Stoneley wave. Williams et al. (1984) showed deterministic correlations between permeability and Stoneley attenuation in field measurements. Theoretical models based on the Biot poroelastic theory were developed by Schmitt et al. (1988) and by Chang et al. (1988). Comparisons with laboratory experiments made by Winkler et al. (1989) provided a validation of these models. Based on these models, a simplified algorithm was developed to estimate permeability from the difference between the Stoneley slowness measured at a given frequency (S) and the slowness calculated using a purely elastic, nonpermeable formation model (Se). This method, called the S-Se technique produces useful estimates in hard rocks but is often unsuccessful in soft rocks. Tang et al. (1991) proposed a simplification of the Biot-Rosenbaum model and developed an inversion technique for the Stoneley wave am-Later, they studied the effect of the plitude. presence of the tool in the borehole (Tang and Chang, 1993) Cassell et al. (1994) proposed a simple technique to extract a permeability indicator from the variation of the Stoneley attenuation with frequency.

These are only a few examples of the numerous attempts made at using the Stoneley wave to measure permeability. From these experiences it has become clear that the problem is complex and that no empirical or simplified approach can provide the desired level of reliability and accuracy which are required by the reservoir engineer. Also, early methods did not account for the presence of mudcake around the borehole in permeable formations The mudcake reduces the communication between the borehole and the formation. and must be accounted for. To be successful, a model must be an accurate representation of all phenomena involved, and the solution should make use of all available data ( i.e., both slowness and attenuation within the available frequency range) The model proposed by Liu and Johnson (1997) implements the full Biot poroelastic theory and includes the effect of the mudcake in the form of an elastic membrane. This model is implemented in a computer program, called PORELAS, that calculates the propagation characteristics of the Stoneley wave as a function of frequency. A multi-frequency inversion technique, which uses this model, was developed by Kimball and Endo (1998). The objective of the present paper is to describe an interpretation procedure, which uses this inversion technique, which does not require external measurements for calibration to determine with sufficient accuracy and reliability in the pore fluid mobility estimated from Stoneley waveforms in clastic rocks. Pampuri et al. (1998) confirmed the validation of this methodology after comparing the results with the other permeability measurements.

### **II** . Model Inversion For Stoneley Mobility

### 1) PORELAS Model

The model used to characterize the borehole configuration that consists of an elastic and flexible mudcake layer of inner radius  $r_a$  and outer radius  $r_b$ , situated between the borehole mud, which is treated as an acoustic fluid, and the formation rock. The rock bulk properties are characterized using the Biot theory (Biot, 1956a, b) An oscillatory pressure wave in the borehole causes fluid to flow in and out through the porous medium, thus contributing to attenuation and dispersion of the Stoneley wave. In the language of the Biot

theory, this effect is described by the coupling of the Stoneley wave to the acoustic slow wave. The mudcake flexibility is introduced by adding a membrane stiffness on the borehole wall to allow the membrane-like mudcake to flex in and out of the pore space (Fig. 2). This mechanism reduces, but does not eliminate, the effects of formation permeability on the Stoneley wave. The theory is described in some detail by Liu and Johnson (1997).

To characterize the Stoneley wave properties, the axially symmetric normal modes that propagate as  $e^{i(kz - \omega t)}$  in a fluid-filled cylindrical borehole surrounded by porous rock are searched for. Here, z indicating the position along the borehole axis and k, the axial wave number, form a complex-valued function of frequency. The solution to the problem is written as a linear combination of eight different solutions to the bulk equations of motion, each of which varies axially in space and time as  $e^{i(kz - \omega t)}$ . These solutions are: a regular solution to the wave equation in the borehole fluid, a cylindrically outgoing and a cylindrically incoming compressional wave in the mudcake, a cylindrically outgoing and a cylindrically incoming shear wave in the mudcake, cylindrically outgoing shear, fast compressional, and slow compressional waves in the porous medium. The relative amplitudes of these constituent solutions are determined by the requirement to satisfy the requisite boundary conditions, of which there are eight in number. These boundary conditions yield a system of eight linear and homogeneous equations in the eight unknown amplitudes. Therefore, a nontrivial solution can exist if and only if the determinant of the matrix of coefficients vanishes. For each frequency  $\omega$ , the corresponding wavenumber for the Stoneley mode  $k(\omega)$  is numerically estimated at zero points of that determinant. The phase slowness  $S(\omega)$  and the specific attenuation  $1/Q(\omega)$  are related to the wavenumber  $k(\omega)$ :



Fig. 2 Elastic membrane mudcake model. The mudcake flexibility is introduced by adding a membrane-line mudcake to flex in and out of the pore space. This reduces pressure communication between the borehole fluid and the formation and reduces the effect of formation permeability on the Stoneley wave.

$$k(\omega) = \omega S(\omega) \left( 1 + \frac{i}{2Q} \right)$$
(1)

The list of the parameters for the forward model is given in Table 1. Among these input parameters, many are obtained from logs or conventional interpretation. Others relate to the mud properties, pore-fluid properties, rock matrix modulus, and mudcake membrane stiffness. One parameter, the fluid mobility, is the one to be evaluated.

Note that all of the formation parameters are determinable from logging measurements. Specifically, nuclear logs provide the porosity and the formation density and allow for the deduction of rock solid grain density. Similarly, the measured compressional and shear slownesses allow for the deduction of the bulk and shear frame moduli from the Gassmann equation, *i.e.*, the low frequency limit of the Biot theory. Fig. 3 illustrates the effect of fluid mobility which increases Stoneley wave

Table 1 - Complete Parameter List						
	Parameters	Unit				
Borehole parameters	Hole diameter	inch				
	Mud slowness	$\mu$ s/ft				
	Mud attenuation	$dB/\lambda$				
	Mud density	g/cm <sup>3</sup>				
Formation parameters	Compressional slowness	$\mu$ s/ft				
	Shear slowness	$\mu$ s/ft				
	Formation density	g/cm <sup>3</sup>				
	Porosity					
	Pore-fluid modulus	GPa				
	Pore-fluid density	g/cm <sup>3</sup>				
	Pore-fluid mobility	md/cp				
	Grain modulus	GPa				
Mudcake parameters	Density	g/cm <sup>3</sup>				
	Bulk modulus	GPa				
	Shear modulus	GPa				
	Thickness	inch				
	Membrane stiffness	GPa/cm				

Table 1 Parameter list for Stoneley wave propagation model.



Fig. 3 Mobility-added slowness, S-Se versus mobility. This shows the variation of S-Se with the mobility for different membrane stiffness (0, 1,5,10 and 50 GPa/cm) computed at the frequency of 600 Hz. Mud slowness  $\Delta t_{mud}$  is assumed to be 203  $\mu$ s/ft. The larger membrane stiffness reduces the effects of formation permeability.

### Phase slowness (µs/ft)

Loss / wavelength (dB)



Fig. 4 Stoneley dispersion for different fluid mobilities. This shows the variation of Stoneley slowness and attenuation with frequency for different fluid mobilities (10, 100 and 1000 mD/cp). Membrane stiffness is assumed to be 5 GPa/cm and mud attenuation is assumed to be 0.5 dB/ $\lambda$ . The mobility effects on slowness are larger for low frequencies. The mobility effects on attenuation are larger for high frequencies.

slowness, the mobility-added slowness, S-Se is plotted as a function of the mobility, where S is a measured Stonelev slowness and Se is a Stoneley slowness computed from an elastic model. One can observe the dependence of S-Se on the mobility. One can also note that the slowness added through a change in mobility from 1 md/cp to 1000 md/cp is only 12  $\mu$ s/ft. This is a small change which has been a cause of the difficulty in evaluating mobility from Stoneley. As a result, the model needs to be very accurate and the input parameters determined precisely to make the evaluation meaningful. As shown in Fig. 3, the presence of mudcake reduces this ef-The mudcake is characterized by its fect. membrane stiffness,  $W_{mc}$  expressed in GPa/cm, and which will be assumed constant over a lithologic zone. The effect of a finite-thickness mudcake is very close to shifting the apparent value of the mud slowness. Accordingly, to reduce the number of parameters to be determined, the mudcake thickness is fixed to a small value and the apparent value of the mud slowness is assumed to adjust for this mudcake thickness effect. The mud slowness determination procedure will be explained in a later section. However, for the membrane stiffness, a finite constant value is assumed for the zones of interest.

The effect on slowness and attenuation as a function of frequency is shown in Fig. 4. Increased permeability leads to increased dispersion and attenuation. One can notice that the effect is larger on slowness at low frequency, where as it is larger on attenuation at high frequencies. From this observation, one can expect that there could be an optimum frequency for simultaneously using slowness and attenuation.

As per Table 1, there are a large number of input parameters for the forward problem, but not all of them are equally important. The situation is clarified by the following lowfrequency analytic solution to the full problem (Liu and Johnson, 1997):

$$k^{2}(\omega) = \omega^{2} \rho_{f0} \left[ \frac{1}{K_{f0}} + \frac{1}{M_{F}} + \frac{1}{W_{mc} + W_{p}} \frac{2}{r_{b}} \frac{(1 - f_{c}) N^{2}}{\left[ N - f_{c} \left[ (\lambda + \mu) / (\lambda + 2\mu) \right] (N - \mu) \right]^{2}} \right]$$
(2)

$$M_F = \frac{N[\lambda + 2\mu] + f_c(\lambda + \mu)(\mu - N)}{\lambda + 2\mu + f_c(N - \mu)}$$
(3)

where  $\rho_{f0}$  is the borehole fluid density,  $K_{f0}$  is the bulk modulus of the borehole fluid, N is the solid shear modulus of the formation,  $W_{h}$ is a frequency-dependent added stiffness due to permeability effects,  $W_{mc}$  is the mudcake membrane stiffness,  $\lambda$  and  $\mu$  are Lames constants of the mudcake, and  $f_c = 1 - r_a^2 / r_b^2$ is the area fraction of the borehole occupied by the mudcake. The first term in Equation (2) is the contribution from the borehole fluid, the second term is the one from formation elasticity effects and the third term is the one from permeability effects. In hard formations, the first borehole fluid term represents approximately 90% of the total contribution. This suggests that in hard formations, the Stoneley wave number, which is related to the Stoneley slowness and attenuation (Equation (1)) depends mostly on the mud properties. Hence in hard formations, Stoneley mobility measurements are very sensitive to mud slowness and attenuation.

In this low frequency limit, permeability effects are contained in a frequency-dependent added stiffness  $W_p$ :

$$W_{p}(\omega) = -\frac{\eta C_{D} k_{c2} H_{0}^{(1)}(k_{c2} r_{b})}{\kappa_{0} H_{1}^{(1)}(k_{c2} r_{b})}$$
(4)

where  $H_0^{(1)}$  and  $H_1^{(1)}$  are Hankel functions of order 0 and 1, respectively,  $\eta$  is the viscosity of the pore fluid,  $\kappa_0$  is the permeability of the formation,  $k_{c2} = \sqrt{i\omega/C_D}$  is the wave number of the Biot slow wave,  $C_D$  is the diffusivity of the slow wave and expressed assuming the formation is very rigid compared with the fluid in the formation:

$$C_D = \frac{\kappa_0 K_{f0}}{\eta \phi}$$

where  $\phi$  is the porosity of the formation.  $C_D$  is an important parameter to control a depth of investigation of Stoneley mobility measurements. In the case of water-bearing formations with a permeability of 100 mD and a porosity of 20%, the depth of investigation is the order of 2 cm at low frequency where Stoneley measurements are made (1,000 to 5,000 Hz)

PORELAS is the implementation of the model described in a forward calculation of the complex-phase slowness at all frequencies. It, therefore, provides the dispersion curves for the Stoneley slowness and attenuation in a borehole drilled in a permeable formation. Following the simplifications made on the mudcake characterization, the list of parameters is now reduced to 13, as shown in Table 2.

### 2)Formation Permeability Determination

The Biot theory depends on viscosity effects only through the ratio  $\kappa_0/\eta$ , the fluid mobility. The formation permeability is, therefore, obtained from the fluid mobility, multiplying it by the viscosity of the pore fluid.

Single-phase fluid viscosity can be determined from the fluid composition, pressure, and temperature. As the goal here is to deduce permeability for hydrocarbon-saturated formations, a formulation is needed to account for partially saturated rocks, especially if one of the phases is gaseous. If one makes the standard assumptions of two-phase relative flows, it is possible to show that the relevant mobility is

$$\frac{\kappa_0}{\eta} = \kappa_0 \left[ \frac{\kappa_1}{\eta_1} + \frac{\kappa_2}{\eta_2} \right] \tag{5}$$

where  $\eta_i$  is the viscosity of the *i*-th phase and  $\kappa_i$  is the relative permeability for that phase.

Table 2 List of input parameters to Stoneley wave propagation model used in the mobility inversion. Except for membrane stiffness, mudcake parameters are fixed in the inversion.

Table 2 - PORELAS 13 Input Parameters						
Parameter	<u>Unit</u>	Source				
Hole diameter	inch	Log measurements				
Compressional slowness	$\mu$ s/ft	Log measurements				
Shear slowness	$\mu$ s/ft	Log measurements				
Formation density	g/cm <sup>3</sup>	Log measurements				
Porosity		Log measurements				
Formation grain modulus	GPa	Formation evaluation				
Pore-fluid density	g/cm <sup>3</sup>	Formation evaluation				
Pore-fluid modulus	GPa	Formation evaluation				
Membrane stiffness	GPa/cm	Estimated				
Mud slowness	$\mu$ s/ft	Pre-evaluation				
Mud attenuation	$\mathrm{dB}/\lambda$	Pre-evaluation				
Mud density	g/cm <sup>3</sup>	Log-header				
Mobility	md/cp	Inversion				



Fig. 5 Diagram for single-parameter Stoneley mobility inversion.

The relative permeabilities are determined from the saturation in the zone measured by the Stoneley wave, usually the invaded zone near the well. In the case of a gas-bearing formation, the saturation in the zone investigated by the sonic logs can be obtained from an analysis of the compressional and shear measurements (Brie *et al.*, 1995)

#### 3) Multi-frequency Mobility Inversion

Obtaining fluid mobility from Stoneley phase slowness and attenuation is an inverse problem. The method is described by Kimball and Endo (1998) First, the dispersion curves of the Stoneley wave are obtained from the PORELAS model. Then, wideband Stoneley waveforms are backpropagated in the frequency domain with these dispersion curves. A least-mean-square-error (LMSE) estimator is used to find the value of the parameter, *i.e.*, in this case, the fluid mobility, to minimize the total error (Fig. 5) This process uses the complex phase slowness (*i.e.*, both slowness and attenuation), over a wide frequency band for optimum results. The other 12 parameters are input to the process. The computation outputs the fluid mobility with its estimation error, and the signal-to-noise ratio (SNR) of the input waveforms.

On the assumption that the PORELAS model is exact, one finds that the error on the mobility computation originates from two different sources. The first one is the error made on the estimation of the 12 other model input parameters. This error increases monotonically with frequency.

The other source of error is the presence of random noise in the waveform. This error varies as the inverse of the signal amplitude (or SNR) and, therefore, goes through a minimum at the peak of the signal amplitude spectrum. The total error is the sum of the two and also goes through a minimum near the peak signal amplitude.

### 3-1) Total Error Analysis

Systematic error estimation based on Stoneley physics and rigorous signal processing theory is implemented in this study. Estimation of physical parameters from dispersive waves is extensively studied by Kimball et al. (1995). They showed that bounds on the errors due to noise for LMSE estimation of physical parameters from dispersive waves are given by the classical Cramer-Rao bounds in a statistical theory. The Cramer-Rao bounds are obtained from the sensitivity of the Stoneley wave to each of its physical parameters and give the variance of the mobility measurement. The bounds are representative of the physics of the Stoneley wave as expressed They apply to any mobility by the model. processing that is unbiased when the other secondary formation and borehole parameters are known exactly. These bounds give error estimates due to random noise (noise error).

If the other 12 secondary parameters are uncertainly known, that is, the parameters have either bias or variance, then the overall mobility estimate will be biased. Consequently, the total error on the estimated parameters will be greater than the variance predicted by the Cramer-Rao bounds. The difference between the total error and the noise error is called the parameter error. Bounds on the total error have been developed so that the relative influence of both noise and parameter errors can be quantitatively examined. This allows informed design of a Stoneley mobility measurement which minimizes the total Consider the case of a Stoneley wave error. propagating in a formation having the properties shown in Table 3. The parameter values have the uncertainties indicated in the Std. Dev column, the signal to noise ratio, SNR is obtained as 25 dB from the mobility inversion. Errors will depend on the specific shape of the wave amplitude spectrum. Here the spectrum is constant over the 1,500 to 4,500 Hz processing band.

In the example given in Table 3, the common logarithm of the minimum total RMS error is 0.662 (about a factor of 5), using both the Stoneley slowness and attenuation (complex-phase slowness) as inputs. The logarithmic contributions are 0.43 for the noise error and 0.505 for the parameter error.

The breakdown of the contributions of the various parameters to the total error shows that the most important contributors are the mud slowness with 56% of the total, mud attenuation with 18.9% and pore-fluid compressibility with 16.2%. Note that this error analysis was made assuming that the standard deviation on the mud slowness determination was 3  $\mu$ s/ft, which is reasonably easy to achieve with a simple evaluation technique. It is certainly possible to improve the determination accuracy of the top three contributors to the total error, in particular, for the determination of the mud slowness. Reducing the standard deviation on mud slowness to  $1 \,\mu s/ft$  would be sufficient to reduce the total parameter error by half, and hence, the error on the mobility determination by one There is, therefore, a strong motivathird. tion to take special care in the determination of the critical parameters. This is the object of interpretation procedure which will be presented next. Although the value of the total

 Table 3
 Error analysis for mobility inversion. Errors for each parameter (<u>Std. Dev</u>) are estimated based on either processing errors or past statistics. Sensitivities (<u>Sens.</u>) are obtained from partial derivatives of Stoneley dispersion curves to each parameter. The signal to noise ratio, SNR is obtained from the mobility inversion.

Table 3 - PORELAS Error Analysis *									
Estimated Parameter	<u>Unit</u>	Value	Noise	Parameter		<u>Total</u>			
Log <sub>10</sub> Mobility	m/cp	2.0	0.43	0.505		0.662			
			41.8%	58.2%		100%			
Parameters	<u>Unit</u>	Value	Std. Dev	Sens.	Error	<u>% Error</u>			
Mud slowness	$\mu$ s/ft	236	3	- 14.86	0.378	56.00%			
Mud attenuation	g/cm <sup>3</sup>	0.9	0.5	- 0.198	0.22	18.90%			
Log pore-fluid modulus	GPa	0.23	0.3	0.078	0.203	16.20%			
Shear slowness	$\mu s/ft$	162	3	- 3.075	0.114	5.10%			
Log membrane stiffness	GPa/cm	0.5	0.3	0.052	0.062	1.50%			
Mud density	g/cm <sup>3</sup>	1.3	0.02	- 1.625	0.05	1.00%			
Hole diameter	inch	8.5	0.25	0.738	0.043	0.70%			
Porosity		0.2	0.02	- 0.142	0.028	0.30%			
Formation density	g/cm <sup>3</sup>	2.5	0.02	1.314	0.021	0.20%			
Compressional slowness	$\mu$ s/ft	90	2	- 0.293	0.013	0.10%			
Grain modulus	GPa	48	5	0.029	0.006	0.00%			
Pore-fluid density	g/cm <sup>3</sup>	1	0.02	- 0.005	0	0.00%			

\* S/N ratio = 25 dB; flat spectrum range 1,500 to 4,500 Hz;

uncertainty shown in the example above may look sizable, it should be noted that the total uncertainty represents the absolute error on the measurement without any calibration using external information. As the error on the parameter determination changes very slowly along the well, the relative variations of the mobility values are determined more precisely and provide valuable information for reservoir engineering applications.

#### **III**. Interpretation Procedure

The following interpretation procedure was designed to determine the critical input parameters as precisely as possible to reduce the uncertainty on the final results.

### 1) Stoneley Waveforms Input

A requirement for Stoneley permeability evaluation is high-fidelity monopole waveforms at low frequency and over a sufficiently wide frequency range. An advanced sonic tool can acquire the low-frequency monopole waveforms, which fulfill these requirements. The latest generation sonic tool (Pistre *et al.*, 2005) has a measurement dedicated for Stoneley data acquisition in which a monopole transmitter is driven by a low-frequency pulse that delivers energy between 1,000 and 5,000 Hz. Thirteen waveforms are acquired across a 6 ft receiver array. In addition, the tool effects are theoretically modeled and corrected during the processing.

### 2) Processing Flow

The evaluation starts with the computation of the apparent mud slowness and attenuation. For this, a two-parameter inversion scheme is used, assuming permeability is nil or a small value. The mud parameters are determined from a plot of mud apparent properties vs. depth or from cross-plots.

In the case of gas sands, the apparent fluid modulus varies widely and should be obtained from logs, after modulus decomposition (Brie  $et \ al.$ , 1995) Then, the Stoneley mobility can



Fig. 6 Diagram of apparent mud properties calculation, two-parameter inversion.

be calculated. The computation requires a few processing parameters in addition to the formation parameters: computation method, frequency band, and time window length.

### 3) Determination of Critical Formation Parameters

The sensitivity analysis described in II 3-1) shows that 90% of the parameter error on the fluid mobility evaluation is caused by the following three critical parameters:

- · mud slowness,
- mud attenuation, and
- · pore-fluid modulus.

Clearly, to evaluate fluid mobility with good accuracy, these parameters must be determined as precisely as possible. Other parameters that have a lesser influence on the total uncertainty are the elastic modulus of the rock matrix (grain modulus), and the mudcake membrane stiffness.

Specific techniques will be proposed to improve the determination of these critical parameters in the following sections.

#### 3-1) Mud Slowness and Attenuation

The least-mean-square-error estimator used for evaluating fluid mobility also has the capability to optimize two model parameters. To determine these mud parameters, an inversion using the PORELAS model is performed, assuming a nonpermeable formation (Fig. 6) The inversion results in apparent values of mud slowness and mud attenuation, which are larger than the true ones when the formation is permeable. Thus, finding the minimum of these curves in low-porosity zones provides a good indication of the mud properties.

The example shown in Fig. 7 uses data from a well in the Adriatic Sea in Italy. One can observe that the  $\Delta t_{mud}$  and the  $Att_{mud}$ change little with depth, making it easy to pick dependable values for the mud parameters.

A more systematic manner of finding the mud parameter values is to crossplot the apparent mud properties vs. porosity (Fig. 8) As the presence of permeability would push the points toward higher values, the values to be used to determine the mud characteristics are the bottom part of the data cloud. Extrapolating the lower part of the trend to the zero porosity axis yields a value at 236  $\mu$ s/ft for  $\Delta t_{mud}$  in the example shown (oil-base mud) The uncertainty on the determination is about 2  $\mu$ s/ft.

A similar crossplot for mud attenuation in Fig. 9 yields a mud attenuation value of 0.9 dB. In general, the apparent mud attenuation varies less than the slowness, and it is easier to determine the mud attenuation than the mud slowness.

#### 3-2) Pore-Fluid Compressibility

The pore-fluid compressibility in the invaded



Fig. 7 Apparent mud slowness and attenuation plot. The right track shows the apparent mud slowness  $\Delta t_{mud-app}$  and apparent mud attenuation  $\operatorname{Att}_{mud-app}$  with compressional ( $\Delta t$ ), shear ( $\Delta tsh$ ), and Stoneley slowness ( $\Delta tSt$ ) from semblance processing and S/N ratio (SNR) from the apparent mud properties inversion. The left track shows the open-hole logs: hole diameter (HD), gamma ray (GR), porosity ( $\phi_e$ ) and density ( $\rho_b$ ).

zone is the third-most important input to the PORELAS model. In cases where the pore fluid is a liquid, the effect of the pore-fluid compressibility is small and the computation of the average fluid modulus from estimated saturation values yields sufficient accuracy.

In the case of gas-bearing reservoirs, the determination of the pore-fluid compressibility is important because gas is orders of magnitude more compressible than liquids. The presence of gas in the pore space, therefore, has a large effect on the effective pore-fluid modulus. The compressibility of the pore-fluid mixture should be done in the zone investigated by the Stoneley measurement at a similar frequency. For this, the most dependable method is to use the apparent pore-fluid modulus,  $K_{fa}$ obtained from the compressional and shear



Fig. 8 Mud slowness determination from apparent mud slowness  $\Delta t_{mud}$  versus effective porosity cross-plot.

measurements, using the modulus decomposition technique described by Brie et~al. (1995)

$$K_{ja} = \phi \frac{K_{ma}}{A_a K_{ma} - 1 + \phi} \tag{6}$$

with

$$A_{a} = \frac{K K_{dry} + K_{ma}^{2} - 2K_{ma} K_{dry}}{K_{ma}^{2} (K - K_{dry})}$$
(7)

$$K_{dry} = N \left[ \left( V_{p} / V_{s} \right)_{ma}^{2} - \frac{4}{3} \right]$$
 (8)

where  $\phi$  is the porosity of the formation, K is the bulk modulus of the formation, N is the shear modulus of the formation,  $K_{ma}$  is the bulk modulus of the rock matrix,  $K_{dry}$  is the bulk modulus of the rock frame, and  $(V_p/V_s)_{ma}$  is the  $V_p/V_s$  ratio of the rock matrix.

The pore-fluid compressibility obtained from this technique is shown in Fig. 10 in the left track. In this example, the apparent fluid modulus is about 3 GPa in water-bearing zones and in shaly sections, and goes down to 1.7 GPa in gas-bearing zones.

Further, the gas volume fraction, in the zone measured by the Stoneley, can be ob-



Fig. 9 Mud attenuation determination from apparent mud attenuation  $Att_{mud}$  versus effective porosity cross-plot.

tained from  $K_{fa}$  and used to calculate the effective viscosity of the pore fluid and to derive formation permeability from mobility.

#### 3-3) Rock Matrix Elastic Modulus

If the lithology is not complex, using a zoned value for the grain modulus is adequate. However, where the lithology is mixed (*e.g.*, sequences of sands and carbonates), the variations of the matrix modulus can be large enough to affect the mud properties and the mobility results. In this case, an effective grain modulus, calculated from the volumetric mineral analysis of the rock, should be used.

### 3-4) Mudcake Membrane Stiffness

In the data sets processed, a value of 3.2 GPa/cm for the membrane stiffness provides values of fluid mobility that match core data [ in the computation,  $\log_{10}(3.2)=0.5$  is used. ] Other than comparing mobility results with core data, no interpretation technique exists at this time to estimate the value of membrane stiffness. It will be an objective of future studies to investigate correlations between membrane stiffness and other parameters such as mudcake thickness and/ or differential pressure between the wellbore and the formation



Fig. 10 Stoneley mobility evaluation results. The fourth track ( right most ) shows the mobility from Stoneley. The first track ( left most ) shows the apparent pore-fluid modulus (  $K_{fa}$  ) evaluated from sonic data, the second track shows the gas saturation (  $V_{xgas}$  ) from the apparent pore-fluid modulus with the porosity (  $\phi_e$  ), and the third track shows compressional (  $\Delta t$  ), shear (  $\Delta tsh$  ), and Stoneley slownesses (  $\Delta tSt$  ) from semblance processing and S/N ratio ( SNR ) from the Stoneley mobility inversion.

with sufficient accuracy.

### 4) Computation Results

The fluid mobility results are shown on the right track of Fig. 10 with the error bar representing the noise error. The SNR is shown on the left and averages 25 dB. This is a rather low value; for optimal results the SNR should be 30 dB or more. To evaluate the formation permeability, the fluid mobility should be multiplied by the effective pore-fluid viscosity. The gas volume seen by the sonic tool was evaluated from the compressional and shear slownesses (Brie *et al.*, 1995) and is shown on the right-hand side of track one. It can be used to calculate the effective porefluid viscosity, when the relative permeability curves in the reservoir are known. In effect, the gas volume is rather small because it is measured in the invaded zone and the original gas was flushed by the mud filtrate. As a consequence, the effective pore-fluid viscosity is not far from that of water and the comparisons between fluid mobility and formation permeability from other sources show good agreement. Additional examples and comparisons with other formation permeability information obtained from core, wireline formation tests and MR (Magnetic Resonance measurements) are given by Pumpuri et al. (1998). Pumpuri showed that the fluid mobility and formation permeability obtained from this technique matched other permeability information within the specified error bar, without the need for additional calibration.

### 5) Limitations

The current limitations of the technique are:

• The noise error increases as the waveform SNR decreases. A minimum SNR is 20 dB while the SNR of high-quality waveforms is in excess of 30 dB. Wideband waveforms are required. The preferred frequency range is from 1,000 to 5,000 Hz.

• The mobility should be in a range of 10 to 1,000 md/cp. As seen on Figs. 3 and 4, below 10 md/cp the effect on the measurement is too small to be evaluated, and above 1,000 md/cp the mobility has little effect on the Stoneley slowness, so that a measurement becomes difficult. It should be noted that in a gas reservoir this may mean that the permeability should be between 1 and 100 md, depending on the viscosity of the gas/liquid pore-fluid mixture.

• Porosities of at least 10% are necessary to achieve acceptable error levels in the mobility computation.

• In particular, this technique is not applicable to fractured reservoirs. Specific techniques exist to evaluate fracture producibility.

• High-porosity, unconsolidated sandstones may cause computational problems and usu-

ally exhibit mobilities above the recommended range.

### **IV**. Conclusion

The methodology described provides quantitative determination of the pore-fluid mobility from Stoneley waves. Unlike previous techniques, this methodology does not require calibration with external information. The following summarizes the conclusions of this study:

• The technique consists of the least-meansquare-error inversion of the full Biot poroelastic model over a wide frequency band. The effect of the mudcake is modeled as an elastic membrane. Estimating the effective pore-fluid viscosity from saturations and relative permeability curves allows the computation of the formation permeability from fluid mobility.

• Three input parameters are responsible for most of the parameter error in the results: mud slowness, mud attenuation, and pore-fluid modulus. An interpretation procedure is proposed to optimize the determination of these parameters.

• The noise error is the other major source of error in the results. Optimum results are obtained with high-quality waveforms with an SNR in excess of 30 dB.

• The model implemented is suitable for distributed permeability in clastic-type rocks. In fractured reservoirs, other techniques are more appropriate.

Stoneley permeability is the only continuous, direct measurement of fluid mobility and formation permeability. With improved accuracy and reliability, it should be proven to be a useful tool for formation evaluation and reservoir engineering applications. In the realm of earth science, other applications are expected to be developed as the use and interest in borehole measurements continues to grow.

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## 音波検層データをつかった地層透水性の評価

遠藤猛\*

坊井内音波検層で観測されるストンレー波は地 層の透水性と相関のあることが知られている。本 論文では、坑井内音波検層で測定されるストン レー波形を用いた波形インバージョンによる、透 水性の評価について述べる。ストンレー波では、 坑井内を伝播する際、地層が透水性をもつ場合、 速度の低下、振幅の減衰が起こる。その現象はビ オの理論により、定量的に記述されており、実験 的にも検証されている。しかし、実データから透 水性の定量的なインバージョンを行うためには、 いくつかの課題がある。ストンレー波に対する透 水性の影響は微小であり、定量的なインバージョ ンのためには、正確な物理モデルとパラメータの 依存性を考慮した最適なインバージョンアルゴリ ズムが必要である。実際の坑井内を伝播するスト ンレー波の物理モデルには坑壁に形成される泥壁 の影響を含める必要がある。泥壁は、坑井内流体 と地層の間の圧力相互作用を減衰させる効果があ り、それゆえ、ストンレー波に対する透水性の影 響を減少させる。この効果は、弾性的な薄膜とし て、モデル化される。本論文で用いるストンレー 波伝播モデルは地層および坑井の13のパラメー タで記述される。それらのパラメータの多くは検 層測定により直接に求めることができるが、いく つかのパラメータは他のデータより間接的に推定 する必要がある。本論文では、ストンレー波分散 の各パラメータに対する依存性を求め、その重要 性を理論的に評価した。その結果、(1)坑井内 流体の速度および(2)減衰、そして(3)地層 内流体の体積弾性率が特に重要であることが示さ れた。これらのパラメータは間接的に推定する必 要があり、その方法について説明する。インバー ジョンアルゴリズムについては、透水性に対する 感度を最適にするために、ストンレー波速度およ び減衰の周波数分散を解析する。

ストンレー波による透水性の評価は連続測定が 可能という特徴があり、また、誤差を理論的に評 価することができる特徴がある。今までは、石油 探査への応用が主であったが、今後は地球科学へ の応用も期待される。

**キーワード**:坑井内ストンレー波,透水性,ビオ理論,泥壁効果,ストンレー波速度および減衰周波数 分散

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