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ADJOINT NUMERICAL APPROACH TO CONVECTION HEAT TRANSFER PROBLEMS

Kazunari Momose and Hideo Kimoto

Graduate School of Engineering Science, Osaka University
1-3 Machikaneyama, Toyonaka 560-8531, Japan

ABSTRACT A numerical method based on adjoint formulation is proposed to generally evaluate boundary condition effects on convection heat transfer characteristics. The main features of the present approach can be summarized as follows: In the forced convection problems, a numerical solution of the adjoint problem gives the heat transfer characteristics, such as the total heat transfer rate or the temperature at a specific location, under arbitrary thermal boundary conditions. In the mixed convection problems, we can construct a kind of sensitivity function from the solutions of the base and adjoint problems. The sensitivity function enables us to predict the change of heat transfer characteristics for arbitrary thermal and flow boundary perturbations.

Keywords: Convection Heat Transfer, Boundary Condition, Integral Equation, Adjoint Problem, Numerical Analysis

1. INTRODUCTION

With recent progress in numerical simulation techniques [1-3], a large number of numerical results for convective heat transfer have been reported under various configurations [4]. Although conventional numerical methods enable us to predict the heat transfer characteristics, each result gives only a particular solution under the specific boundary condition. In other words, the heat transfer characteristics obtained under such a specific boundary condition are no longer meaningful if the boundary condition is modified. For example, even under uniform thermal boundary conditions, the heat transfer characteristics under a uniform heat flux condition are different from that under an isothermal condition [5-7].

In this paper, we firstly propose an adjoint numerical approach to forced convection heat transfer problem to evaluate the heat transfer characteristics under arbitrary thermal boundary conditions. Using the numerical solution of the adjoint problem, which can be derived from the linearity of the energy equation of forced convection heat transfer, we can predict the total heat transfer rate or the temperature at a specific location under arbitrary thermal boundary conditions. Secondly, we extend the adjoint method to mixed convection heat transfer problem. In the mixed convection problem, its adjoint problem cannot be derived directly, because the coupling of the flow and temperature fields makes the energy equation nonlinear. Thus we introduce perturbations from the base boundary conditions, and then derive the adjoint operator for the perturbation problem. Using the numerical solutions of the base and the adjoint problems, we can construct a kind of sensitivity function. The sensitivity function enables us to predict the change of heat transfer characteristics not only for arbitrary thermal boundary perturbations but also for arbitrary flow boundary perturbations.

2. MATHEMATICAL FORMULATION

Consider a convection field Ω with boundary Γ. Using the assumption of an incompressible Boussinesq fluid and adopting an appropriate nondimensionalization, we can write the governing equation of convection heat transfer as

$$\nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

$$(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \Delta \boldsymbol{u} + Gr\theta \boldsymbol{j} \tag{2}$$

$$\boldsymbol{u} \cdot \nabla \theta = \frac{1}{Pr}\Delta\theta \tag{3}$$

where $\boldsymbol{u}$ is the nondimensional velocity vector, p is the nondimensional pressure, $\boldsymbol{j}$ is the unit vector parallel to the gravitational force, θ is the nondimensional temperature, and Gr and Pr are the Grashof and Prandtl numbers. We suppose that the boundary Γ consists of Dirichlet and Neumann boundaries for both thermal and flow fields, namely

$$\Gamma = \Gamma_\theta \cup \Gamma_q = \Gamma_u \cup \Gamma_\sigma \tag{4}$$

where Γ_θ and Γ_q are the Dirichlet and Neumann boundaries for the thermal field, while Γ_u and Γ_σ are those for the flow field. On these boundaries, the boundary conditions are given as

$$\theta = \bar{\theta} \quad \text{on} \quad \Gamma_\theta \tag{5}$$

$$q \equiv \frac{1}{Pr}\frac{\partial \theta}{\partial n} - \theta \boldsymbol{u} \cdot \boldsymbol{n} = \bar{q} \quad \text{on} \quad \Gamma_q \tag{6}$$

$$\boldsymbol{u} = \overline{\boldsymbol{u}} \quad \text{on} \quad \Gamma_u \tag{7}$$

$$\boldsymbol{\sigma} \equiv \frac{\partial \boldsymbol{u}}{\partial n} - p\boldsymbol{n} = \overline{\boldsymbol{\sigma}} \quad \text{on} \quad \Gamma_\sigma \tag{8}$$

where q and $\boldsymbol{\sigma}$ denote the nondimensional heat flux and stress vector, respectively.

Under these assumptions, the purpose of the present study is to predict the heat transfer characteristics, such as total heat transfer rate or temperature at a specific location, under arbitrary boundary conditions.

2.1 Forced Convection Heat Transfer under Arbitrary Thermal Boundary Conditions

In forced convection problem, i.e., $Gr = 0$ in Eq. (2), the thermal field is governed by a linear equation with space-dependent coefficient $\boldsymbol{u}$. Thus, defining a linear differential operator A, we rewrite Eq. (3) as

$$A\theta = 0 \tag{9}$$

where

$$A = \boldsymbol{u}\cdot\nabla - \frac{1}{Pr}\Delta \tag{10}$$

Then the weak solution of Eq. (9) can be expressed as

$$\int_\Omega (A\theta)\theta^* d\Omega = 0 \tag{11}$$

where θ^* is a test function defined in Ω. Applying the divergence theorem to Eq. (11), we obtain the following integral equation:

$$\int_\Omega \theta\left(A^*\theta^*\right) d\Omega = \int_\Gamma \left(q^*\theta - \theta^* q\right) d\Gamma \tag{12}$$

where

$$q^* = \frac{1}{Pr}\frac{\partial \theta}{\partial n} \tag{13}$$

and A^* is the adjoint operator for A. Because the differential operator A is not self-adjoint, the adjoint operator is given as

$$A^* = \boldsymbol{u}\cdot\nabla + \frac{1}{Pr}\Delta \tag{14}$$

From Eq. (12), if we can eliminate the left-hand side integral, we can obtain several boundary integral relationships.

For this reason, we firstly adopt an adjoint problem, such that

$$A^*\theta^* = 0 \tag{15}$$

and set the boundary conditions for the adjoint problem as

$$\theta^* = 1 \quad \text{on} \quad \Gamma_\theta, \quad q^* = 0 \quad \text{on} \quad \Gamma_q \tag{16}$$

Then we have the following boundary integral relationship:

$$Q \equiv \int_{\Gamma_\theta} q\, d\Gamma = \int_{\Gamma_\theta} q^*\theta\, d\Gamma - \int_{\Gamma_q} \theta^* q\, d\Gamma \tag{17}$$

Equation (17) indicates that if we numerically calculate the adjoint equation (15) under the boundary conditions (16) instead of solving the original equation (9) under particular boundary conditions, we can predict the total heat transfer rate under arbitrary thermal boundary conditions. Thus, the adjoint heat flux q^* and the adjoint temperature θ^*, both of which can be obtained from the numerical solution of the adjoint problem, can be regarded as influence functions of the boundary temperature and the boundary heat flux on the total heat transfer.

In a similar fashion, if we choose the adjoint problem as

$$A^*\theta^* = \delta(\xi) \tag{18}$$

under

$$\theta^* = 0 \quad \text{on} \quad \Gamma_\theta, \quad q^* = 0 \quad \text{on} \quad \Gamma_q \tag{19}$$

we get the following relationship:

$$\theta(\xi) = \int_{\Gamma_\theta} q^*\theta\, d\Gamma - \int_{\Gamma_q} \theta^* q\, d\Gamma \tag{20}$$

where ξ is a specific location in the field. Equation (20) implies that if we solve Eq. (18), which can be calculated numerically by setting a point heat source at ξ, we can predict the thermal boundary condition effects on a specific location temperature. Moreover, if we replace the point heat source on the left-hand side of Eq. (18) with heat source distribution with a finite area, we can evaluate the mean temperature within the area under arbitrary thermal boundary conditions.

2.2 Convection Heat Transfer under Arbitrary Thermal and Flow Boundary Perturbations

For natural or mixed convection heat transfer, unfortunately, its adjoint operator cannot be derived directly, because the coupling of the flow and temperature fields causes nonlinear heat transfer characteristics. Thus, we introduce perturbations from the base boundary conditions, and then derive the adjoint operator for the perturbation problem.

Let us suppose that the temperature, heat flux, velocity and stress on their given boundaries change from the base distributions, such that

$$\theta = \overline{\theta} + \widetilde{\theta} \quad \text{on} \quad \Gamma_\theta, \quad q = \overline{q} + \widetilde{q} \quad \text{on} \quad \Gamma_q,$$

$$\boldsymbol{u} = \overline{\boldsymbol{u}} + \widetilde{\boldsymbol{u}} \quad \text{on} \quad \Gamma_u, \quad \boldsymbol{\sigma} = \overline{\boldsymbol{\sigma}} + \widetilde{\boldsymbol{\sigma}} \quad \text{on} \quad \Gamma_\sigma \tag{21}$$

where $\sim$ denotes the perturbation. Then we assume that the velocity, pressure and temperature will also change slightly from their base distributions to

$$\boldsymbol{u} = \overline{\boldsymbol{u}} + \widetilde{\boldsymbol{u}}, \quad p = \overline{p} + \widetilde{p}, \quad \theta = \overline{\theta} + \widetilde{\theta} \quad \text{in} \quad \Omega \tag{22}$$

Substituting Eqs. (22) into Eqs. (1) to (3) and neglecting the second order of the perturbations, we obtain the first-order perturbation equations, which can be expressed in a matrix form as

$$\boldsymbol{A}\widetilde{\boldsymbol{\phi}} = \boldsymbol{0} \tag{23}$$

where

$$\widetilde{\boldsymbol{\phi}} = \left[\widetilde{p}, \widetilde{\boldsymbol{u}}, \widetilde{\theta}\right]^T \tag{24}$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & \nabla & 0 \\ \nabla & \overline{\boldsymbol{u}}\cdot\nabla + (\nabla\overline{\boldsymbol{u}})^T - \Delta & -Gr\,\boldsymbol{j} \\ 0 & \nabla\overline{\theta} & \overline{\boldsymbol{u}}\cdot\nabla - \dfrac{1}{Pr}\Delta \end{bmatrix} \tag{25}$$

Since the governing equations of the perturbations are linear as shown in Eq. (25), we define a test function vector as

$$\widetilde{\boldsymbol{\phi}}^* \equiv \left[\widetilde{p}^*, \widetilde{\boldsymbol{u}}^*, \widetilde{\theta}^*\right]^T \tag{26}$$

and consider the following weak solution of Eq. (23):

$$\int_\Omega \left[\widetilde{\boldsymbol{\phi}}^*\right]^T \boldsymbol{A}\widetilde{\boldsymbol{\phi}}\, d\Omega = 0 \tag{27}$$

Applying divergence theorem to Eq. (27), we arrive at the next integral equation:

$$\int_\Omega \left[\tilde{\phi}\right]^T A^* \tilde{\phi}^* d\Omega = \int_\Gamma \left(\tilde{q}^* \tilde{\theta} - \tilde{\theta}^* \tilde{q} + \tilde{\sigma}^* \cdot \tilde{u} - \tilde{u}^* \cdot \tilde{\sigma}\right) d\Gamma \tag{28}$$

where A^* is the adjoint operator matrix corresponding to A, and A^* possesses the form as

$$A^* = \begin{bmatrix} 0 & \nabla & 0 \\ \nabla & \bar{u}\cdot\nabla + (\bar{u}\nabla)^T + \Delta & \bar{\theta}\nabla \\ 0 & Gr\, j & \bar{u}\cdot\nabla + \dfrac{1}{Pr}\Delta \end{bmatrix} \tag{29}$$

Equations (28) and (29) derived above correspond to Eqs. (12) and (14) in the forced convection problem. Thus, our basic idea presented in the previous section can easily be extended to mixed convection problems for arbitrary thermal and flow boundary perturbations.

According to Eq. (28), if we choose the adjoint problem as

$$A^* \tilde{\phi}^* = \mathbf{0} \tag{30}$$

under

$$\tilde{\theta}^* = 1 \quad \text{on} \quad \Gamma_\theta, \quad \tilde{q}^* = 0 \quad \text{on} \quad \Gamma_q,$$

$$\tilde{u}^* = \mathbf{0} \quad \text{on} \quad \Gamma_u, \quad \tilde{\sigma}^* = \mathbf{0} \quad \text{on} \quad \Gamma_\sigma \tag{31}$$

then we can predict the change of total heat transfer rate on the Dirichlet boundary. The result is

$$\begin{aligned} \tilde{Q} &\equiv \int_{\Gamma_\theta} \tilde{q}\, d\Gamma \\ &= \int_{\Gamma_\theta} \tilde{q}^* \tilde{\theta} d\Gamma - \int_{\Gamma_q} \tilde{\theta}^* \tilde{q} d\Gamma + \int_{\Gamma_u} \tilde{\sigma}^* \cdot \tilde{u} d\Gamma - \int_{\Gamma_\sigma} \tilde{u}^* \cdot \tilde{\sigma} d\Gamma \end{aligned} \tag{32}$$

Equation (32) means that the change of total heat transfer rate can be obtained under arbitrary thermal and flow perturbations by solving numerically the adjoint problem under the boundary conditions (31). Thus, the distributions of $\tilde{q}^*$, $\tilde{\theta}^*$, $\tilde{\sigma}^*$ and $\tilde{u}^*$, all of which will be obtained from a numerical solution of the adjoint problem, can be regarded as sensitivity functions of the perturbations of temperature, heat flux, velocity and stress for the total heat transfer change.

In addition, if we set the adjoint problem as

$$A^* \tilde{\phi}^* = \left[0, \mathbf{0}, \delta(\xi)\right]^T \tag{33}$$

under

$$\tilde{\theta}^* = 0 \quad \text{on} \quad \Gamma_\theta, \quad \tilde{q}^* = 0 \quad \text{on} \quad \Gamma_q,$$

$$\tilde{u}^* = 0 \quad \text{on} \quad \Gamma_u, \quad \tilde{\sigma}^* = 0 \quad \text{on} \quad \Gamma_\sigma \tag{34}$$

then we can predict the temperature change at a specific location ξ, such that

$$\tilde{\theta}(\xi) = \int_{\Gamma_\theta} \tilde{q}^* \tilde{\theta} d\Gamma - \int_{\Gamma_q} \tilde{\theta}^* \tilde{q} d\Gamma + \int_{\Gamma_u} \tilde{\sigma}^* \cdot \tilde{u} d\Gamma - \int_{\Gamma_\sigma} \tilde{u}^* \cdot \tilde{\sigma} d\Gamma \tag{35}$$

under arbitrary thermal and flow boundary perturbations.

It is interesting to note that if we set a point pressure source or a point velocity source instead of the point heat source in Eq. (33), we can predict the change of pressure or velocity at a specific location ξ under arbitrary thermal and flow perturbations.

3. NUMERICAL EXAMPLES

To demonstrate the present approach to the convection heat transfer problems, we present numerical examples for forced convection heat transfer and for mixed convection heat transfer. It should be noted that the numerical examples presented below have no practical meaning, but will give the illustration of the present method.

3.1 Application to Forced Convection Heat Transfer

As an application of the present method to forced convection problems, we computed the adjoint equations defined in Eqs. (15) and (18) in a square cavity, the flow of which is well known as a lid-driven cavity flow as shown in Fig. 1. In the computations, a standard flow and temperature calculation code based on the finite difference method [1] was employed, in which the direction of the velocity vector was reversed to compute Eqs. (15) and (18) instead of Eq. (9).

As the first example, let us consider the total heat transfer rate from the bottom surface under its arbitrary temperature distributions. To obtain the influence function of the bottom-surface temperature on the total heat transfer rate, we calculated the adjoint problem, namely

$$u \cdot \nabla \theta^* = -\frac{1}{Pr} \Delta \theta^* \tag{36}$$

under

$$\theta^* = 1 \quad \text{on} \quad \Gamma_b, \quad \theta^* = 0 \quad \text{on} \quad \Gamma_u,$$

$$q^* = 0 \quad \text{on} \quad \Gamma_l \cup \Gamma_r \tag{37}$$

Then we can get the following relationship from Eq. (17):

$$Q \equiv \int_{\Gamma_b} q\, d\Gamma = \int_{\Gamma_b} q^* \theta\, d\Gamma \tag{38}$$

The influence function, .i.e, the adjoint heat flux on the bottom surface, is indicated in Fig. 2, in which the usual heat flux distribution under an isothermal condition is also indicated by a dashed line. From Fig. 2, the adjoint heat flux on the bottom surface increases from right to left, while the usual heat flux decreases in that direction. This suggests that rais-

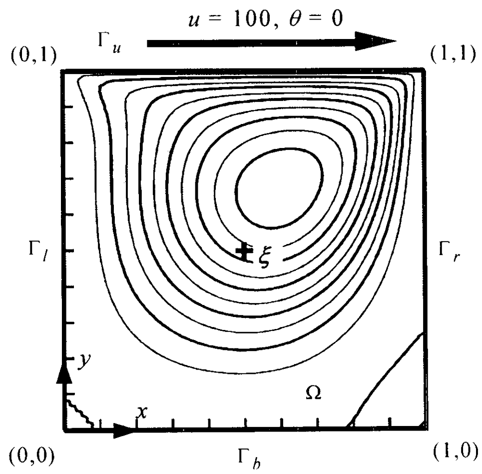


Fig. 1 Configuration of forced convection heat transfer in a square cavity

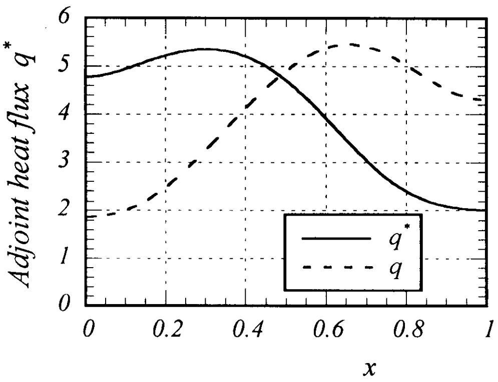


Fig. 2 Influence of bottom surface temperature distribution on total heat transfer

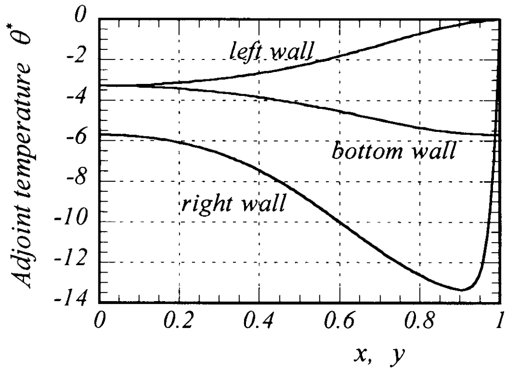


Fig. 3 Influence of wall heat flux distributions on temperature at the center of cavity

ing the surface temperature from right to left, we get a larger heat transfer capability even if the bottom surface has the same average temperature.

As the second example in the same lid-driven cavity, let us evaluate the temperature at the center of cavity under arbitrary surface heat flux distributions on side and bottom walls. This corresponds to the case described in Eq. (18). Thus, we computed the adjoint problem

$$\boldsymbol{u}\cdot\nabla\theta^* = -\frac{1}{Pr}\Delta\theta^* + \delta(\xi) \tag{39}$$

where ξ is the center of the cavity. According to Eq. (19), the boundary conditions for this problem become

$$\theta^* = 0 \quad \text{on} \quad \Gamma_u,$$
$$q^* = 0 \quad \text{on} \quad \Gamma_l \cup \Gamma_r \cup \Gamma_b \tag{40}$$

Then we can predict the temperature at ξ as

$$\theta(\xi) = -\int_{\Gamma_l}\theta^* q\, d\Gamma - \int_{\Gamma_r}\theta^* q\, d\Gamma - \int_{\Gamma_b}\theta^* q\, d\Gamma \tag{41}$$

From the numerical solution of the adjoint problem, the adjoint temperature distributions on the side and bottom walls are indicated in Fig. 3. Figure 3 shows that the heating effects on the temperature at the center of cavity is the largest at the upper portion of the right-side wall. This suggests that if we install a heater on the wall, the upper portion of the right-side wall is the best position to warm the center of the cavity.

3.2 Application to Mixed Convection Heat Transfer

As an application to mixed convection heat transfer, let us consider a square cavity with several inlets and an outlet as shown in Fig. 4. The purpose of this example is to predict the change of temperature at a specific location ξ, where is the center of the cavity in this example, when the inlet temperature and inlet flow are varied from base conditions ($\bar{\theta} = 1$, $\bar{\boldsymbol{u}} = (100,0)^T$). After a computation of the base problem, we numerically solved the adjoint problem, which can explicitly be written as

$$\begin{aligned} &\nabla\cdot\tilde{\boldsymbol{u}}^* = 0 \\ &\left(\bar{\boldsymbol{u}}\cdot\nabla + (\bar{\boldsymbol{u}}\nabla)^T\right)\tilde{\boldsymbol{u}}^* = -\nabla\tilde{p}^* - \Delta\tilde{\boldsymbol{u}}^* - \bar{\theta}\nabla\tilde{\theta}^* \\ &\bar{\boldsymbol{u}}\cdot\nabla\tilde{\theta}^* = -\frac{1}{Pr}\Delta\tilde{\theta}^* - Gr\,\tilde{\boldsymbol{u}}^*\cdot\boldsymbol{j} + \delta(\xi) \end{aligned} \tag{42}$$

and the boundary conditions adopted are

$$\begin{aligned} &\tilde{\theta}^* = 0 \quad \text{on} \quad \Gamma_1\cup\Gamma_2\cup\Gamma_3\cup\Gamma_u\cup\Gamma_b\cup\Gamma_o, \\ &\tilde{q}^* = 0 \quad \text{on} \quad \Gamma_l\cup\Gamma_r, \\ &\tilde{\boldsymbol{u}}^* = \boldsymbol{0} \quad \text{on} \quad \Gamma_1\cup\Gamma_2\cup\Gamma_3\cup\Gamma_u\cup\Gamma_b\cup\Gamma_l\cup\Gamma_r, \\ &\tilde{\boldsymbol{\sigma}}^* = \boldsymbol{0} \quad \text{on} \quad \Gamma_o \end{aligned} \tag{43}$$

Then we get the following boundary integral relationship from Eq.(35):

$$\begin{aligned} \tilde{\theta}(\xi) = &\int_{\Gamma_1}\left(\tilde{q}^*\tilde{\theta} + \tilde{\boldsymbol{\sigma}}^*\cdot\tilde{\boldsymbol{u}}\right)d\Gamma \\ &+\int_{\Gamma_2}\left(\tilde{q}^*\tilde{\theta} + \tilde{\boldsymbol{\sigma}}^*\cdot\tilde{\boldsymbol{u}}\right)d\Gamma \\ &+\int_{\Gamma_3}\left(\tilde{q}^*\tilde{\theta} + \tilde{\boldsymbol{\sigma}}^*\cdot\tilde{\boldsymbol{u}}\right)d\Gamma \end{aligned} \tag{44}$$

Equation (44) implies that if we get adjoint heat fluxes $\tilde{q}^*$ and adjoint stress $\tilde{\boldsymbol{\sigma}}^*$ at the inlets, we can predict the temperature change at ξ for arbitrary temperature and flow perturbations at the inlets. In this example, the base and the adjoint problems were computed by a standard finite difference method[1] as well as that in the previous example.

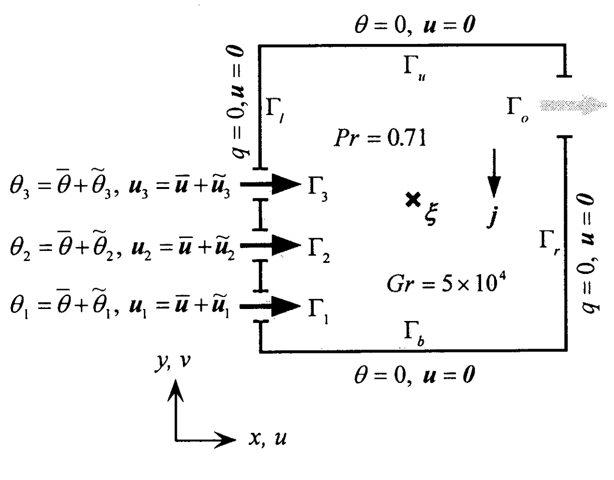
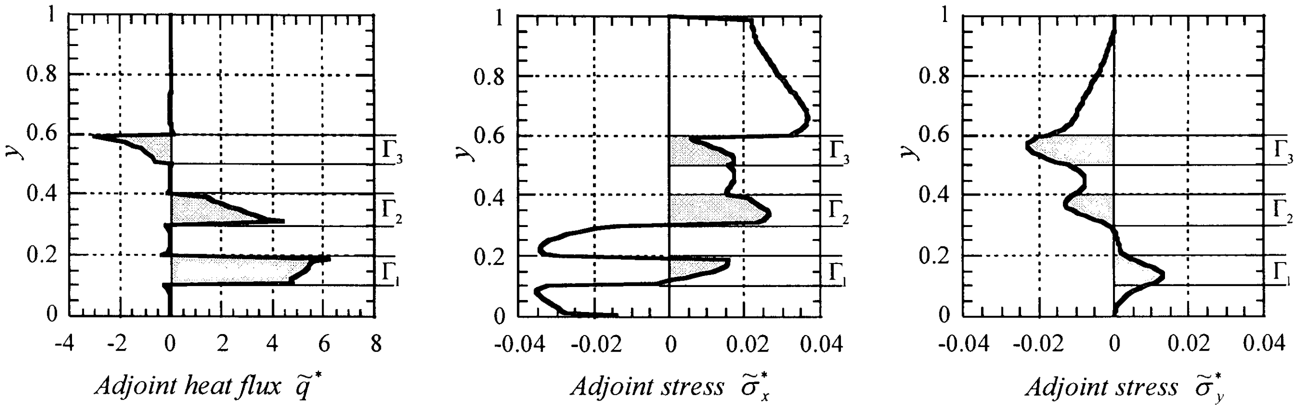


Fig. 4 Configuration of mixed convection heat transfer in a square cavity ($\bar{\theta} = 1$, $\bar{\boldsymbol{u}} = (100,0)^T$)



(a) Influence of temperature

(b) Influence of horizontal velocity component

(c) Influence of vertical velocity component

Fig. 5 Influences of thermal and flow perturbations at inlets on temperature at the center of cavity

Figure 5 shows the adjoint heat flux and the adjoint stress distributions obtained on the left side of the cavity. According to Fig. 5 (a), the adjoint heat flux distributions at lower two inlets are positive. This indicates that if the temperature at lower two inlets increase, the temperature at the center of cavity also increases. On the other hand, the increase of temperature at inlet 3 causes the temperature decrease at the center of cavity, because the adjoint heat flux at this inlet is negative. From Fig. 5 (b), we can raise the temperature at the center of cavity by simply increasing the horizontal velocity components at all inlets. Moreover, Fig. 5 (c) suggests that the vertical velocity components at the upper two inlets should be decreased to increase the temperature at the center of cavity, while that at inlet 1 should be increased.

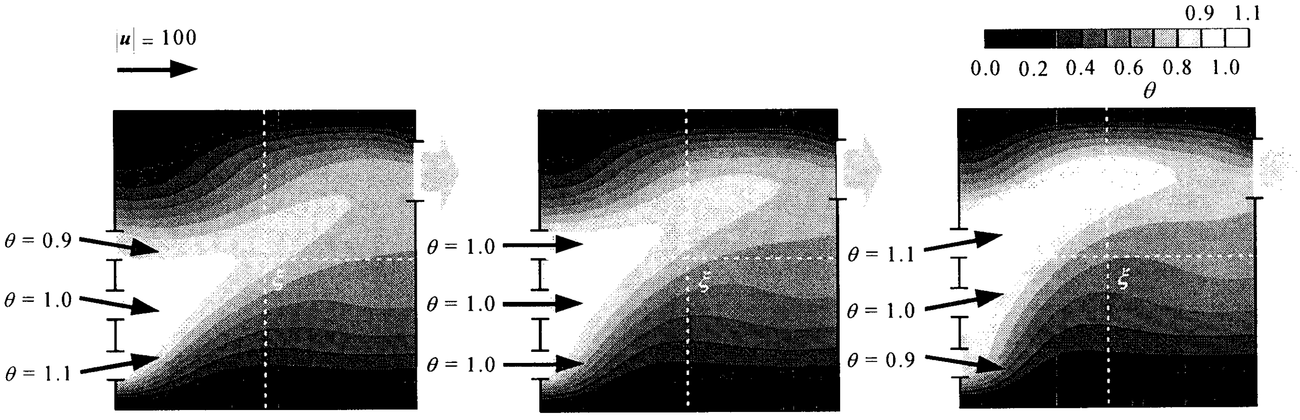
In order to confirm the predictions discussed above, we carried out direct numerical simulations of mixed convection fields with small thermal and flow perturbations at the inlets. Figure 6 shows the results obtained under typical inlet conditions; these are chosen from the predictions in Fig. 5 for the case of temperature increase (a) and for the case of temperature decrease (b), such that

$$\text{(a)}\quad \begin{cases} \theta_1 = 1.1, \quad \theta_2 = 1.0, \quad \theta_3 = 0.9 \\ \varphi_1 = +10°, \quad \varphi_2 = -10°, \quad \varphi_3 = -10° \end{cases}$$

$$\text{(b)}\quad \begin{cases} \theta_1 = 0.9 \quad \theta_2 = 1.0, \quad \theta_3 = 1.1 \\ \varphi_1 = -10°, \quad \varphi_2 = +10°, \quad \varphi_3 = +10° \end{cases}$$

where φ is the inlet flow angle measured from the horizontal line. As shown in Fig. 6, the temperatures at the center of cavity can be well controlled by slightly changing the inlet conditions suggested by the present method. In the figure, the temperatures obtained by the direct simulations are compared with those predicted from the present method, which appear in parenthesis. The agreement is fairly good.

It should be noted again that the adjoint variables obtained in mixed convection problems are sensitivities for thermal and flow perturbations. Although the sensitivity provides no idea for the limitation of the perturbation, it will provide useful information for thermal design, especially when combined with gradient-based optimization strategies.



$\theta(\xi) = 0.847\,(0.858)$ $\theta(\xi) = 0.737$ $\theta(\xi) = 0.609\,(0.603)$

(a) Temperature increase conditions Base conditions (no perturbation) (b) Temperature decrease conditions

Fig. 6 Thermal and flow inlet conditions to increase and decrease temperature at ξ

4. CONCLUSIONS

In this paper, we propose a numerical approach based on adjoint formulation of convection heat transfer to predict the heat transfer characteristics. The main features of the present approach can be summarized as follows:

(1) By numerically solving the adjoint problem for forced convection heat transfer, the total heat transfer rate or the temperature at a specific location can be obtained under arbitrary thermal boundary conditions. The computation time for the adjoint problem is equal to that required in a numerical simulation of forced convection heat transfer under a specific thermal boundary condition.

(2) For mixed convection heat transfer problem, by introducing perturbations from the base boundary conditions, the adjoint system can be derived for the perturbation problem. Thus, by numerically solving the base and the adjoint systems, the change of total heat transfer rate or the change of temperature at a specific location can be predicted not only for arbitrary thermal boundary perturbations but also for arbitrary flow boundary perturbations.

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NOMENCLATURE

A	differential operator defined in Eq. (10)
$\boldsymbol{A}$	differential operator matrix defined in Eq. (25)
Gr	Grashof number
$\boldsymbol{j}$	unit vector parallel to the gravitational force
$\boldsymbol{n}$	unit normal vector to the boundary
Pr	Prandtl number
p	nondimensional pressure
Q	total heat transfer rate
q	nondimensional heat flux
$\boldsymbol{u}$	nondimensional velocity vector
x	horizontal coordinate
y	vertical coordinate
δ	Dirac's delta function
$\boldsymbol{\phi}$	vector defined in Eq. (24)
Γ	boundary
φ	angle measured from horizontal line
θ	nondimensional temperature
σ	nondimensional stress vector
Ω	convection domain
ξ	space vector at a specific location

Subscripts

1, 2, 3	inlet number defined in Fig. 4
b	bottom wall
l	left wall
o	outlet defined in Fig. 4
r	right wall
u	upper wall
q	boundary specified via heat flux
$\boldsymbol{u}$	boundary specified via velocity
θ	boundary specified via temperature
σ	boundary specified via stress

Superscripts

T	transpose
*	adjoint operator or adjoint variable
-	given or base value
~	perturbation from base value