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AUTOMATIC CONSTRUCTION OF TRAIN ARRIVAL AND DEPARTURE SCHEDULES IN TERMINAL STATIONS

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Abstract

Recently, attention to the railway transportation has been revived from the viewpoint of alleviating environmental problems such as the reduction of CO2 emission amount. Moreover, the commuter train services in a city and long distance train services such as the Shinkansen are indispensable in Japan.

In this paper, we formulated the train arrival and departure optimization problem at a terminal station as a 0-1 integer programming problem, and we succeeded in obtaining solutions by using a solver of the case of Tokyo Station of Tohoku Shinkansen. Moreover, we transform this problem in polynomial time to the Maximum matching problem in a bipartite graph when some realistic conditions were assumed.

Keywords: Scheduling , 0-1 Integer Programming, Railway transportation system

1. Introduction

Recently, attention to the railway transportation has been revived from the viewpoint of alleviating environmental problems such as the reduction of CO2 emission amount. Moreover, the commuter train services in a city and long distance train services such as the Shinkansen are indispensable in Japan.

The opening of Kanazawa Station of Hokuriku Shinkansen, and the connection to Shin-Aomori Station of Tohoku Shinkansen, will result in further increase in demand for the Shinkansen transportation. For example, trains of Tohoku Shinkansen in the vicinity of Tokyo Station are operated with a high density i.e. 14 trains per an hour. Therefore, the number of trains that can be operated will reach the limit in the near future. However, the improvement of equipment of terminal stations to cope with this situation is expensive and needs much time. Therefore, we try to increase the number of trains by improving the scheduling.

Moreover, this problem is not limited to the rapid-transit railway. The congestion of the commuter train services in city areas are a serious problem. This problem is especially remarkable in subways. However, construction of new route and/or extension of the station to solve the problem requires a large construction cost.

Therefore, it is desired to increase the number of trains operated by efficiently using existing facilities. For the above-mentioned reasons we studied a maximization problem of the number of trains per unit time(Wakisaka et al., 2010).

Moreover, one of the examples of applying this technology is to use it for the re-scheduling when the operation of trains falls into disorder. For instance, consider the case where two or more trains are delayed due to a bad weather, and the following train assumes that it was able to operate in accordance with the schedule. Then, a late train and the following train might arrive at the station almost the same time. If the train is operated in accordance with the schedule when the most tight schedule is known, the trouble might be overcome.

The number of trains is often considered by "Track capacity" in abroad(see, e.g. Electric Railways Handbook Editorial Board(2007)) . However, a number of trains exceeding this capacity is operated by various plans in Japan.

The scheduling problems of the railway are surveyed in detail in (Tomii, 2008). A number of problems in this field are complex and very large in scale, and requires heavy calculations to solve them.

(Carey et al., 2003) studied the allocation of trains at a busy station to a platform. However, they adopted satisfaction rate to the demand of each company in accordance with circumstances of railways in Europe [1] to be an objective function.

(Billionnet 2003) studied the allocation of the platform to the train that arrives at the station. However, this research assumes a situation from which the arrival time of a train is given. Therefore, the purpose is different from our research.

(Chakroborty et al., 2008) studied the re-assignment of platforms to trains where a large delay is frequently caused. As for these researches, the objective functions are different from our research.

2. The Reason Why We Pay Attention to Terminal Stations

In this paper, the terminal station is defined as follows: The railway track connected from the station to the outside is composed of the arrival track and the departure track, and the train can arrive from and depart for only one direction respectively.

The terminal station becomes a bottleneck in many cases as much work (e.g. the cleaning in the car, the reversal of seats, etc.) is required when a train arrived at a terminal station. This is especially the case for the Shinkansen and express trains. Consequently, longer stoppage time is needed at terminal stations compared with midway stations. Moreover, no interaction among courses of trains is allowed. In many cases, trains must be moved exclusively on the rail. The above-mentioned exclusivity and required stoppage time makes the terminal station a bottleneck.

In this paper, we study an efficient (that can increase the number of trains on service e.g., per an hour) scheduling of trains while considering various problems concerning to a terminal station.

Recently, it has become possible to shorten the operation interval of trains due to the rapid advancement of technology; e.g. acceleration of trains, improvement of traffic signal control systems, etc. However, the interaction of trains in the terminal is still remained to be solved. Therefore, the scheduling at a terminal station is an important factor to increase the number of trains in service e.g., per an hour.

3. The Train Arrival And Departure Optimization Problem at a Terminal Station

In this paper, a terminal station where the traffic volume of the train is stringent is assumed. The objective function to be maximized is the number of trains at a station where

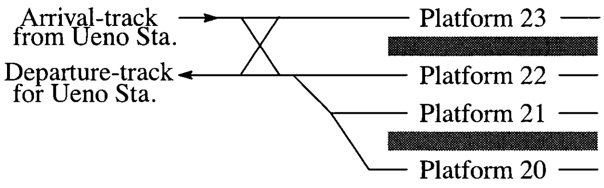

Fig. 1 Station track map (Tokyo Station at Tohoku-Shinkansen line)

trains arrive and depart in a unit time(e.g. an hour). As for the time axis, discrete time is assumed.

In this thesis, we impose the following assumptions on the train and the station:

- The station is connected to only one arrival track and one departure line outside of the station.
- The arrival and the departure of trains are provided only in one direction, respectively, and the number of trains that can move simultaneously is at most two.
- The train type (e.g. express, rapid, local etc.) is not considered, because no passing train exists at a terminal station.
- At most one train simultaneously exists in each platform.
- An arrival train comes from the arrival-track.
- A departure train goes to the departure-track.
- An arrival train moves from the arrival-track to the platform in just one unit-time.
- A departure train moves from the platform to the departure-track in just one unit-time.
- The train can arrive from the arrival-track to any platform, and it can depart from any platform to the departure-track.
- Two or more trains never arrive at the station simultaneously. Similarly, two or more trains never depart from the terminal station simultaneously.
- The arriving train and the departure train can move if there is no interaction among their courses. Only one train can move when an interaction of courses among trains happens.
- The train cannot depart before the fixed time has passed since it arrived at the platform.

4. Formulation as a 0-1 Integer Programming problem

We formulate the train arrival and departure optimization problem at a terminal station as a 0-1 integer programming problem.

4.1 Formulation

Notations

- **L**: The set of platforms.

[1] The government own the railway track, and, a lot of railway companies make a train run there.

- t_{max}: The number of units of time (Note that we assume discrete time)
- $\mathbf{T} = \{0, 1, \cdots, t_{\max} - 1\}$: The set of discrete time instants.
- s: Stoppage time duration at the terminal station.
- $c_{a,d}$:$a, d \in \mathbf{L}$, The upper bound of the events that can be done simultaneously, i.e. the event of the train arrival to platform a and the event of the train departure from platform $d (a \neq d)$. If they are executable simultaneously, it has value 2, otherwise, value 1.

Table 1 The upper bound of the events(Tokyo Station at Tohoku-Shinkansen line)

	$c_{a,d}$	d: #20	#21	#22	#23
a	Track #20	1	1	1	1
	Track #21	1	1	1	1
	Track #22	1	1	1	1
	Track #23	2	2	2	1

Decision variable $x^a_{t,l}$ is assumed to be 1 when the train arrives at platform l at a certain time t. (otherwise, 0). Similarly, decision variable $x^d_{t,l}$ is assumed to be 1 when the train departs at platform l at a certain time t. (otherwise, 0).

The objective function and constraints are expressed as follows. We assumed that trains do not exist at each platform as an initial state at time 0.

$$\text{Maximize} \sum_{t \in \mathbf{T}, l \in \mathbf{L}} x^a_{t,l} \tag{1}$$

subject to

$$\sum_{l \in \mathbf{L}} x^a_{t,l} \leq 1, t \in \mathbf{T} \tag{2}$$

$$\sum_{l \in \mathbf{L}} x^d_{t,l} \leq 1, t \in \mathbf{T} \tag{3}$$

$$\sum_{t=0}^{j} \left(x^a_{t,l} - x^d_{t,l} \right) \leq 1, j \in \mathbf{T}, l \in \mathbf{L} \tag{4}$$

$$x^d_{t,l} = 0, 0 \leq t < s, l \in \mathbf{L} \tag{5}$$

$$\sum_{t=0}^{j-s} x^a_{t,l} - \sum_{t=0}^{j} x^d_{t,l} \geq 0, s \leq j \leq t_{max}, l \in \mathbf{L} \tag{6}$$

$$x^a_{t,l_1} + x^d_{t,l_2} \leq C_{l_1,l_2}, t \in \mathbf{T}, l_1, l_2 \in \mathbf{L} \tag{7}$$

$$x^a_{t,l} \in \{0, 1\}, t \in \mathbf{T}, l \in \mathbf{L} \tag{8}$$

$$x^d_{t,l} \in \{0, 1\}, t \in \mathbf{T}, l \in \mathbf{L} \tag{9}$$

Expression (1) is the objective function. Expression (2) means that two or more trains cannot arrive simultaneously. Expression (3) means that two or more trains cannot depart simultaneously. Expression (4) means that two or more trains cannot exist in each platform. Expressions (5) and (6) are the constraints that ensure that the stoppage time of the train is at least s units of time. Expression (7) is a constraint that a train that might collide cannot move simultaneously. Expressions (8) (9) mean constraints that variables must be either 0 or 1.

4.2 Computational Experiments

We solved the arrival and departure optimization problem for the terminal station at Tokyo Station of Tohoku Shinkansen as an example by using Solver GLPK4.39. Discrete time is assumed, where the unit time is defined as two minutes[2]. The minimum stoppage time is defined as 12 minutes, which is an actual value. We calculate during 30 units of time (During one hour in actual time).

Fig. 2 shows the obtained train arrival and departure pattern. We obtain an answer where 18 trains arrive in 30 units of time. The obtained pattern repeats the same arrival and departure every 7 units of time. All trains arrive at and depart from in the shortest stoppage time, and each platform is used without idle time, and the best movement pattern is achieved.

When this pattern is repeated, four trains for every 14 minutes can be operated, specifically, 17.1 trains per an hour can be operated. 14 trains per an hour are operated now at Tokyo station of Tohoku Shinkansen, thus three more trains (about 4,800 people can be on board) can be operated by the solution.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
#20						A						D	A						D	A						D	A			
#21		A						D	A						D	A						D	A						D	A
#22				A						D	A						D	A						D	A					
#23	A						D	A						D	A						D	A						D	A	

Fig. 2 Obtained train plan

5. Polynomial Time Transformation to Maximum Matching in the Bipartite Graph

An 0-1 integer programming problem is NP-hard, and believed to be a difficult problem. In this section, we convert the problem into the bipartite graph maximum matching problem which can be solved in polynomial time. To do this, we introduce "Cycle", and add two conditions.

[2]This value is used based on actual values of train intervals and travel time from/to the platform of Tohoku Shinkansen at Tokyo Station.

When the following two conditions are satisfied, the train arrival and departure optimization problem at a terminal station can be transformed to the maximum matching problem in the bipartite graph.

condition 1 The constraint concerning the stoppage time is relaxed. The train can leave at the next time of its arrival at the platform.

condition 2 During "One cycle" the train arrives at and departs from all platforms exactly once.

In this section, we relax the constraint of the stoppage time. However, if the stoppage time is not secured, the train cannot be safely operated. This problem can be solved by appropriately inserting the gap time after the algorithm described later is applied.

The station where the platform is exclusively used by some types of trains (express, limited express, local, etc.) exists. However, all platforms are often evenly used in the station where a large number of trains exist.

As nodes of the bipartite graph, a node at one side in the bipartite graph corresponds to a departure event of the train from each platform, and a node at the other side corresponds to an arrival event, and the combination of events that can arrive and depart simultaneously is connected by an edge. The maximum matching is obtained on the bipartite graph. The arrival and departure events mutually connected by the edge is done simultaneously, and the event not connected cannot be done simultaneously, thus, we can obtain the train arrival and departure pattern in one cycle.

Thus, we have the following algorithm which works in polynomial time:

Train operation pattern generation algorithm

Step 1 Make the "arrival and departure competition table".

Step 2 Generate the bipartite graph as follows:

- Make the node, and let them correspond to a departure event of each platform.
- Make the node on an opposite side, and let them correspond to an arrival event of each platform.
- Connect the event that can be executed simultaneously by an edge.

Step 3 Solve the bipartite graph maximum matching problem.

Step 4 Decide the arrival and departure event executed simultaneously in accordance with the order.

- Execute the event at both ends of the selected edge simultaneously.
- Execute the event not connected by an edge alone.

Step 5 Sort it so that the existence time of the train may become the maximum. (The next train should arrive at each platform immediately after the train.)

□

For the station of the layout like Fig. 3, the "Arrival and departure competition table" is shown in Table 2. In this table, "P"(possible) denotes the combination of the arrival and departure events that can be executed simultaneously and "I"(Impossible) denotes the arrival and departure combination that cannot be executed simultaneously. [3]

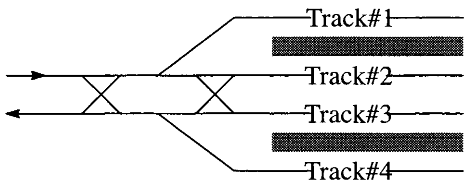

Fig. 3 An example of the track layout of a terminal station

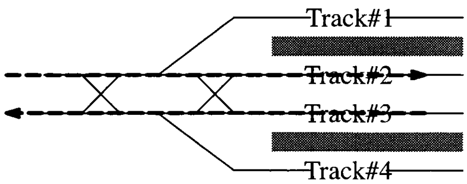

Fig. 4 Situation where train can move simultaneously (arrival at Track #2 and departure from Track #3)

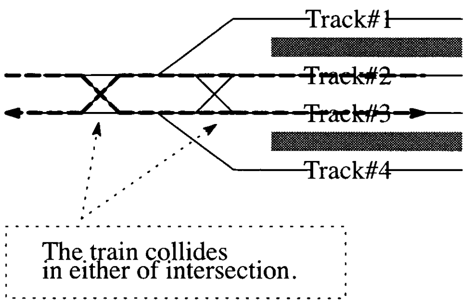

Fig. 5 Situation where two trains collide when train moves simultaneously (arrival at Track #3 and departure from Track #2)

The bipartite graph generated from this table is shown in Fig. 6. The edge shown by a heavy line is one selected by the maximum matching for this bipartite graph.

Next, events executed simultaneously is decided from the obtained matching. Each of five rectangles in the upper row of Fig. 7 correspond to an event executed at a certain one unit time, and, these rectangles correspond to fragments to which the Gantt chart obtained as an execution result was sliced vertically with the time axis every unit of time. Moreover, in order to lengthen the stop time of each train in the station, these fragments are permutated.

[3]For instance, it is not possible to move simultaneously in the arrival at track #3 and the departure from track #2. Because the course of the train collides where the railway tracks cross intersects. (fig.5)

Table 2 An example of "Arrival and departure competition table"

		Depart #1	#2	#3	#4
Arrival	Track #1	I	P	P	P
	Track #2	I	I	P	P
	Track #3	I	I	I	P
	Track #4	I	I	I	I

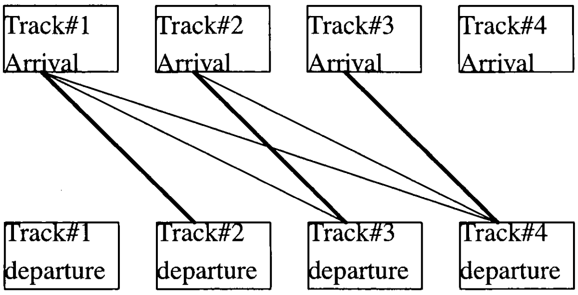

Fig. 6 The generated bipartite graph

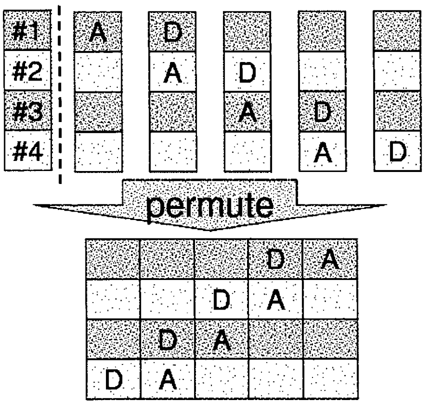

Fig. 7 Example of timetable of generated "One cycle"

Consequently, the pattern of one cycle like the figure in the lower of Fig. 7 can be obtained. This "One cycle" can be repeatedly applied. A very tight schedule of trains, where the interval from the departure of a train to arrival of the next train at a platform is as short as possible, can be obtained.

The maximum matching in the bipartite graph can be obtained in $O(mn)$ time where m is the number of nodes, and, n is the number of edges, and other steps can be done in polynomial time. Therefore, it is possible to make train operation pattern in polynomial time, when conditions 1 and 2 are satisfied.

However, it is difficult to operate trains safely if the stoppage time is insufficient. Therefore, the constraints of the stoppage time cannot be actually disregarded.

If the stoppage time is insufficient, the pair of the arrival and departure done simultaneously is executed not simultaneously. When all arrival and departure is solitarily executed, the gap of which any train doesn't arrive and depart is inserted. Then, the stoppage time of all trains in one cycle increases at one unit time.

We experimented on some existing terminal stations in Japan.

The number of trains that can operate are greatly influenced by the size of the station (e.g., the number of platforms) and the stoppage time. Thus, the discussion only about the number of the trains is insufficient. Therefore, we verified "Whether the stoppage time of the train was minimum or not?" and "Whether extra time exists or not?". Any case was able to obtain a very tight schedule.

However, we assume that the stoppage time of all trains is the same in this paper. The stoppage time of the train that becomes a forwarding train after it arrives at the station might be short. Moreover, there is a possibility that an efficient schedule can be obtained by choosing the strategy that allows some platforms not to be used. It is desirable to analyze the computational complexity by considering these factors.

6. Conclusion

In this paper, we formulated the train arrival and departure optimization problem at terminal stations to the 0-1 integer programming problem, and we succeeded in solving it by using a solver for the case of Tokyo Station of Tohoku Shinkansen. Moreover, we showed polynomial time transformation from this problem to the maximum matching problem in a bipartite graph when some realistic conditions were assumed.

Acknownledgement

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References

[1] Tomii N. (2008). Scheduling problem of railway: Difficulty and interest. Operations Research 53(8). Page.427-432(in Japanese).

[2] Wakisaka K. Masuyama S. (2010). Train arrival and departure scheduling aimed at railhead of Shinkansen, Operations Research Society of Japan Research presentation forums 2010 spring. pp.24-25(in Japanese).

[3] Malachy Carey. Sinead Carville. (2003). Scheduling and platforming trains at busy complex stations. Transportation Research Part A: Policy and Practice. Volume 37, Issue 3. pp.195-224.

[4] Alain Billionnet. (2003). Using Integer Programming to Solve the Train-Platforming Problem. TRANSPORTATION SCIENCE. Vol. 37, No. 2. pp. 213-222.

[5] Partha Chakroborty. Durgesh Vikrama. (2008). Optimum assignment of trains to platforms under partial schedule compliance. Transportation Research Part B: Methodological. Volume 42, Issue 2. pp.169-184.

[6] Train schedule research group. (2008). Train schedule and operation management. seizando press(in Japanese).

[7] Electric Railways Handbook Editorial Board. (2007). Electric Railways handbook. pp.426. CORONA PUBLISHING. Tokyo(in Japanese).