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DEFORMATION ANALYSIS AND BEARING CAPACITY OF A TWO-LAYERED SOIL DEPOSIT WITH A SURFACE CRUST CONSIDERING COUPLE STRESSES

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ABSTRACT

This paper provides a numerical analysis of the bearing capacity of a two-layered soil deposit: a crust on saturated soft clay. The surface crust is modeled as a Cosserat medium, considering couple stresses for the bending deformation, and the underlying clay layer is treated as an elasto-plastic material. The formulation of the Cosserat-FE for the two phase media is presented briefly, and a numerical comparison between the Cosserat and an ordinary continuum, for the example of a simple beam, demonstrates the effect of couple stresses. Asaoka's method allows us to predict the bearing capacity of such two-layered deposits by considering the settlement or horizontal deformation beneath the embankment. The numerical results are then discussed.

Key words: bearing capacity, (Cosserat continuum), (couple stress), deformation, finite element method, (surface crust) (**IGC**: E1/E3)

INTRODUCTION

A surface crust often forms on soft clay and its existence may have an important impact on the bearing capacity of two-layered deposits. Several researchers have investigated this subject experimentally (Brown and Meyerhof, 1969) and numerically (Button, 1953; Reddy and Srinivasan, 1967; Purushothamaraj et al., 1974; Takemura, 1993). They have provided an upper bound solution for the value of the bearing capacity.

From an analytical viewpoint, some difficulties can be pointed out in the adoption of a failure mechanism, including a certain thickness of the surface crust. We are also confronted with lacking much potential for an FE modeling of the surface crust in the framework of an ordinary continuum. A very large value must be adopted for the elastic modulus in the modeling.

In order to overcome these difficulties, a Cosserat continuum is introduced herein to model the surface crust by considering couple stresses. The consideration of couple stresses can provide a bending effect for the layer as a beam.

An increased interest in the use of a Cosserat continuum (Cosserat, E. and F., 1909) has been shown. It takes into account the so-called couple stresses for analyzing localization and bifurcation problems (for example, *see* Mühlhaus, 1986, 1987a, 1987b; de Borst, 1991a, 1991b; and Tejchman, 1989, 1992). In applying such a continuum to various problems in the field of geomechanics, it is necessary to find an appropriate example where couple stresses play an important role. Considerations of couple stresses for a layer, for example, can provide the bending effect as a beam. Such a bending effect on the deformational behavior and the bearing capacity of a layered soil deposit is reviewed and discussed.

To begin with, the formulation of FEM for a soil-water mixture is derived. Its numerical performance will be examined through a comparison with the exact solution for the problem of an infinite shear layer. The influence of the couple stresses on the deformation and pore pressure behavior will then be pointed out.

A numerical profile of the deformation in the problem of a simple beam is also examined through a comparison with a solution based on an ordinary continuum, and the influence of couple stresses on the deformation is discussed. The bearing capacity of a layered deposit and the effect of couple stresses on that of a layered deposit with a surface crust is discussed.

In what follows, second section refers to the formulation of the finite elements based on Cosserat media for a soil-water mixture. Third section demonstrates a numerical solution comparing a closed solution, in the case of

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an infinite shear layer, and points out the bending effect, in the case of a simple beam, based on the consideration of couple stresses. In the fourth section, the procedure proposed by Asaoka and Ohtsuka (1986) for estimating the undrained bearing capacity by observing the deterioration of the stiffness factor is presented. Fifth section provides an estimate of the bearing capacity of a two-layered deposit by the strategy presented in the preceding sections. Finally, the last section concludes with the results.

FORMULATION OF FINITE ELEMENTS BASED ON COSSERAT MEDIA FOR A SOIL-WATER MIXTURE

This section refers to the finite element formulation. Figure 1 describes a set of governing equations to be solved under the boundary conditions shown in Fig. 2.

By integrating both Eqs. (1) and (3), multiplied by an arbitrary function, which is prescribed to be zero on the geometric boundary, over volume V, and by adopting the Gauss theorem, we obtain a weak form of the equilibrium equation based on the boundary conditions, as seen in Fig. 3(a).

On the other hand, the continuity equation for pore water is discretized in the same manner as that proposed by Akai and Tamura (1978). The resultant equation is shown in Fig. 3(b).

As a result, we summarize the flow of the coupled FE formulation of the Cosserat media in Fig. 4. The program, DACSAR, was developed by Iizuka and Ohta (1987) and is herein extended to perform the above strategy. The revised program therefore has common features with DACSAR in its numerical schemes.

Balance equation of momentum

$$\sigma_{ij,j} + b_i = 0 \quad \text{in } V, \tag{1}$$

Principle of effective stress

$$\sigma_{ij} = \sigma'_{ij} + p_w \delta_{ij}, \tag{2}$$

Balance equation of angular momentum

$$m_{ij,j} + \nu_i - e_{ijk}\sigma_{jk} = 0 \quad \text{in } V, \tag{3}$$

Kinematics

$$\varepsilon_{ij} = u_{i,j} + e_{ijk}\phi^c_k, \quad \kappa_{ij} = \phi^c_{i,j}, \tag{4}$$

Constitutive equations

$$\sigma'_{ij} = D_{ijkl}\varepsilon_{kl}, \quad m_{ij} = \mu l_c^2 \kappa_{ij}, \tag{5}$$

Continuity condition

$$\dot{\varepsilon}_{kk} = v_{i,i}, \tag{6}$$

Darcy's law

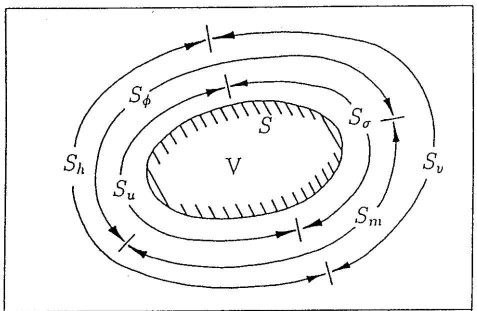
$$v_i = -kh_{,i}, \quad h = p_w/\gamma_w + \Omega, \tag{7}$$

Initial conditions

$$\sigma'_{ij} = \sigma'_{ij}|_{t=0} \quad \text{in } V, \quad h = h|_{t=0} \quad \text{in } V, \tag{8}$$

where σ_{ij}: Cauchy stress tensor, σ'_{ij}: effective stress tensor, b_i: body force, p_w: pore water pressure, δ_{ij}: Kronecker's delta, m_{ij}: couple stress tensor, ν_i: body couple, e_{ijk}: permutation symbol, ε_{ij}: strain tensor, ϕ^c_i: Cosserat rotation, κ_{ij}: curvature, μ_c: Cosserat shear modulus, D_{ijkl}: elastic moduli, l_c: characteristic length, γ_w: unit weight of water, k: permeability, h: total head, Ω: head, n_i: normal vector.

Fig. 1. Governing equations



Stress boundary $\sigma_{ij}n_i = \bar{t}_i$ on $S_\sigma \subset S$ $m_{ij}n_j = \bar{r}_i$ on $S_m \subset S$	Soil
Geometric boundary $u_i = \bar{u}_i$ on $S_u \subset S$ $\phi^c_i = \bar{\phi}^c_i$ on $S_\phi \subset S$	
Head boundary $h = \bar{h}$ on $S_h \subset S$	Water
Velocity boundary $v_i = \bar{v}_i$ on $S_v \subset S$	
$S_\sigma + S_u = S_m + S_\phi = S_h + S_v = S$	

Fig. 2. Boundary conditions

NUMERICAL PERFORMANCE

To check the performance of the procedure described in the preceding section, the example of an infinite shear layer as described in the reference (Choi and Mühlhaus, 1991) is initially analyzed. A description of the problem is found in Fig. 5. For this problem, we have an exact solution and a numerical solution by FEM that can be compared.

Problem-1: Infinite Shear Layer

We deal with the case of an infinite shear layer represented in Fig. 5 where a shear layer is treated as an elastic solid (Murakami and Oda, 1992). At the boundary of $|x_2| = h/2$, it is assumed that $\phi_3 = 0$ to suppress grain rotations and that traction $\bar{\sigma}_{12}$ acts on the boundary surface. We have an exact solution for this problem (Schaefer, 1962) and in Fig. 6 details of the solution procedure according to Schaefer (1962) are shown. This solution, however, is only for solids and pore water is not considered in this example.

Dealing with the case of horizontal infinity, it is sufficient for us to incorporate a one-dimensional differentiation into the equation. Therefore, a set of governing equations is reduced to a set of ordinary differential equations, as seen in Fig. 6, after substituting the constitutive relation into the balance equations. This set of equations can easily be solved under the boundary conditions and we obtain the solution for deformations and stresses in the same figure.

Figure 7 compares numerical deformations by FEM with theoretical ones based on the above equations. Figure 8 also describes a comparison between numerical

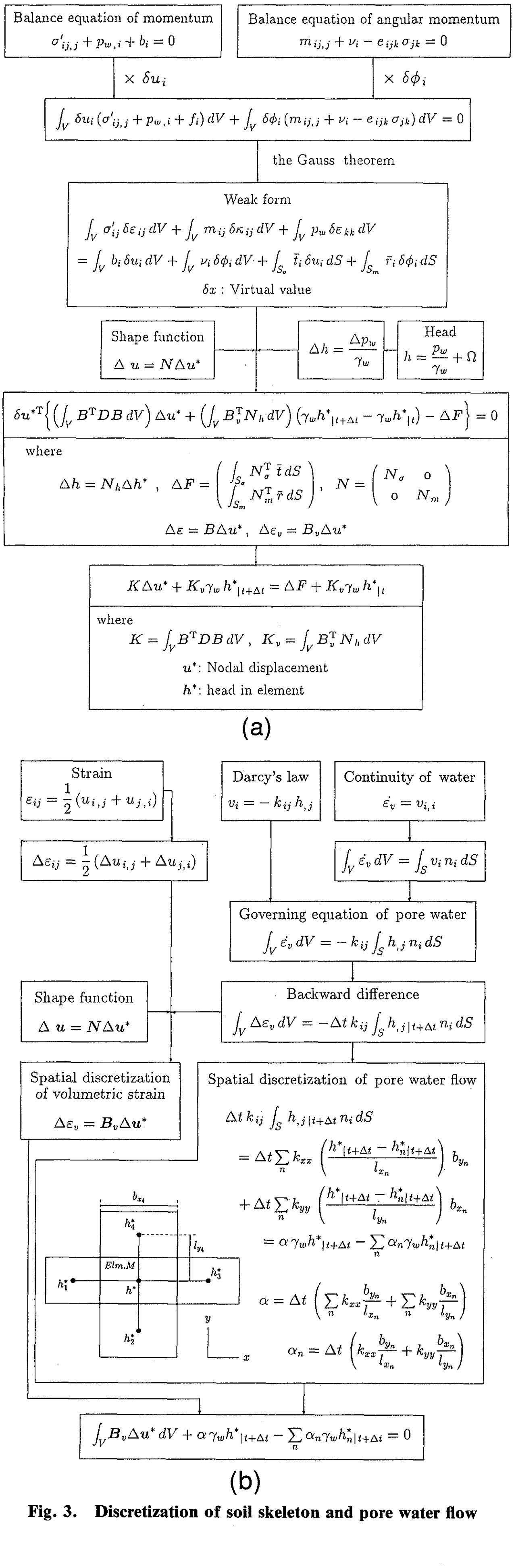


Fig. 3. Discretization of soil skeleton and pore water flow

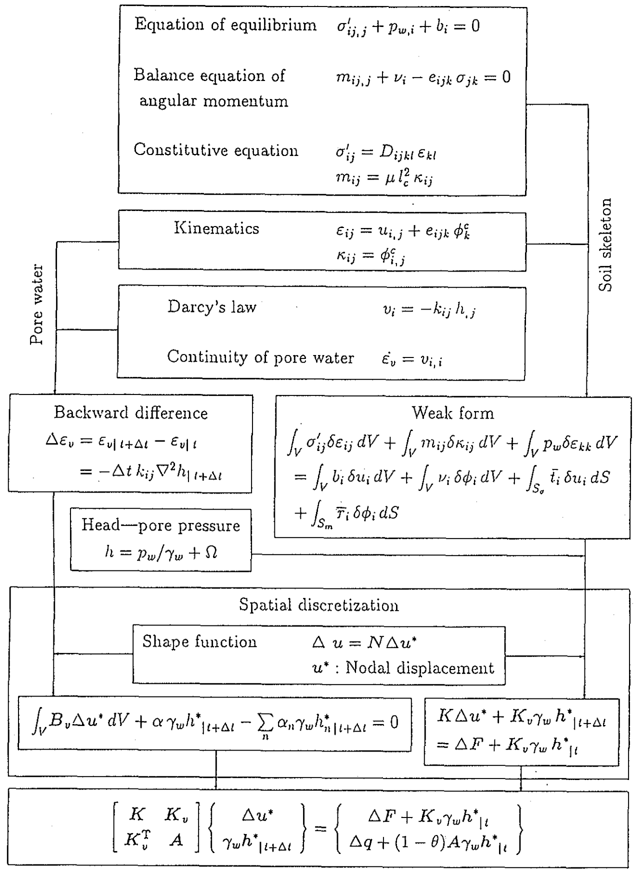


Fig. 4. Formulation of coupled FE

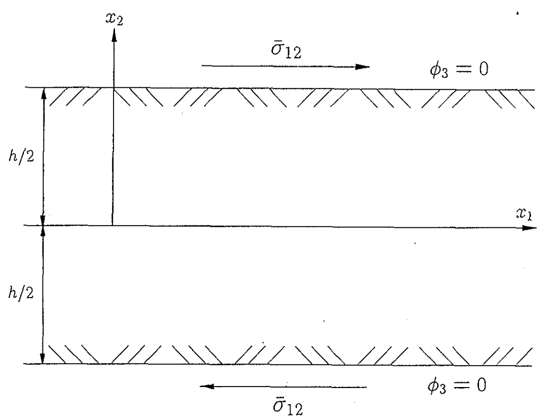


Fig. 5. Description of the problem

stresses by FEM and theoretical ones. There is an excellent agreement between the analytical and the FEM results, as seen in these figures.

Problem-2: Simple Beam

In order to demonstrate the present formulation, another typical example is solved. As carried out by Oka, Yashima and Hirata (1994), a simple beam loaded at the center is analyzed to capture the bending effect of an elastic Cosserat material. For this problem, we have a numerical solution which does not consider couple stresses and a solution by the current FE which can be compared to

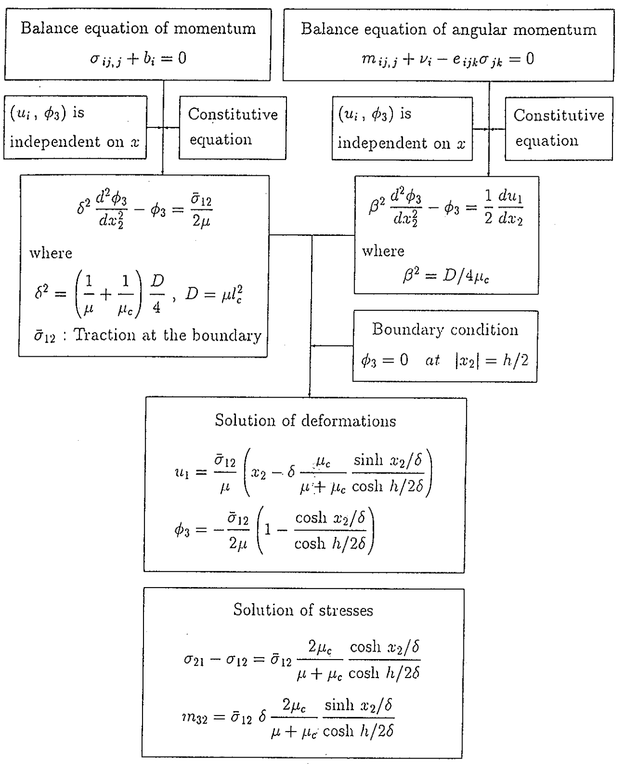


Fig. 6. Solution procedure

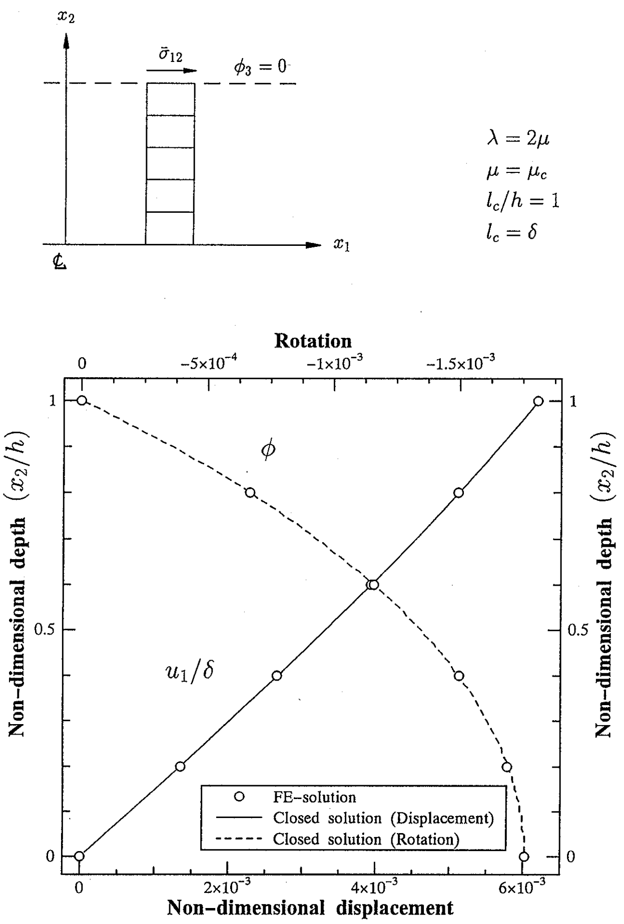


Fig. 7. Comparison of deformations

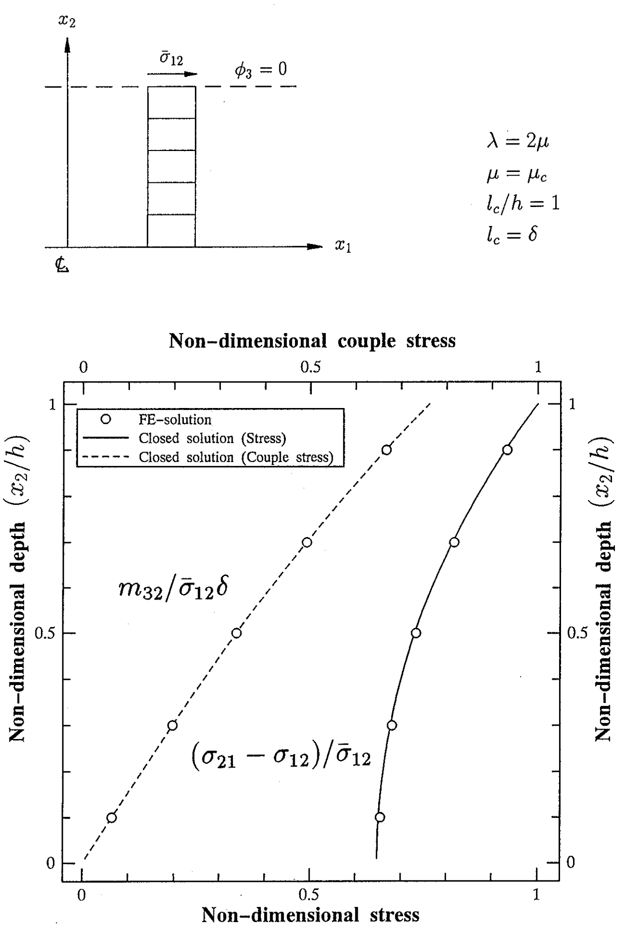


Fig. 8. Comparison of stresses

Table 1. Parameters adopted in the Problem-2

	λ (kPa)	μ (kPa)	ν	l_c (m)
Cosserat	9800	2450	0.4	1.0
Classical	9800	2450	0.4	0.0

that one. Material properties for both cases are listed in Table 1.

In Fig. 9, the deformed meshes for cases of (a) ordinary continuum (without considering couple stresses) and (b) Cosserat continuum (considering couple stresses) are presented. As shown in Fig. 9, the profiles of deformation are quite different from each other. As expected, displacements in the case of the Cosserat continuum have smaller values than those in the case of an ordinary continuum, and therefore, will transfer a lighter load toward the subsurface.

BEARING CAPACITY OF A TWO-LAYERED DEPOSIT

It is assumed that a crust layer exists which has a greater strength than that of the underlying saturated soil deposit. In such a case, the crust behaves like a beam and may influence the undrained bearing capacity of the en-

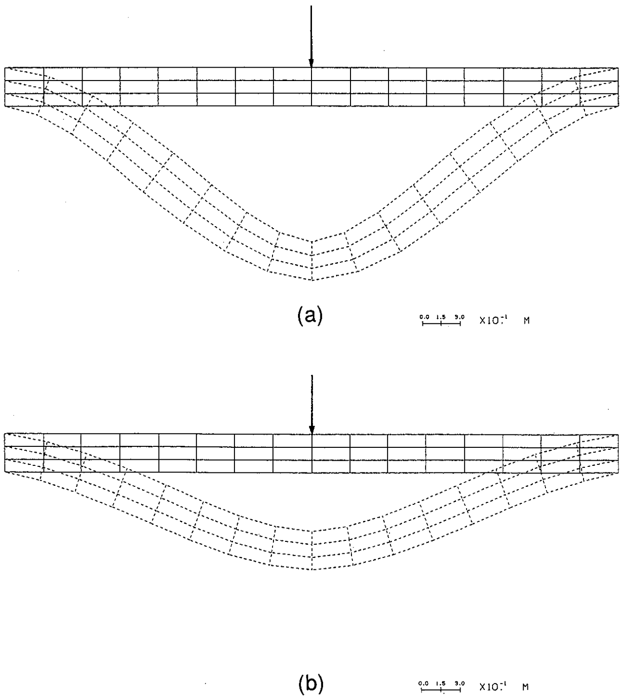


Fig. 9. Deformed profile for simple beam

tire deposit. Several researchers have provided solutions for the undrained bearing capacity of soft clay deposits with a surface crust both numerically (Button, 1953; Reddy and Srinivasan, 1967; Purushothamaraj et al., 1974; Takemura, 1993) and experimentally (Brown and Meyerhof, 1969).

The strategy adopted herein is as follows:

(i) The influence of a surface crust on the undrained bearing capacity of an entire soil deposit is evaluated by considering the surface crust as a Cosserat material which behaves like a beam on a soft foundation.

(ii) The undrained bearing capacity of an entire deposit is estimated through the method proposed by Asaoka and Ohtsuka (1986). It involves monitoring the deterioration of the tangential stiffness factor of the foundation based on the settlement under the embankment or the horizontal deformation below the toe of the embankment. Asaoka and Ohtsuka introduced the stiffness factor of clay foundation in terms of effective stresses from which the definition of total failure of clay foundation is made (Asaoka and Ohtsuka, 1986). They proposed a procedure to predict the ultimate load intensity from observations of elasto-plastic consolidation behavior in the early loading stages. They identified the deterioration of the tangential stiffness factors of the clay foundation, $\tilde{K}_p$, $\tilde{K}_\delta$, and estimated the bearing capacity of clay foundation by extrapolating the deterioration curve. A procedure of prediction, for example, is shown in Fig. 10.

NUMERICAL ANALYSIS AND DISCUSSION

Hypothetical Soil Deposit

To illustrate the analysis discussed above, the example problem shown in Fig. 11 is solved. It deals with the behavior of the deformation and the pore water pressure of a saturated soil deposit, which has a surface crust or surface sand, under the external action of continuous embankment loading, and it compares the undrained bear-

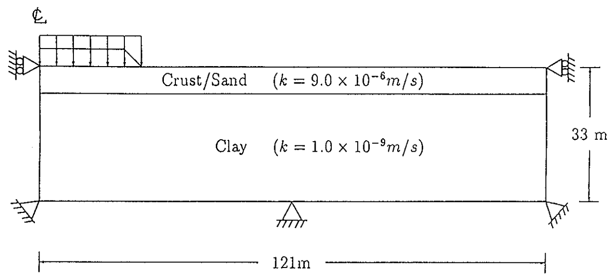


Fig. 11. Two-layered soil deposit

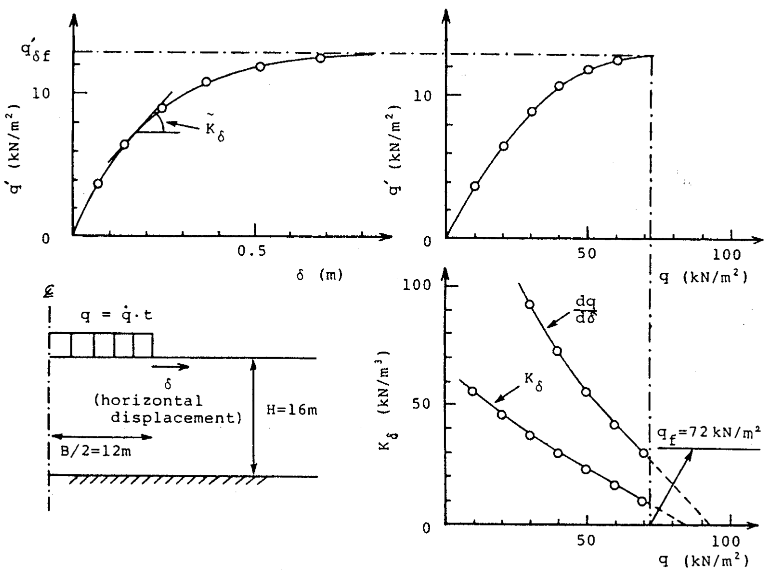


Fig. 10. Prediction procedure for bearing capacity of saturated soil deposit (Asaoka and Ohtsuka, 1986)

Table 2. Parameters adopted in the numerical analysis

	λ (kPa)	μ (kPa)	D	Γ	M	ν	l_c (m)
I	9800	2450	—	—	—	0.4	3.0
II	9800	2450	—	—	—	0.4	0.0
III	1900	950	0.02	0.7	1.4	0.333	—

I: Crust, II: Sand, III: Clay

ing capacities among them. Table 2 lists the set of parameters adopted in the analysis of this hypothetical ground.

In this example, a comparative analysis is performed for the following two cases: 1) a surface layer is considered as a sand which is modeled by an elastic material without considering couple stresses; 2) a surface layer is treated as a crust which complies with an elastic Cosserat material characterized by the characteristic length as the thickness of the layer.

In the case of the layered material, Mühlhaus (Mühlhaus, 1992) stated that the Cosserat theory can be used to model the influence of the bending stiffness for each layer on the material response by introducing the internal length of the layer thickness; the bending stiffness, B_s, is defined as $B_s=Gl_c^2$ seen in Mindlin (Mindlin, 1963). A layered material is modeled as a conventional orthotropic continuum only when the layer thickness is zero. However, the characteristic length can hardly be determined in general and the effect of couples stress becomes to be negligibly small in this example, if d_{50} is adopted as the characteristic length.

The subsoil is a soft clay foundation modeled by Sekiguchi-Ohta's constitutive equation. After obtaining FE solutions for both cases, we point out the bending effect on the lateral deformation below the toe of the embankment. This is done by comparing the responses of different materials where the settlements under the embankment are similar to each other.

Undrained Bearing Capacity of a Two-Layered Deposit

The thickness of a crust, $l_c=H$, is assumed to vary with the width of the embankment, B. Figure 12 compares each deformation profile based on different modeling of a surface layer. Figure 13 depicts a profile of the deterioration of the tangential stiffness factors, $\tilde{K}_\rho$, of the entire foundation by monitoring the settlement beneath the embankment. Figure 13 also describes the estimation of undrained bearing capacity based on different modeling of a surface layer. In this figure, it can be seen that the soil deposit with a stiff surface carries a higher capacity against surface load within a range of smaller value of H/B. When the thickness of a surface layer shrinks, estimated bearing capacity based on both modeling corresponds to each other. It reveals that the crust layer provides a higher bearing capacity than the surface sand layer for the case of lateral displacement observations. This is due to its flexural deformation profile.

Figure 14 summarizes an estimated curve for the bearing capacity based on the relative thickness of the layer, H/B, obtained by Asaoka-Ohtsuka's procedure. It can be seen that the estimated bearing capacity decreases when the width of the embankment, B, increases or the thickness of the crust, H, decreases. It should also be pointed out that the estimated bearing capacity, when

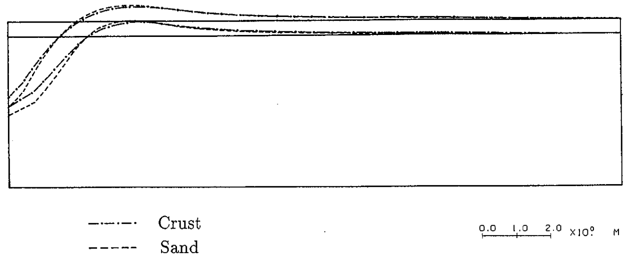


Fig. 12. Deformation profiles based on different modeling of surface layer

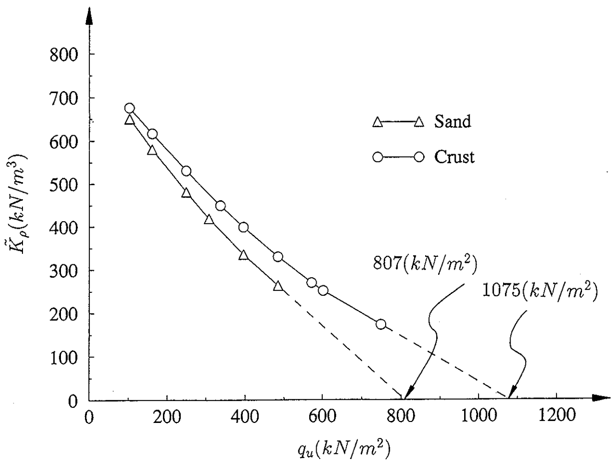


Fig. 13. Deterioration of tangential stiffness factors

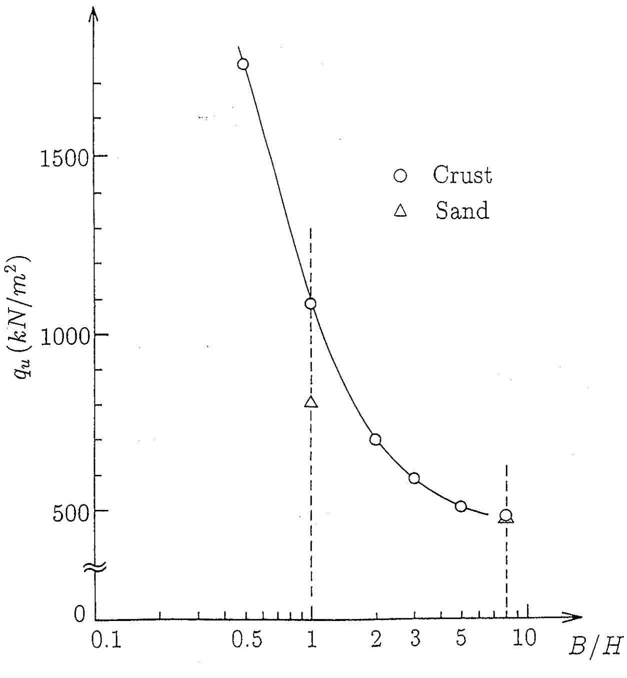


Fig. 14. Estimated bearing capacity

modeled as a Cosserat beam, is quite a bit larger than that based on an ordinary continuum for a smaller value of H/B. Estimated bearing capacities by both modeling are almost the same when H/B has a larger value as seen in Fig. 14.

CONCLUSIONS

The analysis of a two-layered ground, under the embankment-like loading presented above, can provide an estimate of the undrained bearing capacity of a saturated soil deposit below a surface crust. These calculations require two steps: a deformation analysis of the layered deposit by FEM incorporating a Cosserat beam and an estimation of the bearing capacity by observing the deterioration of the stiffness factor of the entire foundation. It has been shown that the bending effect of a stiff layer, i.e., a surface crust, has a significant influence on the undrained bearing capacity of a two-layered saturated deposit. The numerical results indicate that the estimated value of the undrained bearing capacity of such a deposit is related to the relative thickness of the layer, H/B. The use of such modeling may produce accurate estimates of the effect of a stiff layer within a surface on the bearing capacity of an overall soil stratum.

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