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# AN APPROACH FOR ESTIMATING DEFORMATION MODULI FROM SELF-BORING PRESSUREMETER TEST DATA

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## ABSTRACT

During a self-boring pressuremeter test (SBPMT) a cylindrical cavity is expanded from a finite radius. To determine undrained shear strength, $c_u$, of a saturated clay, SBPMT data is analyzed using the cylindrical cavity expansion theory, and curve fitting methods. At present, there is no completely consistent and reliable method to estimate the value of modulus of elasticity. In this paper, an alternative method has been presented to first estimate $c_u$ and limit pressure, $p_L$, using a logarithmic model and then determine initial tangent modulus, $E_i$, secant modulus at failure, $E_{sf}$, and secant modulus at half the value of $(\sigma_1-\sigma_3)_f$, $E_{50}$, using a hyperbolic-model. Values of $c_u$ determined from this method compare well with those determined from other methods. The predicted values of $E_i$ and $E_{50}$ compare well with those determined from the triaxial tests. The values of $E_{50}$ also compare well with the values of modulus determined from unload-reload cycle of SBPMT.

**Key words:** modulus of deformation, pressuremeter test, shear modulus, undrained shear strength (**IGC**: E2/C3)

## INTRODUCTION

The theory of expansion of cavities in a semi-infinite soil mass (Gibson and Anderson, 1961; Vesic, 1972; Baguelin et al., 1978; Ladanyi, 1972; Palmer, 1972) is being widely used for the solution of a number of geotechnical problems such as (i) the analysis of the pressuremeter tests, (ii) estimating excess pore pressure distribution around cone penetrometers (Baligh and Levadoux, 1980; Gupta and Davidson, 1986), and (iii) the bearing capacity of deep foundations (Vesic, 1972). A self-boring pressuremeter is introduced into a soil deposit with minimal disturbance. When it is expanded, it simulates a cylindrical cavity expansion starting from a finite radius. It is generally agreed that the existing methods for the analysis of the SBPMT data provide a reasonable estimate of the undrained shear strength, $c_u$, and the coefficient of lateral earth pressure at rest, $K_0$. However, these methods do not provide a reasonable estimate of the modulus of elasticity, $E$. Some researchers (Wroth, 1984) use the value of $E$ obtained from unload-reload loops, but others (Huang et al., 1991) have found that the determination of the loop slope or gradient as a secant is sensitive to "noise" in the data, especially when the hysteresis is large. Considering all the uncertainties involved in the determination of $E$, Chameau et al. 1987 has suggested not to use the SBPMT to determine this soil parameter. An alternative method to provide a reasonable estimate of initial tangent modulus, $E_i$, secant modulus at failure, $E_{sf}$, and secant modulus at half the value of $(\sigma_1-\sigma_3)_f$, $E_{50}$, is presented in this paper.

## CYLINDRICAL CAVITY EXPANSION

When the internal cavity pressure is increased, a cylindrical zone around the cavity passes into the state of equilibrium. The plastic zone expands until the pressure reaches an ultimate value. In SBPMT, the expansion test is stopped prior to reaching ultimate state to prevent bursting the membrane of the device. For the analysis, it is assumed that the cavity expansion occurs under radial plane strain and undrained conditions. A brief summary of the theory is given below (For details, *see* Gibson and Anderson, 1961; Baguelin et al., 1978).

*(a) Elastic Zone*: Consider the expansion of a cylindrical cavity of initial radius $R_0$ in an elastic, homogeneous and isotropic medium defined by an undrained modulus, $E$, Poisson's ratio, $\nu$, and undrained shear strength, $c_u$. For the elastic phase of the test, the constitutive equations are as follows:

$$\varepsilon_{rr}=\frac{1}{E}[(\sigma_{rr}-\nu(\sigma_{\theta\theta}+\sigma_{zz})],$$

$$\varepsilon_{\theta\theta}=\frac{1}{E}[(\sigma_{\theta\theta}-\nu(\sigma_{rr}+\sigma_{zz})],$$

$$\varepsilon_{zz}=\frac{1}{E}[(\sigma_{zz}-\nu(\sigma_{rr}+\sigma_{\theta\theta})]=0. \tag{1a}$$

Where at any instant of time, $\varepsilon_{rr}$, $\varepsilon_{\theta\theta}$, $\varepsilon_{zz}$, $\sigma_{rr}$, $\sigma_{\theta\theta}$, and $\sigma_{zz}$ are the radial displacement, principle radial, circumferen-

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tial, and axial strains and stresses, respectively. At any instant of time, $R(t)$, $r(t)$, $\xi_r$, and $\xi_{rc}$ are radius of the cavity, radial distance of a particle from the axis of the cavity, radial displacement of the particle at radial distance $r(t)$, and radial displacement at the face of cavity, respectively. Substituting for $\sigma_{zz}$ in expressions for $\sigma_{rr}$ and $\sigma_{\theta\theta}$, the compatibility equations for plane strain are:

$$E\,\varepsilon_{rr}=-E\frac{d\xi_r}{dr(t)}=(1-\nu^2)\sigma_{rr}-\nu(1+\nu)\sigma_{\theta\theta},$$

$$E\,\varepsilon_{\theta\theta}=-E\frac{\xi_r}{r(t)}=(1-\nu^2)\sigma_{\theta\theta}-\nu(1+\nu)\sigma_{rr} \tag{1b}$$

$$\sigma_{rr}=\sigma_m+\Delta\sigma_{rr},$$

$$\sigma_{\theta\theta}=\sigma_m+\Delta\sigma_{\theta\theta}. \tag{1c}$$

The equations of equilibrium are

$$\frac{d\sigma_{rr}}{dr(t)}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r(t)}=0. \tag{1d}$$

If stresses are eliminated using the above equations, the elastic equation satisfies the following.

$$r^2(t)\frac{d^2\xi_r}{dr^2(t)}+r(t)\frac{d\xi_r}{dr(t)}-\xi_r=0 \tag{1e}$$

General solutions of this equation are: $\xi_r=r^n(t)$, where $n$ must satisfy $(n+1)(n-1)=0$, and therefore has roots of $n=-1$ or $n=1$. Then $\xi_r=Ar(t)+Br^{-1}(t)$; for $r(t)=\alpha$, $\xi_r=0$, therefore $A=0$; at the cavity face, $r(t)=R(t)$, and $\xi_r=\xi_{rc}$, therefore $B=\xi_{rc}R(t)$. Then,

$$\xi_r=\frac{\xi_{rc}R(t)}{r(t)},$$

$$\varepsilon_{rr}=\frac{d\xi_r}{dr(t)}=-\frac{\xi_{rc}R(t)}{r^2(t)},$$

$$\varepsilon_{\theta\theta}=\frac{\xi_r}{r(t)}=\frac{\xi_{rc}R(t)}{r^2(t)}. \tag{1f}$$

Similarly, it can be shown (Beguelin et al., 1978):

$$\sigma_{rr}=\sigma_m+[p(t)-\sigma_m]\frac{R^2(t)}{r^2(t)},$$

$$\sigma_{\theta\theta}=\sigma_m-[p(t)-\sigma_m]\frac{R^2(t)}{r^2(t)},$$

$$\xi_{rc}=\frac{(1+\nu)R(t)}{E}[p(t)-\sigma_m]. \tag{1g}$$

Where

(a) $\sigma_m=\sigma_h=K_0\sigma_v'+\gamma_w h_w=K_0\sigma_v'+u_w$; (1h)

(b) prior to the expansion of the cavity, $\sigma_h=\sigma_m$=mean horizontal stress, $K_0$=coefficient of lateral earth pressure at rest, $\sigma_v'$=effective vertical stress, $\gamma_w$=unit weight of water, $h_w$=depth of water below water table, and $u_w$=pore water pressure due to ground water table, $\nu$=Poisson's ratio (0.5 for saturated clay); and (c) at any instant of time $(t)$, $p(t)$=internal cavity pressure.

*(b) Plastic Zone*: In the plastic zone, at any instant of time, $\sigma_{rr}-\sigma_{\theta\theta}$ is equal to $2c_u$. Then Eq. (1d) reduces to $d\sigma_{rr}/dr(t)+2c_u/r(t)=0$. Using the condition that $\sigma_{rr}=p(t)$ when $r(t)=R(t)$, the solution of the differential equation is

$$\sigma_{rr}=p(t)-2c_u\ln\left(\frac{r(t)}{R(t)}\right) \tag{2}$$

$$\sigma_{\theta\theta}=\sigma_{rr}-2c_u, \tag{3a}$$

$$\sigma_{mp}=\frac{\sigma_{rr}+\sigma_{\theta\theta}}{2} \tag{3b}$$

where at time $t$, $\sigma_{mp}$ is the mean horizontal stress in plastic zone. From (2), at $r(t)=r_p$=radius of plastic zone at time $t$, $\sigma_{rr}=\sigma_{rp}=\sigma_m+c_u=p(t)-2c_u\ln[r_p/R(t)]$, then

$$\frac{r_p}{R(t)}=e^{(p(t)-\sigma_m-c_u)/2c_u}, \tag{4a}$$

$$\xi_{rc}=R(t)-\sqrt{R^2(t)-r_p^2+(r_p-\xi_{rp})^2}. \tag{4b}$$

The equation for no volume change in the plastic zone and the radial displacement at the interface of plastic and elastic zones, $\xi_{rp}$, are given by

$$R^2(t)-R_0^2=r_p^2-(r_p-\xi_{rp})^2, \tag{5a}$$

where

$$\xi_{rp}=\frac{(1+\nu)r_pc_u}{E}=\frac{r_p}{2I_r}. \tag{5b}$$

Where, rigidity index

$$(I_r)=\frac{E}{2(1+\nu)c_u}. \tag{5c}$$

Substituting values of $r_p/R(t)$ and $\xi_{rp}$ from (4a) and (5b), respectively, in (5a) and re-arranging terms, we get

$$1-\frac{R_0^2}{R^2(t)}=\frac{r_p^2}{R^2(t)}-\left(\frac{r_p}{R(t)}-\frac{\xi_{rp}}{R(t)}\right)^2$$

$$=\frac{r_p^2}{R^2(t)}\left[1-\left(1-\frac{1}{2I_r}\right)^2\right]$$

$$=\frac{r_p^2}{R^2(t)}\left(\frac{4I_r-1}{4I_r^2}\right)$$

or

$$\frac{r_p^2}{R^2(t)}=\left(1-\frac{R_0^2}{R^2(t)}\right)\left(\frac{4I_r^2}{4I_r-1}\right)=e^{(p(t)-\sigma_m-c_u)/c_u}$$

or

$$p(t)=\sigma_m+c_u+c_u\ln\left[\left(1-\frac{R_0^2}{R^2(t)}\right)\left(\frac{4I_r^2}{4I_r-1}\right)\right].$$

For $\sigma_r>\sigma_{rp}$. (6)

Where $\sigma_{rp}$=the radial stress at a radial distance equal to $r_p$. The expression $4I_r^2/(4I_r-1)$ can be expressed as $1/(1/I_r-1/4I_r^2)$. The value of the term $1/4I_r^2$ is a very small and can be neglected, then (6) reduces to the following well-known Gibson and Anderson (1961) formula.

$$p(t)=\sigma_m+c_u+c_u\ln\left[\left(1-\frac{R_0^2}{R^2(t)}\right)\left(\frac{E}{2(1+\nu)c_u}\right)\right]$$

For $\sigma_r>\sigma_{rp}$. (7)

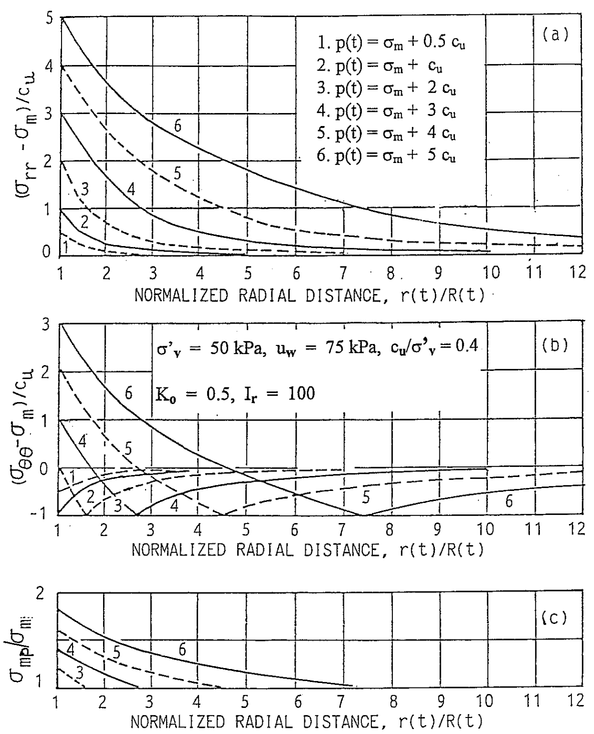


**Fig. 1. During expansion of cylindrical cavity in saturated clay, radial distributions of (a) normalized radial stress, (b) normalized circumferential stress, and (c) mean horizontal stress**

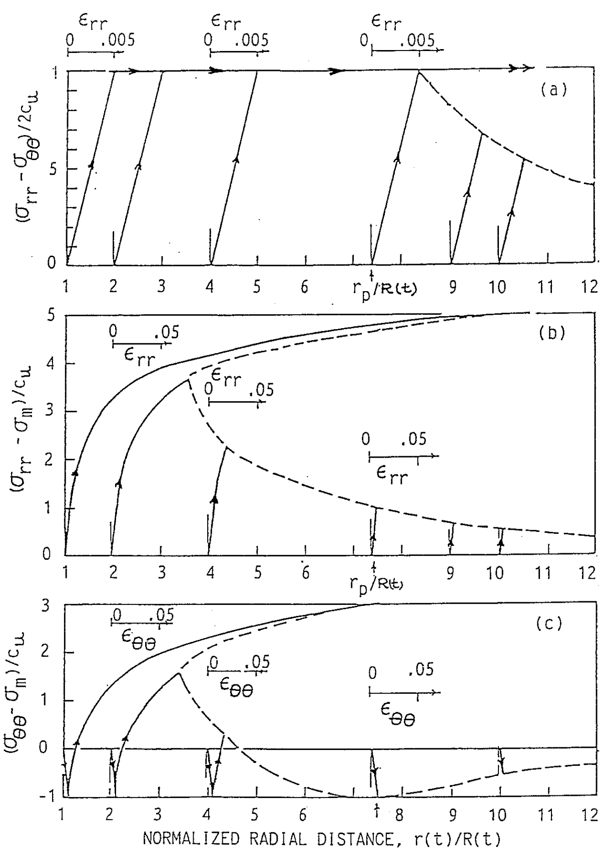


**Fig. 2. During expansion of cylindrical cavity in saturated clay, relationships between (a) normalized maximum shear stress and radial strain, (b) normalized radial stress and radial strain, and (c) normalized circumferential stress and circumferential strain**

Gibson and Anderson formula has been used extensively to determine the soil properties of clayey soils using the pressuremeter or self-boring pressuremeter data. The value of $c_u$ is determined by selecting two points on the curve, say, using internal cavity pressure values at $\Delta V/V=0.1$ and 0.3.

Figures 1 and 2 show the stress-strain relationship at various stages of expansion (i.e. at $p(t)=\sigma_m+0.5c_u$ to $\sigma_m+0.5c_u$), when values of $\sigma_v'$, $u_w$, $c_u/\sigma_v'$, $K_0$ and $I_r$ of the saturated clay are 50 kPa, 75 kPa, 0.4, 0.5 and 100, respectively. At various normalized radial distances, $r(t)/R(t)$, values of $(\sigma_{rr}-\sigma_m)/c_u$, $(\sigma_{\theta\theta}-\sigma_m)/c_u$, $\sigma_{mp}/\sigma_m$, $(\sigma_{rr}-\sigma_{\theta\theta})/c_u$, and $(\sigma_{rr}-\sigma_m)/c_u$ are calculated using (1g) for the elastic zone, and using (2), (3a), and (3b) for the plastic zone, respectively. Values of $\varepsilon_{rr}$ and $\varepsilon_{\theta\theta}$ are calculated using (1f) for the elastic zone around the cavity, and using (8b) for the plastic zone around the cavity. As shown in Fig. 1(b), for each stage of expansion, the value of $(\sigma_{\theta\theta}-\sigma_m)/c_u$ begins to decrease from the face of cavity and decreases to $-1$ at the interface of plastic and elastic zones; thereafter it begins to increase in the elastic zone. In Fig. 2, the normalized stress-strain relationships that may develop at various radial distances have been shown. For this purpose, horizontal axis has been used both for $\varepsilon_{rr}$ and $\varepsilon_{\theta\theta}$ and for $r(t)/R(t)$. At distances of $r(t)<r_p$, the value of $(\sigma_{rr}-\sigma_{\theta\theta})/c_u$ increases linearly until it reaches a peak value of 1, indicating that a state of equilibrium has reached, thereafter, $\varepsilon_{rr}$ increases at the peak value. Prior to reaching a state of equilibrium, the increase in the value of $\sigma_{rr}$ is equal to the decrease in the value of $\sigma_{\theta\theta}$, however, both the $\sigma_{rr}$ and $\sigma_{\theta\theta}$ increase after the state of equilibrium has been reached. Therefore, prior to reaching a state of equilibrium, values of $(\sigma_{rr}-\sigma_m)/c_u$ and $((\sigma_{\theta\theta}-\sigma_m)/c_u$ vary linearly to 1 and $-1$, respectively, thereafter, $\varepsilon_{rr}$ and $\varepsilon_{\theta\theta}$ increase rapidly with increase in the values of $(\sigma_{rr}-\sigma_m)/c_u$ and $(\sigma_{rr}-\sigma_m)/c_u$, respectively.

In the cavity expansion theory for undrained conditions, it is assumed that the finite plastic zone is incompressible and semi-infinite zone around the plastic zone is elastic. Another case can be considered where the infinitely long cylindrical cavity expansion occurs from finite radius in an infinite, incompressible and homogeneous medium; the cavity has a radius $R$ at time zero, and radius $R(t)$ at time $t$. At any time $t$, a soil element located at radial distance $r(t)$, is related to its initial position $r$ at time $t=0$ by a simple expression (8a), and the radial and the circumferential strains with respect to the displaced position of the particle are defined by Eq. (8b), (Gupta, 1991).

$$r(t)=[r^2+R^2(t)-R_0^2]^{1/2}, \tag{8a}$$

$$\xi_r=r(t)-r=r(t)-[r^2(t)+R_0^2-R^2(t)]^{1/2},$$

$$\varepsilon_{rr}=\frac{d\xi_r}{dr(t)}=1-\frac{r(t)}{[r^2(t)+R_0^2-R^2(t)]^{1/2}}=-\frac{\xi_r}{r},$$

$$\varepsilon_{\theta\theta}=\frac{\xi_r}{r(t)}. \tag{8b}$$

For the incompressible finite plastic zone surrounded by semi-infinite elastic zone, $R^2(t)-R^2$ is equal to $r_p^2-(r_p-\xi_{rp})^2$. Equations (8a) and (8b) will also work for incompressible finite plastic zone, if $R^2(t)-R^2$ is replaced by $r_p^2-(r_p-\xi_{rp})^2$. It may be noticed that the circumferential strain in the infinite incompressible medium is equal to the ratio $\xi_r/r(t)$, while the radial strain is equal to the ratio $-\xi_r/r$. On the contrary, the absolute values of both the radial and circumferential strains with respect to the displaced position of the particle in the elastic zone are the same, i.e. equal to $\xi_r/r(t)$, *see* (1f). Displacements, strains, and stresses in the semi-infinite elastic zone around the plastic zone can be calculated assuming that (i) the interface of the plastic-elastic zones is the face of the cavity, (ii) the radius of the cavity is $r_p$, and (iii) displacement at the face of the cavity is $\xi_{rp}$. Thus the analysis is based on the continuity of the displacement at the interface of the elastic-plastic zones. Strains in the plastic zone for distances less than $r_p$ are calculated from (8b). Strains in the elastic zone for distances equal to or greater than $r_p$ are calculated after replacing $\xi_{rc}$ by $\xi_{rp}$ in (1f). Thus to maintain continuity, the interface of elastic-plastic zones is considered to be a part of elastic zone.

## ANALYSIS OF PRESSUREMETER TEST DATA

Several methods are in use for the analysis and the interpretation of the pressuremeter test data. For the Gibson and Anderson method, either (1) an iterative procedure (Ladd et al., 1980; Benoit and Clough, 1986), or (2) a procedure based on matching the relationship obtained from measurements both during loading and unloading with that computed from the assumed values of $G$, $c_u$, $\sigma_h$ (Jefferies, 1988), is used. More advanced and superior methods which have provided useful results are those proposed by Baguelin et al. 1978, Ladanyi, 1972, Palmer, 1972 and Prevost and Hoeg, 1975. Several curve fitting techniques have also been developed such as those proposed by Wroth and Hughes, 1973, Denby, 1978, Ladd et al., 1980, Arnold, 1981, and Huang et al., 1991.

## ALTERNATIVE METHOD FOR ANALYSIS OF SBPMT TESTS

Expressing (6) in the following form can develop an alternative method for the analysis of the SBPMT data

$$
\begin{aligned}
p(t)&=\sigma_m+c_u+c_u\ln\left(\frac{4I_r^2}{4I_r-1}\right)+c_u\ln\left(1-\frac{R_0^2}{R^2(t)}\right)\\
&=a_1+a_2\ln\left(1-\frac{R_0^2}{R^2(t)}\right)\\
&=a_1+a_2\ln\left(\frac{\Delta V}{V}\right)\\
&=a_1+a_2\ln\varepsilon_v. \quad \text{For} \quad \sigma_r>\sigma_{rp} \qquad (9)
\end{aligned}
$$

$$a_1=p_L=\sigma_m+c_u+c_u\ln\left(\frac{4I_r^2}{4I_r-1}\right), \qquad (10a)$$

$$a_2=c_u. \qquad (10b)$$

Where $V$=volume of cavity at time $t$, $\Delta V=V-V_0$=change in volume of the cavity$=\pi[R^2(t)-R_0^2]$, $V_0$=initial volume of cavity$=\pi R_0^2$, $\varepsilon_v=\Delta V/V=\pi[R^2(t)-R_0^2]/\pi R^2(t)=1-R_0^2/R^2(t)$, and $p_L$=limit pressure when the value of $\Delta V/V$ is equal to 1. It may be noted that $\Delta V/V$ becomes 1, when $\Delta V$ is equal to $V_0$; this condition is almost impractical to achieve in a test. Mathematically, the importance of $p_L$ is similar to a y-intercept of a straight line, signifying that when $x=0$, the value of the y-coordinate is equal to y-intercept. Similarly, $p_L$ is a y-intercept of $p(t)$ versus $\ln(\Delta V/V)$ linear relationship, when the value of $x=\ln(\Delta V/V)$ is equal to zero. Therefore, using this property, the value of $p_L$ is always theoretically calculated from $p(t)$ versus $\ln(\Delta V/V)$ relationship. As shown by (10a), the value of $p_L$ is a property of clay and has a specific value which depends upon $\sigma_m$, $c_u$ and $I_r$.

Equation (9) can be transformed as a straight line, by substituting as $x=\ln(\varepsilon_v)$ and $y=p(t)$. Using the method of least squares for a straight line, values of $a_1$ and $a_2$ are given by (11a) and (11b), respectively (Harr, 1977).

$$a_1=\frac{\Sigma x^2\Sigma y-\Sigma x\Sigma xy}{n\Sigma x^2-(\Sigma x)^2},\quad a_2=\frac{n\Sigma xy-\Sigma x\Sigma y}{n\Sigma x^2-(\Sigma x)^2} \qquad (11a)$$

or

$$a_1=\frac{\Sigma(\ln\varepsilon_v)^2\Sigma p(t)-\Sigma\ln\varepsilon_v\Sigma p(t)\ln\varepsilon_v}{n\Sigma(\ln\varepsilon_v)^2-(\Sigma\ln\varepsilon_v)^2},\quad a_2=\frac{n\Sigma\ln\varepsilon_v p(t)-\Sigma\ln\varepsilon_v\Sigma p(t)}{n\Sigma(\ln\varepsilon_v)^2-(\Sigma\ln\varepsilon_v)^2}. \qquad (11b)$$

Where $n$=number of pressure increments at which measurements of pressure and volume are made during the test, and $p(t)$ and $\varepsilon_v$ are the measured readings at each pressure increment.

The value of $a_2$ is equal to the value of the undrained shear strength, $c_u$ (Wroth, 1984). When the value of $c_u$ is substituted in (10a), the sum of terms containing $I_r$ and $\sigma_m$ is obtained. Values of $I_r$, $\sigma_m=\sigma_h$, and $K_0$ can be determined from any of the following three procedures.

*(a) Procedure 1*: If the clay behaves as a perfectly linear elastic material in an unloading-reloading cycle performed in a SBPMT test, the slope of the straight line relationship between $p(t)$ and circumferential strain will be equal to twice the value of shear modulus, $G$, *see* Wroth (1984). Using Eq. (1g), this is also illustrated below:

$$\varepsilon_{\theta\theta}=\frac{\xi_{rc}}{R(t)}=\frac{(1+\nu)(p(t)-\sigma_m)}{E},$$

$$Slope=\frac{p(t)-\sigma_m}{\varepsilon_{\theta\theta}}=\frac{E}{1+\nu}=2G.$$

$I_r$ is equal to $G/c_u$ and $G$ is equal to $E/2(1+\nu)$. The value of $I_r$ is substituted in (10a) to determine values of $\sigma_h$ and $K_0$.

*(b) Procedure 2*: At the start of the expansion test the

membrane fits tightly over the instrument and has the same diameter as the cutting shoe. In theory no expansion of the membrane should be detected until the applied pressure is equal to the in situ total lateral stress in the ground in contact with the pressuremeter. In reality there will be some small compliance of the instrument itself, until at a point, known as the lift-off pressure on the expansion curve, the soil starts to deform under increasing lateral stress. The lift-off pressure is in situ total lateral stress, for details *see* Ghionna, Jamiolkowski and Lancellotta, 1982 and Lacasse and Lunne, 1982. When the information about the ground water table and the unit weights of the subsurface layers is available, the value of $K_0$ can be readily determined from the known value of $\sigma_h$. Values of $I_r$, and $E$ are determined by substituting the value of $\sigma_m=\sigma_h$ in (10a).

*(c) Procedure 3*: The ratio $c_u/\sigma_v'$ for a normally consolidated (NC) clay, overconsolidation ratio (OCR) of overconsolidated (OC) clay, and $K_0$ can be estimated from the following equations (Skempton, 1957; Ladd et al., 1977; Schmertmann, 1978).

$$\left(\frac{c_u}{\sigma_v'}\right)_{\mathrm{NC}}=0.11+0.0037(\mathrm{P.I.}),$$
$$(\mathrm{OCR})^{0.8}=\frac{(c_u/\sigma_v')_{\mathrm{OC}}}{(c_u/\sigma_v')_{\mathrm{NC}}},$$
$$K_0=0.45(\mathrm{OCR})^{0.42}. \qquad (12)$$

Where P.I. = Plasticity index of clay. Using the value of $c_u$ obtained from (10b), $K_0$ is determined from (12) and then values $I_r$ and $E$ are determined from (10a).

*(d) Initial Tangent Modulus*: Results of the triaxial tests have shown that the stress-strain relationship of both clay and sand is non-linear. It is quite difficult to determine the initial tangent modulus, $E_i$, accurately from such tests, since the slope of the stress-strain curve changes rapidly even at very small strains. The following hyperbolic model (Duncan and Chang, 1970) can approximate the nonlinear stress-strain curves.

$$(\sigma_1-\sigma_3)=\frac{\varepsilon_{rr}}{\dfrac{1}{E_i}+\dfrac{R_f\varepsilon_{rr}}{(\sigma_1-\sigma_3)_f}}$$
$$\text{For} \quad (\sigma_1-\sigma_3)\leq(\sigma_1-\sigma_3)_f. \qquad (13)$$

Where $(\sigma_1-\sigma_3)$ = stress difference at any instant of loading; $(\sigma_1-\sigma_3)_f=2c_u$ = compressive strength at failure, or stress difference at failure; and $R_f$ = failure ratio = $(\sigma_1-\sigma_3)_f$/Asymptotic value of $(\sigma_1-\sigma_3)$.

As shown in Fig. 3, initial tangent modulus, $E_i$, equals the slope of the $\sigma_1-\sigma_3$ versus normal strain ($\varepsilon$) plot at the start of the test. Secant modulus at failure, $E_{sf}$, equals the slope of the straight line between the origin and the point of failure (i.e. the first instant when peak shear strength is reached) on the plot of $\sigma_1-\sigma_3$ versus $\varepsilon$. Secant modulus, $E_{50}$, equals the slope of the straight line between the origin and the point at half of the value of $(\sigma_1-\sigma_3)_f$.

The theory for expansion of a cylindrical cavity (Gibson and Anderson, 1961; Vesic, 1972) is based on the assumption that (i) Prior to the formation of the plastic zone, the stress-strain relationship during expansion of the cavity is linear in the elastic zone, Figs. 3 and 4, and (ii) the saturated clay in the plastic zone behaves as an incompressible plastic solid, defined by the undrained shear strength, $c_u$. As shown in Fig. 3, these assumptions imply that the clay during the expansion of the cavity follows path ACD for $R_f=0.7$, or AC′D′ for $R_f=0.8$, or AC″D″ for $R_f=0.9$, in lieu of path ABCC′C″E. Therefore, the value of $E$ used in the analysis of this theory is the value of the secant modulus at failure, $E_{sf}$, equal to the slope of secant AC for $R_f=0.7$, secant AC′ for $R_f=0.8$, and secant AC″ for $R_f=0.9$. The stress-strain relationship defined by Eq. (6) is applicable, when the value of $p(t)$ exceeds the value of $\sigma_{rp}$, therefore this method is based on the response of expanding plastic zone. Since the undrained shear strength is dependent only upon the initial conditions existing before shear and are independent of the way in which shear is applied (Lambe and Whitman, 1969), assuming path as secant AC in lieu of hyperbolic curve ABC should not significantly affect the results of the analysis expressed by Eq. (6).

The radial strain, $\varepsilon_{rp}$, at the interface of the elastic-plastic interface is equal to $\xi_{rp}/r_p$, where $\xi_{rp}$ is defined by Eq. (5b). $E_{sf}$ is equal to $(\sigma_1-\sigma_3)_f/\varepsilon_{rp}$. Substituting $c_u=0.5(\sigma_1-\sigma_3)_f$, the following relationship for $\varepsilon_{rp}$ is obtained

$$\varepsilon_{rp}=\frac{\xi_{rp}}{r_p}=\frac{(1+\nu)c_u}{E}=\frac{(1+\nu)(\sigma_1-\sigma_3)_f}{2E_{sf}},$$

or

$$\frac{\varepsilon_{rp}}{(\sigma_1-\sigma_3)_f}=\frac{1+\nu}{2E_{sf}}. \qquad (14)$$

At radial distance $r_p$, $\varepsilon_{rr}$ is equal to $\varepsilon_{rp}$ and $\sigma_1-\sigma_3$ is equal to $(\sigma_1-\sigma_3)_f$, then substituting these values in Eq. (13), we get

$$(\sigma_1-\sigma_3)_f=\frac{\varepsilon_{rp}}{\dfrac{1}{E_i}+\dfrac{R_f\varepsilon_{rp}}{(\sigma_1-\sigma_3)_f}},$$

or

$$\frac{\varepsilon_{rp}}{(\sigma_1-\sigma_3)_f}=\frac{1}{E_i}+\frac{R_f\varepsilon_{rp}}{(\sigma_1-\sigma_3)_f},$$

or

$$\frac{1+\nu}{2E_{sf}}=\frac{1}{E_i}+\frac{R_f(1+\nu)}{2E_{sf}},$$

or

$$E_i=\frac{2E_{sf}}{(1+\nu)(1-R_f)}=\frac{4E_{sf}}{3(1-R_f)}. \qquad (15)$$

(Note: For saturated clay, $\nu$ is equal to 0.5.)

$E_{50}$ is equal to half the value of $(\sigma_1-\sigma_3)_f$ divided by radial strain, $\varepsilon_{rr(0.5f)}$ at half the value of $(\sigma_1-\sigma_3)_f$. Substituting $(\sigma_1-\sigma_3)=0.5(\sigma_1-\sigma_3)_f$ and $\varepsilon_{rr}=\varepsilon_{rr(0.5f)}$ in Eq.

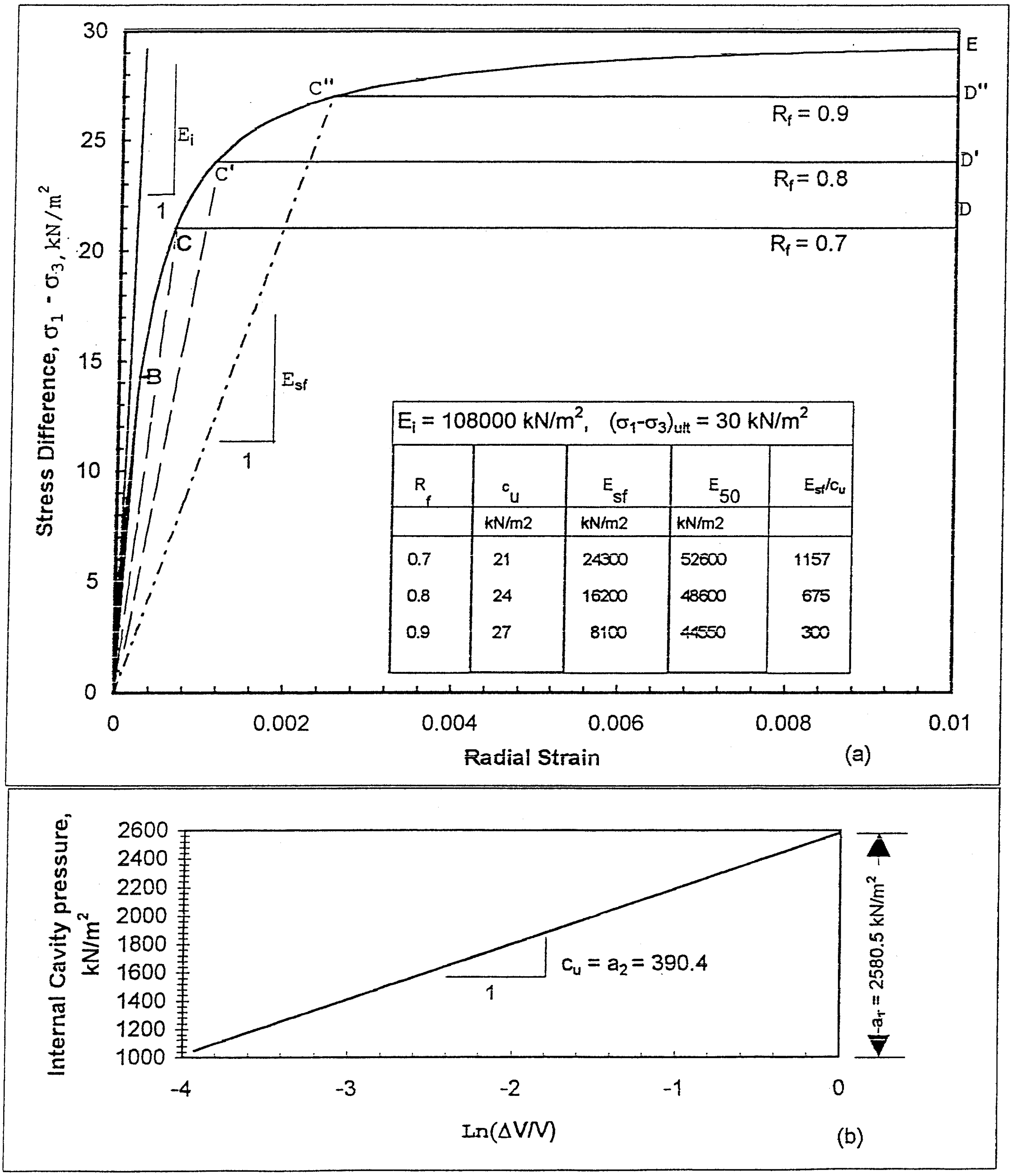


**Fig. 3. (a) Typical hyperbolic stress-strain relationship during expansion of a cylindrical cavity, (b) relationship between internal cavity pressure and volumetric strain for Bartoon clay using data of Wroth (1984)**

(13) following relationship between $E_{50}$ and $E_{sf}$ is obtained.

$$0.5(\sigma_1-\sigma_3)_f=\frac{\varepsilon_{rr(0.5f)}}{\dfrac{1}{E_i}+\dfrac{R_f\,\varepsilon_{rr(0.5f)}}{(\sigma_1-\sigma_3)_f}},$$

or

$$\frac{1}{E_{50}}=\frac{1}{E_i}+\frac{0.5R_f}{E_{50}},$$

or

$$E_{50}=E_i(1-0.5R_f)=\frac{4E_{sf}(1-0.5R_f)}{3(1-R_f)}. \qquad (16)$$

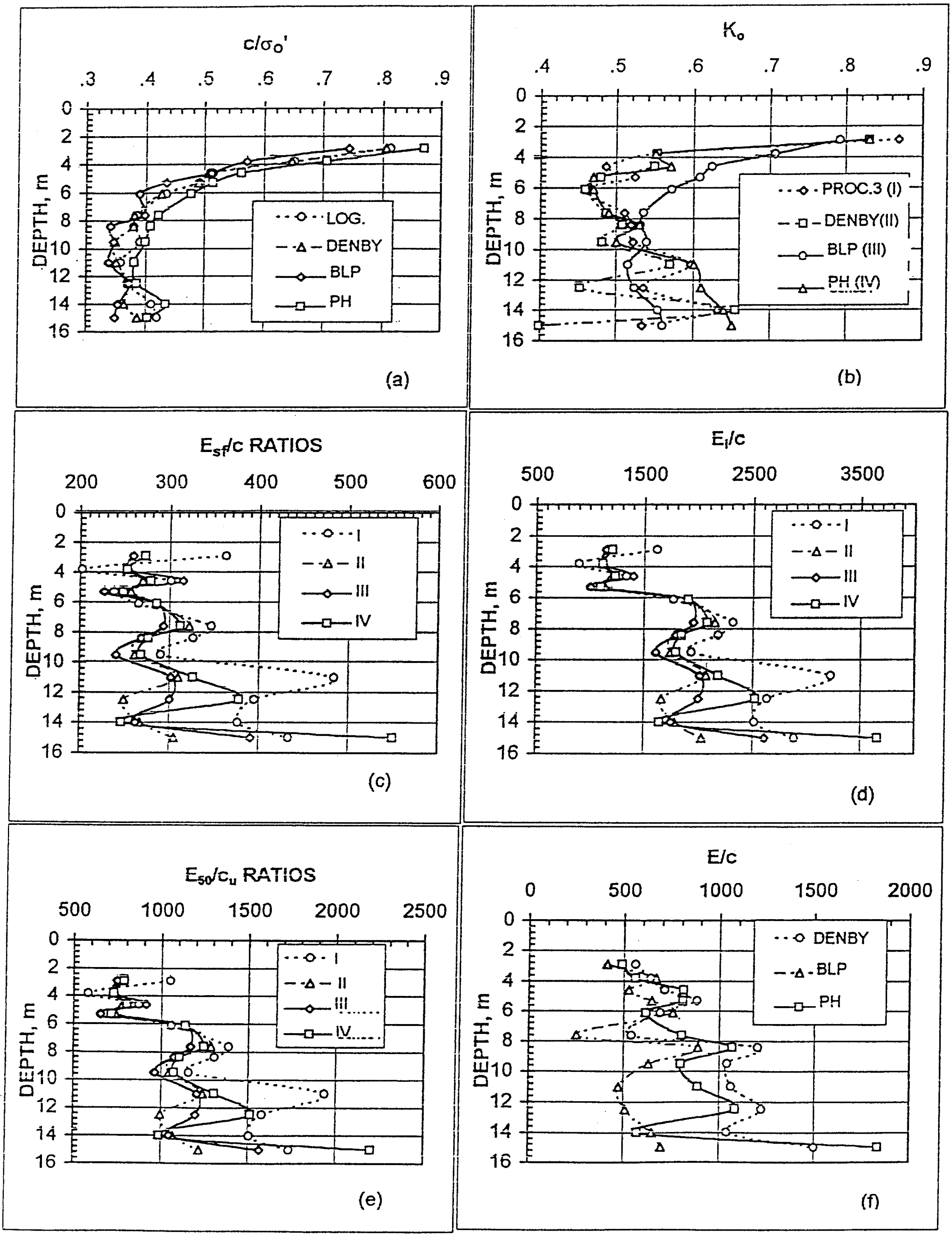


**Fig. 4. For San Francisco Bay Mud, using alternative method, and methods of Denby, Baguelin et al., (1978), Ladanyi (1972), Palmer (1972), and Prevost and Hoeg (1975), values of (a) $c_u/\sigma_v'$, (b) $K_0$, (c) $E_{sf}/c_u$, (d) $E_i/c_u$, (e) $E_{50}/c_u$, and (f) $E/c_u$**

The failure factor, $R_f$, which always has the value less than unity accommodates the fact that at the failure strain, the soil no longer follows a hyperbolic response and at strains greater than the failure strain, the soil deforms at constant value of $(\sigma_1-\sigma_3)_f$, i.e. at peak deviator stress. Values of $R_f$ have been found to vary from approximately 0.9 for soft plastic clays to 0.6 for overconsolidated clays (Denby, 1978). Using Eqs. (15) and (16), relationship between $E_i$ and $E_{sf}$ or $E_{50}$ for various values of $R_f$ have been shown in Table 1. As previously stated the value of $E$ determined from the alternative method is equal to the value of $E_{sf}$. Therefore, after determining the value of $E_{sf}$, values of $E_i$ and $E_{50}$ can be calculated by using Eqs. (15) and (16) or from Table 1.

## EXAMPLES

The use and application of the alternative method as described above shall be explained by the following examples.

*(a) Bartoon Clay*: Wroth (1984) analyzed the SBPMT data obtained at a depth of 43.4 m (142.4 ft) of the Bartoon clay by plotting the internal cavity pressure, $p(t)$ on a logarithmic scale and volumetric strain on a normal scale, and found that the data can be approximated by a linear relationship. The slope of the linear relationship was found to be equal to 386 kN/m$^2$, and compared well with the value of $c_u$ of the Bartoon clay. Wroth (1984) also determined the value of $E$ from the unload-reload cycle and found it to be equal to 141 MN/m$^2$. The same data is reproduced in Table 2, and have been analyzed using the alternative method, described in the previous paragraphs. Values of $a_1$ and $a_2$ by using Eq. (11b) are found to be equal to 2580.5 kN/m$^2$ and 390.4 kN/m$^2$, respectively, *see* Fig. 3(b). The value of the coefficient of correlation is 0.99. The value of $a_2$ is equal to the value of $c_u$ and compares well with the value determined by Wroth (1984). When ln ($\Delta V/V$) is equal to 0, the value of y-intercept is equal to $a_1$, which is equal to the limit pressure.

Values of $a_1$ and $a_2$ are then used to determine the value of $I_r$ or $E$ and $\sigma_h$ using Procedures 1, 2, and 3, and the results of the analysis are shown in Table 3. According to Procedure 1, the value of $I_r$ based on the values of $E_r$ determined from the unload-reload cycle and shown in Table 3, is substituted in (10a) and a value of $\sigma_h$ equal to 320 kN/m$^2$ is obtained. This value of $\sigma_h$ is about one-half of the measured $\sigma_h$ value of 646 kN/m$^2$ and is even less than the hydrostatic water pressure of 383 kN/m$^2$, created by the ground water table, *see* Table 3. Thus, Procedure 1 using the value of $E$ (i.e. $E_r$) from unload-reload cycle provides an inaccurate value of $\sigma_h$. This result indicates that the $p(t)-\Delta V/V$ relationship expressed by Eq. (6) does not depend up on the value of $E_{50}$, instead depends on $E_{sf}$.

As shown in Table 3, the measured value of $\sigma_h=\sigma_m$ is 646 kN/m$^2$ and the value of $c_u=a_2$ from (11) is 390 kN/m$^2$. These two values are used for Procedure 2, and the value of $I_r$ is determined from Eq. (10a). The value of $E_{sf}=E$ is then determined from Eq. (5c). For the overconsolidated Bartoon clay, the value of $R_f$ is approximately equal to 0.6. Then, $E_i$ and $E_{50}$ are determined from Eqs. (15) and (16), respectively. The value of $E_{50}$ is found equal to 143 MN/m$^2$ and compares well with the value of $E_r$ equal to 141 MPa which was obtained from the unload-reload cycle, *see* Table 3. This indicates that a linear stress-strain relationship during unload-reload cycle follows path of secant AB, *see* Fig. 3(a). This result also shows that the assumption that the $p(t)-\Delta V/V$ relationship depends on secant modulus, $E_{sf}$, at failure and that a hyperbolic stress-strain relationship for the elastic zone shall provide a reasonable estimate of $E_{50}$, appears to be correct.

For Procedure 3, the value of $\sigma_h=K_0\sigma_v'$ estimated from Eq. (12) is 670 kN/m$^2$ and compares well with the measured value of 646 kN/m$^2$, *see* Table 3. By substitut-

**Table 1.** $E_i$, $E_{sf}$, $E_{50}$ **for various values of failure ratio,** $R_f$

| Failure ratio $R_f$ | $E_i/E_{sf}$ | $E_{50}/E_{sf}$ |
|---|---|---|
| 0.6 | 3.33 | 2.33 |
| 0.7 | 4.44 | 2.89 |
| 0.8 | 6.67 | 4.00 |
| 0.9 | 13.33 | 7.33 |

**Table 2. SBPMT data for Bartoon clay (from Wroth, 1984)**

| $\varepsilon_{rr}$, %= | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\Delta V/V$= | .020 | .039 | .057 | .075 | .093 | .110 | .127 | .143 | .158 | .174 | .188 |
| $p(t)$, kN/m$^2$= | 1095 | 1265 | 1440 | 1555 | 1660 | 1725 | 1770 | 1840 | 1870 | 1890 | 1940 |

**Table 3. Analyzed results for Bartoon clay**

| SBPMT data (Wroth, 1984) | | | | Alternative method | | Procedure 1 | Procedure 2 | | |
|---|---|---|---|---|---|---|---|---|---|
| $c_u$ | $\sigma_h$ | $u_w$ | $E_r$ | $c_u$ | $a_1$ | $\sigma_h$ | $E_{sf}$ | $E_i$ | $E_{50}$ |
| * | * | * | ** | * | * | * | ** | ** | ** |
| 386 | 646 | 383 | 141 | 390 | 2580 | 320 | 61.2 | 272 | 143 |

| Procedure 3 | | | | | | | |
|---|---|---|---|---|---|---|---|
| PI | $c_u/\sigma_v'$ for NC | $c_u/\sigma_v'$ for OC | $K_0$ | $\sigma_h$ | $E_{sf}$ | $E_i$ | $E_{50}$ |
| | | | | * | ** | ** | ** |
| 45 | 0.276 | 1.465 | 1.08 | 670 | 57 | 190 | 143 |

* in kN/m$^2$, ** in MN/m$^2$

ing the estimated of value of $\sigma_m=\sigma_h$ and $c_u$, value of $I_r$ is obtained from (10a), and value of $E=E_{sf}$ is obtained from Eq. (5c). Then the value of $E_{50}$ is calculated from Eq. (16), which is found to match with the measured value of $E$ from unload-reload cycle, *see* Table 3.

*(b) San Francisco Bay Mud*: Using Procedures 2 and 3, the SBPMT tests conducted in San Francisco Bay Mud (Denby, 1978) are analyzed and the results are shown in Fig. 4. In this figure, values determined by Denby (1978) method, Baguelin, Ladanyi and Palmer (BLP) method, and Prevost and Hoeg (PH) method are also shown. In Denby method, $(\sigma_{rr}-\sigma_{\theta\theta})/2$ versus $\varepsilon_{rr}$ curve obtained from pressuremeter data is transformed as a modified hyperbolic curve to determine the value of $G$ and $c_u$. BLP method developed by Baguelin (1972), Ladanyi (1972), and Palmer (1972) lead to the same expression, i.e. $(\sigma_{rr}-\sigma_{\theta\theta})/2=\varepsilon_{rr}\varphi'(\varepsilon_{rr})$, where $\varphi'(\varepsilon_{rr})$ is the slope of the pressure versus $\varepsilon_{rr}$ curve at any radial strain, $\varepsilon_{rr}$. Using sub-tangent method, a curve with above expression is constructed from the pressuremeter curve. Initial slope of this constructed curve is equal to two times the shear modulus, while peak value of this curve is the value of the shear strength of the material. The PH method analyzes strain softening by incremental plasticity theory, which assumes that the soil undergoes elastic and plastic strains from the very beginning of the cavity expansion.

As shown in Fig. 4(a), values of the ratio $c_u/\sigma_0'$ determined from the various methods compare well with each other. Between depths of 2 and 11 m, the $K_0$ values predicted by Procedure 3 and Denby and PH methods compare well with each other, *see* Fig. 4(b). Values of $K_0$ predicted by various methods differ by about only 0.1 between 11 and 15 m.

In Fig. 4(c), values of the $E_{sf}/c_u$ ratios determined by using (a) Procedure 3 (*see* Curve I), (b) Procedure 2 with $K_0$ values determined by Denby method (*see* Curve II), (c) Procedure 2 with $K_0$ values determined by BLP method (*see* Curve III), and (d) Procedure 2 with $K_0$ values determined by PH method (*see* Curve IV), have been shown. It can be seen that $E_{sf}/c_u$ determined using values of $K_0$ estimated from various methods compare well up to a depth of 11 m. This shows that small errors in estimating $K_0$ values does not significantly affect the values of $E_{sf}/c_u$.

$R_f$ is either determined based on values available in literature (Duncan and Chang, 1970; Denby, 1978), or by obtaining stress difference $(\sigma_1-\sigma_3)$ and axial strain $(\varepsilon_{rr})$ data from the triaxial consolidated undrained tests performed on high quality undisturbed samples. Using a hyperbolic model the asymptotic value of $(\sigma_1-\sigma_3)$ is determined as shown below (Duncan and Chang, 1970):

$$\sigma_1-\sigma_3=\frac{\varepsilon_{rr}}{b+a\varepsilon_{rr}} \quad \text{or} \quad \frac{\varepsilon_{rr}}{\sigma_1-\sigma_3}=b+a\varepsilon_{rr}$$

$$\text{or} \quad y=a_1+a_2x, \quad \text{i.e.,} \quad x=\varepsilon_{rr} \quad \text{and} \quad y=\frac{\varepsilon_{rr}}{\sigma_1-\sigma_3}.$$

From Eq. (11a), using $x$ and $y$ values, determine the value of $a_1$ and $a_2$. Then $E_i$ is equal to $1/a_1$, and the asymptotic value of $(\sigma_1-\sigma_3)$ is equal to $1/a_2$. The value of stress difference $[(\sigma_1-\sigma_3)_f]$ is determined from the observed peak value at failure from triaxial tests. $R_f$ is the ratio between $[(\sigma_1-\sigma_3)_f]$ and the asymptotic value of $(\sigma_1-\sigma_3)$. $R_f$ is approximately equal to 0.7 for the upper desiccated, overconsolidated Bay mud, which extends to 5 m depth and 0.8 for the underlying NC to slightly OC Bay mud. As shown in Figs. 4(d) and 4(e), values of $E_i/c_u$ and $E_{50}/c_u$ are determined by substituting $E_{sf}/c_u$ values of Curves I through IV of Fig. 4(c) in Eqs. (15) and (16), respectively. As shown in Fig. 4(d), the average values of $E_i/c_u$ for the lower NC to slightly OC Bay mud (i.e. from 6 to 15 m) varies between 1890 and 2430, and compare well with values of $E_i/c_u$ between 2200 and 2600 determined from the triaxial shear tests performed by Denby, 1978. The estimated values of $E_{50}/c_u$, as shown in Fig. 4(e), compare well with values of the $E/c_u$ determined by Denby (1978), using the Denby and PH methods, as shown in Fig. 4(f)] in a depth interval of 6 to 15 m. This shows that the $E$ values determined by Denby and PH methods are approximately egual to $E_{50}$ values.

*(c) Kaolinite*: Using Procedure 2, the strain-controlled model pressuremeter tests performed in a large calibration chamber by Huang et al. 1991 are analyzed, *see* Table 4. The value of $c_u$ determined from the logarithmic model compares well with the value determined by Huang et al. 1991 and also with plain strain undrained shear strength. The value of $E_{50}/c_u$ ratio determined from pressuremeter data using Procedure 2, compares well with the value of $E_{50}/c_u$ determined from the triaxial shear tests.

## SPHERICAL CAVITY EXPANSION

When a balloon of spherical shape is expanded in a soil

**Table 4. Analyzed results of kaolinite**

| Data of Huang et al. 1991 | | | | | Alternative method | | | |
|---|---|---|---|---|---|---|---|---|
| Pressuremeter test | | | Triaxial test | | Procedure 2 | | | |
| Strain rate | $c_u$ | $E$ | $c_u$ | $E_{50}$ | $c_u$ | $E_{sf}$ | $E_i$ | $E_{50}$ |
| %/minute | * | ** | * | ** | * | ** | ** | ** |
| 0.1 | 65 | 22.5 | 64 | 108 | 68.6 | 19.3 | 172 | 99 |

* in kN/m$^2$, ** in MN/m$^2$

deposit, it simulates a spherical cavity from finite radius. The problem of the spherical cavity is analogous to the problem of the cylindrical cavity, with the difference that it is spherically symmetrical instead of axially symmetrical. Using the equations described in Appendix I, a similar analysis for the spherical cavity expanded from a finite radius can be performed.

## CONCLUSIONS

Following conclusions are based on the findings of this study: (1) The alternative method consisting of (a) cavity expansion method, (b) curve fitting with a logarithmic model, and (c) hyperbolic stress-strain model for the elastic zone provides a rational procedure to estimate soil parameters of saturated clays. (2) The curve fitting method based on a logarithmic model for values of $p(t)$ greater than $\sigma_{rp}$ provides an excellent match with the SBPMT stress-strain curve. (3) The value of the undrained shear strength, $c_u$, compares well with values determined by other methods. (4) The undrained cavity expansion theory from finite radius provides a reasonable estimate of secant modulus at failure. (5) Using the hyperbolic model and value of secant modulus at failure, the estimated values of initial tangent modulus compares well with those determined from the triaxial tests. Values of secant modulus at half the value of $(\sigma_1-\sigma_3)_f$ determined using the alternative method compare well with those determined from the triaxial tests and from the unload-reload cycle of the SBPMT tests.

## NOTATIONS

The following symbols are used in this paper:

$c_u$=undrained shear strength of saturated clay;
$E$=Young's modulus of elasticity;
$E_i$=initial tangent modulus;
$E_{sf}$=secant modulus at failure;
$E_{50}$=secant modulus at half the value of $(\sigma_1-\sigma_3)_f$;
$E_r$=unload-reload modulus;
$G$=shear modulus;
$K_0$=coefficient of earth pressure at rest;
$h_w$=height of ground water table over test location;
$I_r$=rigidity index;
$R_0$=radius of cavity at time $t=0$;
$R(t)$=radius of cavity at time $t$;
$r$=radial distance of particle from axis of cavity in its original position;
$r(t)$=radial distance of particle from axis of cavity in its deformed position at time $t$;
$r_p$=radius of plastic zone at time $t$;
$R_f$=failure ratio;
$p(t)$=internal cavity pressure at time $t$;
$u_w$=pore water pressure due to ground water table;
$\nu$=Poisson's ratio;
$\Delta V$=change in volume of cavity at time $t$;
$V_0$=initial volume of cavity;
$V$=volume of cavity at time $t$;
$\varepsilon_v$=volumetric strain;
$\varepsilon_{rr}$=radial strain;
$\varepsilon_{\theta\theta}$=circumferential strain in cylindrical or spherical polar coordinates;
$\varepsilon_{\phi\phi}$=circumferential strain in spherical coordinates;
$\varepsilon_{zz}$=normal strain in vertical direction;
$\xi_r$=radial displacement of particle;
$\xi_{rc}$=radial displacement of particle at face of cavity;
$\xi_{rp}$=radial displacement of particle at the interface of elastic and plastic zones;
$\sigma_{rr}$=radial stress;
$\sigma_{\theta\theta}$=circumferential stress in cylindrical and spherical coordinates;
$\sigma_{\phi\phi}$=circumferential stress in spherical coordinates;
$\sigma_{zz}$=normal stress in vertical direction;
$\sigma_h$=total horizontal stress prior to cavity expansion;
$\sigma'_v$=effective vertical stress;
$\sigma_m$, $\sigma'_m$=total and effective mean stress in elastic zone;
$\sigma_{mp}$, $\sigma'_{mp}$=total and effective mean stress in plastic zone;
$\sigma_{oct}$, $\sigma'_{oct}$=total and effective octahedral stress in elastic zone;
$\sigma_{octp}$=octahedral stress in plastic zone;
$\sigma_{rp}$=radial stress at interface of elastic and plastic zones;
$\sigma_1-\sigma_3$=stress difference at failure;
$\tau$=maximum shear stress; and
$\gamma_w$=unit weight of water.

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## APPENDIX I—SPHERICAL CAVITY EXPANSION

*(a) Cavity Expansion Prior to Formation of State of Plastic Equilibrium at Face of Cavity*

$$\sigma_{rr}=\sigma_{oct}+[p(t)-\sigma_{oct}]\frac{R^3(t)}{r^3(t)},$$

$$\sigma_{\theta\theta}=\sigma_{\phi\phi}=\sigma_{oct}+[p(t)-\sigma_{oct}]\frac{R^3(t)}{2r^3(t)} \tag{17}$$

$$\sigma_{oct}=\gamma_w+\sigma_v'(1+2K_0)/3,$$

$$\xi_{rc}=\frac{(1+\nu)}{2E}R(t)[p(t)-\sigma_{oct}]. \tag{18}$$

*(b) For Plastic Zone, i.e. For $r(t) \Leftarrow r_p$*

$$\sigma_{rr}=p(t)-4c_u\ln\frac{r(t)}{R(t)},$$

$$\sigma_{rp}=\sigma_{oct}+\frac{4c_u}{3},$$

$$\frac{r_p}{R(t)}=e^{((p(t)-\sigma_{oct})/4c_u)-(1/3)} \tag{19}$$

$$\sigma_{\theta\theta}=\sigma_{\phi\phi}=p(t)-4c_u\ln\frac{r(t)}{R(t)}-2c_u,$$

$$\sigma_{octp}=p(t)-4c_u\ln\frac{r(t)}{R(t)}-\frac{4c_u}{3}. \tag{20}$$

$$R^3(t)-R^3=r_p^3-(r_p-\xi_{rp})^3, \tag{21a}$$

where

$$\xi_{rp}=\frac{2(1+\nu)r_pc_u}{3E}=\frac{r_p}{3I_r} \tag{21b}$$

$$\xi_{rc}=R(t)-\sqrt[3]{R^3(t)-r_p^3+(r_p-\xi_{rp})^3}. \tag{22}$$

Substituting values of $r_p/R(t)$ and $\xi_{rp}/R(t)$ from Eqs. (19) and (21b) in (21a), the cavity pressure $p(t)$ at any instant $t$ is given by

$$p(t)=\sigma_{oct}+\frac{4}{3}c_u+\frac{4}{3}\ln\left(1-\frac{R^3}{R^3(t)}\right)\left(\frac{27I_r^3}{27I_r^2-9I_r+1}\right). \tag{23}$$