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# MAXIMUM AND MINIMUM VOID RATIO CHARACTERISTICS OF SANDS

MISKO CUBRINOVSKI[i] and KENJI ISHIHARA[ii]

## ABSTRACT

Characteristics of the maximum and minimum void ratios of sands and their possible use for material characterization have been investigated in this study. Data of over 300 natural sandy soils including clean sands, sands with fines and sands containing small amount of clay-size particles have been used to examine the influence of fines, grain-size composition and particle shape on $e_{max}$, $e_{min}$ and void ratio range ($e_{max}-e_{min}$). A set of empirical correlations are presented which clearly demonstrate the link between these void ratios and material properties of sands. The key advantage of ($e_{max}-e_{min}$) over-conventional material parameters such as $F_C$ and $D_{50}$ is that ($e_{max}-e_{min}$) is indicative of the overall grain-size composition and particle characteristics of a given sand and that it shows off the combined influence of relevant material factors. The void ratio range provides a general basis for comparative evaluation of material properties over the entire range of cohesionless soils.

Important issues related to the laboratory procedures used for determination of $e_{max}$ and $e_{min}$ as well as their applicability to fines-containing sands are also addressed. Three distinct linear correlations were found to exist between $e_{max}$ and $e_{min}$ for clean sands, sands with 5-15% fines and sands with 15-30% fines respectively, thus illustrating that the standard JGS procedures for minimum and maximum densities of sands can provide reasonably consistent $e_{max}$ and $e_{min}$ values for sands with fines content of up to 30%. The importance of the grain-size distribution and presence of gaps in the grading of composite soils or mixtures of sands with fines produced in the laboratory is also discussed.

**Key words:** fines, grain shape, grain size, sand, void ratio (**IGC**: D2/D3)

## INTRODUCTION

The stress-strain behaviour of sand depends on two key factors: one is the physical nature of the sand and the other is the physical state of the sand. The nature of the sand is related to the constitution of the sand as a granular material and is commonly described by material properties such as the grain-size distribution, fines content, grain shape and mineralogy among others. The state of the sand, on the other hand, indicates the physical conditions under which the sand exists. Typical variables describing the state of the sand are the relative density $D_r$, the effective stress state and fabric. It is well known that both material properties and state variables have a governing influence on the deformation and strength characteristics of sands.

In the generalized charts that are used to identify sand behaviour, the material properties are often represented either by a single or a pair of parameters, most commonly by the fines content $F_C$ and mean grain size $D_{50}$. In these charts, it has essentially been assumed that sands with similar fines content have also similar deformational behaviour irrespective of their differences in the grain-size distribution, grain shape, nature of fines or any other material property. There is abundant experimental evidence, however, showing that even when two sands have identical fines content, they still can have remarkably different deformation and strength characteristics, thus indicating the need to better characterize sands and allow for the effects of various material properties (e.g. Selig and Ladd, 1973; Aberg, 1992; Miura et al., 1997). In an effort to provide more general representation of the material properties of sands, it was suggested by Cubrinovski and Ishihara (1999) that the void ratio range ($e_{max}-e_{min}$), which is the difference in the void ratio between the loosest and densest states of packing of a sand, would be an appropriate parameter for this purpose. For example, Fig. 1(a) shows an empirical correlation between the normalized SPT blow count, $N_1$, and relative density, $D_r$, where the void ratio range is used to quantify

i) Senior Research Engineer, Kiso-Jiban Consultants Co., Ltd., 1-11-5 Kudan-kita, Chiyoda-ku, Tokyo 102-8220, e-mail: misko.cubrinovski@kiso.co.jp
ii) Professor, Department of Civil Engineering, Science University of Tokyo, 2641 Yamazaki, Noda, Chiba 278-0022.
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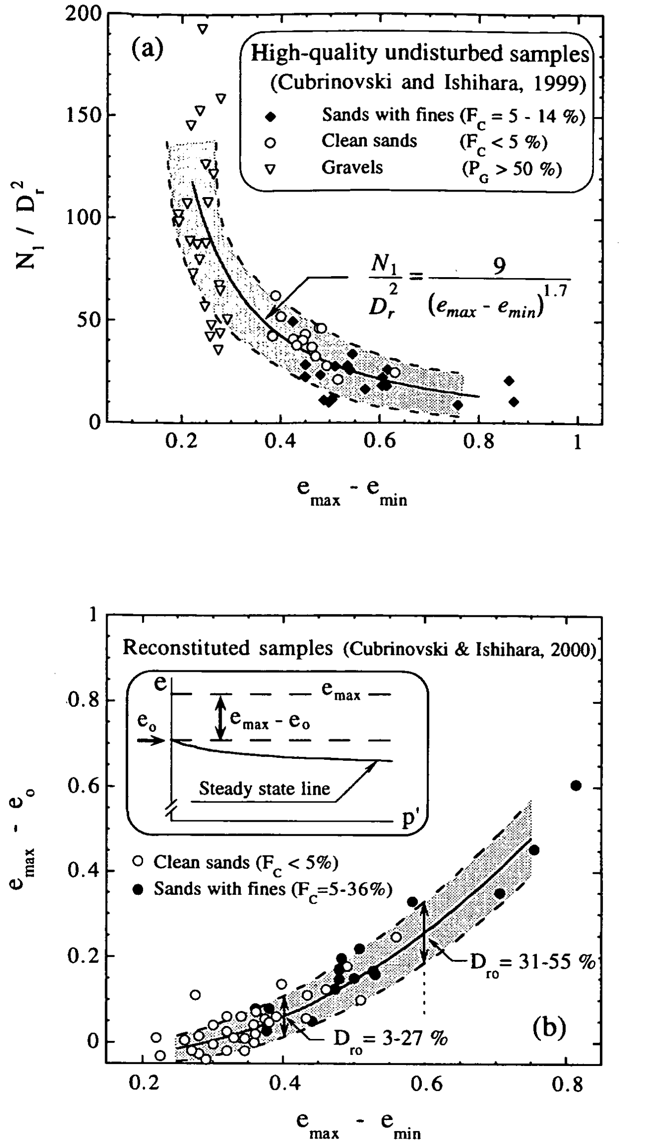


**Fig. 1. Void ratio range as a parameter for sand characterization: (a) empirical correlation between $N_1/D_r^2$ and $(e_{max}-e_{min})$ and (b) position of steady state line as a function of $(e_{max}-e_{min})$**

the grain-size effects on the penetration resistance of cohesionless soils (Cubrinovski and Ishihara, 1999). Similarly, Fig. 1(b) shows the variation in the position of the steady state line with $(e_{max}-e_{min})$ of sandy soils where the void ratio range is also used as a measure indicative of the effects of grain-size distribution and fines content on the vertical position of the steady state line in the *e-p'* plot (Cubrinovski and Ishihara, 2000). In both figures, there is clear correlation between the subject behavioral parameter and $(e_{max}-e_{min})$ for a range of cohesionless soils including clean sands, sands with fines and gravels. The key feature of the void ratio range utilized in these correlations is that $(e_{max}-e_{min})$ reflects the overall grain composition and particle characteristics of cohesionless soils. In other words, $(e_{max}-e_{min})$ appears to embody the combined influence of the grain sizes and grain shapes of all fractions constituting a given soil.

The void ratio range is used as a denominator in the conventional expression for the relative density, but it is important to recognize the essential difference between $D_r$ and $(e_{max}-e_{min})$, as parameters. Namely, $D_r$ is a state parameter indicating how dense a given sand sample or deposit is with respect to its range of possible densities. On the other hand, $(e_{max}-e_{min})$ is a material parameter indicative of the inherent properties of the sand such as the grain-size composition, fines content and grain shape. In the same way, there is a principal difference between $(e_{max}-e_{min})$ and the parameter $(e-e_{min})$, which was used by Ishihara and Watanabe (1976) as a measure for the volume decrease potential in liquefaction evaluation. Apparently, $(e-e_{min})$ is a state parameter and represents an index density for a given state of the sand.

Previous studies on the maximum and minimum void ratios have mainly been focused on clean sands. In particular, characteristics of $e_{max}$ and $e_{min}$ have been investigated in the context of the accuracy and applicability of the relative density of sands, since $e_{max}$ and $e_{min}$ are used in the calculation of relative density (e.g. Tavenas and La Rochelle, 1972; Selig and Ladd, 1973). Other studies have examined the relation between the material properties and $e_{max}$ or $e_{min}$ of sands (Miura et al., 1997) and the use of the void ratio range for sand classification (Shimobe and Moroto, 1995; Fukumoto and Sumisaki, 1999). Recently, effects of fines on $e_{max}$ and $e_{min}$ of sands have been studied to examine the influence of fines on undrained behaviour of sands (e.g. Lade and Yamamuro, 1997; Lade et al., 1998; Thevanayagam, 1998). As illustrated above and discussed in previous papers (Cubrinovski and Ishihara, 1999, 2000), the void ratio range seems to permit characterization of the overall grain-size composition and particle characteristics of cohesionless soils with a reasonable level of credibility, and therefore, it can be used as an index of the material properties of sands.

The objective of this study is to examine the void ratio characteristics for a wide range of natural sandy soils including various clean sands, sands with fines and sands containing small amount of clay-size particles. In particular, the study highlights the effects of fines, grain-size distribution and grain shape on $e_{max}$, $e_{min}$ and $(e_{max}-e_{min})$ of sandy soils. In what follows, background information is first given by considering some idealized packings of spherical grains and by examining void ratio characteristics of sand-fines composites created by mixing different soils in the laboratory. Subsequently, the maximum and minimum void ratios as well as the void ratio range of over 300 natural sandy soils are presented and their correlation with the material properties is examined and discussed.

## IDEALIZED PACKINGS OF SPHERICAL GRAINS

### *Single-Sized Spheres*

Before examining the packing characteristics of sands, let a pair of idealized packings of spherical particles be considered (e.g. Graton and Fraser, 1935; White and Walton, 1937; Lade et al., 1998). For single-sized spheres, the loosest possible packing is illustrated in

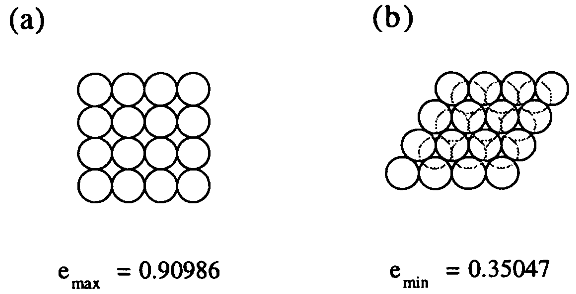


**Fig. 2. Schematic illustration of packing of single-sized spheres: (a) loosest state and (b) densest state**

Fig. 2(a), with the corresponding maximum void ratio being defined as

$$e_{max}=\frac{6-\pi}{\pi}=0.90986 \qquad (1)$$

On the other hand, the void ratio of the densest possible packing is

$$e_{min}=\frac{3\sqrt{2}-\pi}{\pi}=0.35047 \qquad (2)$$

Arrangement of particles for the densest packing is schematically shown in Fig. 2(b). It is evident from the above expressions that the maximum and minimum void ratios are independent of the size of the spheres.

*Mixtures of Two Grain Sizes*

When mixing spherical particles of two different sizes, the packing will be affected by the proportion of large-size and small-size spheres in the total volume of solids as well as by the relative size of the large and small spheres. We will first examine the change in the densest packing ($e_{min}$) with the percentage of the small-size fraction.

Figure 3(a) schematically shows how the volumes of solids and voids vary with a change in the percentage of the small-size particles. Here, point L denotes the densest possible packing of the larger spheres. Initially, adding smaller size particles into the densest packing of large spheres leads to a decrease in the volume of voids since the small spheres fill in the voids among the larger particles. This filling-of-voids phase is indicated in the diagram with the path L-T. Upon adding small particles beyond a certain percentage corresponding to point T, a reverse trend is observed in which the volume of voids increases with the percentage of the small-size fraction. In this so-called replacement-of-solids phase, the large-size particles are pushed apart and gradually replaced by the small-size spheres until the entire volume of solids is comprised of smaller particles (point S). The corresponding change in the minimum void ratio with the percentage of the small-size fraction is shown in Fig. 3(b), where it can be seen that $e_{min}$ decreases in the course of the filling-of-voids process and reaches its minimum value of $e_{min(T)}$ at the threshold percentage corresponding to T. Subsequently, $e_{min}$ steadily increases during the replacement-of-solids process, as shown with the path T-S in Fig. 3(b).

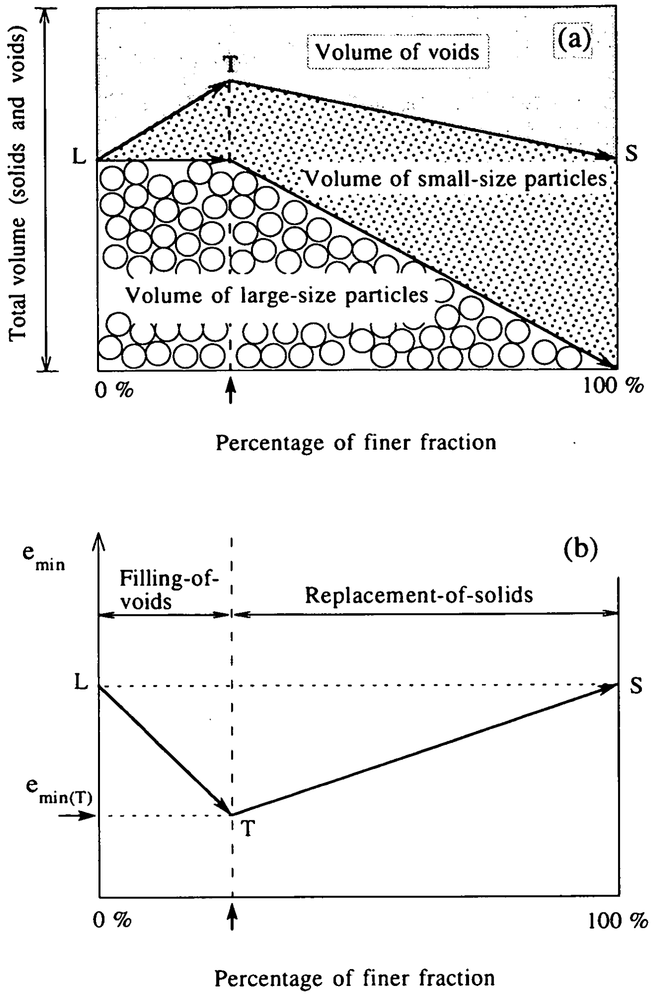


**Fig. 3. Effects of fines on binary packing of spherical particles: (a) variation in the volume of voids and solids and (b) variation in $e_{min}$**

One observation which is of importance for fines-containing sands is that the threshold percentage of fines at which the filling process is reversed into the replacement process is significantly smaller than 50%.

As mentioned earlier, the relative size of the large and small spheres is the other factor influencing the binary packing of spheres. It is apparent that the small particles can be enclosed by the large particles only if the diameter of the small spheres, $d$, is at least 6.5 times smaller than that of the large spheres, $D$, i.e. $d<D/6.5$, as illustrated by the geometrical considerations in the inset of Fig. 4. Lade et al. (1998) examined in details the packings of spherical particles and presented experimental data obtained by McGeary (1961) illustrating the change in the minimum void ratio $e_{min(T)}$ with the ratio $D/d$, as shown in Fig. 4. Note that here, the void ratio for $D/d=1$ corresponds to the void ratio at point L of Fig. 3. As shown in Fig. 4, the decrease in the minimum void ratio depends on the diameter ratio $D/d$ or relative size of the large and small particles. Initially, $e_{min(T)}$ decreases sharply as the diameter ratio $D/d$ increases from 1 to approximately 7, whereas beyond the ratio of 7, the influence of $D/d$ is much smaller. It is important to note that the mixtures of

**Table 1. Properties of sands and fines used for producing soil mixtures in the laboratory**

| No. | Soil | $e_{max}$ | $e_{min}$ | Grain shape | $F_C$ (%) | $D_0$ (mm) | $D_{50}$ (mm) | $D_{100}$ (mm) | Ref. | Remarks |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Cambria sand | 0.767 | 0.538 | R | 0 | 0.83 | 1.50 | 2.0 | R1 | NS |
| 2 | Nevada sand 50/80 | 0.858 | 0.581 | SA-A | 0 | 0.175 | 0.211 | 0.3 | | NS |
| 3 | Nevada sand 80/200 | 0.940 | 0.617 | SA-A | 0 | 0.075 | 0.12 | 0.175 | | NS |
| 4 | Nevada fines | 1.178 | 0.754 | SA-A | 100 | 0.0133 | 0.05 | 0.075 | | NS |
| 5 | Ottawa sand 50/200 | 0.805 | 0.550 | A | 0 | 0.075 | 0.202 | 0.3 | R2 | NS |
| 6 | Ottawa sand F-95 | 0.865 | 0.580 | SR | 0 | 0.075 | 0.163 | 0.245 | | NS |
| 7 | Host sand A2 | 0.980 | 0.600 | | 2 | <0.075 | 0.25 | 0.83 | R3 | NS |
| 8 | KS-fines | | | | 100 | <0.001 | 0.009 | 0.075 | | NS |
| 9 | Ottawa sand 20-30 | | | SR-SA | 0 | 0.075 | 0.747 | 1.2 | R4 | ASTM |
| 10 | Silt | | | | 95 | <0.001 | 0.015 | 0.2 | | LPF, ASTM |
| 11 | Toyoura sand | 0.988 | 0.616 | SR-SA | 0 | 0.075 | 0.17 | 0.44 | R5 | JGS |
| 12 | Silt | | | | 100 | <0.003 | 0.0093 | 0.075 | | JGS |
| 13 | Ottawa sand | 0.780 | 0.480 | R-SR | 0 | 0.075 | 0.39 | 1.0 | R6 | ASTM |
| 14 | Silt | | | | 100 | 0.0007 | 0.023 | 0.075 | | ASTM |
| 15 | Ottawa sand C-109 | 0.830 | 0.500 | SR | 0 | | 0.39 | | R7 | |
| 16 | Silica sand | | | SR | 0 | 0.075 | 0.15 | 0.25 | | |
| 17 | Silica fines | | | | 100 | <0.005 | 0.012 | 0.075 | | |
| 18 | Kaolinite | | | | 100 | <0.0012 | 0.0012 | 0.075 | | PF |

Grain shape: R-round, SR-subround, SA-subangular, A-angular
Remarks: LPF-low plasticity fines, PF-plastic fines, NS-Non-standard procedures, ASTM-procedures, JGS-procedures for $e_{max}$ and $e_{min}$
References: R1—Lade et al. (1998), R2—Lade and Yamamuro (1997), R3—Thevanayagam (1998), R4—Amini and Qi (2000), R5—Zlatovic (1994), R6—Salgado et al. (2000), R7—Pitman et al. (1994)

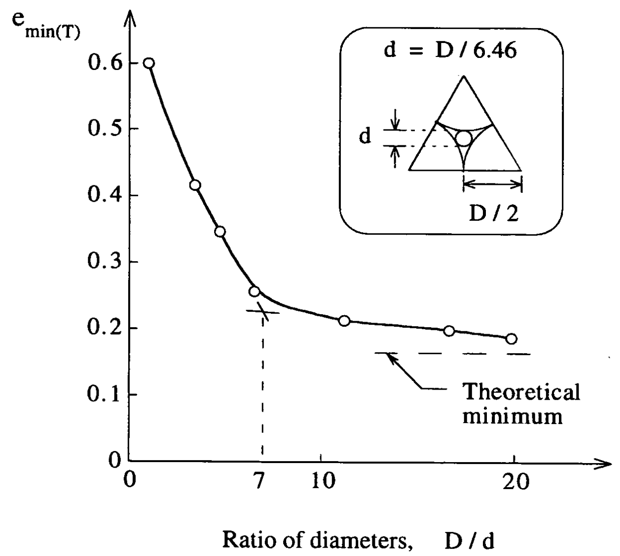


**Fig. 4. Dependence of the minimum void ratio $e_{min(T)}$ on the relative size of spherical particles in binary packing (after Lade et al., 1998)**

two aggregates of spheres are gap-graded, and that the gap in the grain-size composition increases with the ratio $D/d$.

It will be shown in the following that $e_{max}$ of sands may change with the fines content in a pattern similar to that of $e_{min}$ in Fig. 3, but $e_{max}$ may also remain nearly constant or may even slightly increase with an addition of a very small amount of fines. Because of this diversity in the maximum void ratio of sands and practical insignificance of the idealized loosest packing of spheres, the $e_{max}$ characteristics will be directly examined on sands.

## SAND-FINES COMPOSITES

Void ratio characteristics of natural sands are influenced by several other factors in addition to those introduced for the idealized packings of spherical particles. Natural sands have various gradings and practically infinite number of grain compositions; the grain shape can be round, subround or more or less angular; and also, in addition to the gravitational forces, small soil particles in the region of fines can be affected by interacting surface forces. To illustrate the effects of some of these factors and gradually bridge the difference between the packing of spherical grains and natural sands, investigated in the following are the void ratio characteristics of sand-fines composites or mixtures of sands and fines produced in the laboratory.

### *Compiled Data*

In several experimental studies regarding the effects of fines on undrained behaviour of sand (e.g. Lade and Yamamuro, 1997; Thevanayagam, 1998), a selected sand has been used as a basic material, and mixtures of this sand with different amount of fines, say 10%, 20%, 30% etc., have been produced in the laboratory so as to comparatively examine these soils and evaluate the effects of fines on the subject sand behaviour. Data were compiled from this kind of studies in which, among others, the maximum and minimum void ratios of the sand-fines mixtures have been determined. Information on the compiled data is listed in Table 1 and grain-size curves of the soils that have been used to produce the soil mixtures are displayed in Fig. 5. The investigated sands are uniform and include coarse, medium and fine sands. The fines ($D<0.075$ mm), on the other hand, have various degrees

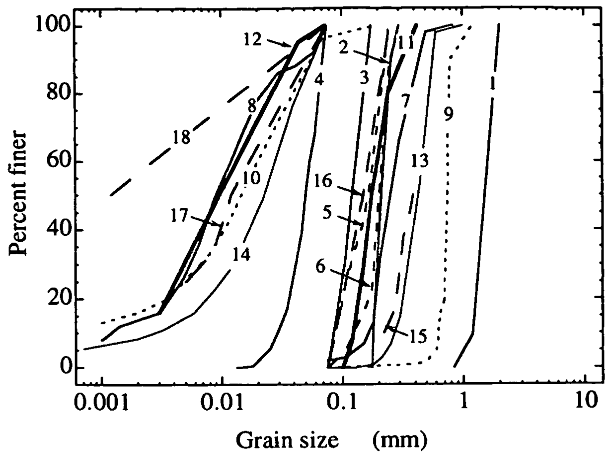


**Fig. 5. Grain-size curves of sands and fines used for producing composite soils (soils listed in Table 1)**

**Table 2. Characteristic grading properties of composite soils**

| Composite soil | $D_{50}/d_{50}$ | $D_0/d_{100}$ | $D_{100}/d_0$ | Remarks |
|---|---|---|---|---|
| 1–2 | 7.1 | 2.8 | 11.4 | (1, 3) |
| 1–3 | 12.5 | 4.7 | 26.7 | (1, 3) |
| 1–4 | 30 | 11.1 | 150.4 | (1) |
| 2–4 | 4.2 | 2.3 | 22.5 | (1) |
| 3–4 | 2.4 | 1 | 13.1 | |
| 2–3 | 1.8 | 1 | 4.0 | (3) |
| 2 and 3–4 | 3.3 | 1 | 22.5 | (4) |
| 5–4 | 4.0 | 1 | 22.5 | |
| 6–4 | 3.3 | 1 | 18.4 | |
| 7–8 | 27.7 | 1 (1.4) | >830 | (2) |
| 9–10 | 49.8 | 2.7 | >747 | (1) |
| 11–12 | 18.3 | 1 (1.4) | >147 | (2) |
| 13–14 | 16.9 | 1.7 | 1428 | |
| 15–16 | 2.6 | — | — | (3) |
| 15–17 | 32.5 | — | — | (1) |
| 15–18 | 325 | — | — | (1) |

Remarks: (1) gap-graded, (2) practically gap-graded, (3) sand-sand composite, (4) mixture of three soils, $D$ = diameter of larger fraction, $d$ = diameter of smaller fraction, $D_{50}/d_{50}$ = mean grain size ratio, $D_0/d_{100}$ = gap ratio, $D_{100}/d_0$ = ratio used by Aberg (1996)

of uniformity and commonly contain some percentage of clay-size particles ($D<0.005$ mm). Except for two cases which are indicated in Table 1, the fines are nonplastic. Three different sets of testing procedures have been used for determination of $e_{max}$ and $e_{min}$ of the compiled soils, i.e., procedures stipulated by the Japanese Geotechnical Society for sands with less than 5% fines (JGS procedures), procedures of the American Society for Testing of Materials for soils that contain up to 15% cohesionless fines (ASTM procedures) and non-standard procedures (NS). The minimum void ratio has been determined using densification procedures either by tapping the mold (JGS procedures and NS procedures employed by Lade et al., 1998) or by vertically vibrating the specimen (ASTM procedure). The maximum void ratio has been determined, on the other hand, by carefully depositing dry soil in the mold with a zero height of fall (JGS procedure) and

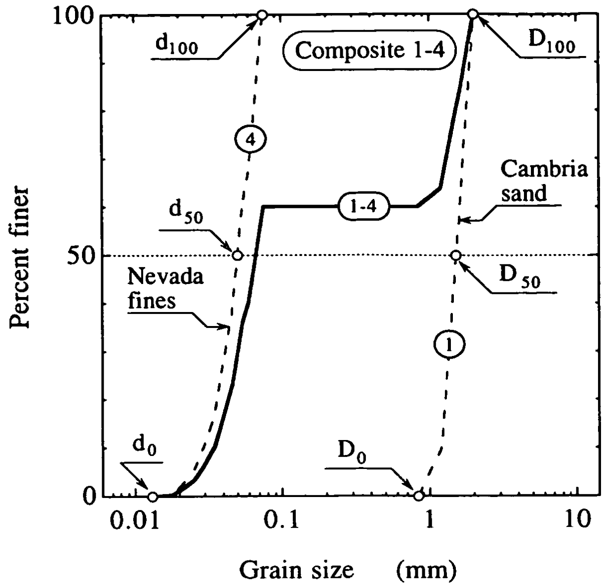


**Fig. 6. Grain-size curve of the mixture consisting of 40% Cambria sand and 60% Nevada fines (data after Lade et al., 1998)**

slowly rotating the mold several times to achieve a very loose state (ASTM and NS procedures). The above procedures for $e_{max}$ and $e_{min}$ differ in details regarding the amount of soil used, method of deposition, densification technique, etc.

In each of the studies referenced in Table 1, a number of sand-fines or sand-sand composites have been produced by blending two soils in different proportions. For example, Cambria sand (1) was mixed with 10, 20, 30, 40, 50, 60 and 80% of Nevada fines (4) to produce 7 soils with different grain compositions (Lade et al., 1998). In the following, all these mixtures are referred to as the composite 1–4 according to the soil numeration given in Table 1. A complete list of the sand-fines and sand-sand composites is given in Table 2. It is important to note that many of the mixed materials are gap-graded soils, as illustrated in Fig. 6 where the grain-size distribution curve of the 1–4 composite consisting of 40% Cambria sand and 60% Nevada fines is shown as an example.

*Effects of Fines and Relative Grain Size on $e_{max}$ and $e_{min}$*

For each of the composite soils, the maximum and minimum void ratios have been determined according to the adopted laboratory procedures (Table 1). Thus, it is possible to plot the values of $e_{max}$ and $e_{min}$ as a function of the fines content for each of the composite soils, as shown in Fig. 7 for the gap-graded mixtures consisting of Cambria sand and Nevada fines. It is observed that both $e_{max}$ and $e_{min}$ initially decrease as the fines content increases from 0% to about 20%. Within the range of 20 to 40% fines, the relationships show a change of pattern indicating a transition from the filling-of-voids to the replacement-of-solids process. Above 40% fines, the maximum and minimum void ratios are seen to steadily increase until they eventually reach the highest values at 100% fines content. It is to be noticed that the variation

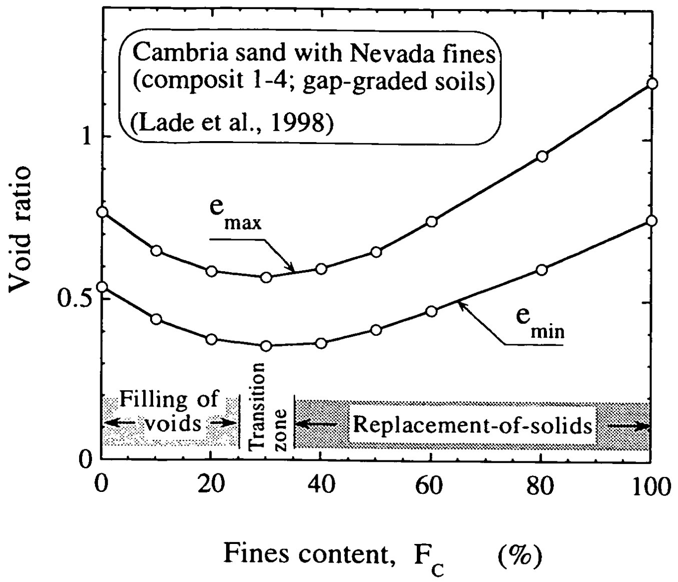


**Fig. 7. Variation in $e_{max}$ and $e_{min}$ with fines content for mixtures of Cambria sand and Nevada fines**

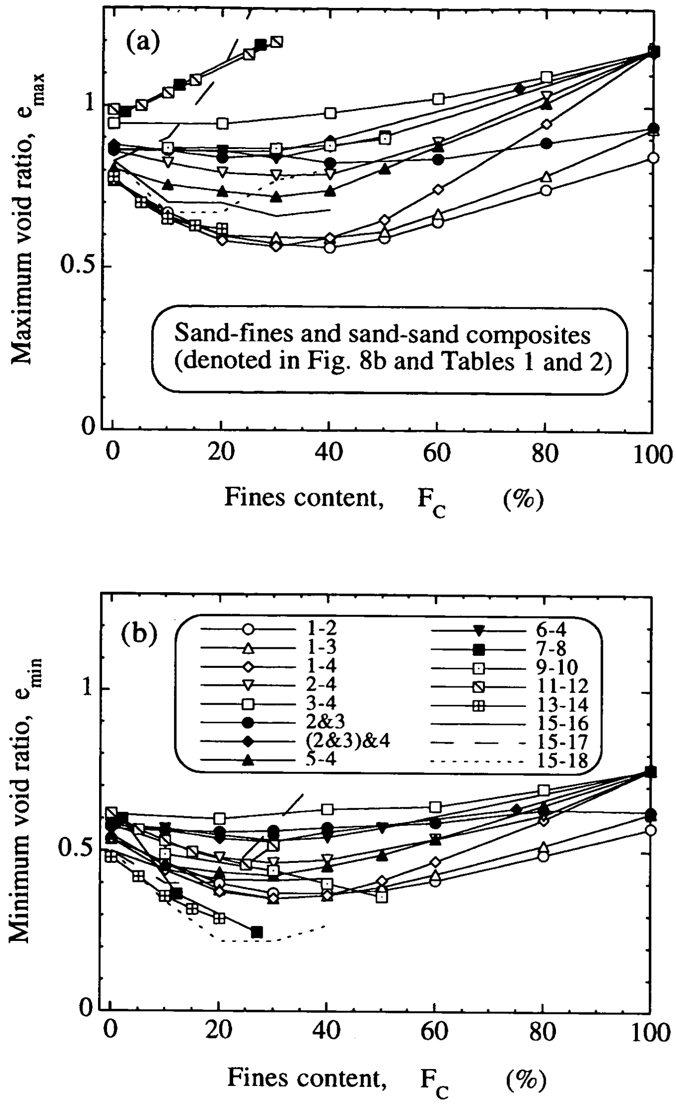


**Fig. 8. Variation in $e_{max}$ and $e_{min}$ with fines content of composite soils: (a) $e_{max}$ vs. $F_C$ and (b) $e_{min}$ vs. $F_C$**

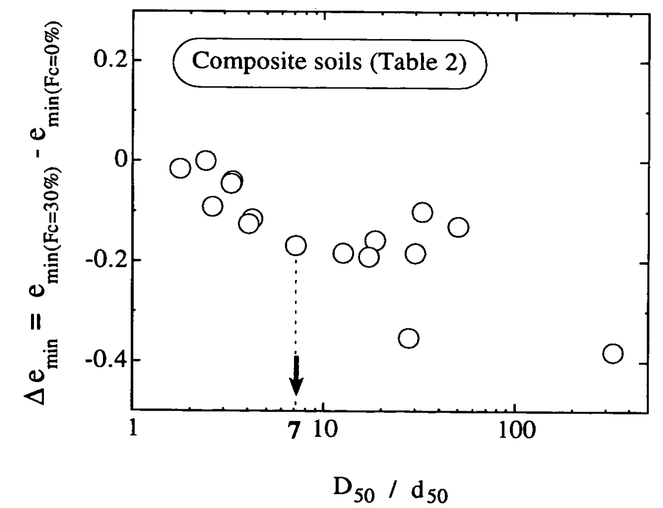


**Fig. 9. Change in the minimum void ratio with an increase in the fines content from 0% to 30% as a function of mean grain-size ratio of composite soils**

of $e_{min}$ with the fines content in Fig. 7 closely resembles that of the spherical particles shown in Fig. 3(b). It will be shown in the following that this similarity in the $e_{min}$ versus $F_C$ relationship is attributed to the fact that both the examined sand-fines composite and binary mixture of spheres are gap-graded.

Variations of the maximum and minimum void ratios with the fines content are shown in Figs. 8(a) and 8(b) respectively, for all sand composites considered. In the case of sand-sand composites (mixtures of two sands), the fines content in these figures denotes the percentage of the finer sand. It is evident in Fig. 8 that in all cases where $e_{max}$ or $e_{min}$ initially decreases with the fines content, the minimum value of $e_{max}$ or $e_{min}$ is obtained for a fines content somewhere between 20 and 40%, and most commonly at $F_C=30\%$. While the majority of materials show more or less an initial decrease in the minimum void ratio, the value of $e_{min}$ remains nearly constant over the range of 0 to 60% fines, for several sand composites. A close scrutiny of these data reveals that the two fractions mixed to produce each of these composites are of similar grain sizes. On the other hand, the largest drop in $e_{min}$ is obtained for composites produced by mixing two soils that have very different grain sizes. Clearly, the relationship of $e_{max}$ or $e_{min}$ with the fines content is affected by the grain-size distribution and in particular by the relative size of the grains of the two soils used to produce the composite. To quantify these results, the net change in $e_{min}$ between 0% and 30% fines content is plotted against the mean grain-size ratio $D_{50}/d_{50}$ in Fig. 9, where $D_{50}$ denotes the mean grain size of the sand and $d_{50}$ is the corresponding particle size of the fines of a given composite soil (*see* Fig. 6). Note that in the case of mixtures of two sands, $d_{50}$ represents the mean grain size of the finer sand. It may be seen in Fig. 9 that the minimum void ratio is nearly unchanged for small $D_{50}/d_{50}$ ratios, but it is reduced as the

**Table 3. Properties of standard Toyoura sand as defined in 13 independent studies ($e_{max}$ and $e_{min}$ have been determined according to the JGS procedures)**

| Reference | $e_{max}$ | $e_{min}$ | $D_{50}$ (mm) | $U_C$ | $e_{max}-e_{min}$ | $D_r$ at $e=0.75$ (%) |
|---|---|---|---|---|---|---|
| 30(4): 90–104 | 0.988 | 0.615 | 0.18 | 1.79 | 0.373 | 63.81 |
| 31(3): 60–76 | 0.973 | 0.635 | 0.18 | 1.20 | 0.338 | 65.98 |
| 31(4): 121–132 | 0.977 | 0.597 | 0.17 | | 0.380 | 59.74 |
| 32(1): 149–160 | 0.977 | 0.605 | | | 0.372 | 61.02 |
| 33(2): 47–59 | 0.977 | 0.605 | 0.16 | 1.46 | 0.372 | 61.02 |
| 33(3): 92–104 | 0.970 | 0.618 | 0.18 | 1.50 | 0.352 | 62.50 |
| 36(1): 39–50 | 0.985 | 0.611 | 0.15 | | 0.374 | 62.83 |
| 36(4): 119–126 | 0.976 | 0.611 | 0.18 | 1.30 | 0.365 | 61.87 |
| 37(2): 51–59 | 0.973 | 0.635 | 0.18 | | 0.338 | 65.98 |
| 38(1): 163–179 | 0.973 | 0.612 | 0.16 | 1.46 | 0.361 | 61.77 |
| 38(2): 93–99 | 0.980 | 0.610 | 0.16 | | 0.370 | 62.16 |
| 38(3): 115–127 | 0.988 | 0.616 | 0.19 | 1.70 | 0.372 | 63.98 |
| 40(3): 101–110 | 0.961 | 0.601 | | | 0.360 | 58.61 |
| Mean value: | 0.9768 | 0.6131 | 0.17 | 1.487 | 0.364 | 62.41 |
| Standard deviation: | 0.0075 | 0.0114 | 0.01 | 0.2066 | 0.013 | 2.162 |

Reference: Soils and Foundations, Volume (Number): pp.

ratio $D_{50}/d_{50}$ increases from 2 to approximately 7. The largest reduction in $e_{min}$ is observed for sands having $D_{50}/d_{50}>7$ and gap-graded distribution ($D_0/d_{100}>1$), where $D_0/d_{100}$ indicates the width of the gap in the grain-size distribution curve (Fig. 6). Similar results as above have been addressed in recent experimental and theoretical studies by Barton et al. (2001) and Aberg (1996) respectively.

It may be seen in Fig. 8(a) that, for three sands, the value of $e_{max}$ increases initially with the addition of fines (composites 7–8, 11–12 and 15–17) indicating that the volume of voids increases even when a very small amount of fines, say 5% or 10% fines, is added to these sands. In contrast to the previously introduced filling-of-voids process, the increase in $e_{max}$ as above implies that these composites have a potential to produce very loose and highly unstable particle structure. It is of interest to note that the shape of the $e_{max}-F_C$ relationships in Fig. 8(a) is somewhat related to the value of the maximum void ratio of the clean sand ($e_{max}$ at $F_C=0\%$), though quantification of this trend was not attempted herein since different test methods have been used to determine the $e_{max}$ data of Fig. 8(a).

## MAXIMUM AND MINIMUM VOID RATIOS OF NATURAL SANDS

### *Evaluating $e_{max}$ and $e_{min}$ of Sands*

Before examining the maximum and minimum void ratios of natural sands, it is necessary to address several important issues related to the determination procedures and applicability of $e_{max}$ and $e_{min}$ of sands. As it is well known, $e_{max}$ and $e_{min}$ of sands are determined from laboratory tests using two independent test methods for the minimum and maximum densities respectively, such as the procedures specified by the Japanese Geotechnical Society (JGS), for example. There are many other laboratory procedures for evaluating $e_{max}$ and $e_{min}$ of sands which differ more or less from the standard procedures adopted in the Japanese practice. Even though these procedures tend to identify the limiting densities of a given soil, it would be difficult to determine the minimum and maximum densities in the true sense of the word. Thus, $e_{max}$ and $e_{min}$ of sands are not the two extreme void ratios, but rather they are void ratios at the loosest and densest states produced by a certain set of laboratory test procedures. Another important consequence is that the maximum and minimum void ratios of a given sand are not unique, but rather they depend on the test procedures used for their determination. For this reason, when comparing maximum or minimum void ratios of various soils, it is a prerequisite that $e_{max}$ of all soils are determined using the same test method for the minimum density, and respectively, that all $e_{min}$ values are determined using the same test method for the maximum density.

In fact, $e_{max}$ and $e_{min}$ of a given sand are not uniquely determined even when using a specific set of test methods since these laboratory tests do allow a certain degree of variation in the details of the procedures. To illustrate this variation in the maximum and minimum void ratios, data on $e_{max}$ and $e_{min}$ were compiled for the standard Toyoura sand from 13 independent studies that have been reported in Soils and Foundations over the past decade (Table 3). The $e_{max}$ and $e_{min}$ values reported in these studies as determined according to the JGS procedures are plotted in Fig. 10. It may be seen that the data are well grouped and show reasonably small scatter around the mean values of $e_{max}$ and $e_{min}$, with a standard deviation of 0.8% and 1.14%, respectively. The corresponding maximum deviations are 1.6% and 3.6%.

One important limitation of the JGS test methods for maximum and minimum densities of sands is that they are established as standard test methods for clean sands (soils with more than 95% of the grains in the range between 0.075 mm and 2 mm). Due to the lack of standard procedures for sands with fines, however, the JGS methods for clean sands have been used to evaluate $e_{max}$ and $e_{min}$ of fines-containing sands. Thus, there are abundant data of $e_{max}$ and $e_{min}$ values of fines-containing sands obtained by the JGS procedures for clean sands. On the other hand, because of potential difficulties in determining $e_{max}$ and $e_{min}$ of sands with fines as well as the lack of established standard procedures for these soils, it has been tacitly accepted in the geotechnical practice that $e_{max}$ and $e_{min}$ values of fines-containing sands are not reliable. In view of the state of the practice as above, it is of interest to examine the influence of fines on the maximum and minimum void ratios, and to try to clarify the issues of reliability and applicability of $e_{max}$ and $e_{min}$ for sands containing more than 5% fines.

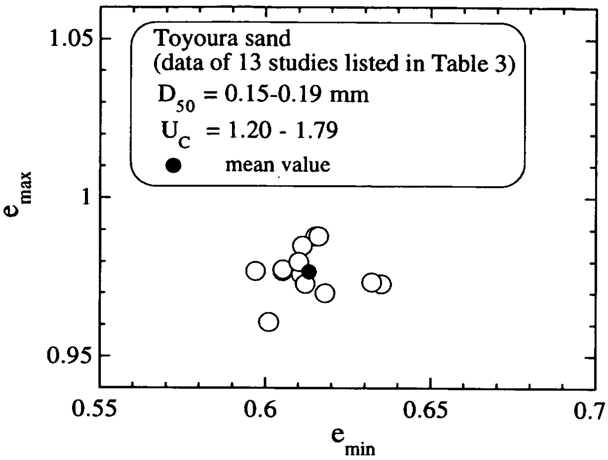


**Fig. 10.** $e_{max}$ **and** $e_{min}$ **values of Toyoura sand determined in 13 independent studies using the JGS procedures**

*Correlation between* $e_{max}$ *and* $e_{min}$

In order to investigate the maximum and minimum void ratios of sands, data were collected of over 300 soils from natural deposits in Japan including clean sands, sands with fines and silty soils. To organize the data systematically and examine the effects of fines on $e_{max}$ and $e_{min}$, the data were classified in the following four groups: (1) clean sands ($F_C$=0–5%); (2) sands with fines ($5<F_C\leq 15\%$); (3) sands with fines and clay ($15<F_C\leq 30\%$, $P_C$=5–20%), and (4) silty soils ($30<F_C\leq 70\%$, $P_C$=5–20%). Here, the fines content $F_C$ and clay-size content $P_C$ denote grain sizes smaller than 0.075 mm and 0.005 mm respectively. Importantly, all $e_{max}$ and $e_{min}$ data of the above soils have been obtained by using the Test Methods for Minimum and Maximum Densities of Sands stipulated in the JGS Standards (JGS, 2000).

Figure 11 shows the correlation between the maximum and minimum void ratios of the compiled soils. In order to extend the correlation over the range of coarse-grained soils, data of gravels, gravelly sands and coarse sands are also indicated in the plot. Evidently, there is well-defined correlation between $e_{max}$ and $e_{min}$ of the compiled soils. As expected, the void ratios increase with the decreasing grain size of the soil, with gravels having the smallest $e_{max}$ and $e_{min}$ values, whereas gravelly sands, clean sands and sands with fines being gradually positioned upward in the correlation until eventually the largest values of $e_{max}$ and $e_{min}$ are encountered for silty soils.

To further discriminate the effects of fines on $e_{max}$ and $e_{min}$, regression analyses were carried out separately for each of the four soil groups. Figures 12(a)–12(d) show the respective data of these soils together with the best-fit

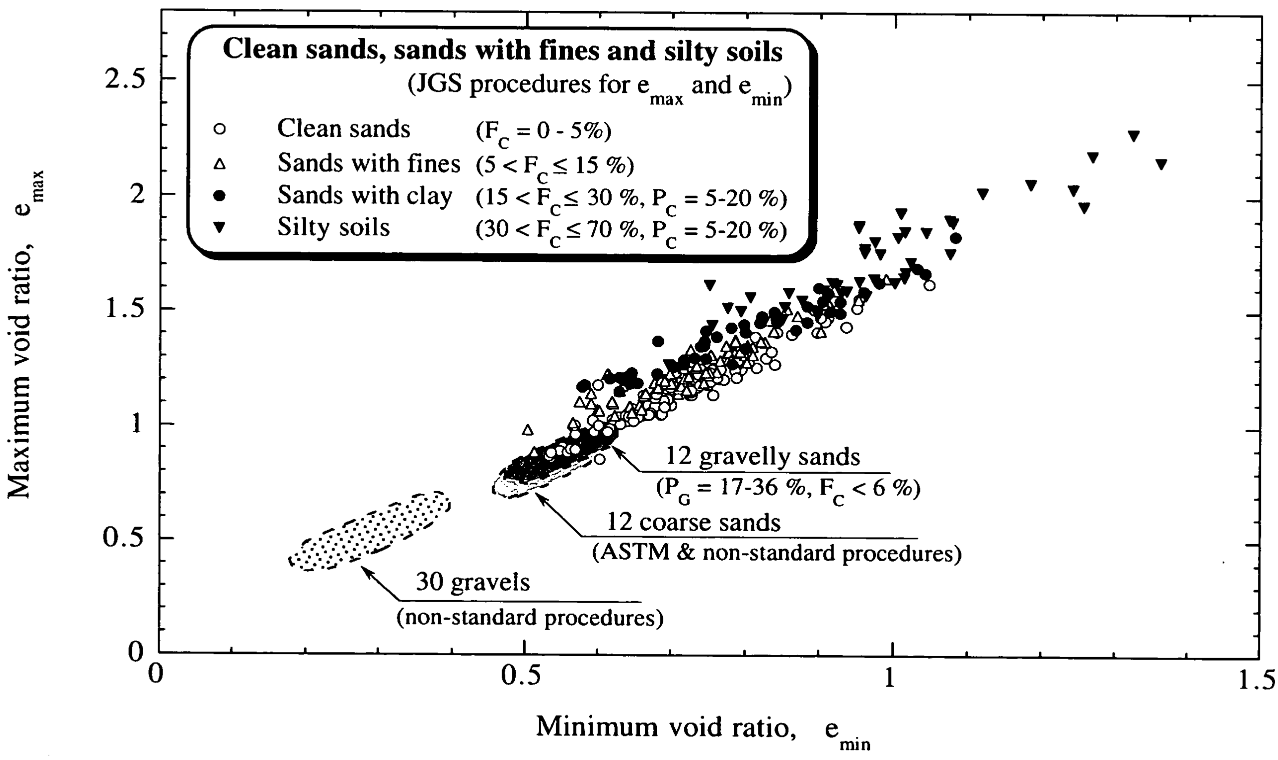


**Fig. 11. Correlation between** $e_{max}$ **and** $e_{min}$ **of sandy soils from natural deposits**

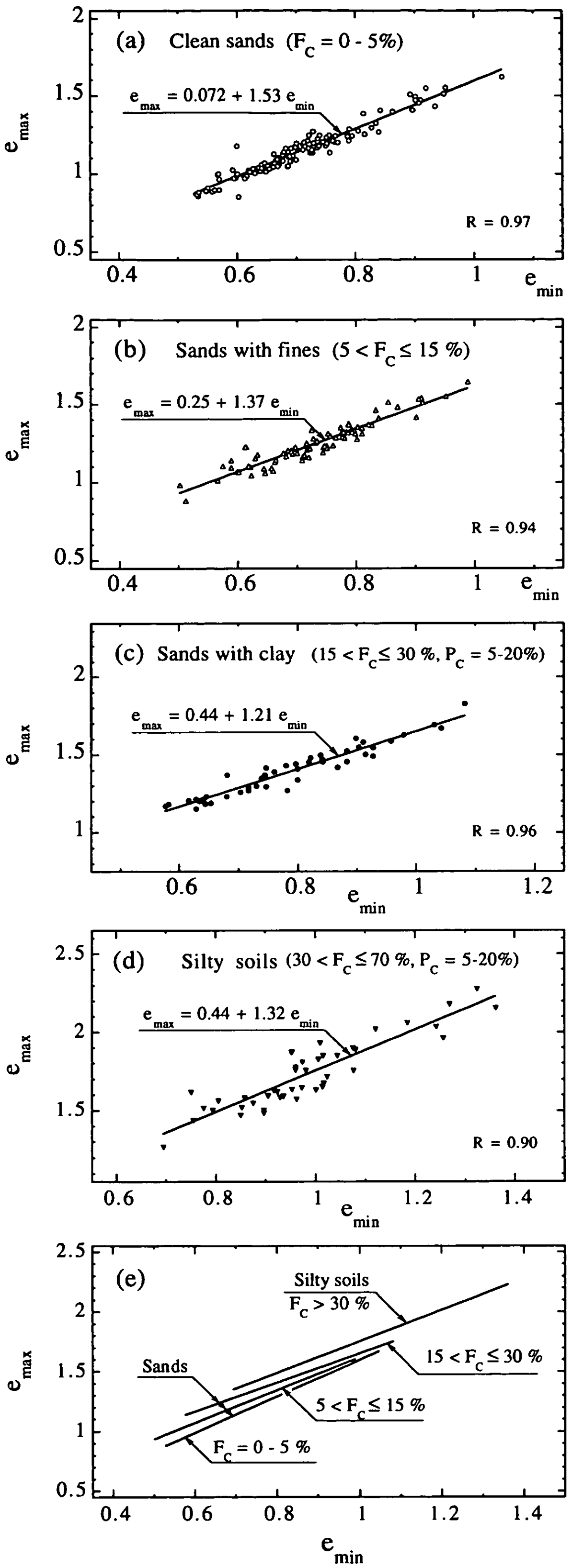


**Fig. 12. Relationships between $e_{max}$ and $e_{min}$ of sandy soils: (a) clean sands, (b) sands with fines, (c) sands with fines and clay, (d) silty soils and (e) summary plot for all soils**

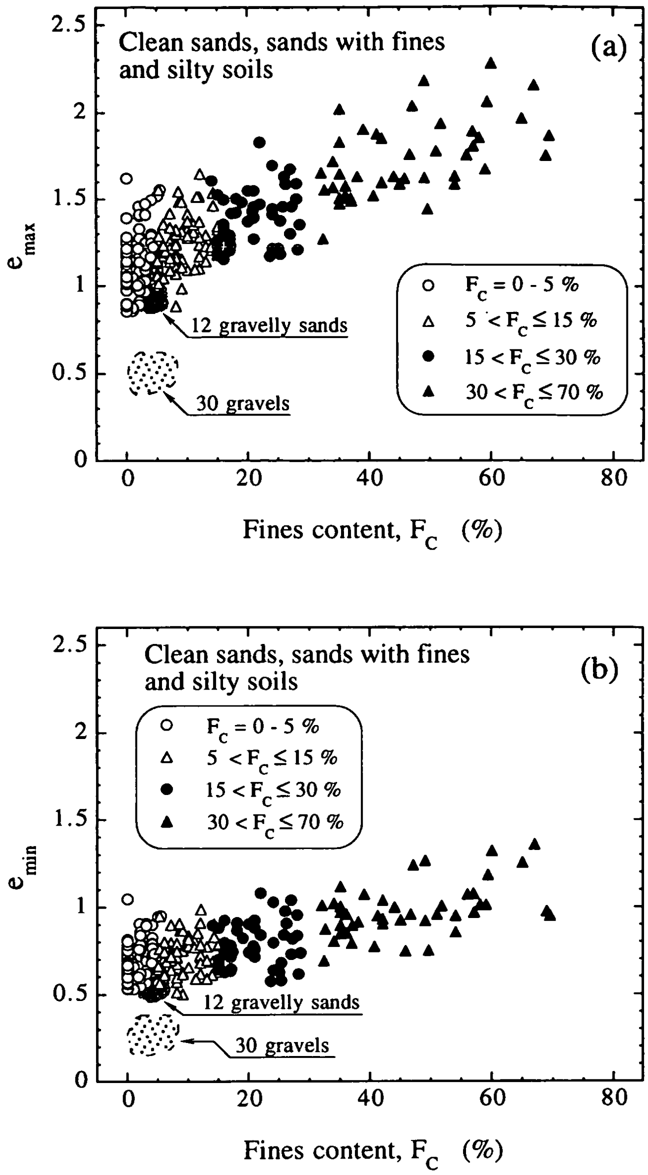


**Fig. 13. Effects of fines on $e_{max}$ and $e_{min}$ of sandy soils: (a) $e_{max}$ vs. $F_C$ and (b) $e_{min}$ vs. $F_C$**

linear regression lines as defined below:

(a) Clean sands ($F_C$=0–5%):

$$e_{max}=0.072+1.53e_{min} \tag{3}$$

(b) Sands with fines ($5<F_C\leq 15\%$):

$$e_{max}=0.25+1.37e_{min} \tag{4}$$

(c) Sands with fines and clay ($15<F_C\leq 30\%$, $P_C$=5–20%):

$$e_{max}=0.44+1.21e_{min} \tag{5}$$

(d) Silty soils ($30<F_C\leq 70\%$, $P_C$=5–20%):

$$e_{max}=0.44+1.32e_{min} \tag{6}$$

Apparently, very good linear correlation between $e_{max}$ and $e_{min}$ is observed for clean sands, sands with 5–15% fines and sands with 15–30% fines, as exemplified by the high correlation coefficients of 0.97, 0.94 and 0.96 respec-

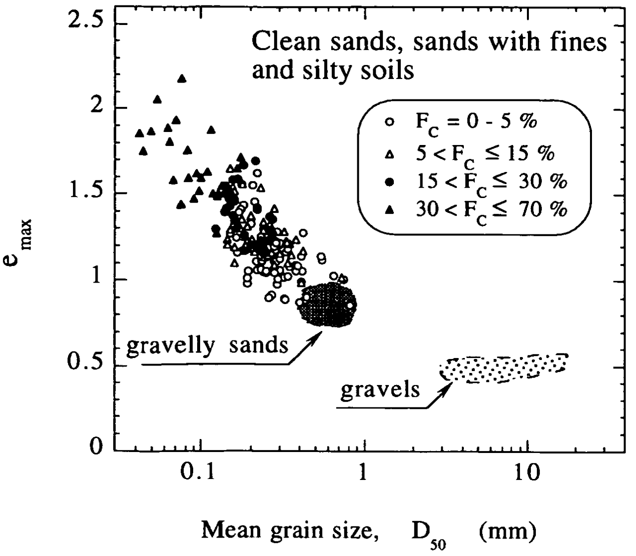


**Fig. 14. Influence of mean grain size on maximum void ratio**

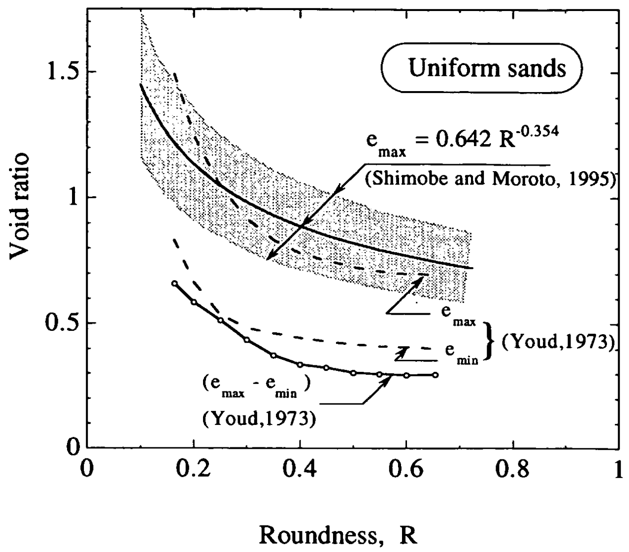


**Fig. 15. Influence of particle shape on $e_{max}$, $e_{min}$ and ($e_{max}-e_{min}$) of uniform sands**

tively. For silty soils, some scatter in the data is observed resulting in a lower correlation coefficient of 0.90. It is of importance that the correlation for fines-containing sands is as good as that of clean sands. Since $e_{max}$ and $e_{min}$ are determined from two independent tests that have opposite targets, namely to produce the loosest state and the densest state of a sand respectively, the high degree of correlation as observed in Fig. 12 indicates that standard JGS procedures can provide reasonably consistent maximum and minimum void ratios for sands with up to 30% fines content. It might be further argued that the correlations shown in Fig. 12 are an evidence confirming the applicability of the maximum and minimum void ratios even for the range of fines-containing sands. The expressions given in Eqs. (3)–(5) can be used to approximately evaluate $e_{max}$ from $e_{min}$ and vice versa, for clean sands and sands with less than 30% fines, under the assumption that the standard JGS procedures are adopted for determination of the maximum and minimum densities. It is to be noted that most of the considered soils with clay-size particles have a clay content of less than 15%, and that effects of clay-size fractions and plasticity of fines could not be evaluated in this study. Therefore, caution is needed when applying the above expressions to sands containing either plastic fines or predominantly clay-size fines.

The best fit linear correlations of the four soil types are comparatively plotted in Fig. 12(e) where it is observed that a distinct correlation exists for each soil type with an apparent trend in the correlation to gradually shift upwards as the fines content increases in the sand. In other words, for a given $e_{min}$ value, the maximum void ratio increases with the fines content. This in turn suggests that the effects of fines on $e_{max}$ and $e_{min}$ are different.

*Effects of Fines, Grain Size and Grain Shape*

The maximum and minimum void ratios are plotted against the fines content of the sands in Figs. 13(a) and 13(b). Over the entire range of fines, from 0% to approximately 70%, there is nearly a proportional increase in $e_{max}$ with the fines content. The minimum void ratio, on the other hand, shows only a slight increase with $F_C$ from 0 to 30% fines, while a higher rate of increase in $e_{min}$ is observed for soils with more than 30% fines content. This change in the relationship between $e_{min}$ and $F_C$ will be more clearly illustrated in the following section through the use of the void ratio range ($e_{max}-e_{min}$).

Comparing Figs. 13(a) and 13(b) with the corresponding Figs. 8(a) and 8(b) of the composite soils, it is apparent that unlike the gap-graded sand-fines mixtures, natural sands do not show any notable drop in $e_{max}$ or $e_{min}$ as the fines content increases from 0 to 30%. This appears to reflect a relatively gradual change in the grain-size distribution between the coarser and finer grain fractions for majority of the natural sands. The absence of gaps in the grading of natural sands has to be taken into account when interpreting the behaviour of sand-fines mixtures produced in the laboratory. In this context, it is conceivable that sand-fines composites with a wide gap in the grading may not be representative of the actual packing structure of natural sands with fines.

Whereas a clear trend for the increase in $e_{max}$ and $e_{min}$ with increasing fines content is observed in Fig. 13, the large scatter of data indicates that the maximum and minimum void ratios are significantly affected by material properties other than the fines content. For example, grain-size effects on the maximum void ratio are clearly seen in Fig. 14 where $e_{max}$ is plotted against the mean grain size $D_{50}$ of the examined soils. Evidently, $e_{max}$ increases with a decrease in the mean grain size of the soil with the tendency being more pronounced for fine-grained soils indicating the importance of fines-content and plasticity of fines in the packing of fine soils. It is apparent from the previously established proportional relationships between $e_{min}$ and $e_{max}$ that a similar correlation exists between $e_{min}$ and $D_{50}$.

It is seen in Figs. 12(a) and 13(a) that $e_{max}$ of clean sands takes values over a wide range between 0.85 and 1.5. A significant part of the scatter in the maximum void ratio is due to the fact that the void ratio is affected by the grain shape in a way that it increases with increasing angularity or decreasing roundness of the particles. This influence is illustrated in Fig. 15 where correlation between $e_{max}$ and roundness of particles $R$ of more than 40 uniform sands ($U_C<2.0$) is shown quoted from the data of Shimobe and Moroto (1995). Superimposed in this figure are $e_{max}-R$ and $e_{min}-R$ relationships of uniform clean sands ($U_C=1.4$) defined by Youd (1973). Here, the roundness $R$ is defined as the ratio of the average radii of the corners of a sand grain image to the radius of the maximum circle that can be inscribed within the grain image (Youd, 1973). All relationships in Fig. 15 show a clear tendency for decrease in the void ratio with increasing roundness of the particles.

In view of the evidence presented above, it may be concluded that the scatter of the data seen in Fig. 13 is primarily due to differences in the angularity or roundness of the grains as well as differences in the grain-size distribution of the considered soils. Generally, uniform angular soils will tend to be located in the upper zone of the correlation in Fig. 13, and conversely, well-graded and gap-graded round-grained soils will have relatively low $e_{min}$ and $e_{max}$ values, for any given fines content.

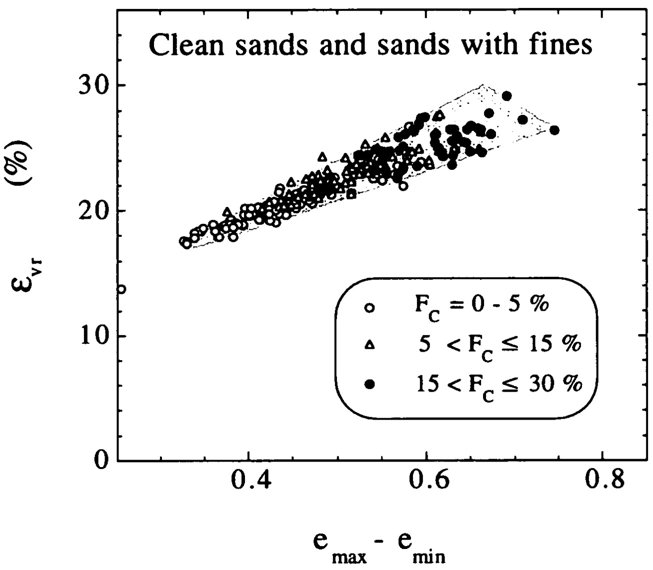


**Fig. 16. Volumetric strain potential as a function of ($e_{max}-e_{min}$) of sands**

## VOID RATIO RANGE

By definition, the void ratio range specifies the difference between the void ratio of the loosest state and that of the densest state of a sand, ($e_{max}-e_{min}$), and therefore it is indicative of the degree of possible variation in the packing of the sand. Alternatively, this variation in the packing of a sand can be expressed by the volumetric strain range $\varepsilon_{vr}$ or volumetric strain induced in the soil when densifying it from its loosest state ($e_{max}$) to the densest state ($e_{min}$), as defined by:

$$\varepsilon_{vr}=\frac{e_{max}-e_{min}}{1+e_{max}} \tag{7}$$

By rearranging the data of the compiled natural sands, the volumetric strain range is plotted against ($e_{max}-e_{min}$) in Fig. 16. It is seen that $\varepsilon_{vr}$ increases with increasing ($e_{max}-e_{min}$) of sands, which is a consequence of the previously introduced relationship between $e_{max}$ and $e_{min}$, and the link between $\varepsilon_{vr}$ and ($e_{max}-e_{min}$) given in Eq. (7). The characteristics of the void ratio range as above bring its capacity to correlate with the compressibility or contractiveness of

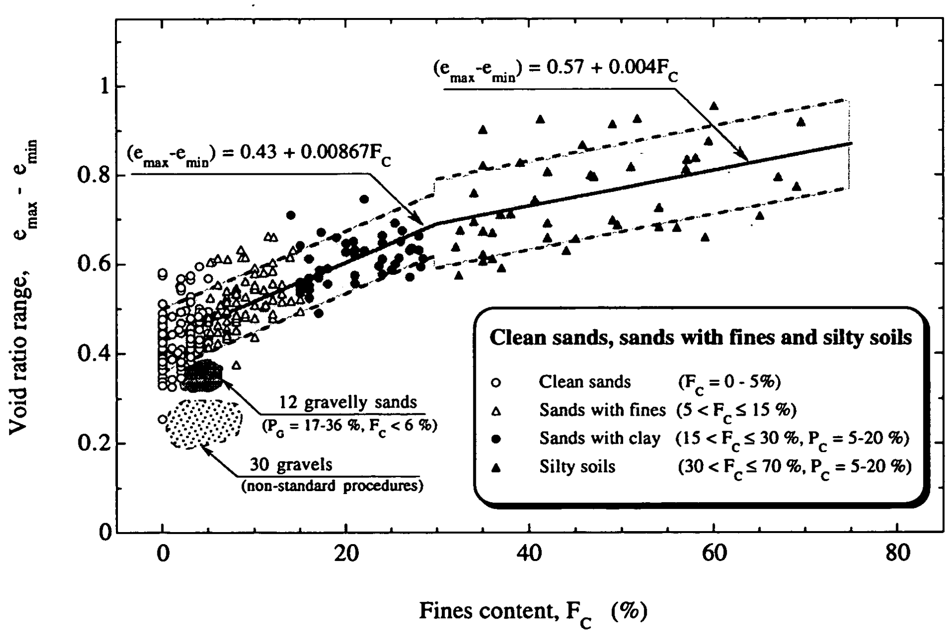


**Fig. 17. Relationship between void ratio range and fines content of sandy soils**

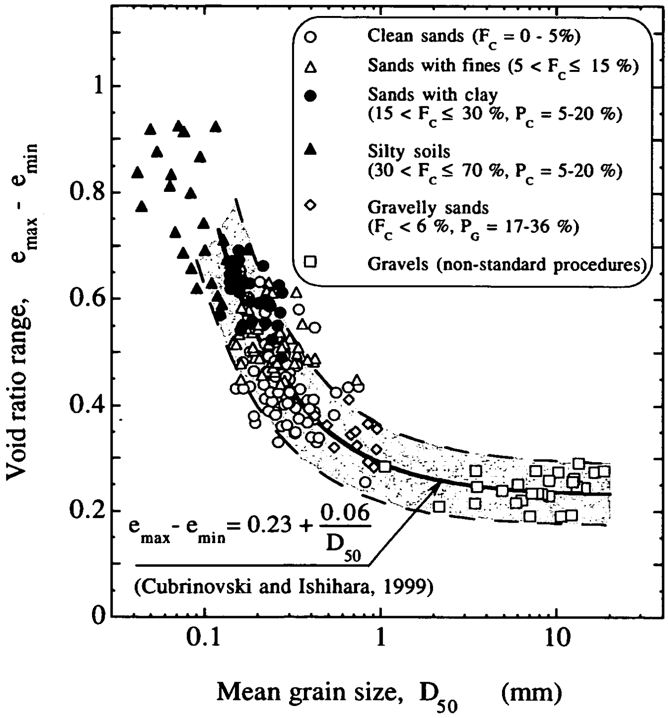


**Fig. 18. Relationship between void ratio range and mean grain diameter of sandy and gravelly soils**

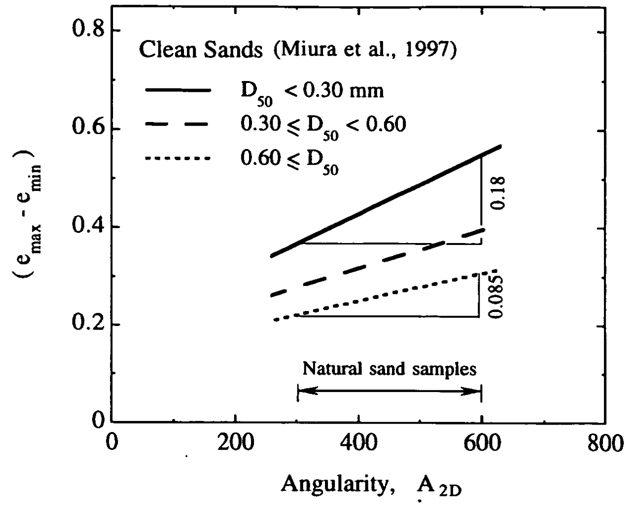


**Fig. 19. Effects of angularity on $(e_{max}-e_{min})$ of clean sands**

soils, as illustrated in the relationships shown in Fig. 1.

To examine the link between the void ratio range and fines content, $(e_{max}-e_{min})$ is plotted against $F_C$ for the compiled soils in Fig. 17. For purpose of comparison, this figure also shows the range of data for 30 gravels and 12 gravelly sands. As far as the clean sands and sands with fines are concerned, there is a steady increase in the void ratio range as the fines content increases from 0 to 30%. A much smaller rate of increase in $(e_{max}-e_{min})$ with $F_C$ is seen for soils with more than 30% fines. Significantly, a change of pattern in the relationship is again observed at approximately 30% fines. This threshold fines content of about 30% can be explained by the key difference in the particle structure of sands with 0–20% fines as compared to sands with more than 30% fines. Namely, experimental findings suggest that for sands with more than 30% fines, the fines are the controlling grain fraction in the particle structure and deformational behaviour of these soils. On the other hand, for fines content below 20%, the packing structure is obviously dominated by the sand matrix, but importantly, the role of the fines in the densest packing can be essentially different from the role of the fines in the loosest state of packing. Thus, it is conceivable that for a fines content of 0 to 20%, the effects of $F_C$ on $e_{max}$ and $e_{min}$ could be different while in the case of $F_C>30\%$, $e_{max}$ and $e_{min}$ are similarly influenced by the fines content. The correlation in Fig. 17 where relatively small change in $(e_{max}-e_{min})$ is seen for $F_C>30\%$ is in accordance with this kind of reasoning.

The relatively wide variation of $(e_{max}-e_{min})$ for a given fines content in Fig. 17 is not surprising in view of the presented evidence that both $e_{max}$ and $e_{min}$ are significantly affected by the grain size distribution and grain shape of soils. Focusing on the data with less than 5% fines, it is apparent that gravels have the smallest $(e_{max}-e_{min})$ values, gravelly sands have slightly higher void ratio ranges while $(e_{max}-e_{min})$ of clean sands are the highest in the plot. This dependence of the void ratio range on the grain size of soils is illustrated in Fig. 18 where $(e_{max}-e_{min})$ is plotted against $D_{50}$ of the soils. The difference in the angularity or roundness of the particles of different soils is another major factor causing the scatter of the correlation in Fig. 17. Miura et al. (1997) have shown that the void ratio range is affected by the grain shape in a way that $(e_{max}-e_{min})$ increases with increasing angularity of sands. They used a two-dimensional angularity $A_{2D}$ as a measure for quantifying the grain shape, where $A_{2D}=0$ for round grains, and $A_{2D}$ increases with increasing angularity of the grain. Most of the natural sand samples used in their study were found to have an angularity in the range of $A_{2D}=300$–$600$. On the basis of comprehensive data, Miura et al. (1997) defined three representative linear relationships between $(e_{max}-e_{min})$ and $A_{2D}$ for clean sands with a mean grain size of $D_{50}<0.30$ mm, $0.30\le D_{50}<0.60$ mm and $D_{50}\ge 0.60$ mm respectively. These relationships are shown in Fig. 19 where it is evident that the void ratio range increases with increasing angularity of the grains. It is apparent from the slopes of the relationships that the increase in the void ratio range is more pronounced for fine sands. An identical tendency in the effects of grain shape on $(e_{max}-e_{min})$ is evident in Fig. 15 where the void ratio range derived from the data of Youd (1973) is seen to decrease with the roundness $R$. Note that angularity and roundness are indices for the grain shape that have negative correlation, i.e., increasing angularity implies decreasing roundness and vice versa. Quantification of the grain shape effects on $(e_{max}-e_{min})$ was not attempted herein since the data compiled in the present study do not contain information about the particle shape.

The effects of fines content, mean grain size and particle shape on the void ratio range are jointly indicated in

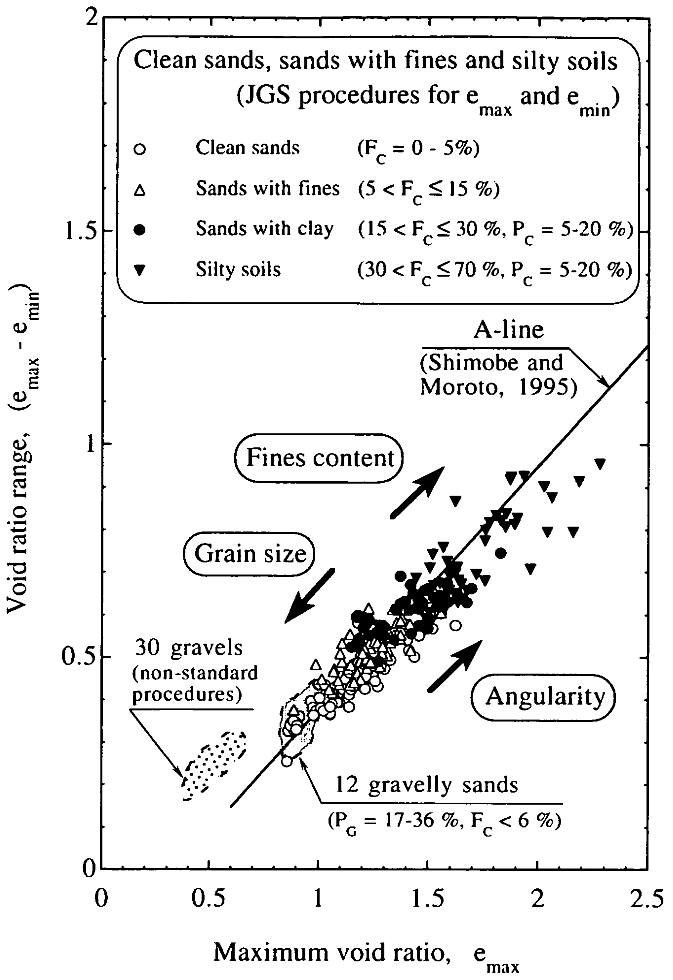


**Fig. 20. Illustration of effects of material properties on ($e_{max}-e_{min}$)**

Fig. 20 where ($e_{max}-e_{min}$) is plotted against $e_{max}$ of the compiled soils. It is seen that the increase in the mean grain size of soils has an opposite influence on ($e_{max}-e_{min}$) as compared to that of an increasing fines content or angularity of particles. The indicators in this figure approximately show the general trends of influence, but it should be recognized that the influence of each of these factors is more or less affected by the remaining factors considered. It is interesting to notice the analogy between this chart and the plasticity chart for clays where the plasticity index $I_P=w_L-w_P$ is plotted against $w_L$. Here, $w_L$ and $w_P$ are the liquid limit and the plastic limit respectively. Shimobe and Moroto (1995) noted this analogy and defined the so-called A-line based on data of uniform sands for the purpose of classification of cohesionless soils. In view of the evidence presented in this study, it appears difficult to provide rational classification of cohesionless soils based on the chart of Fig. 20 in an efficient way as the plasticity chart works for clays, and therefore, further studies on the possible use of the ($e_{max}-e_{min}$) vs. $e_{max}$ chart for this purpose are needed.

## DISCUSSION

The presented experimental correlations clearly illustrate the link between the characteristic void ratios and material properties of sands. In order to facilitate the use of $e_{max}$, $e_{min}$ and ($e_{max}-e_{min}$) for material characterization, it is useful to summarize the advantages as well as to point out the shortcomings of the void ratio range as compared to conventional parameters such as the fines content and mean grain size, as given below:

(a) Unlike the partial and practically single-point description of the grain-size distribution curve provided by $F_C$ and $D_{50}$, the void ratio range expresses the combined influence of all the grain sizes that constitute a given sand.

(b) Since the void ratio range is affected by the angularity or roundness of the grains, it also permits consideration of the particle shape characteristics of sands. Thus, the key advantage of the void ratio range as opposed to $F_C$ and $D_{50}$ is that ($e_{max}-e_{min}$) reflects the overall grain-size composition and particle characteristics of a given sand.

(c) The void ratio range can be applied to clean sands, sands with fines and gravelly sands thus providing a common basis for comparative evaluation of a wide range of cohesionless soils. Another point of importance is that ($e_{max}-e_{min}$) permits distinguishing different material properties among soils belonging to a same generic group, say clean sands ($F_C<5\%$), soils for which apparently the fines content is not an effective material parameter.

(d) An obvious disadvantage of the void ratio range would be that it is more difficult to determine than $F_C$ and $D_{50}$, parameters which are readily available from the gradation curve.

The fact that ($e_{max}-e_{min}$) is affected by various material properties such as the mean grain size, grain-size distribution, fines content and particle shape might be considered as a shortcoming in certain cases, because ($e_{max}-e_{min}$) manifests the combined effects of all these factors and separation of individual contributions could be difficult. This is particularly important for cases in which two factors, for example the fines content and angularity, have opposite effects on the subject behaviour of sand. Thus, when using ($e_{max}-e_{min}$) as an index property or normalizing parameter for sands, one should examine what would be the most effective use of the void ratio range by taking into account its correlation with the material properties as well as the characteristics of the sand behaviour being considered. The authors are of the opinion that, by and large, ($e_{max}-e_{min}$) can provide valuable and unique information about the material properties of sandy soils. In particular, ($e_{max}-e_{min}$) can be effective in discriminating among different potentials of compressibility and contractiveness of cohesionless soils.

## CONCLUDING REMARKS

Data of over 300 natural sandy soils were used to examine the link between characteristic void ratios and material properties of sands. The principal findings can be summarized as follows:

(1) Good correlation exists between the maximum and minimum void ratios of sands with fines content of less than 30%. To clearly discriminate the effects of fines, three distinct linear correlations between $e_{max}$ and $e_{min}$ were established for clean sands, sands with 5–15% fines and sands with 15–30% fines ($P_C$ = 5–20%), respectively. The correlations have high correlation coefficients of 0.94–0.97 and can be used to approximately evaluate $e_{max}$ from $e_{min}$ and vice versa, under the assumption that the standard JGS procedures for sands are adopted for determination of the minimum and maximum densities. The presented evidence suggests that the standard JGS procedures for clean sands can provide reasonably consistent $e_{max}$ and $e_{min}$ values for sands with a fines content of up to 30%.

(2) The effects of fines on $e_{max}$, $e_{min}$ and ($e_{max} - e_{min}$) are essentially related to the role of the fines in the particle structure. It was found that, for sands with less than 30% fines, the effects of fines on $e_{max}$ and $e_{min}$ are different, as indicated by the notable increase in ($e_{max} - e_{min}$) as the fines content increases from 0 to 30%. On the other hand, sands with more than 30% fines show fairly similar influence of the fines on $e_{max}$ and $e_{min}$. A threshold fines content of about 30% is seen for both composite sands and natural sandy soils indicating a transition from a sand dominated particle structure to a fines controlled particle structure.

There is a significant difference in the variation of $e_{max}$ and $e_{min}$ with $F_C$ between gap-graded sand-fines composites and natural sandy soils. Wide gaps in the grain-size distribution may significantly affect the particle structure of sands and these effects should be carefully considered when producing composite soils by mixing sands with fines in the laboratory.

(3) The maximum and minimum void ratios as well as the void ratio range are significantly affected by the particle shape in a way that these void ratios increase with increasing angularity or decreasing roundness of the particles. Thus, for a known fines content and grain-size characteristics of a sand, $e_{max}$, $e_{min}$ or ($e_{max} - e_{min}$) enable us to indirectly assess the particle shape characteristics of the sand.

(4) The void ratio range embodies the combined effects of mean grain size, grain-size distribution, fines content and particle shape, and therefore, it reflects the overall grain-size composition and particle characteristics of a sand. In general, ($e_{max} - e_{min}$) can provide valuable and unique information about the material properties of sandy soils and it can be particularly effective in evaluating the potential of compressibility and contractiveness of cohesionless soils. It can also serve as a general basis for comparative evaluation of overall material properties over the entire range of cohesionless soils.

The void ratio characteristics discussed in this study should not be looked upon as an isolated feature of the packing of sands, but rather they should be viewed through the fact that the particle structure is directly reflected in the mechanical behaviour of sands.

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