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STRESS-HISTORY EFFECTS ON SHEAR MODULUS OF SOILS

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ABSTRACT

The effects of time duration of the confining pressure and stress-history patterns were studied through resonant column tests of soil samples. For each soil the dynamic shear modulus obtained at small shearing strain amplitudes was the quantity evaluated as the testing conditions were changed.

Under uniform confining pressures an increase in shear modulus occurred which was approximately linear with log time after about 1000 minutes of pressure application. This rate of shear modulus increase per log cycle was expressed as a per cent of the numerical value of the shear modulus at 1000 minutes, to minimize the influence of the pressure magnitude for each test. For soils with median grain size larger than about 0.04 mm the per cent increase in shear modulus per log cycle was about 3% or less. For dry and saturated samples of kaolinite, this increase averaged about 6% to 13%. The effect of overconsolidation was to reduce slightly this rate of shear modulus increase per log cycle.

Soils with median grain size of 0.04 mm or greater showed little change in the small amplitude shear modulus as the time of applied confining pressure increased, or little influence of previous preconsolidation. For finer soils ($D_{50} \ll 0.04$ mm) the effects of time of loading and of preconsolidation were significant. Consequently, extrapolation of laboratory shear moduli of fine-grained soils to field applications requires careful study.

Key words: dynamics, overconsolidation, resonant column, shear modulus, time effects

IGC: D7/D6

INTRODUCTION

The shear moduli of soils determined from small amplitude dynamic tests are often needed in studies of dynamic soil-structure interaction. Hardin and Black (1968) have noted some of the parameters which may influence the shear modulus. These include the void ratio, effective octahedral normal stress, soil structure, saturation, amplitude of vibration, frequency of vibration, temperature, stress history and vibration history, and secondary effects that are functions of time and stress patterns. Of course, several of these variables may be interrelated, but during the past decade continuous study has been directed toward evaluating the effects of each parameter. The investigation reported in this paper was directed toward evaluation of the stress history and time effects which caused variations in the low amplitude shear modulus of seven soils.

The low amplitude dynamic shear modulus was evaluated from solid cylindrical soil

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samples tested in a resonant column device. In this device the cell pressure can be changed readily to provide the appropriate confining pressure during each stage of the test. Pressure-time patterns were chosen for study of the influence of pressure levels, overconsolidation ratio, and secondary time effects which developed in the samples. The normal duration of each pressure increment on the fine-grained soils was from two days to one week, which required total testing times as noted in Table 1.

Table 1. List of tests

Stress history No.	Range of void ratio	Material	Range of pressure, psi	Duration of test, days
1	0.48	Air-dry Ottawa sand (30-50)	30	112
1	0.48		30	22
1	0.49		30	88
1	0.71	Air-dry Agsco No. 1	20	70
1	0.71		30	4
1	1.06	Air-dry Agsco No. 2	10	430
1	0.88	Air-dry Agsco No. 4	20	50
1	0.87-0.86	Air-dry Agsco No. 1250	20	140
1	2.04-1.95	Air-dry kaolinite HUF	20	35
1	0.99-0.95	Saturated kaolinite EPK	10	8
1	1.00-0.92		10	2
1	0.75-0.91		20	8
1	0.97-0.95		10	5
1	0.98-0.95		10	10
1	0.97-0.95		10	7
1	0.95-0.92		20	43
1	1.01		10	1
2	0.48	Air-dry Ottawa sand (30-50)	20-40	90
2	0.49		20-40	10
2	0.47		20-40	112
2	0.67-0.66		20-40	10
2	0.70	Air-dry Agsco No. 1	20-40	26
2	0.69		20-40	10
2	1.07	Air-dry Agsco No. 2	20-40	9
2	1.01		20-40	12
2	0.99-0.97	Air-dry Agsco No. 4	20-40	9
2	0.87-0.83	Air-dry Agsco No. 1250	30-40	42
2	1.00-0.96		30-40	46
2	0.70-0.69		20-40	135
2	1.27-1.30	Air-dry kaolinite EPK	20	803
			30	30
			40	20
			20	210
2	1.26-1.21		20-40	187
2	1.44-1.29		20-40	112
2	1.00-0.84	Saturated kaolinite EPK	20-40	68
3	0.71-0.69	Air-dry Agsco No. 1250	20-60	34
3	1.28-1.22	Air-dry kaolinite EPK	20-60	34
3	1.30-1.20		20-60	37
3	2.01-1.89	Air-dry kaolinite HUF	20-60	32
3	0.99-0.85		20-60	28
Oedometer tests	1.62-1.44	Air-dry kaolinite EPK	5-40	20
	1.44-1.36	Air-dry kaolinite EPK	10-40	20
	0.82-0.72	Saturated kaolinite EPK	2.5-50	8
	0.92-0.82	Air-dry Agsco No. 1250	5-40	8

REVIEW OF PREVIOUS WORK

Studies of soil properties by the resonant column method have been summarized by Richart, Hall, and Woods (1970) covering the three decades preceding 1969. Most of the research activity occurred in the 1960's.

Hardin and Richart (1963) determined that for clean cohesionless soils the low amplitude shear modulus was primarily a function of the void ratio and the octahedral (or average) normal confining stress. The results of this study can be expressed as empirical equations

$$\underset{(\mathrm{lb/in^2})}{G} = \frac{2630(2.17-e)^2}{1+e} \underset{(\mathrm{lb/in^2})}{(\bar{\sigma}_0)^{0.5}} \qquad (1)$$

for round-grained sands ($0.3<e<0.8$) and

$$\underset{(\mathrm{lb/in^2})}{G} = \frac{1230(2.97-e)^2}{1+e} \underset{(\mathrm{lb/in^2})}{(\bar{\sigma}_0)^{0.5}} \qquad (2)$$

for angular-grained materials ($0.6<e<1.3$). In Eqs. (1) and (2) the shear modulus, G, and the octahedral normal stress (or average effective confining pressure), $\bar{\sigma}_0$, are expressed in lb/in^2, and e is the void ratio. Hardin and Black (1968) also showed that Eq. (2) could be used to establish a first estimate of G for normally consolidated clays of low activity. However, they suggested that further studies were needed to determine the time-dependent effects for cohesive materials. Humphries and Wahls (1968) also found that Eq. (2) was useful for estimating G for normally consolidated kaolinite samples, but it was not appropriate for bentonite samples.

Hardin and Black (1966, 1968) demonstrated that the low amplitude shear modulus was essentially independent of the octahedral shearing stress, τ_0, but it depended only on the octahedral normal stress, $\bar{\sigma}_0$.

The effect of amplitude of shearing strain on the effective shear modulus and damping of soil samples has been found significant. Hall (1962), Hall and Richart (1963), Drnevich (1967), Drnevich and Richart (1970), and Hardin and Drnevich (1972a, 1972b) have found that shearing strain amplitudes greater than about 10^{-5} cause a decrease in shear modulus and an increase in hysteretic damping. Hardin and Drnevich (1972a, 1972b) introduced a slightly modified hyperbolic shearing stress vs. shearing strain curve which adequately represented the changes in shear modulus and damping with increasing shearing strain amplitude. Seed and Idriss (1970) compared the available data on the influence of shearing strain amplitude (including the Hardin and Drnevich test results) and confirmed the applicability of a modified hyperbolic stress strain relation to represent the effects of this parameter. Thus the amplitude extrapolation procedure is adequately established, and the key value becomes the low amplitude shear modulus ($G_{\mathrm{max.}}$) which must be determined for each soil under its local conditions.

The effect of time of load duration is one of the testing parameters which has not been completely evaluated. Afifi (1970) and Afifi and Woods (1971) have presented test data which help to answer some of these questions. The paper by Afifi and Woods (1971) treated tests on air-dry soils. This present paper covers the tests on some saturated clay samples and relates these test results to those for air-dry samples.

TEST EQUIPMENT

The four resonant column devices used in these tests were of the fixed-free type, having the base of the sample fixed and the top end subjected to the torsional excitation.

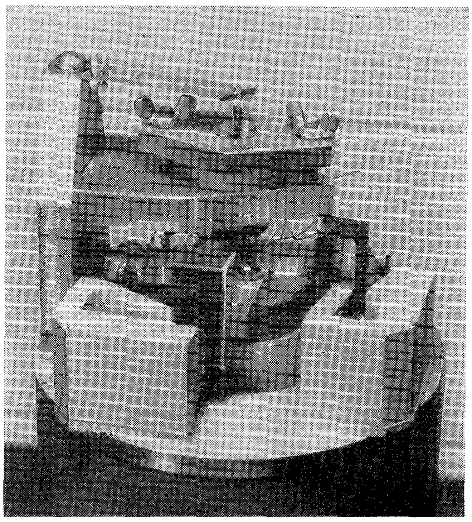


Photo. 1. Driver-pickup system of the resonant column device

They were designed by Dr. J. R. Hall, Jr. and are described in Hall (1962), and Richart, Hall, and Woods (1970). Solid cylindrical soil samples were encased in a rubber membrane and mounted on a pedestal attached to the base of the pressure cell. The air-dry samples were 4 cm diameter and 27 cm high, while the saturated samples had a diameter of 3.6 cm and were 10 cm long. Air pressure was used to apply a uniform confining pressure, $\bar{\sigma}_0$, to samples tested in the drained condition.

Photo. 1 shows the driving and pickup coils as well as the length-measuring system which was attached to the upper end of the sample. The permanent magnets are attached to the supporting frame and the lightweight coils are carried on the vibrating cap. The rectangular shape of the coils and arrangement of the magnets permitted a change in length of the sample up to 0.5 in. without developing mechanical interference between the coils and magnets.

The driver-pickup system is excited by a sinusoidally varying emf applied to the driver coil, which causes the top cap and sample to vibrate in a torsional mode. The torsional vibration produces a sinusoidally varying emf in the pickup coil (which is oriented at 90° to the driving coil to minimize any induced voltage, and to eliminate measurement of any vibration amplitudes which might be caused by bending of the sample) which is proportional to the amplitude of the velocity response of the top of the sample. This motion could produce shearing strain amplitudes of 10^{-6} to 2×10^{-5} in the 27 cm long samples and about three times these amplitudes in the 10 cm long samples. However, in the tests reported herein the shearing strain amplitudes were 10^{-5} or smaller.

The length-measuring system also shown in Photo. 1 consists of a cantilever beam of 0.2 mm thick steel feeler gage stock on which strain gages were mounted to measure bending strains. With this equipment, sample length changes as small as 10^{-4} in. could be measured.

The air pressure system which provided continuous confining pressures to the sample depended primarily on the laboratory supply system, with a nominal 100 psi pressure. A standby pressure system depended upon a gas-powered motor to drive the auxiliary compressor during periods of power failures or other emergencies. This standby system was occasionally needed during long-duration tests.

Figure 1 is a schematic diagram of the test setup required for the resonant column tests. The exciting frequency was varied until the maximum amplitude of response was indicated. This frequency is then the "resonant frequency", which, with the length of the sample and dynamic characteristics of the top cap and sample, permits evaluation of the shear wave velocity in the sample under the testing conditions. Then the shear modulus was calculated from

$$G=v_s^2\rho \tag{3}$$

In Eq. (3), v_s represents the shear wave velocity, and ρ is the mass density $\left(=\frac{\gamma}{g}\right)$ of the sample.

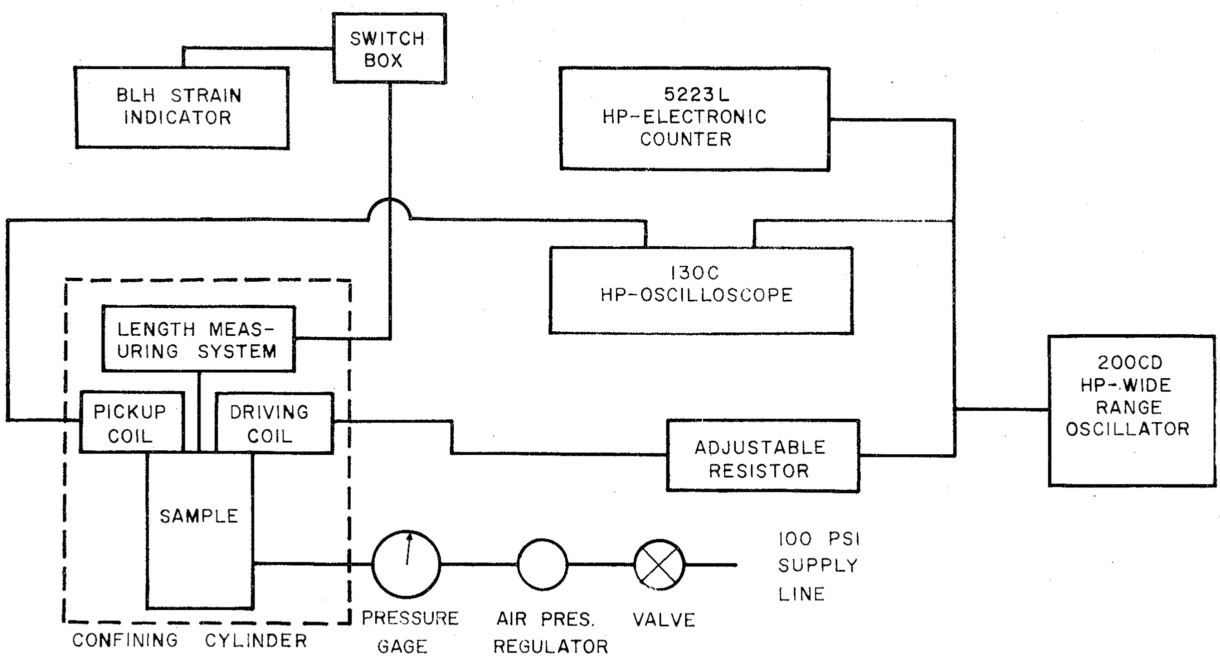


Fig. 1. A schematic diagram of the test setup

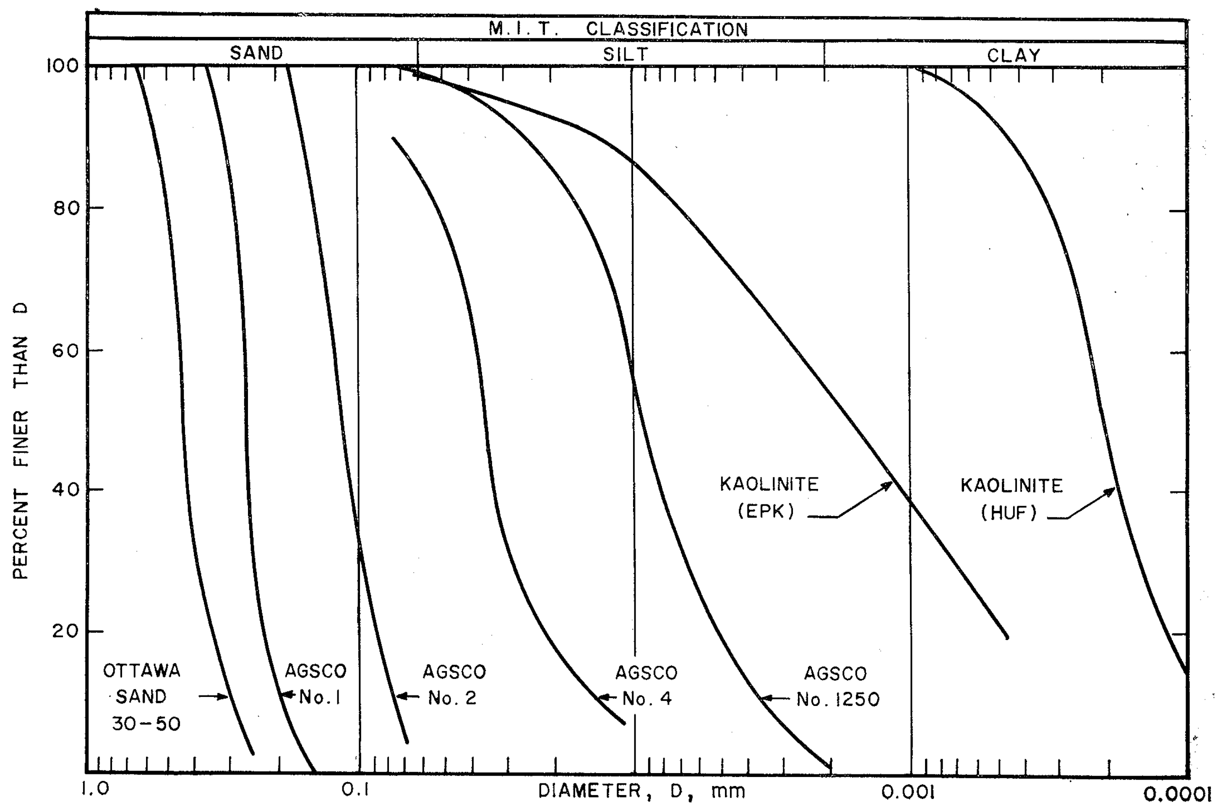


Fig. 2. Grain size distribution curves

MATERIALS

Seven soils having grain size distributions as shown in Fig. 2 were chosen for the study. The sand and silt-sized materials were quartz particles having $G_s=2.66$, the kaolinite EPK had $G_s=2.62$, and kaolinite HUF had $G_s=2.64$. Uniform physical and chemical properties of each of these soils permitted reasonably identical samples to be constructed in the laboratory. These soils had also been selected by previous investigators who studied the effects of other parameters on the dynamic behavior of resonant column samples. For this investigation, however, the principal reason for selecting these soils depended upon the wide ranges of void ratios, densities, grain sizes and grain shapes which could be obtained.

SAMPLE PREPARATIONS

Dry samples were prepared by placing the material inside a rubber membrane held by vacuum against the sides of a mold. Sand samples were "rained" into place by a 45 in. free fall through a glass tube to obtain the dense condition, or allowed to roll into position from the bottom of a sand-filled rubber tube to obtain the loose condition. Air-dry samples of silt and clay sized particles were prepared by compacting the soils in the mold using a special tamper.

The saturated clay samples were prepared in a Vac-Aire extrusion machine. This machine is capable of extruding circular bars of clay having a high degree of saturation and structural uniformity. However, a helical structure is imposed on the sample by the action of the auger of the extrusion machine. This helical structure has been considered acceptable because of its reproducibility in all samples (Schmertmann and Osterberg, 1960).

Preparation of the saturated clay samples began by mixing the clay powder with distilled water to provide a water content of about 35%. The Edgar Plastic Kaolin (EPK—from Croxall Chemical and Supply Company) had a liquid limit of 52% and a plastic limit of 31%. The Hydrite-UF (HUF—from Georgia Kaolin Company) had a liquid limit of 57% and a plastic limit of 27%. The Vac-Aire Extrusion Machine was provided with a circular orifice which controlled the diameter of the samples to 3.6 cm. The extruded bars of clay were cut into 12 cm lengths, and coated immediately with several layers of Gulf Petrowax. Samples were stored at 72°F for a period at least 30 days to insure completion of thixotropic effects (Kashmeeri, 1969).

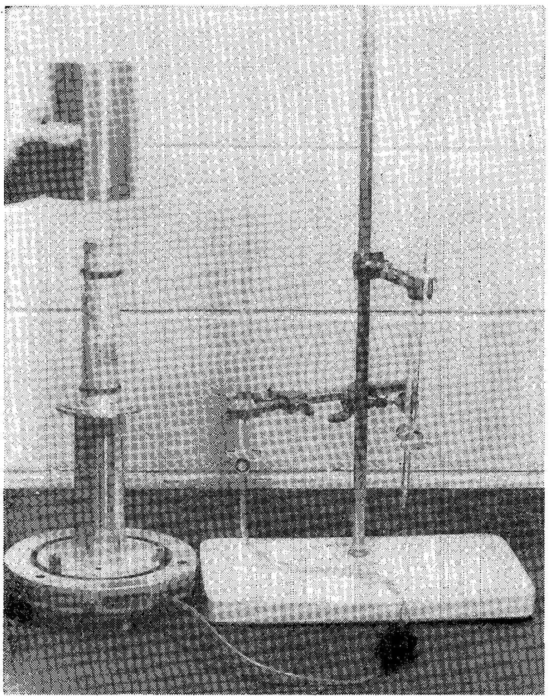


Photo. 2. Installation of fluid jacket cylinder around a sample of saturated clay

When the storage period was completed, the wax was removed, and the samples were trimmed to the correct legth. A wool drain saturated with distilled water was inserted through the longitudinal axis of the sample, filter paper and porous disks were placed on the ends, and four filter paper strips each 8 mm wide were placed at 90° intervals along the height of the sample. Filter paper rings 8 mm wide overlapped the vertical strips and the porous disk at each end of the sample. All filter paper drains were saturated with distilled water before being placed on the sample. The sample was then confined by two rubber membranes with a high-vacuum silicone grease between. Photo. 2 shows the saturated sample in position on the fixed pedestal, with the fluid jacket cylinder ready to be installed. The jacket provided an annular ring of fluid around the sample in order to reduce the leakage of air through the membrane. Initially water was used as the fluid but glycerine was used in a few tests. In tests on saturated soils the samples were sliced and the distribution of moisture content was evaluated. This procedure showed that the moisture content was statistically uniform throughout the samples in most cases.

TESTING PROCEDURES

This investigation considered primarily the influences of confining pressures and length of time each pressure was applied upon the low amplitude shear modulus of the particular sample. Therefore, in testing cohesive materials it was necessary to monitor the progress of primary consolidation to establish the secondary effects which continued after primary consolidation was completed.

Figure 3 shows the stress-history patterns which were applied to each type of sample. The behavior of air-dry samples subjected to stress-history pattern No. 1 (Fig. 3a) was described by Afifi and Woods (1971) and only results needed for comparison with stress-

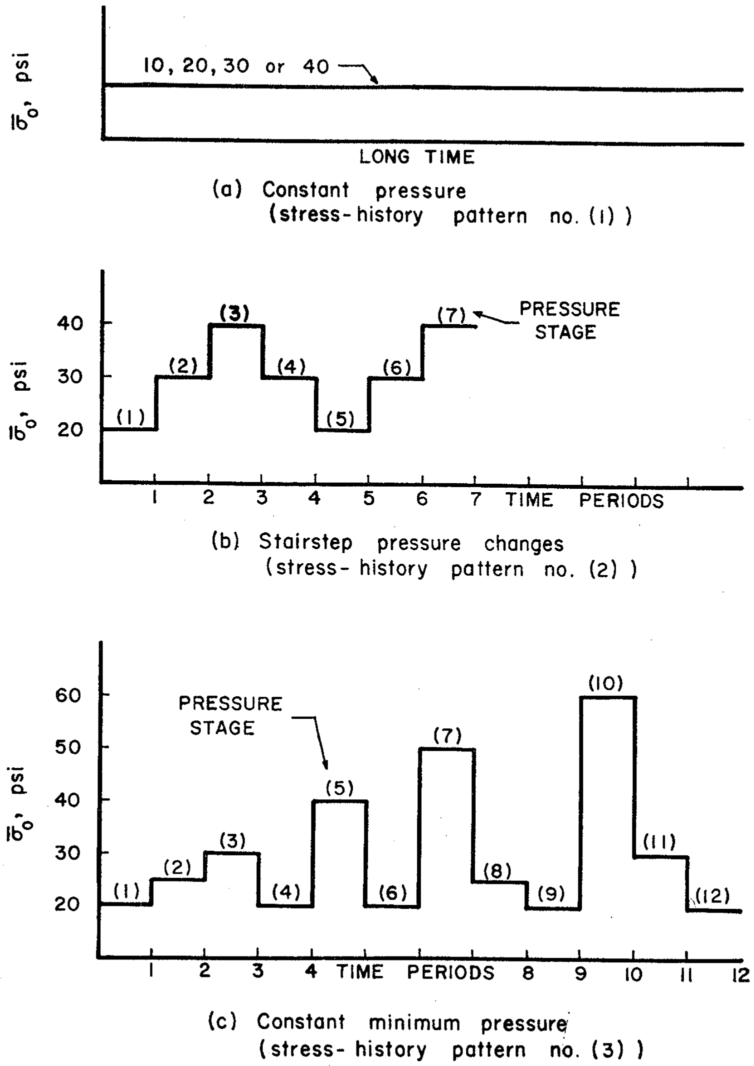


Fig. 3. Stress-history patterns

history patterns No. 2 and No. 3 will be included herein. The stress-history patterns No. 2 and No. 3 (Figs. 3b and 3c) were selected to introduce the effects of different confining stress levels and the influence of the overconsolidation ratio (OCR). Each pressure step illustrated in Fig. 3b was initially sustained for a period of one week to ascertain that secondary time effects were being measured. After a few tests had been completed it became evident that the effects of overconsolidation ratio could be evaluated from pressure steps which lasted one to two days. However, to include the effects of time, a few tests were continued with the one-week duration of pressure steps. The pressure steps in stress-history pattern No. 3 (Fig. 3c) were applied for a maximum period of four days.

In the resonant column tests, the pressure was maintained at the prescribed level throughout the testing interval and intermittent low amplitude vibrations were applied to monitor the time-dependent changes in the resonant frequency. Measurements of the change in sample length were also recorded periodically throughout the time interval to indicate changes of sample volume. In some tests on saturated clay the volume of water squeezed out of the sample was also measured by a burette to indicate the change of sample volume with time.

Standard consolidation tests were conducted on the saturated kaolinite samples to determine the void ratio vs. vertical pressure relationships. Figure 4 illustrates this relationship for the saturated EPK kaolinite. From Fig. 4 it is indicated that the preconsolidation pressures created during extrusion of the samples was on the order of 15 psi. Thus samples tested at pressures less than 15 psi should behave as overconsolidated samples.

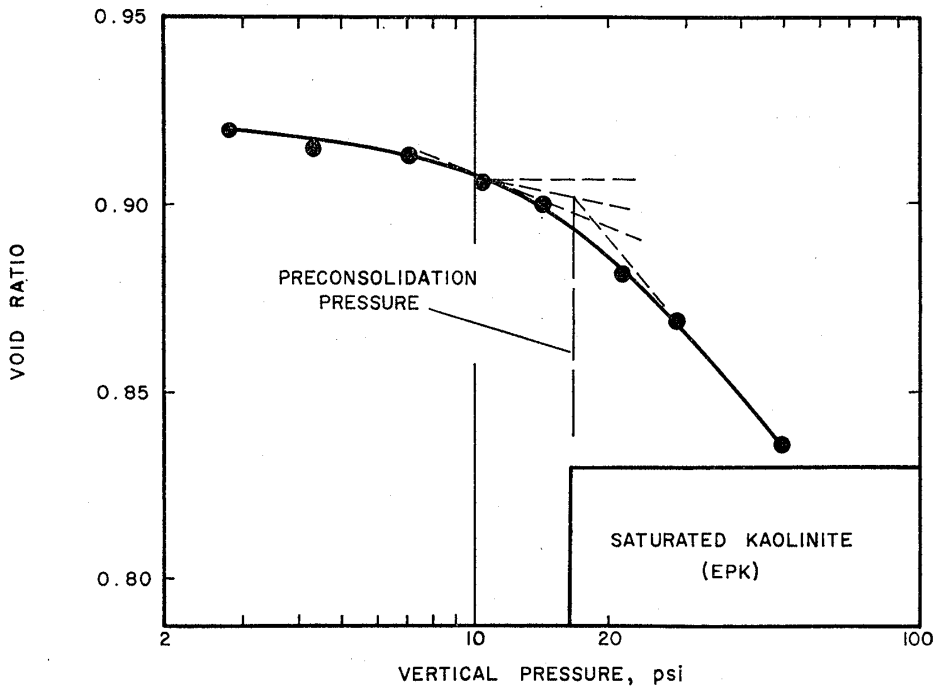


Fig. 4. Void ratio vs. pressure for saturated kaolinite clay

TEST RESULTS

A total of 38 resonant column tests of the seven soils required a testing time of 3145 days. Two tests of air-dried soils used up almost 1500 days of this time. Even with four resonant column devices in operation these tests involved an appreciable length of time. The details of the types of soils, stress patterns, and duration of tests are included in Table 1.

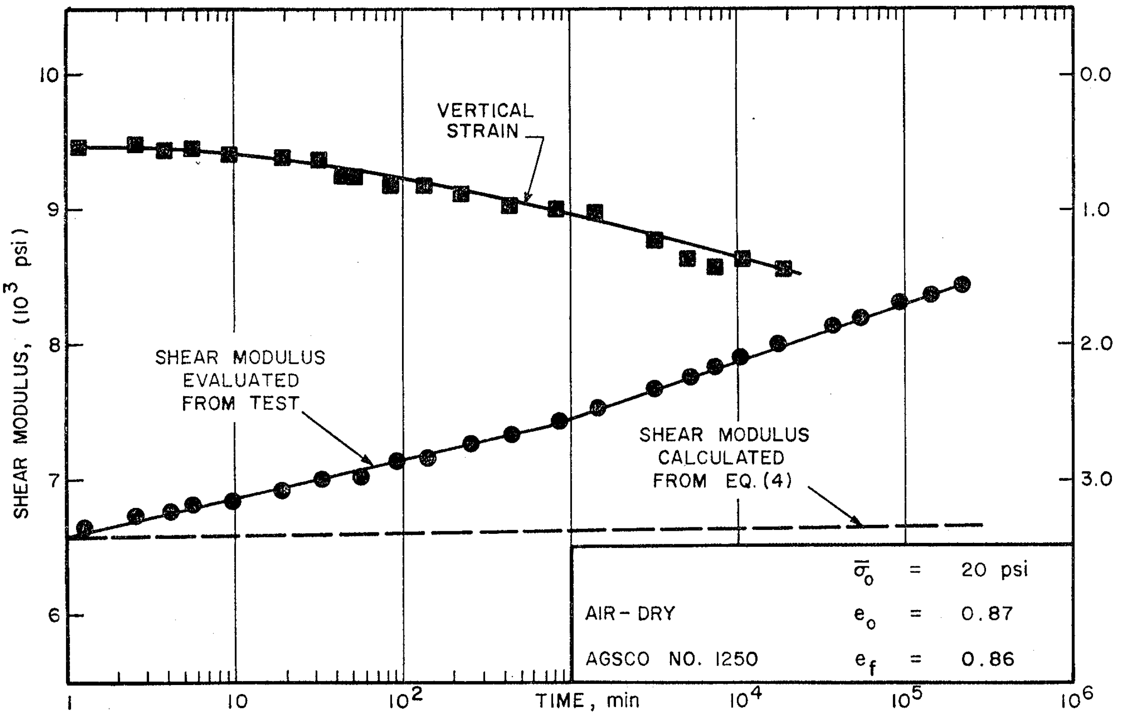


Fig. 5. Variation in shear modulus and vertical strain with time for a sample of air-dry crushed-quartz silt subjected to constant confining pressure

Figure 5 shows the time-dependent increase in shear modulus for a sample of Air-Dry Agsco No. 1250 which was tested under a constant pressure of 20 psi. On this figure the variation of vertical strain with time is also shown to illustrate that secondary compression does occur. These relationships are shown by solid curves on Fig. 5. The dashed line in this figure shows that the time-dependent increase in the shear modulus G cannot be fully accounted for by the time-dependent decrease in void ratio (increase in density) caused by secondary compression. This dashed line was obtained using the procedure described by Hardin and Black (1968) which employs the more general form of Eq. (2),

$$G=A(T)\frac{(2.97-e)^2}{1+e}(\bar{\sigma}_0)^{0.5} \tag{4}$$

To obtain the dashed line in Fig. 5, a value of the constant $A(T)$ was first calculated from Eq. (4) using values of G, e and $\bar{\sigma}_0$ at the end of primary consolidation (1 min for dry soil and 100 min for saturated kaolinite EPK). Then, with the computed value of $A(T)$ assumed constant, Eq. (4) was used again to calculate the change in the shear modulus with time caused by the change in void ratio with time.

A second phenomenon illustrated by Fig. 5 is the break in the G vs. log time curve at approximately 1000 minutes. After this break the straight line relationship between G and log time appears to continue indefinitely, although some difficulties in extended tests prevented a confirmation of this straight line relation beyond about 10^5 minutes.

Effects of Consolidation

All samples of fine-grained soils consolidated after each pressure increment was applied. The curves on Fig. 6 show the changes in sample volume with time as measured by the amount of water squeezed out of the sample (from measurements with burette) and as calculated from measurements of changes in sample height with time. The burette measurements indicate that primary consolidation was completed after about 100 minutes, whereas the height change measurements would suggest that it took about 1000 minutes for completion of primary consolidation (height change measurements also reflect some secondary compression).

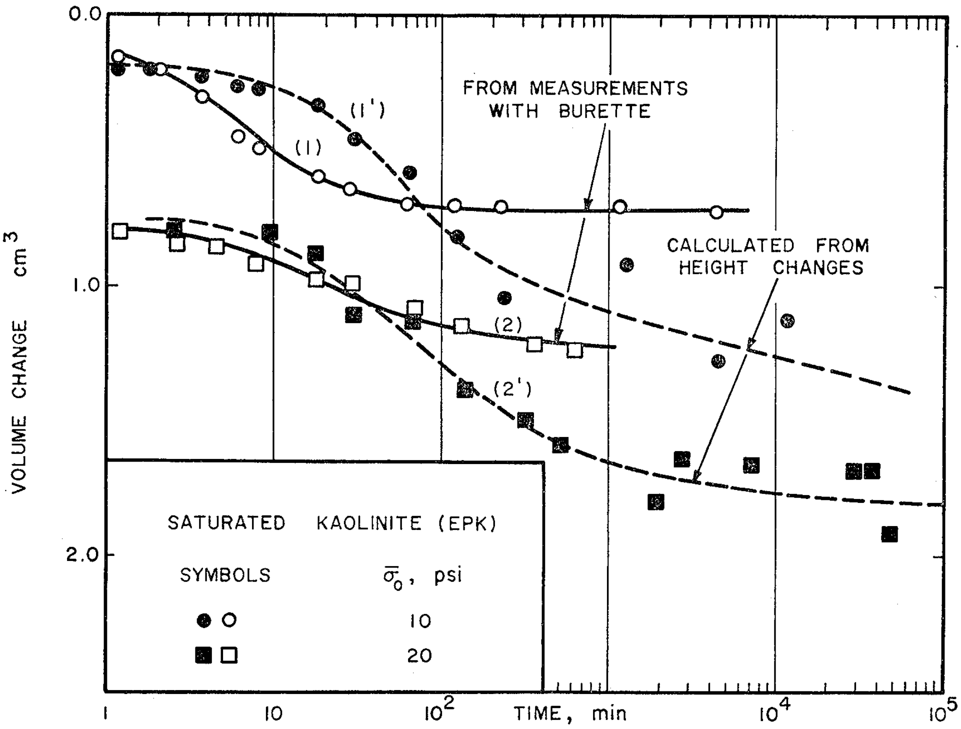


Fig. 6. Comparison between measured and calculated changes in volume with time for a sample of water-saturated clay subjected to constant confining pressures

Figure 7 illustrates the influence of pressure levels on the time-dependent volume change of a sample of saturated kaolinite EPK tested with stress-history pattern No. 2. The top three curves correspond to the behavior of the sample under the normally consolidated conditions in stages (1), (2), and (3) in which the load was increased. In these cases the primary consolidation was completed in less than 1000 minutes. The two lower curves corresponed to stages (6) and (7) in which the sample was reloaded. In these cases the sample volume remained essentially constant.

Effects of Sustained Pressure

The preceding section has demonstrated that primary consolidation was completed in the kaolinite samples before 1000 minutes of load duration. For samples of air-dry sands, silts, and clay size particles the primary consolidation (when it could be measured) was completed in a much shorter time. Each pressure increment was applied to all samples for a length of time much greater than the primary consolidation time in order to study the dynamic behavior during the period of secondary compression.

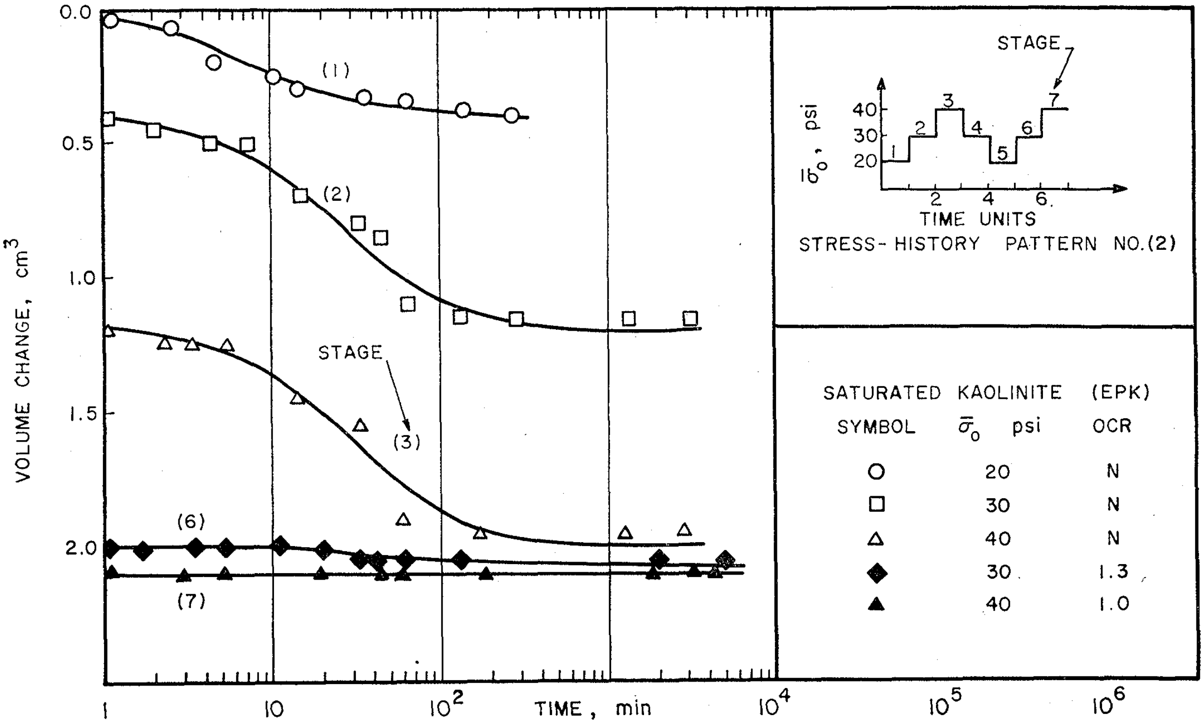


Fig. 7. Variation of volume with time and confining pressure for water-saturated kaolinite clay

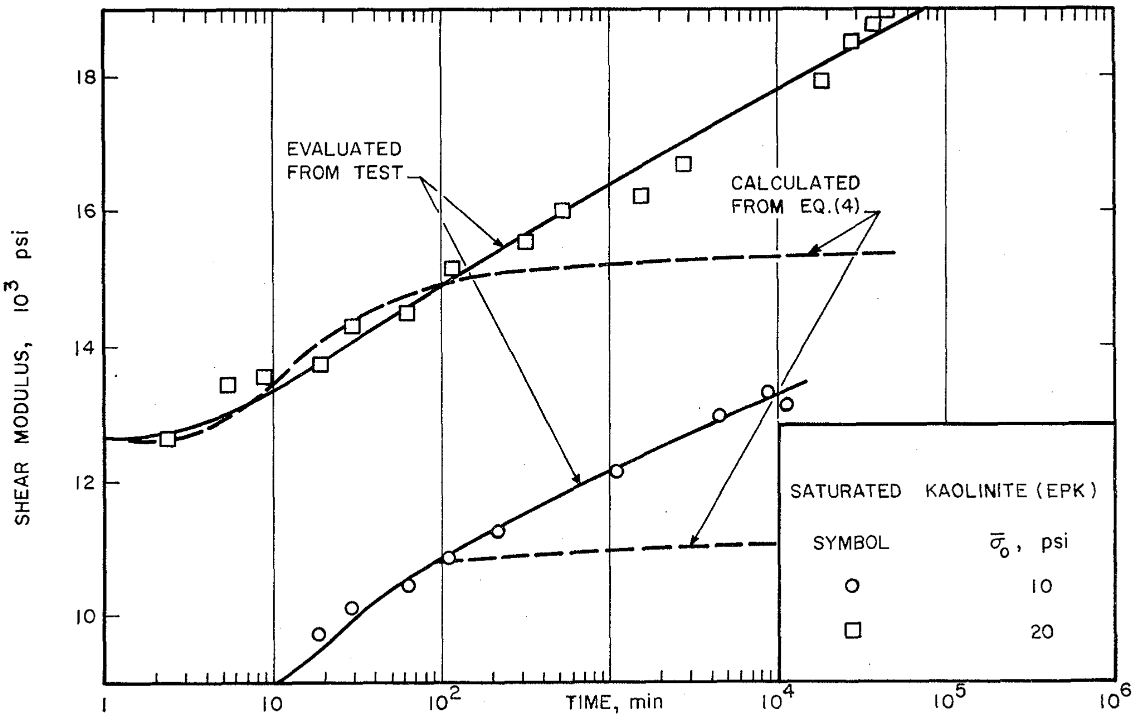


Fig. 8. Variation of shear modulus with time for saturated kaolinite clay

Figure 8 represents the behavior of a sample of saturated kaolinite EPK under two values of increasing load. For this sample the straight line portion of the curve was reached at approximately 100 minutes. Again it is seen that the calculated change of G which depended upon the change in void ratio, e, did not provide an explanation for the secondary time-dependent change in G.

Undoubtedly there are several parameters which contribute to this secondary time effect of increase of G with log time. However, it is convenient to minimize the influence of confining pressure and void ratio by forming a ratio of $(\Delta G)_{\text{per log cycle}}/(G)_{1000\text{ min}}$ Then from Fig. 5 by taking $(G)_{1000\text{ min}}=7500$ psi, and $(G)_{100,000\text{ min}}=8300$ psi, the increase per log cycle is $(\Delta G)_{\text{per log cycle}}=\dfrac{8300-7500}{2}=400$ psi. The ratio then becomes

$$\frac{(\Delta G)_{\text{per log cycle}}}{(G)_{1000\text{ min}}}=\frac{400}{7500}=0.053=5.3\%$$

In the remaining discussion this ratio will be discribed as $\Delta G/G_{1000}$. From Fig. 8 this ratio amounts to $\Delta G/G_{1000}=10\%$ for $\bar{\sigma}_0=10$ psi, and $\Delta G/G_{1000}=9\%$ for $\bar{\sigma}_0=20$ psi. These values include an amount of 1% caused by the decrease in void ratio with time. This amount is relatively small and will be included as a part of the overall increase in modulus with time in the remaining discussion.

As shown on Fig. 9, range and average values of the ratio $\Delta G/G_{1000}$ (as ordinate) are plotted against the D_{50} particle size (see Fig. 2) for the normally consolidated samples. These data were obtained from results of stress-history pattern No. 1 and the stages of stress-history patterns No. 2 and No. 3 which represent normally consolidated conditions. The data in this figure correspond to a range of the pressure $\bar{\sigma}_0$ of 10–60 psi (see Fig. 3); each of these pressures was reached through a net increment $\Delta\bar{\sigma}_0$ of 10 psi (in stress-history pattern No. 3 the pressures of 40, 50 and 60 psi were reached by sudden increments of 20, 30 and 40 psi; however, in each case the pressure was increased above its previous high value only by 10 psi).

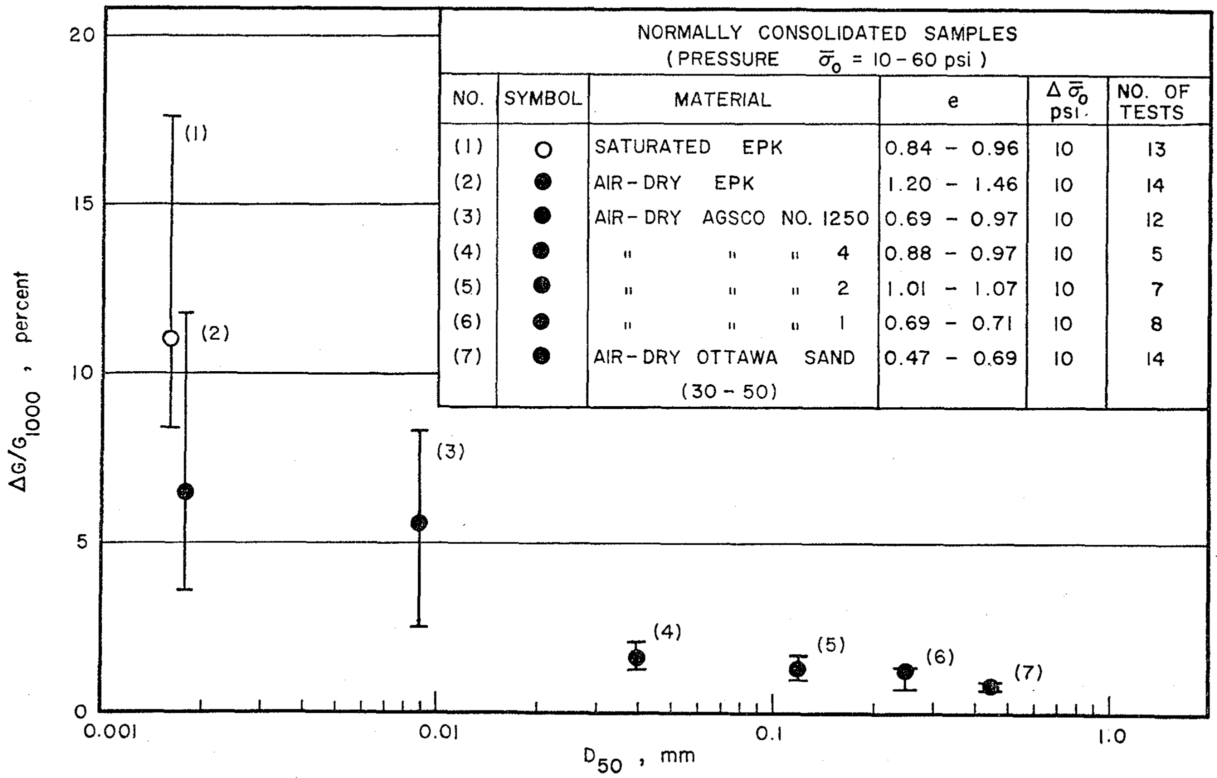


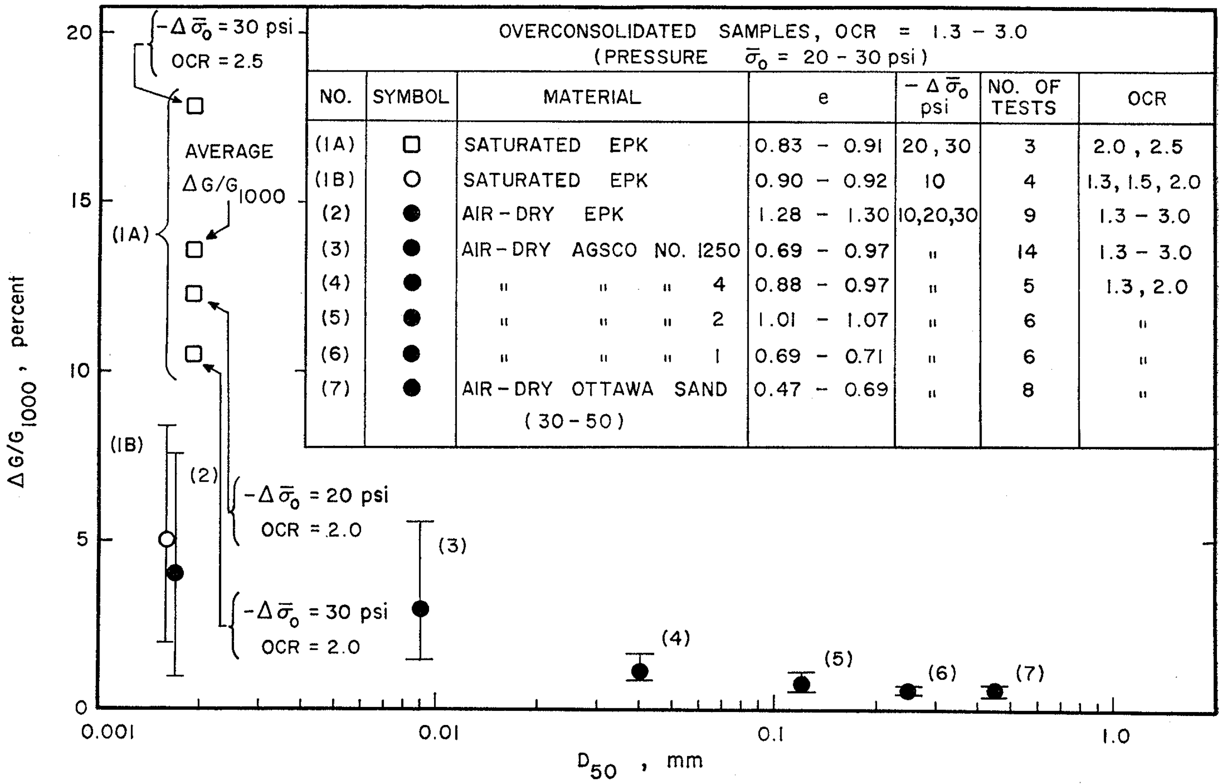
Fig. 9. Summary diagram illustrating ranges and average values of time-dependent modulus increase for different soils—normally consolidated condition

When individual test points were plotted on a similar diagram, no systematic variation was observed for the influence of test confining pressure $\bar{\sigma}_0$, void ratio e, or pressure increment $\Delta\bar{\sigma}_0$. Consequently, on Fig. 9 only ranges and average values are given. Ranges are given by the length of the vertical lines, average values for the air-dry samples are represented by the solid circles, and the average value for saturated kaolinite EPK is represented by an open circle. The table in Fig. 9 shows the range of sample void ratios and the number of samples tested from each material. Each sample was homogeneous, but samples with different void ratios were lumped together in calculating the average $\Delta G/G_{1000}$ value for each material.

It is observed from Fig. 9 that the average value (11%) of $\Delta G/G_{1000}$ for the thirteen tests on saturated kaolinite EPK was higher than the average value (6%) from fourteen tests on air-dry kaolinite EPK. Also note that the void ratios for the air-dry samples were appreciably higher than those for the saturated samples. The average values of $\Delta G/G_{1000}$ of 6 to 11% per log cycle of time correspond to the values of $\Delta G/G_{1000}$ of approximately 10% found by Marcuson (1970) from resonant column tests on samples of saturated Burgess Pigment No. 10 kaolinite (tested at e=1.3 to 1.1).

Effects of Previous Stress-History

From tests utilizing stress-history patterns No. 2 and No. 3 it was possible to establish the influence of overconsolidation ratio (OCR) on the time-dependent increase in modulus described by the ratio $\Delta G/G_{1000}$. Figure 10 is a plot of $\Delta G/G_{1000}$ versus D_{50} for samples with overconsolidation ratios (OCR) of 1.3–3.0, pressures $\bar{\sigma}_0$ of 20–30 psi and pressure changes ($\Delta\bar{\sigma}_0$) of −10, −20 and −30 psi. All these data are represented in terms of ranges and average values in the same manner employed in Fig. 9, except for data of



NO.	SYMBOL	MATERIAL	e	$-\Delta\bar{\sigma}_0$ psi	NO. OF TESTS	OCR
(1A)	□	SATURATED EPK	0.83 − 0.91	20, 30	3	2.0, 2.5
(1B)	○	SATURATED EPK	0.90 − 0.92	10	4	1.3, 1.5, 2.0
(2)	●	AIR-DRY EPK	1.28 − 1.30	10,20,30	9	1.3 − 3.0
(3)	●	AIR-DRY AGSCO NO. 1250	0.69 − 0.97	"	14	1.3 − 3.0
(4)	●	" " " 4	0.88 − 0.97	"	5	1.3, 2.0
(5)	●	" " " 2	1.01 − 1.07	"	6	"
(6)	●	" " " 1	0.69 − 0.71	"	6	"
(7)	●	AIR-DRY OTTAWA SAND (30-50)	0.47 − 0.69	"	8	"

Fig. 10. Summary diagram illustrating ranges and average values of time-dependent modulus increase for different soils—overconsolidated condition

saturated kaolinite EPK with $(\Delta\bar{\sigma}_0)=-20$ and -30 psi where individual data points are also shown. The table in Fig. 10 shows the range of sample void ratios and the number of samples tested from each material.

For saturated kaolinite EPK, comparing the data labeled (1) in Fig. 9 (all had $\Delta\bar{\sigma}_0=10$ psi) with the data labeled (1B) in Fig. 10 (all had $\Delta\bar{\sigma}_0=-10$ psi), it is seen that the overconsolidation ratio had the effect of reducing $\Delta G/G_{1000}$. In fact, Fig. 10 shows that the average values of the ratio $\Delta G/G_{1000}$ for both dry and saturated EPK samples were nearly equal, while as noted before, Fig. 9 shows that the average value of this ratio for saturated EPK was higher than that for dry EPK. Figure 10 further shows that, for saturated EPK, values of the ratio $\Delta G/G_{1000}$ for tests with $(\bar{\sigma}_0)=-20$ and -30 psi were higher than those for tests with $(\Delta\bar{\sigma}_0)=-10$ psi. This indicates that the pressure change might have a significant effect on the time-dependent increase in G.

Comparison of Figs. 9 and 10 also shows that overconsolidation had the effect of reducing $\Delta G/G_{1000}$ for air-dry EPK and Agsco No. 1250. In this case the effects of the pressure change $(\Delta\bar{\sigma}_0)$ were not significant and values of the ratio $\Delta G/G_{1000}$ were lumped together for all values of $(\Delta\bar{\sigma}_0)$. Figures 9 and 10 further show that the overconsolidation reduced values of $\Delta G/G_{1000}$ for coarse-grained soils (Agsco No. 2 through Ottawa sand), also.

In general, it is evident from Figs. 9 and 10 that the greatest influence of overconsolidation on $\Delta G/G_{1000}$ occurred for the fine-grained soils, and that there is relatively little influence of overconsolidation on fine or medium sands.

Figure 11 shows the time history increase of shear modulus for steps 2, 3, 4 and 5 of stress-history pattern No. 2 applied to a sample of air-dry Agsco No. 1250. Step 1 is given in Fig. 5. At each stress level there is a break in the G vs. log time curve, generally at about 1000 minutes for the normally consolidated samples and at a time

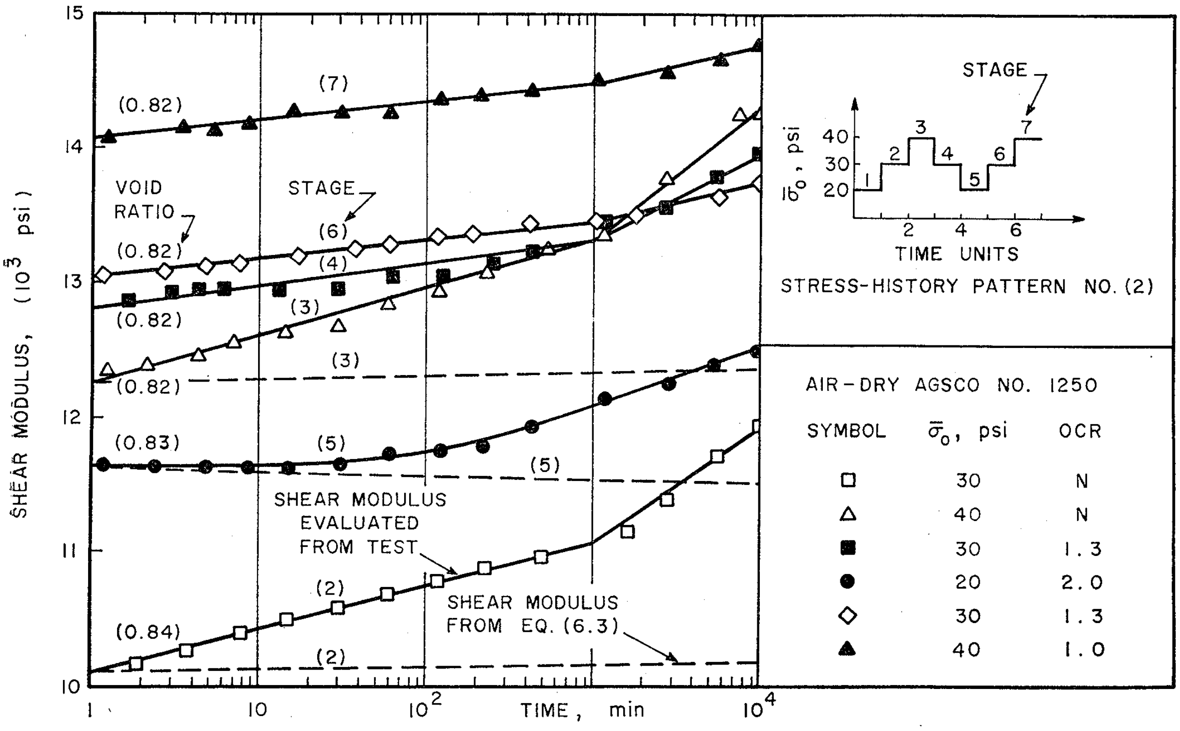


Fig. 11. Variation of shear modulus with time and confining pressure for air-dry crushed-quartz silt

between 100 and 1000 minutes for the overconsolidated samples. The test results for air-dry kaolinite EPK gave curves similar to those shown on Fig. 11, while saturated kaolinite EPK gave curved relationships for times less than 100 to 1000 minutes and straight line relationships thereafter (see Fig. 8). It is evident from Fig. 11 that the slope of the line is flatter for the overconsolidated than it is for the normally consolidated samples, for times greater than 1000 minutes. It is the slope of this line, (ΔG) per log cycle divided by G at 1000 minutes which was used as the ordinate for Figs. 9 and 10.

As seen from Fig. 11 the effects of overconsolidation pressures acting for given time periods are (a) to increase the value of G_{1000} and (b) to decrease the secondary time increase $\Delta G/G_{1000}$. The increase in G_{1000} is caused by the time duration of these pressures and the effect of overconsolidation ratio. In order to minimize the time effects in evaluating the influence of the overconsolidation ratio, a procedure described in Fig. 12 was adopted. The open circles on Fig. 12 represent the shear modulus at different times for a normally consolidated sample of saturated kaolinite EPK ($\bar{\sigma}_0$=20 psi, e=0.98). Point A represents the shear modulus at a time of 100 minutes when normally consolidated under a pressure of $\bar{\sigma}_0$=20 psi. The irregular curve above the line $A-B'$ represents the changes in G with time and pressure as stress-history pattern No. 2 (shown in the top left corner of Fig. 12) was applied. At Point A, the first stage ($\bar{\sigma}_0$=20 psi) was applied for 100 minutes. At Point B, stage 5 of the stress-history (the stage with $\bar{\sigma}_0$=20 psi for the second time) has been applied for 100 minutes. It is seen from Fig. 12 that, between the times t_A-t_B, the increase in G can be considered to be composed of an increase $B-B'$ caused by the overconsolidation ratio of 2, and an increase $B''-B'$ caused by the duration of time (time-dependent increase).

Figure 13 includes this comparison of the effect of overconsolidation on shear modulus on a summary diagram of shear modulus vs. void ratio for the various soils tested.

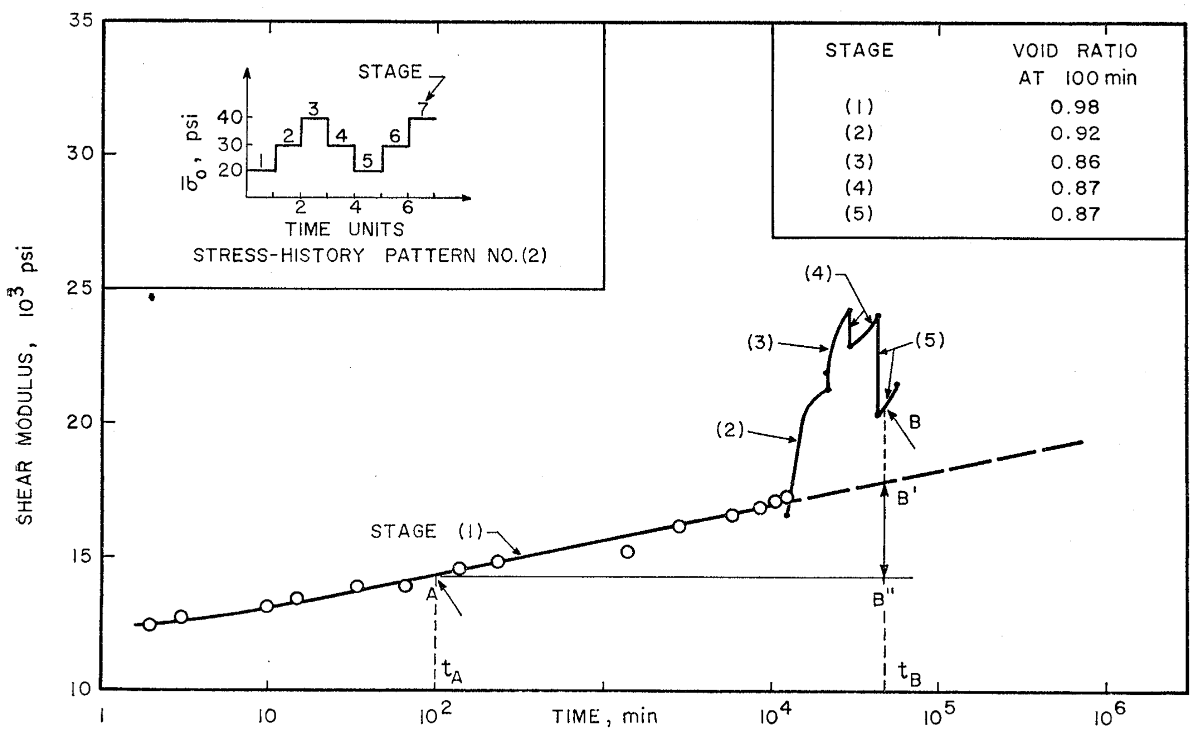


Fig. 12. Method of separating effects of overconsolidation ratio and time between stages (1) and (5) of stress-history pattern No. 2

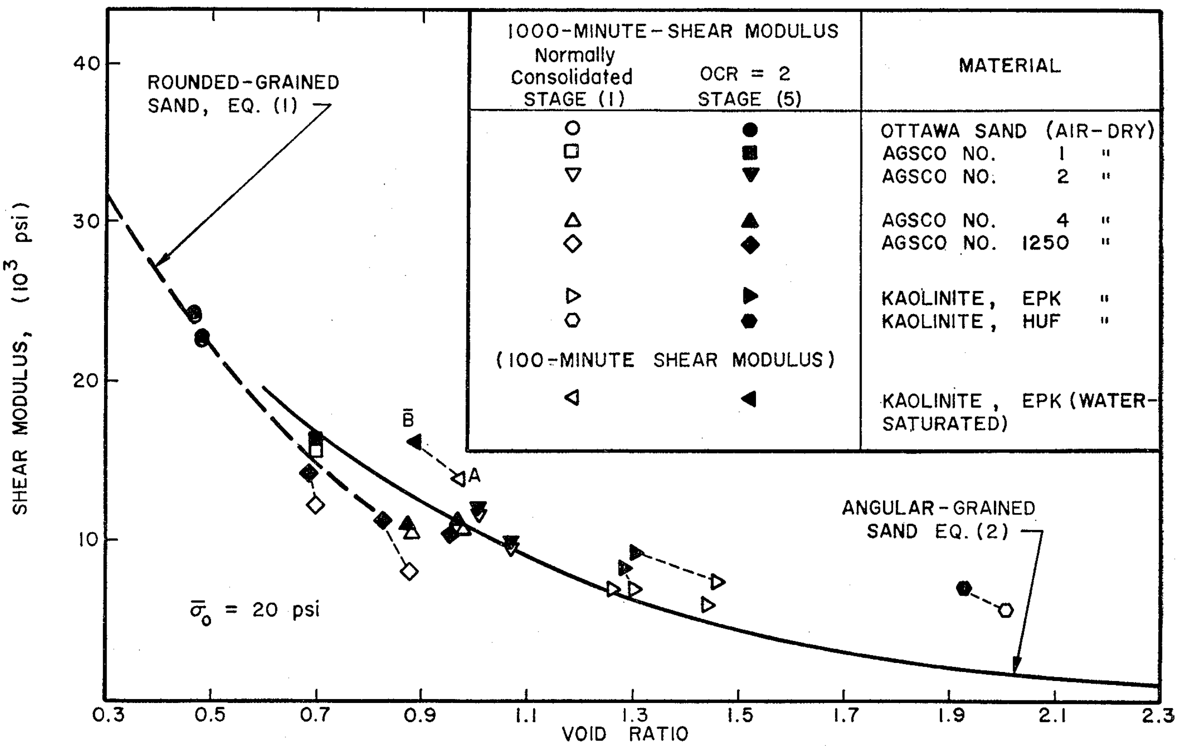


Fig. 13. Comparison between results of stages (1) and (5) of stress-history pattern No. 2 after accounting for effects of time

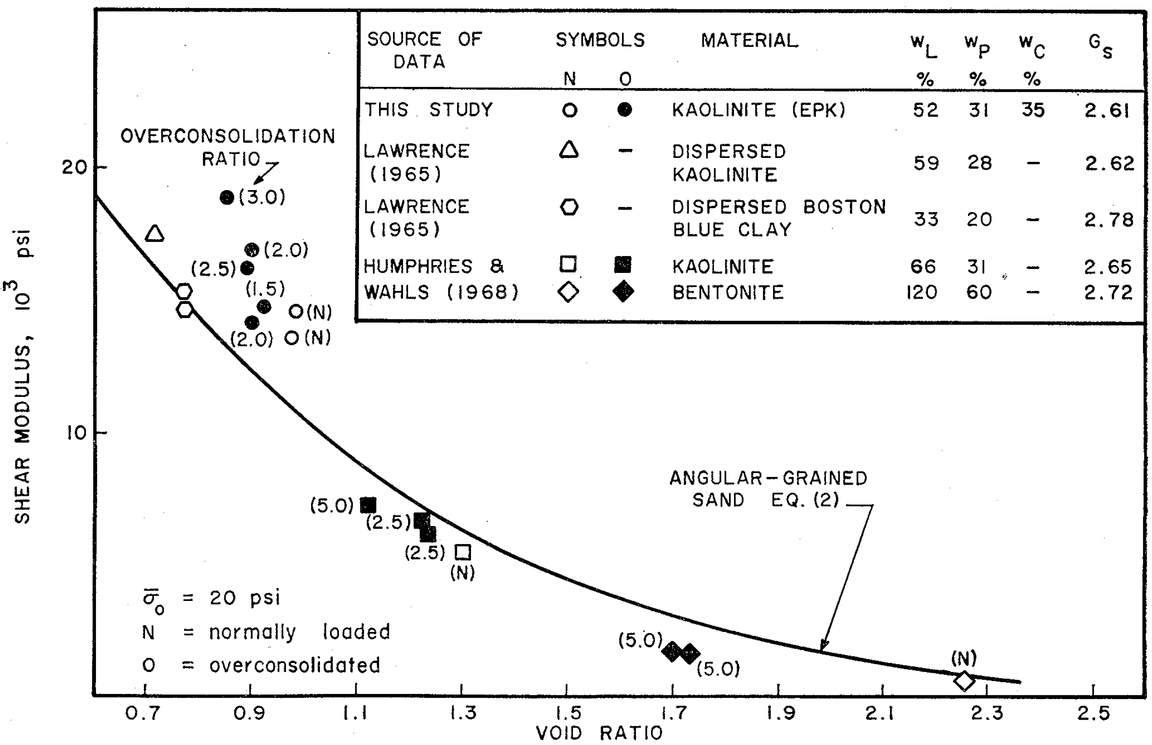


Fig. 14. Effect of overconsolidation ratio on shear modulus of saturated clays

Values of G_{1000} for air-dry soils and of G_{100} for saturated EPK obtained from stage 5 of stress-history pattern No. 2 were corrected to eliminate the time effects by the procedure noted in the preceding paragraph. In this figure open symbols represent values of G at the normally consolidated condition and solid symbols represent values of G at the overconsolidated condition. Thus, for example, Point A represents the shear modulus for a normally consolidated sample of saturated kaolinite EPK and Point $\bar{B}$ represents the corrected value for an overconsolidation ratio of 2.0. Note that these two points are essentially parallel to the solid curve in Fig. 13 which was obtained from Eq. (4). Similar behavior was noted for other fine-grained soils, as shown by the dashed lines connecting open and solid symbols in Fig. 13. It is concluded, therefore, that the effect of an overconsolidation ratio as high as 2.0 on fine-grained soils was primarily to reduce the void ratio, thereby increasing G in conformance with Eq. (4).

Figure 14 presents corrected values of the shear modulus of saturated EPK samples tested with stress-history pattern No. 2 and stress-history pattern No. 3. The figure also includes data from resonant column tests and from ultrasonic pulse tests by other investigators who considered the effects of overconsolidation. Most of these data were based on one-day tests. Again note that the effect of overconsolidation appears to be well taken care of by the change in void ratio in conformance with Eq. (4).

SUMMARY AND CONCLUSIONS

This investigation considered the effects of time of loading and overconsolidation on the low amplitude shear modulus, G, of seven soils. The soils were chosen to include a wide range of void ratios and grain sizes.

The influence of overconsolidation on G was found to be insignificant for air-dry samples of Ottawa sand (30–50), Agsco No. 1, Agsco No. 2, and Agsco No. 4. Therefore, it would be anticipated that overconsolidation would not affect the shear modulus of other uniform dry soils which had a grain size, D_{50}, larger than about 0.04 mm. For samples of air-dry Agsco No. 1250 and air-dry and saturated kaolinite, the influence of overconsolidation was exhibited primarily by a reduction of the void ratio of the sample. The increase in G developed by reducing the void ratio could be anticipated through the use of Eq. (4).

The length of time the confining pressure was applied introduced the most important changes in the shear modulus. The data in Figs. 9 and 10 indicate that the time dependent increase in shear modulus G is relatively unimportant for soils having $D_{50}>0.04$ mm. For these soils the percent increase per log cycle, $\Delta G/G_{1000}$, was less than about 3%. However for both air-dry and saturated samples of soils having $D_{50}\ll 0.04$ the time dependent increase in G can be significant and must be evaluated. For the kaolinite samples tested in both the air-dry and saturated conditions the secondary time rate of increase ($\Delta G/G_{1000}$) was on the order of 6 to 11% per log cycle for normally consolidated samples subjected to a pressure change $\Delta\bar{\sigma}_0=10$ psi, and on the order of 5% per log cycle for overconsolidated samples subjected to a pressure change $(\Delta\bar{\sigma}_0)=-10$ psi. Three tests on saturated kaolinite showed values of the ratio $\Delta G/G_{1000}$ on the order of 13% per log cycle for overconsolidated samples subjected to pressure change $(\Delta\bar{\sigma}_0)=-20$ and -30 psi. Therefore we must extrapolate the laboratory test data for application to field conditions.

It is possible that some of the secondary time-dependent increase in G of the saturated kaolinite samples was caused by the method of preparing the samples, the pressure increment applied, the ratio of vertical to lateral pressures (K_0 was 1.0 for all the tests in this study), the temperature of testing (72°F), or other laboratory test conditions. Field evaluations of the shear wave propagations in cohesive soils before and for an extended time after preloading would provide valuable information for interpretation of laboratory test results. The investigation described by Yamamoto, Seki, and Suzuki (1971), for example, is a significant contribution to the understanding of the changes in shear wave propagation, shear modulus, and damping after field preloading of cohesive soils. It is hoped that more investigations of this type will be conducted, and that they will include measurements continued for significant periods of time after preloading was completed. The cross-hole shooting technique described by Stokoe and Woods (1972) represents one convenient method for obtaining this information.

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NOTATION

$A(T)$ = positive constant given by Eq. (4) $(\text{lb/in}^2)^{0.5}$
e = void ratio
g = acceleration of gravity (ft/sec^2)
D_{50} = diameter at 50% finer
G = shear modulus (lb/in^2)
ΔG = change in shear modulus per log cycle of time after the first 1000 minutes of the test (three log cycles)
G_{100} = shear modulus at 100 minutes after the application of a confining pressure
G_{1000} = shear modulus at 1000 minutes after the application of a confining pressure
$\Delta G/G_{1000}$ = normalized quantity to express the relative variation of shear modulus with log time
$G_{\max.}$ = maximum shear modulus, which is the shear modulus at low amplitude vibration (shearing strain less than 10^{-5})
G_s = specific gravity
N = normally loaded condition
O = overconsolidated condition
OCR = overconsolidation ratio
w_L = liquid limit (%)
w_P = plastic limit (%)
v_s = shear wave velocity (ft/sec)
γ = unit weight (lb/ft^3)
$\bar{\sigma}_0$ = average effective confining pressure (lb/in^2)
$\Delta\bar{\sigma}_0$ = change in $\bar{\sigma}_0$
$\bar{\sigma}_1, \bar{\sigma}_2, \bar{\sigma}_3$ = major, intermediate and minor effective principal stresses
τ_0 = octahedral shearing stress
ρ = mass density $(=\gamma/g)$

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