fairly many samples.

The fourth problem is that the reading the contact angles of the grains on which no force acts cannot be avoided. The writer's method employing aluminum rods also faces this problem, however, it has been improved by using a photoelastic material for the rods.[11)~13)]

Some of the above discussions may result from the difference between the writer and the author with respect to the method of the experiments.

References

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STIFFNESS MATRIX FOR BEAMS ON ELASTIC FOUNDATION BY VIRTUAL WORK PRINCIPLE*

Discussion by HIROAKI NAGAOKA**

The writer wish to ask the author about the assumptions made in the present technical note and the displacement function employed for subregion of ground whose surface a beam does not rest on.

In the technical note it is assumed that only vertical component of displacement is nonzero while the other two horizontal components are zero, soil is in the state of plane strain, and displacement is nonzero only in the region of width b and depth H across the boundary of which displacement is discontinuous (cf. Fig. 4). Since these assumptions, which were also made in the paper by Harr *et al* (1969), seem to the writer very bold, the writer wishes to know the reason by which the author judged that under these assumptions an approximate solution of good accuracy could be obtained.

Since the method in the technical note is a special form of finite element method, the displacement function in the region of nonzero displacement must be prescribed. In cases of numerical examples (2) a, b, and d, since the vertical component of displacement at beam end G in Fig. 5 is zero, the vertical component of displacement in the region $FGJK$ of Fig. 5 whose surface a beam does not rest on is zero. In cases of numerical examples (1) and (2) c, since the vertical component of displacement in the region $FGJK$ is nonzero because of nonzero vertical component of displacement at G, the displacement

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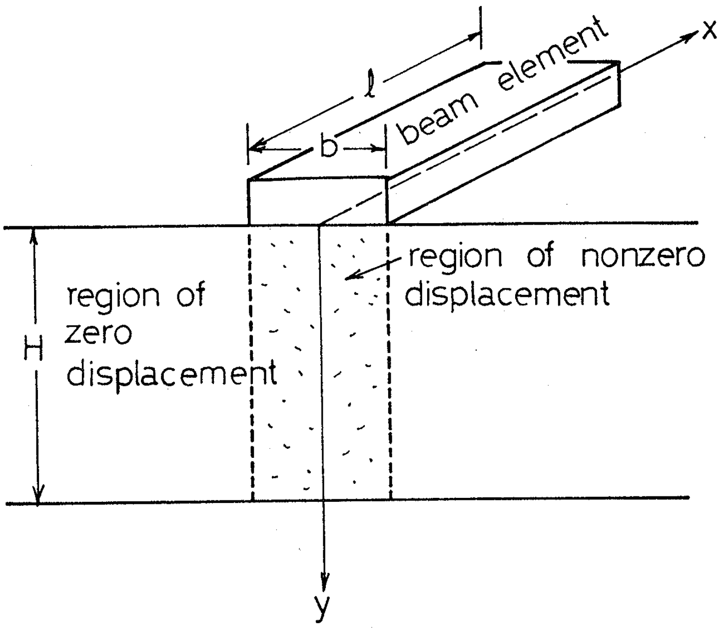

Fig. 4.

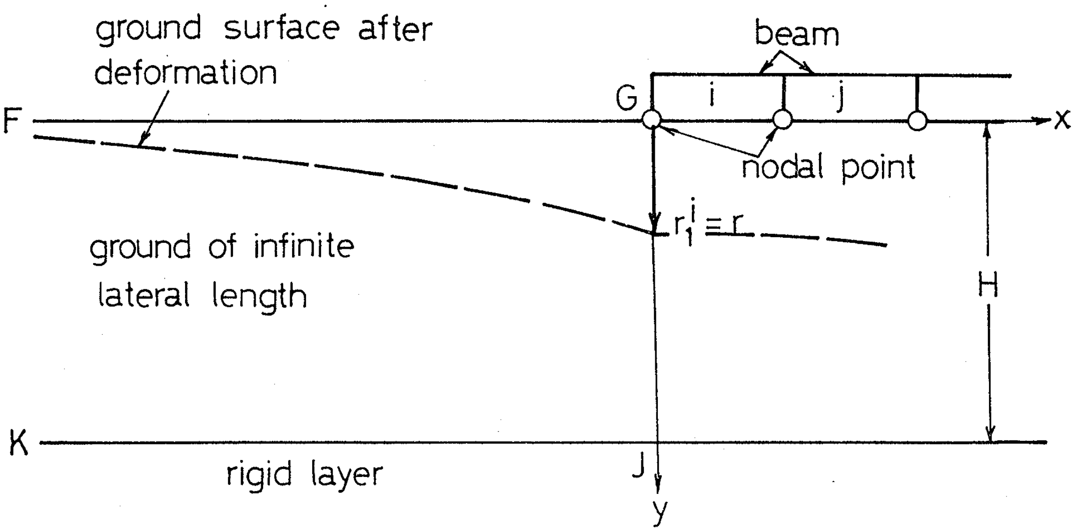

Fig. 5.

function must be prescribed which determines displacement in the region *FGJK* by finite number of generalized displacements at nodal points and is not shown in the technical note. The writer wishes to know the displacement function in the region *FGJK* which the author employed in numerical examples (1) and (2)c.

Questions which the writer has in his mind are the above two and in the following the writer is going to show a displacement function in the region *FGJK*, which can be employed in numerical examples (1) and (2)c, under the assumptions made by the author. The region *FGJK* of infinite lateral length is considered as one element, the displacement of which is determined by only a vertical component of displacement at the nodal point *G* (cf. Fig. 5). Governing differential equation in the region is (Haar *et al.*, 1969)

$$2t\frac{d^2v(x)}{dx^2}-kv(x)=0 \tag{34}$$

where

$$t=\frac{E_s}{4(1+\nu_s)}\int_0^1\phi^2(\eta)bHd\eta \tag{35}$$

$$k=\frac{E_s}{1-\nu_s^2}\int_0^1\frac{b}{H}\left(\frac{d\phi}{d\eta}\right)^2d\eta \tag{36}$$

and $v(x)$ corresponds to $v(\xi)$ in Eq. (5). By boundary condition that

$$v(\infty)=0 \tag{37}$$

$$v(0)=r \tag{38}$$

where r is vertical displacement at G, the following solution can be obtained.

$$v(x)=re^{\alpha x} \tag{39}$$

$$V(x,\eta)=re^{\alpha x}\phi(\eta) \tag{40}$$

$$\alpha=\sqrt{\frac{k}{2t}} \tag{41}$$

Strain vector is

$$e=\begin{Bmatrix}0\\ \dfrac{1}{H}\dfrac{d\phi(\eta)}{d\eta}\\ \alpha\phi(\eta)\end{Bmatrix}e^{\alpha x}r=fe^{\alpha x}r \tag{42}$$

$$f=\begin{Bmatrix}0\\ \dfrac{1}{H}\dfrac{d\phi(\eta)}{d\eta}\\ \alpha\phi(\eta)\end{Bmatrix} \tag{43}$$

Internal virtual work is

$$\int_{-\infty}^0 dx\int_0^1 e^TDeHd\eta=\delta r\frac{H}{2\alpha}\int_0^1 f^TDfd\eta r=\delta rR \tag{44}$$

where D is defined in Eq. (21) and R is the generalized nodal force corresponding to r.

$$R=\frac{H}{2\alpha}\int_0^1 f^TDfd\eta r \tag{45}$$

At nodal point G inclination of ground surface is discontinuous. When no external load acts on beam element i one of nodal points of which is G, equilibrium equations at G are

$$R+R_1^i=0 \tag{46}$$

$$R_2^i=0 \tag{47}$$

where superscript i implies element i and subscripts 1 and 2 correspond to those in Eq. (26)

Reference

8) Harr, M. E., *et al.* (1969): "Euler beams on a two parameter foundation model," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 95, No. SM4, Proc. Paper 6651, pp. 933-948.