infinite integrals involving Bessel functions," Mathematical Tables and Aids to Computations, Vol. 11, pp. 166-180.
36) Nielsen, J.P. and Wilcoxson, W.L. (1965): "Numerical analysis for two-layered pavement systems," U.S. Naval Civil Engineering Laboratory, Port Hueneme, California, U.S.A., Technical Report R-378.

APPENDIX

$$N^*=\frac{e^{2\frac{h}{r}t}+4K\frac{h}{r}t-KLe^{-2\frac{h}{r}t}}{e^{2\frac{h}{r}t}-\left(L+K+4K\frac{h^2}{r^2}t^2\right)+KLe^{-2\frac{h}{r}t}}$$

$$K=\frac{1-n}{1+n(3-4\mu_1)}$$

$$L=\frac{(3-4\mu_2)-n(3-4\mu_1)}{(3-4\mu_2)+n}$$

$$n=\frac{E_2}{E_1}\frac{1-\mu_1}{1+\mu_2}$$

ERRORS IN SIMULATING EXCAVATIONS IN ELASTIC MEDIA BY FINITE ELEMENTS*

Discussion by J. H. LaVerne Palmer**

The authors should be commended for a useful contribution toward a more satisfactory simulation of the process of excavation using the finite element method. In the process of developing a computer program to simulate the construction of a braced excavation*** the writer faced the same problem of checking the validity of the excavation procedure and applied a similar checking procedure to that suggested by the authors.

The writer's program entitled FESSE is based upon an assumption that it is reasonable to consider a supported excavation as composed of three major systems, i.e. an excavated or passive soil mass, a supporting system, and a supported or active soil mass (Fig. 9). The approach taken in the development of FESSE was to use the most suitable solution available at the time for each component and to force compatibility between these individual and essentially independent solutions by an iterative technique which seeks compatibility between the deformation profile of the supporting system and the lateral deformation of the soil masses.

In the excavation procedure the forces equivalent to the tractions between elements are calculated on the basis of the nodal forces equivalent to the initial stresses at the centre of each element similar to approach number 1 suggested by the authors, plus the forces which would be required at each node to cause a displacement of each node equivalent to the displacements computed in the normal finite element solution.

* By John T. Christian and Ing Hieng Wong, Vol. 13, No. 1, Mar. 1973, pp. 1-10.

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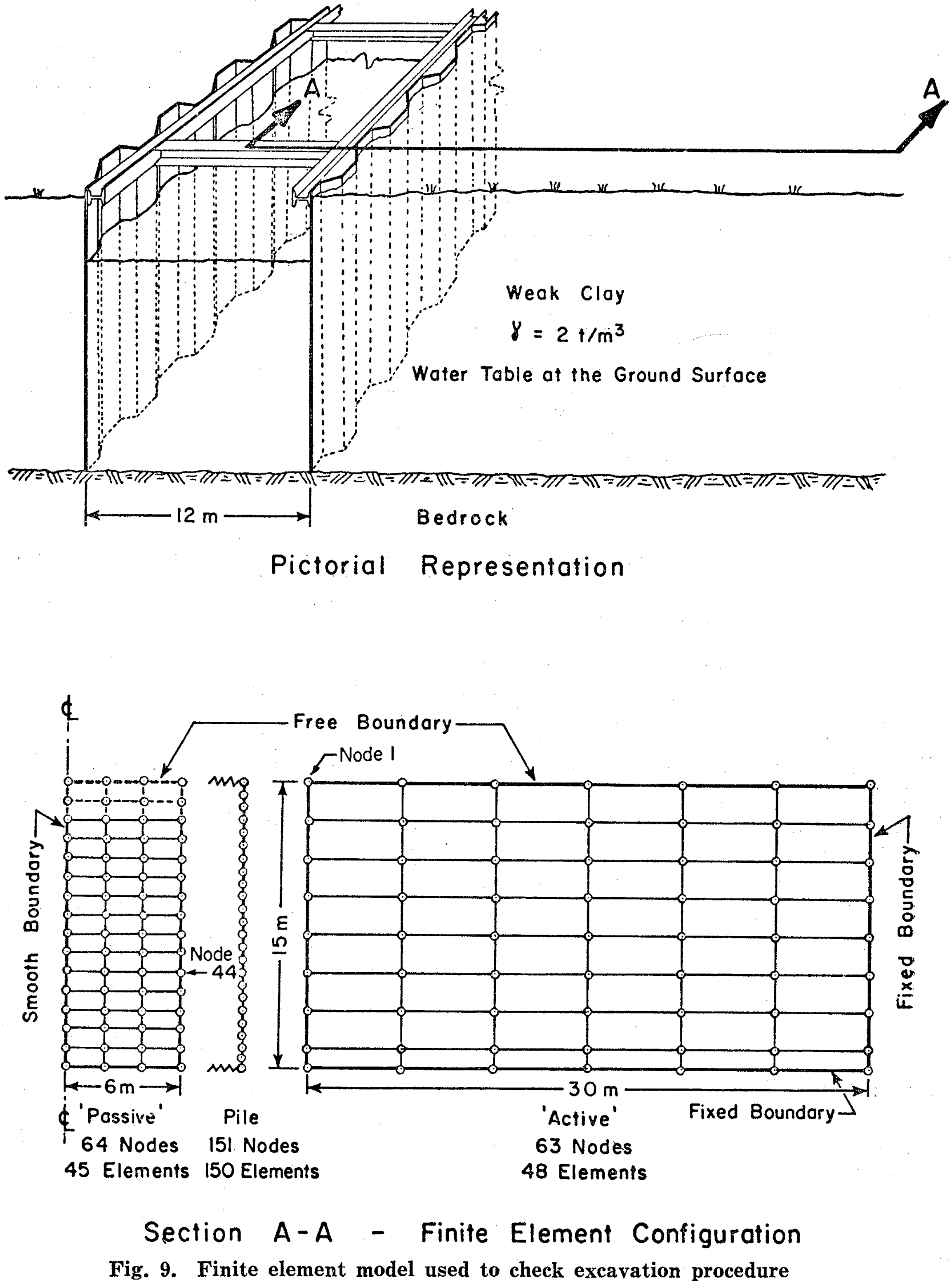

Fig. 9. Finite element model used to check excavation procedure

The problem used for checking the excavation procedure is illustrated in Fig. 9. The primary soil and structural parameters used for the problem are listed in Table 1 and the results of computations for an excavation completed in one step, five steps and ten steps are summarized in Table 2. Since the solution is also dependent upon the iterative limits, results are also shown for acceptable closure limits on the deformations of 0.20 and 0.01

Table 1.

Soil parameters		Structural parameters	
Deformation modulus (E)	$500 t/m^2$	Pile stiffness (EI)	6.24×10^{10} kg-cm^2
Poisson's ratio	0.49	Effectve strut stiffness	1×10^5 kg/cm
Initial stress ratio (K_0)	0.5	Restraint at pile tip	1×10^6 kg/cm
Undrained shear strength (Cu)	$1.0+0.2p' t/m^2$		
Soil-to-pile adhesion	0		

Table 2.

		No. of levels				
		1(0.100)**	5(0.100)	10(0.100)	1(0.010)	1(0.200)
Max. pile deflection		17.48 *	17.30	17.23	17.44	17.52
Passive node 44	Hor.	15.01 *	15.12	15.19	15.03	15.02
	Vert.	11.35 *	11.43	11.51	11.36	11.34
Active node 1	Hor.	0.508*	0.418	0.354	0.514	0.502
	Vert.	4.404*	4.343	4.325	4.413	4.401

* all displacements are in centimetres
** acceptable iterative closure in brackets

centimetres. Within the constraints of an iterative type of solution, the results indicate a satisfactory excavation procedure.

These results are in apparent contradiction with the authors. An earlier version of FESSE in which the traction forces were computed entirely on the basis of the stresses at the centre of the elements gave almost identical results.

The major difference between the authors' computer programs and the writer's program is in the basic solution technique. It is possible, although certainly not conclusively proven, that the separation of the solution as done in FESSE considerably improves the accuracy of the excavation procedure. The current version of FESSE is valid for situations where the supporting system penetrates to a stiff layer or to bedrock (i.e. fully-penetrating piles); unfortunately, it is not valid for partially-penetrating pile systems.