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METHODS TO ESTIMATE LATERAL FORCE ACTING ON STABILIZING PILES

Tomio Ito* and Tamotsu Matsui**

ABSTRACT

The growth mechanism of lateral force acting on stabilizing piles in a row due to the surrounding ground undergoing plastic deformation is discussed, and its theoretical analysis is carried out considering the interval between the piles. In this analysis, it is assumed that two types of plastic states occur in the ground just around piles, one of which is a plastic state satisfying Mohr-Coulomb's yield criterion and the other a plastic state where the soil is considered as a visco-plastic solid. The former is called the theory of plastic deformation and the latter the theory of plastic flow. Then, the characteristics of both theoretical equations are clarified by examining the effects of various parameters on the lateral force. Comparing between the theoretical and the observed lateral forces which are obtained in the stabilizing piles against landslides, the order of magnitude of the theoretical values due to both theories agrees with that of the observed values. Whereas, the theoretical values due to the Hennes' equation underestimate and the equation of the Public Works Research Institute overestimate the observed lateral force. Especially, it is clarified that under the condition of restrained pile top, the lateral force acting on stabilizing piles may be estimated by the theory of plastic deformation.

Key words: deformation, earth pressure, horizontal load, landslide, pile, rheology, slope stability

IGC: E4/E5/E6

INTRODUCTION

When piles are placed through such plastically deforming ground as in landslides, slope failures or lateral flow, it is considered that piles have a preventive effect against the plastic deformation in the ground. Especially, it may be sure that this effect is significant in the case where piles stand in a row. One of such examples will be the stabilizing pile against landslide, which is often used as a landslide control work (Yamada, Watari and Kobashi, 1971; Fukumoto, 1972, 1973; Ito and Matsui, 1974). On the other hand, piles in a row may often pass through a sliding surface in the problem of slope stability, such as pile foundation of landing piers in a harbor (Kitazima and Kishi, 1967; Leussink and Wenz, 1969; De Beer and Wallays, 1972), bridge abutments (Nicu, Antes and Kessler, 1971; Marche and Lacroix, 1972), buildings (Heyman and Boersma, 1961; Heyman, 1965) and so on. In these cases, the effect of piles preventing the plastic deformation in the

* Professor, Department of Civil Engineering, Osaka University, Yamadakami, Suita, Osaka.
** Assistant Professor, Department of Civil Engineering, Osaka University, Yamadakami, Suita, Osaka.

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ground will also be expected. Moreover, for the purpose of controlling lateral flow induced in a soft soil layer due to a surface load, it is considered that piles in a row are driven on both sides of it. These piles also have the same effect as mentioned above.

All phenomena described above are common in respect that the piles are subjected to lateral force due to plastically deforming ground. While, the problems of lateral resistance in piles have been solved merely for the case of known lateral force to date. Therefore, it is very important to estimate the lateral force acting on piles due to plastically deforming ground. But, the growth mechanism of the lateral force, which occurs by the interaction between piles and plastically deforming ground, may be very complicated, and it may not be clarified at all at the present time.

The past methods of estimating the lateral force may be summarized as follows (Yamada, Watari and Kobashi, 1971). Lamb and White have experimentally obtained the lateral force acting on a single pile and piles in a row, respectively, assuming that the ground around piles is a viscous fluid. Because of this assumption, these methods can not be applied except to a landslide as a mud flow. Hennes have obtained the lateral force, assuming that the ground is a plastic solid with cohesion only and that piles are infinitely thin plates put parallel to the direction of landslide. Because of the latter assumption, this method neglects the compressive and the shear deformations of the soil between the piles. Then, it is supposed that this method will underestimate the lateral force. The Public Works Research Institute (P. W. R. I), the Ministry of Construction, has derived a theoretical equation for the lateral force, assuming that the passive Rankine failure occurs in the ground situated in front of piles. Because of this assumption, this method may estimate the lateral force in the limiting state of landslide movement. However, the required value is not such a limiting value, but a value in the initial state of landslide movement. Therefore, it is supposed that this method will overestimate the lateral force. Recently, an analysis of piles in soils undergoing lateral movement has been described by Poulos (1973). In this method, the effect of interval between piles is not considered.

There are some problems in each method described above. That is, the interval between piles is not accurately considered, and the soil condition and the plastic state of the surrounding ground are not reasonably assumed.

Herein, a theoretical analysis of the lateral force acting on piles in a row through plastically deforming ground is described considering the interval between piles. In this analysis, it is assumed that two types of plastic states occur in the surrounding ground just around piles, one of which is a plastic state satisfying Mohr-Coulomb's yield criterion and the other a plastic state considering the ground as a visco-plastic solid. The former is called the theory of plastic deformation and the latter the theory of plastic flow in the following. A quantitative examination is made varying some of the parameters. Several comparisons are then carried out between the observed lateral force acting on stabilizing piles against landslides and that given by the theories.

THEORETICAL ANALYSIS OF LATERAL FORCE ACTING ON STABILIZING PILES

General Remarks

As shown in Fig. 1, piles of diameter d are placed in a row with a spacing D_1 through plastically deforming ground. When a lateral deformation occurs in a soil layer of thickness H in a direction perpendicular to the direction of the row of piles, a lateral force acts on the piles as an interaction between the piles and the soil layer. In the analysis, it is sufficient to deal with the behavior of a soil layer between two piles, as shown by the shadowed portion in Fig. 1. Two types of analyses are carried out, which are based on the theories of plastic deformation and plastic flow. It is supposed that the analyses based on the former and the latter theories may be applied to the plastic deformation in a comparatively hard soil layer and the creep deformation in a soft soil layer, respectively, according to the assumptions in the two theories.

Theory of Plastic Deformation

Following assumptions are set up in order to analyze the lateral force by the theory of plastic deformation. The soils between two piles ACDFF′D′C′A′ are considered, as shown in Fig. 2.

Assumptions:

a) When the soil layer deforms, two sliding surfaces occur along the lines AEB and A′E′B′, in which the lines $\overline{EB}$ and $\overline{E'B'}$ make an angle $(\pi/4+\varphi/2)$ with the x-axis.

b) The soil layer becomes plastic only in the soil AEBB′E′A′ just around piles, in which the Mohr-Coulomb's yield criterion is applied. Then, the soil layer is represented as plastic solids with an angle of internal friction φ and a cohesion c.

c) The soil layer is in a plane-strain condition in the direction of depth.

d) Even if frictional forces act on the surfaces AEB and A′E′B′, the stress distribution

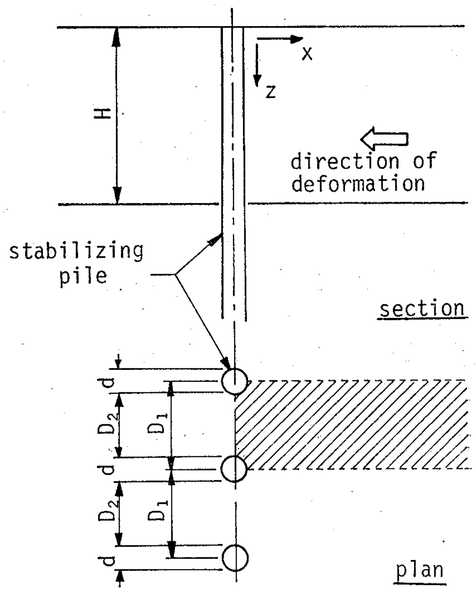


Fig. 1. Stabilizing piles in a row through plastically deforming ground

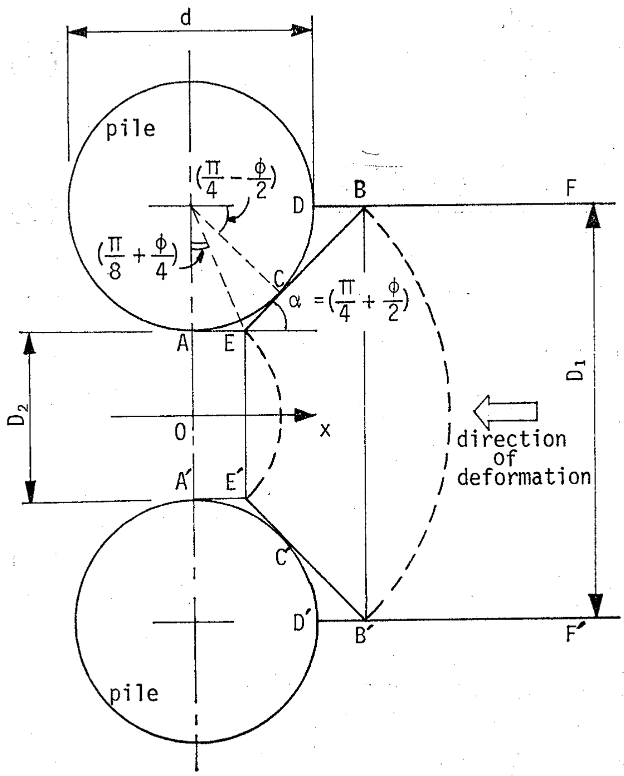


Fig. 2. State of plastic deformation in the ground just around piles

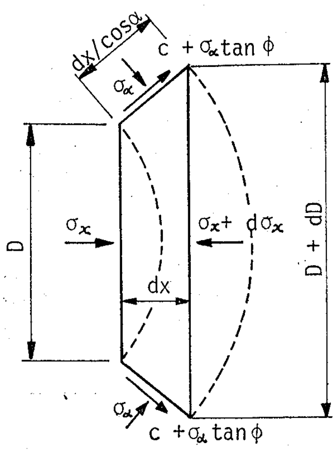


Fig. 3. Small element of plastically deforming ground (EBB′E′)

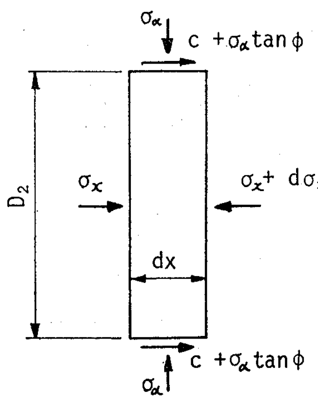


Fig. 4. Small element of plastically deforming ground (AEE′A′)

in the soil AEBB'E'A' is almost the same as that in the case of no frictional forces on those surfaces.

e) Piles are rigid.

First, in the zone EBB'E', the equilibrium of forces acting on a small soil element will be considered in the x direction, as shown in Fig. 3.

$$-D\mathrm{d}\sigma_x-\sigma_x\mathrm{d}D+2\mathrm{d}x\left\{\sigma_\alpha\tan\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)+\sigma_\alpha\tan\varphi+c\right\}=0 \tag{1}$$

As a normal stress σ_α on the surface EB (E'B') may be approximately a principal stress corresponding to a principal stress σ_x from the assumption d), the following equation is given as the yield criterion of soil layer from the assumption b).

$$\sigma_\alpha=\sigma_x N_\varphi+2c\sqrt{N_\varphi} \tag{2}$$

where, $N_\varphi=\tan^2(\pi/4+\varphi/2)$. From the geometrical conditions,

$$\mathrm{d}x=\frac{\mathrm{d}\left(\frac{D}{2}\right)}{\tan\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)} \tag{3}$$

Substituting Eqs. (2) and (3) into Eq. (1), the following equation holds.

$$D\mathrm{d}\sigma_x=\mathrm{d}D\times\{(N_\varphi^{1/2}\tan\varphi+N_\varphi-1)\sigma_x+c(2\tan\varphi+2N_\varphi^{1/2}+N_\varphi^{-1/2})\} \tag{4}$$

As Eq. (4) is a differential equation with separated variables, it can readily be integrated. Then,

$$\sigma_x=\frac{(C_1D)^{(N_\varphi^{1/2}\tan\varphi+N_\varphi-1)}-c(2\tan\varphi+2N_\varphi^{1/2}+N_\varphi^{-1/2})}{N_\varphi^{1/2}\tan\varphi+N_\varphi-1} \tag{5}$$

where, C_1 is an integration constant.

Next, in the zone AEE'A', the equilibrium of forces acting on a small soil element will be considered in the x direction, as shown in Fig. 4.

$$D_2\mathrm{d}\sigma_x=2(\sigma_\alpha\tan\varphi+c)\mathrm{d}x \tag{6}$$

Substituting Eq. (2) into Eq. (6), and integrating it,

$$\sigma_x=\frac{C_2\exp\left(\frac{2N_\varphi\tan\varphi}{D_2}x\right)-c(2N_\varphi^{1/2}\tan\varphi+1)}{N_\varphi\tan\varphi} \tag{7}$$

where, C_2 is an integration constant.

On the other hand, assuming that the active earth pressure acts on the plane AA' in Fig. 2, the normal stress on the plane AA', i. e., the $x=0$ plane, is represented by the following equation:

$$|\sigma_x|_{x=0}=\gamma zN_\varphi^{-1}-2cN_\varphi^{-1/2} \tag{8}$$

where, z is an arbitrary depth from the ground surface and γ the unit weight of soil. The constant C_2 in Eq. (7) may be obtained considering Eq. (8) as the boundary condition. Then,

$$C_2=\gamma z\tan\varphi+c \tag{9}$$

Using Eqs. (7) and (9), the normal stress acting on the plane EE' in Fig. 2 is obtained by the following equation:

$$|\sigma_x|_{x=\{(D_1-D_2)/2\}\tan(\pi/8+\varphi/4)}=\frac{1}{N_\varphi\tan\varphi}\left\{(\gamma z\tan\varphi+c)\cdot\exp\left(\frac{D_1-D_2}{D_2}\times N_\varphi\tan\varphi\tan\left(\frac{\pi}{8}+\frac{\varphi}{4}\right)\right)-c(2N_\varphi^{1/2}\tan\varphi+1)\right\} \tag{10}$$

In regard to the zone EBB'E', considering Eq. (10) as the boundary condition on the plane

EE′ (where $D=D_2$), in Eq. (5) the constant C_1 may be obtained as follows:

$$(C_1\cdot D_2)^{(N_\varphi^{1/2}\tan\varphi+N_\varphi-1)}=\frac{(N_\varphi^{1/2}\tan\varphi+N_\varphi-1)}{N_\varphi\tan\varphi}\left\{(\gamma z\tan\varphi+c)\exp\left(\frac{D_1-D_2}{D_2}N_\varphi\tan\varphi\right.\right.$$
$$\left.\left.\times\tan\left(\frac{\pi}{8}+\frac{\varphi}{4}\right)\right)-c(2N_\varphi^{1/2}\tan\varphi+1)\right\}+c(2\tan\varphi+2N_\varphi^{1/2}+N_\varphi^{-1/2}) \tag{11}$$

Using Eqs. (5) and (11), the lateral force $p_{BB'}$ acting on the plane BB′ per unit thickness of layer in the direction of x-axis is obtained as follows:

$$p_{BB'}=D_1\{\sigma_x\}_{D=D_1}=D_1\left(\frac{D_1}{D_2}\right)^{(N_\varphi^{1/2}\tan\varphi+N_\varphi-1)}\left[\frac{1}{N_\varphi\tan\varphi}\left\{(\gamma z\tan\varphi+c)\right.\right.$$
$$\left.\times\exp\left(\frac{D_1-D_2}{D_2}N_\varphi\tan\varphi\tan\left(\frac{\pi}{8}+\frac{\varphi}{4}\right)\right)-c(2N_\varphi^{1/2}\tan\varphi+1)\right\}$$
$$\left.+c\frac{2\tan\varphi+2N_\varphi^{1/2}+N_\varphi^{-1/2}}{N_\varphi^{1/2}\tan\varphi+N_\varphi-1}\right]-c\cdot D_1\frac{2\tan\varphi+2N_\varphi^{1/2}+N_\varphi^{-1/2}}{N_\varphi^{1/2}\tan\varphi+N_\varphi-1} \tag{12}$$

The lateral force acting on a pile p per unit thickness of layer in the direction of x-axis is the difference between lateral forces acting on the plane BB′ and the plane AA′. Then, it is obtained from Eqs. (8) and (12) as follows:

$$p=p_{BB'}-D_2\{\sigma_x\}_{x=0}$$
$$=cD_1\left(\frac{D_1}{D_2}\right)^{(N_\varphi^{1/2}\tan\varphi+N_\varphi-1)}\left[\frac{1}{N_\varphi\tan\varphi}\left\{\exp\left(\frac{D_1-D_2}{D_2}N_\varphi\tan\varphi\right.\right.\right.$$
$$\left.\left.\left.\times\tan\left(\frac{\pi}{8}+\frac{\varphi}{4}\right)\right)-2N_\varphi^{1/2}\tan\varphi-1\right\}+\frac{2\tan\varphi+2N_\varphi^{1/2}+N_\varphi^{-1/2}}{N_\varphi^{1/2}\tan\varphi+N_\varphi-1}\right]$$
$$-c\left\{D_1\frac{2\tan\varphi+2N_\varphi^{1/2}+N_\varphi^{-1/2}}{N_\varphi^{1/2}\tan\varphi+N_\varphi-1}-2D_2N_\varphi^{-1/2}\right\}+\frac{\gamma z}{N_\varphi}\left\{D_1\left(\frac{D_1}{D_2}\right)^{(N_\varphi^{1/2}\tan\varphi+N_\varphi-1)}\right.$$
$$\left.\times\exp\left(\frac{D_1-D_2}{D_2}N_\varphi\tan\varphi\tan\left(\frac{\pi}{8}+\frac{\varphi}{4}\right)\right)-D_2\right\} \tag{13}$$

Therefore, the total lateral force, induced on a stabilizing pile due to the plastically deforming soil layer, will be obtained by the integration of Eq. (13) along the depth of the soil layer.

Cohesionless soil: The cohesion will be considered as zero in cohesionless soils. Replacing the cohesion c by zero in Eq. (13), the lateral force p is obtained as follows:

$$p=\frac{\gamma z}{N_\varphi}\left\{D_1\left(\frac{D_1}{D_2}\right)^{(N_\varphi^{1/2}\tan\varphi+N_\varphi-1)}\cdot\exp\left(\frac{D_1-D_2}{D_2}N_\varphi\tan\varphi\tan\left(\frac{\pi}{8}+\frac{\varphi}{4}\right)\right)-D_2\right\} \tag{14}$$

Cohesive soil: The angle of internal friction φ will be negligible in cohesive soils. Replacing the angle of internal friction φ by zero, equations are derived as described before. In Eq. (4), let the angle of internal friction be zero.

$$\frac{dD}{D}=\frac{d\sigma_x}{3c} \tag{15}$$

Integrating Eq. (15),

$$\sigma_x=3c\log D+C_3 \tag{16}$$

where, C_3 is an integration constant. Substituting Eq. (2) into Eq. (6), and letting the angle of internal friction to vanish,

$$\frac{d\sigma_x}{c}=\frac{2}{D_2}dx \tag{17}$$

Integrating Eq. (17),

$$\sigma_x=\frac{2c}{D_2}x+C_4 \tag{18}$$

where, C_4 is an integration constant.

On the other hand, assuming that the active earth pressure acts on the plane AA′ in Fig. 2, the normal stress on the plane AA′ is represented as follows, referring to Eq. (8):

$$|\sigma_x|_{x=0}=\gamma z-2c \tag{19}$$

The constant C_4 in Eq. (18) may be obtained considering Eq. (19) as the boundary condition. Then, the normal stress acting on the plane EE′ in Fig. 2 is obtained as follows:

$$|\sigma_x|_{x=\{(D_1-D_2)/2\}\tan(\pi/8)}=c\left(\frac{D_1-D_2}{D_2}\tan\frac{\pi}{8}-2\right)+\gamma z \tag{20}$$

In regard to the zone EBB′E′, considering Eq. (20) as the boundary condition, the constant C_3 in Eq. (16) is obtained as follows:

$$C_3=c\left(\frac{D_1-D_2}{D_2}\tan\frac{\pi}{8}-3\log D_2-2\right)+\gamma z \tag{21}$$

Using Eqs. (16) and (21), the lateral force $p_{BB'}$ acting on the plane BB′ per unit thickness of layer in the direction of x-axis is obtained as follows:

$$\begin{aligned} p_{BB'}&=D_1\{\sigma_x\}_{D=D_1} \\ &=D_1\left\{c\left(3\log\frac{D_1}{D_2}+\frac{D_1-D_2}{D_2}\tan\frac{\pi}{8}-2\right)+\gamma z\right\} \end{aligned} \tag{22}$$

Then, as for the cohesive soils, the lateral force p acting on a pile per unit thickness of layer in the direction of x-axis is obtained as follows:

$$\begin{aligned} p&=p_{BB'}-D_2\{\sigma_x\}_{x=0} \\ &=c\left\{D_1\left(3\log\frac{D_1}{D_2}+\frac{D_1-D_2}{D_2}\tan\frac{\pi}{8}\right)-2(D_1-D_2)\right\}+\gamma z(D_1-D_2) \end{aligned} \tag{23}$$

Theory of Plastic Flow

Following assumptions are set up in order to analyze the lateral force by the theory of plastic flow. The soil between two piles ACDFF′D′C′A′, which flows in at a velocity v_1, is considered as shown in Fig. 5.

Assumptions:

a) A visco-plastic flow occurs in the zone AEBB′E′A′ just around piles, and the direction of the flow is always centripetal towards the center 0 in the zone EBB′E′, where the lines $\overline{EB}$ and $\overline{E'B'}$ make an angle of $\pi/4$ with the x-axis.

b) The soil layer is in a steady state, and it is represented as visco-plastic solids, i.e., Bingham solids with a yield stress τ_y and a plastic viscosity η_p.

c) The soil layer flows uniformly in the direction of depth.

d) Forces exerted on small portions $\overline{GH}$ and $\overline{G'H'}$ by a small clay element GHH′G′ at a certain radius r can be obtained as the sum of a force due to earth pressure and a viscous force obtained by assuming the flow of this element to be a visco-plastic flow in a channel of width $\overset{\frown}{GG'}$ with smooth base.

e) Piles are rigid.

First of all, a channel of width B, length L and unit depth is considered as shown in Fig. 6. No friction acts on its base, because it is assumed to be smooth. The steady visco-plastic flow of Bingham solids is dealt with in such a channel. When a pressure $\Delta p'$ is applied, the total shearing force p_0 acting on side walls may be obtained by the same manner as in the analysis of pipe flow (Nakagawa and Kanbe, 1959), as follows:

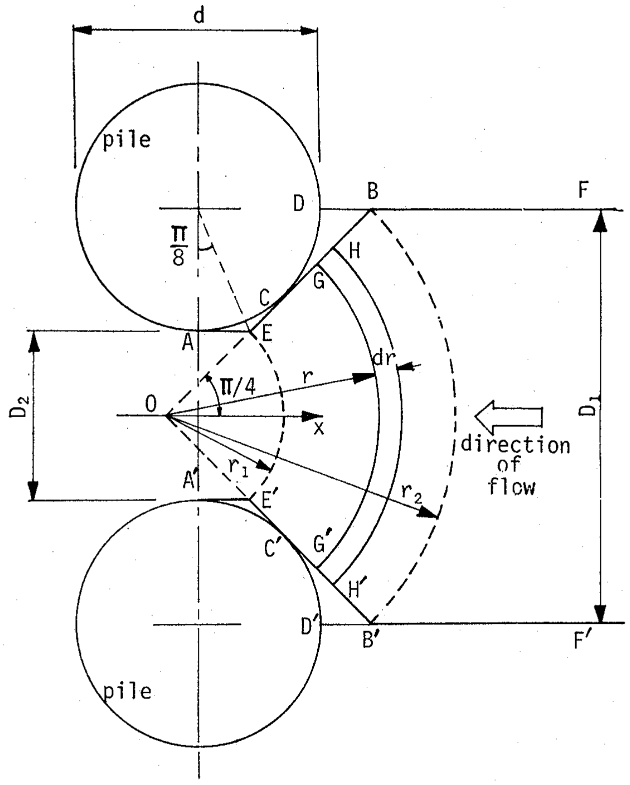


Fig. 5. State of plastic flow in the ground just around piles

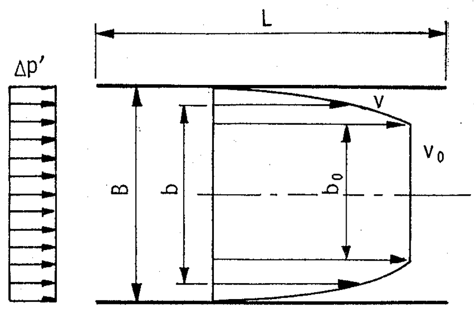


Fig. 6. Plastic flow in a channel with smooth base

$$p_0=\frac{2L}{B}\{2\eta_p v_0+B\tau_y+\sqrt{(2\eta_p v_0+B\tau_y)^2-B^2\tau_y^2}\} \tag{24}$$

where, v_0 denotes a velocity of plug in a channel.

In regard to the zone EBB′E′ in Fig. 5, using Eq. (24) under the assumption d), a viscous force exerted on small portions $\overline{\mathrm{GH}}$ and $\overline{\mathrm{G'H'}}$ by a small clay element GHH′G′ may be derived. In this small clay element, the velocity of plug is described by v_p and the x-axis component of shearing force acting on small portions $\overline{\mathrm{GH}}$ and $\overline{\mathrm{G'H'}}$ by dp_1. Carrying out such replacements of notation as $p_0=\sqrt{2}\,\mathrm{d}p_1$, $B=\pi r/2$, $L=\mathrm{d}r$ and $v_0=v_p$ in Eq. (24), the following equation is obtained:

$$\mathrm{d}p_1=\frac{\sqrt{2}}{\pi r}\mathrm{d}r\left\{4\eta_p v_p+\pi r\tau_y+\sqrt{(4\eta_p v_p+\pi r\tau_y)^2-(\pi r\tau_y)^2}\right\} \tag{25}$$

It is considered that the soil flows uniformly at average velocities v_1, v_r and v_2 through surfaces $\overset{\frown}{\mathrm{BB'}}$, $\overset{\frown}{\mathrm{GG'}}$ and $\overset{\frown}{\mathrm{EE'}}$ in Fig. 5, respectively. From continuity,

$$v_r=\frac{2D_1}{\pi r}v_1 \tag{26}$$

$$v_2=\frac{D_1}{D_2}v_1 \tag{27}$$

Assuming that a velocity of plug v_p approximately equals v_r, when a distance between the surface EB (E′B′) and the plug is small, the x-axis component of viscous force p_1 acting on surfaces EB and E′B′ is obtained by substituting Eq. (26) into Eq. (25) and integrating it, as follows:

$$p_1=\int_{D_2/\sqrt{2}}^{D_1/\sqrt{2}}\frac{1}{\sqrt{2}}\left\{m\frac{1}{r^2}+2\tau_y+2\sqrt{m\tau_y}\frac{\sqrt{r^2+\frac{m}{4\tau_y}}}{r^2}\right\}\mathrm{d}r$$
$$=\tau_y(D_1-D_2)-m(D_1^{-1}-D_2^{-1})+\sqrt{2m\tau_y}\left\{\sqrt{1+\frac{m}{2\tau_yD_2^2}}-\sqrt{1+\frac{m}{2\tau_yD_1^2}}\right.$$
$$\left.+\log\frac{D_1\left(1+\sqrt{1+\frac{m}{2\tau_yD_1^2}}\right)}{D_2\left(1+\sqrt{1+\frac{m}{2\tau_yD_2^2}}\right)}\right\} \tag{28}$$

where, $m=16\eta_p v_1 D_1/\pi^2$.

On the other hand, the x component of total force p_2 due to earth pressure acting on surfaces EB and E′B′ is obtained, considering the earth pressure to be the active one. That is,

$$p_2=(\gamma z-2c)(D_1-D_2) \tag{29}$$

where, z is an arbitrary depth from the ground surface, γ the unit weight of soil and c the cohesion of soil. It is supposed that in this case the cohesion of soil c approximately equals the yield stress of soils τ_y. Next, the x component of the force p_3 acting on surfaces AE and A′E′ is only the viscous force, because surfaces AE and A′E′ are parallel to the x-axis. Therefore, carrying out such replacements of notation as $p_0=p_3$, $B=D_2$, $L=[(D_1-D_2)/2]\tan(\pi/8)$ and $v_0=v_2$ in Eq. (24), and using Eq. (27), the following equation is obtained:

$$p_3=(\sqrt{2}-1)(D_1-D_2)\left\{\frac{\pi^2 m}{8D_2^2}+\tau_y+\sqrt{\left(\frac{\pi^2 m}{8D_2^2}\right)^2+\frac{\pi^2 m\tau_y}{4D_2^2}}\right\} \tag{30}$$

Then, the lateral force acting on a pile p per unit thickness of layer in the direction of x-axis is obtained by summing up p_1, p_2 and p_3. That is,

$$p=p_1+p_2+p_3$$
$$=\sqrt{2m\tau_y}\left\{\sqrt{1+\frac{m}{2\tau_yD_2^2}}-\sqrt{1+\frac{m}{2\tau_yD_1^2}}+\log\frac{D_1\left(1+\sqrt{1+\frac{m}{2\tau_yD_1^2}}\right)}{D_2\left(1+\sqrt{1+\frac{m}{2\tau_yD_2^2}}\right)}\right\}$$
$$+(D_1-D_2)\left\{\frac{(\sqrt{2}-1)\pi^2 m}{8D_2^2}+(\sqrt{2}-1)\sqrt{\left(\frac{\pi^2 m}{8D_2^2}\right)^2+\frac{\pi^2 m\tau_y}{4D_2^2}}+\frac{m}{D_1D_2}\right.$$
$$\left.+\sqrt{2}\tau_y-2c+\gamma z\right\} \tag{31}$$

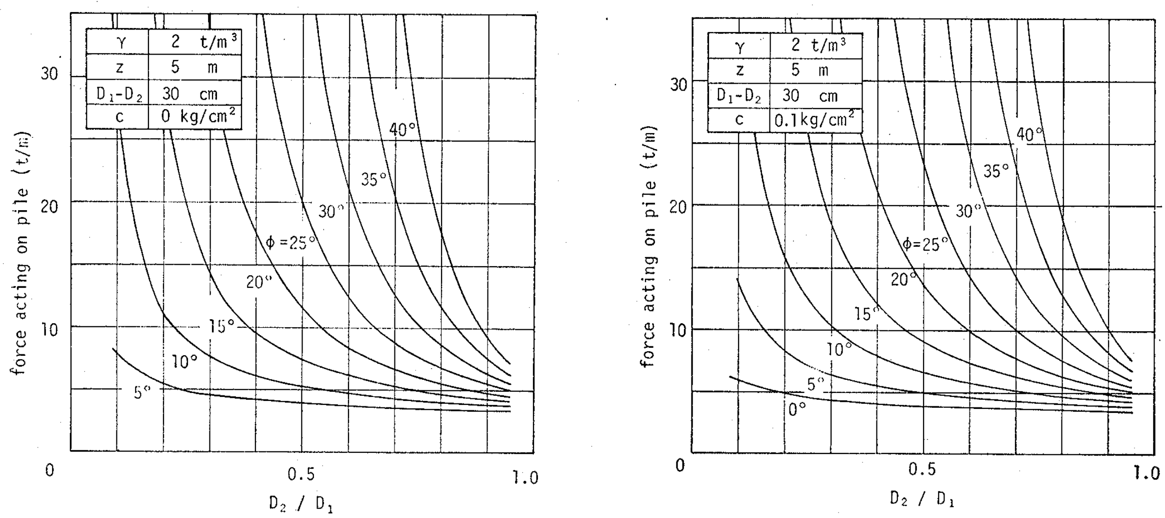
Finally, the total lateral force, induced on a stabilizing pile due to the plastic flow of soil layer, will be obtained by the integration of Eq. (31) along the depth of the soil layer.

CHARACTERISTICS OF THEORIES

The theoretical equations of the lateral force induced on a stabilizing pile include many parameters in both the theories of plastic deformation and plastic flow, as described before. Therefore, it may be necessary that the characteristics of theoretical equations are clarified by examining the effects of various parameters on the lateral force. The common parameters in both theoretical equations are the unit weight γ, the depth from the ground surface z, the center-to-center interval between piles D_1 and the clear interval between piles D_2. Moreover, there are some other parameters representing mechanical properties of soils. That is, in the theory of plastic deformation, the angle of internal friction φ and the cohesion c, and in the theory of plastic flow, the yield stress τ_y ($\fallingdotseq c$), the plastic viscosity η_p and the flow velocity of soil layer v_1.

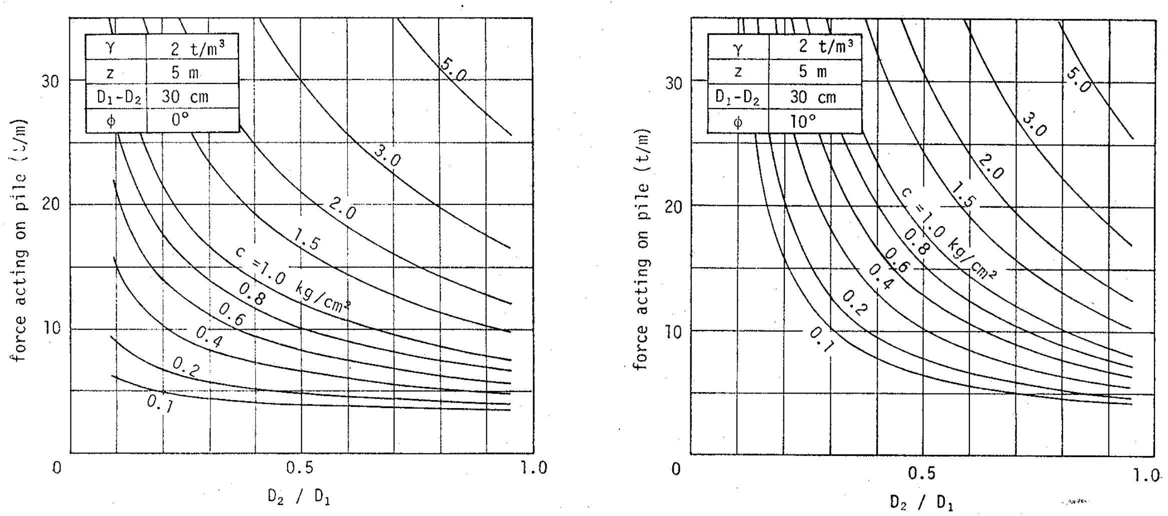
In regard to the parameters γ and z, the lateral force p increases linearly as γ or z increases, because both theoretical equations of lateral force are linear expressions of γ or z. On the other hand, the effects of parameters D_1, D_2 and mechanical properties of soils are complicated, and very important at that. Therefore, the characteristics of theoretical equations are examined taking notice of these parameters in the following. In order to examine the effects of D_1 and D_2, the ratio D_2/D_1 and the diameter of pile (D_1-D_2) are chosen as parameters. And for the effects of mechanical properties of soils, in the theory of plastic deformation φ and c are chosen as parameters, and in the theory of plastic flow τ_y and $(v_1 \cdot \eta_p)$. The reason why $(v_1 \cdot \eta_p)$ is chosen as a parameter is due to the fact that v_1 and η_p are necessarily included as the product in the theoretical equation of plastic flow.

The characteristics of theoretical equation for the theory of plastic deformation are shown in Figs. 7(a) and (b), 8(a) and (b), and 9. All these figures show the relation



(a) $c=0$ kg/cm²　　(b) $c=0.1$ kg/cm²

Fig. 7. The effect of angle of internal friction ϕ on the theory of plastic deformation



(a) $\phi=0°$　　(b) $\phi=10°$

Fig. 8. The effect of cohesion c on the theory of plastic deformation

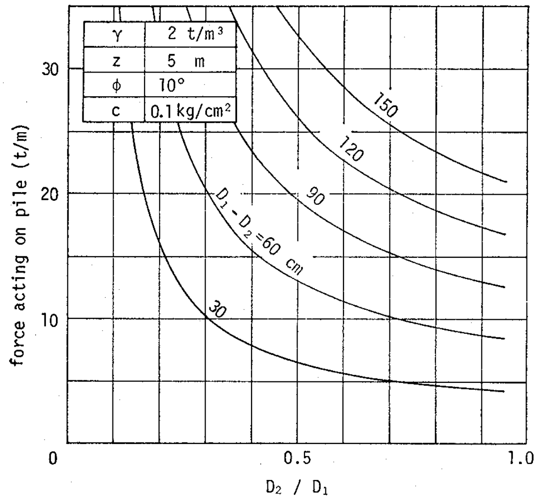


Fig. 9. The effect of pile diameter (D_1-D_2) on the theory of plastic deformation

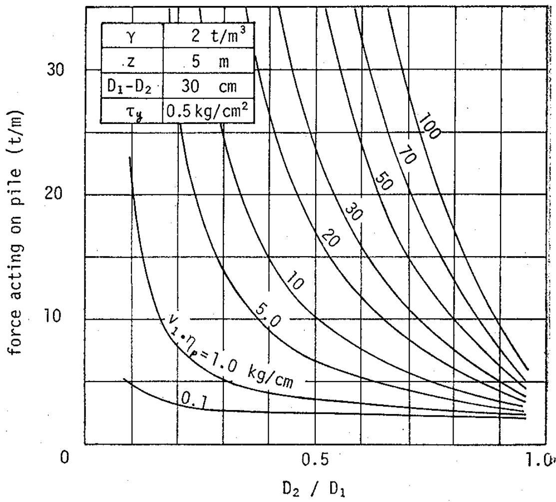


Fig. 10. The effect of product of flow velocity and plastic viscosity ($v_1 \cdot \eta_p$) on the theory of plastic flow

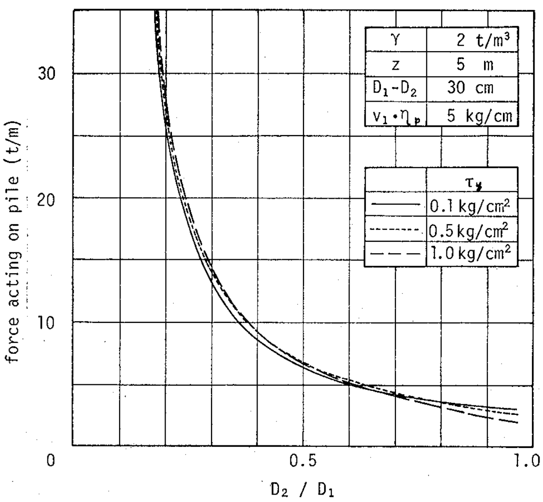


Fig. 11. The effect of yield stress τ_y on the theory of plastic flow

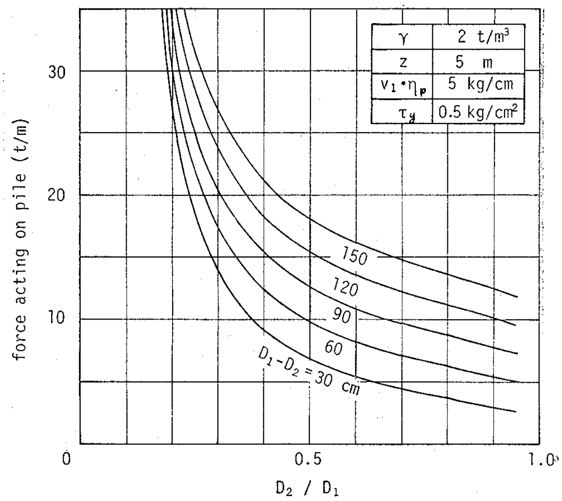


Fig. 12. The effect of pile diameter (D_1-D_2) on the theory of plastic flow

between the lateral force per unit depth p and the ratio D_2/D_1. Generally speaking of these figures, when the diameter of pile is constant, the lateral force p increases as the ratio D_2/D_1 decreases, and it increases rapidly as D_2/D_1 decreases still more. In other words, it increases as the interval between piles becomes narrow relatively. Fig. 7(a) and (b) show the effect of φ for constant c, and Fig. 8(a) and (b) the effect of c for constant φ. From these figures, it is seen that the lateral force p increases as φ or c increases. These tendencies are reasonable, because, when φ or c in soils is larger, the soils just around piles are harder to pass through between two piles, and then the lateral force p becomes larger in a plastic state satisfying Mohr-Coulomb's yield criterion. When the diameter and the interval of piles are constant, the lateral force p increases nonlinearly as φ increases and linearly as c increases. Fig. 9 shows the effect of the diameter of pile (D_1-D_2). The lateral force p increases as (D_1-D_2) increases. This relation is linear when D_2/D_1 is constant.

Next, the characteristics of theoretical equation for the theory of plastic flow are shown in Figs. 10, 11 and 12. All these figures also show the relation between the lateral force

p and the ratio D_2/D_1. Generally speaking of these figures, when the diameter of pile is constant, the lateral force p increases as the ratio D_2/D_1 decreases, in the same manner as the theory of plastic deformation. Fig. 10 shows the effect of $(v_1 \cdot \eta_p)$ for constant τ_y, and Fig. 11 the effect of τ_y for constant $(v_1 \cdot \eta_p)$. From these figures, it is seen that the lateral force p increases as $(v_1 \cdot \eta_p)$ increases, and that it does not change very much as τ_y changes. In other words, the lateral force p is hardly influenced by τ_y, but significantly by $(v_1 \cdot \eta_p)$. Fig. 12 shows the effect of the pile diameter (D_1-D_2). The lateral force p increases as (D_1-D_2) increases. This relation is linear when D_2/D_1 is constant.

COMPARISONS WITH FIELD MEASUREMENTS

The observed lateral forces acting on stabilizing piles in landslide areas are compared with the theoretical ones. The observations are made in the typical Tertiary landslide areas of Niigata in Japan, which are Katamachi, Higashitono and Kamiyama landslide areas (Fukumoto, 1972, 1973). In these landslides, the clay layers of several meters thick, mixed with pieces of mudstone, slide slowly. The stabilizing piles used are hollow reinforced concrete piles (diameter 300 mm, wall thickness 60 mm) in Katamachi landslide area, and steel pipe piles (diameter 318.5 mm, wall thickness 6.9 mm) in the other landslide areas. These piles are set up zigzag in two rows at 4 m intervals, and the interval between the rows is 2 m. Electric strain gages are installed in piles, and the lateral force is analyzed from the measured strains induced in piles. Piles which become the object of comparison between the observed and the theoretical lateral force, are the Katamachi B pile, the Higashitono No. 2 and No. 3 piles, and the Kamiyama No. 1 and No. 2 piles. The conditions of surrounding ground and the observed lateral forces (dotted lines) acting on these five piles are shown in Fig. 13 (a)∼(e).

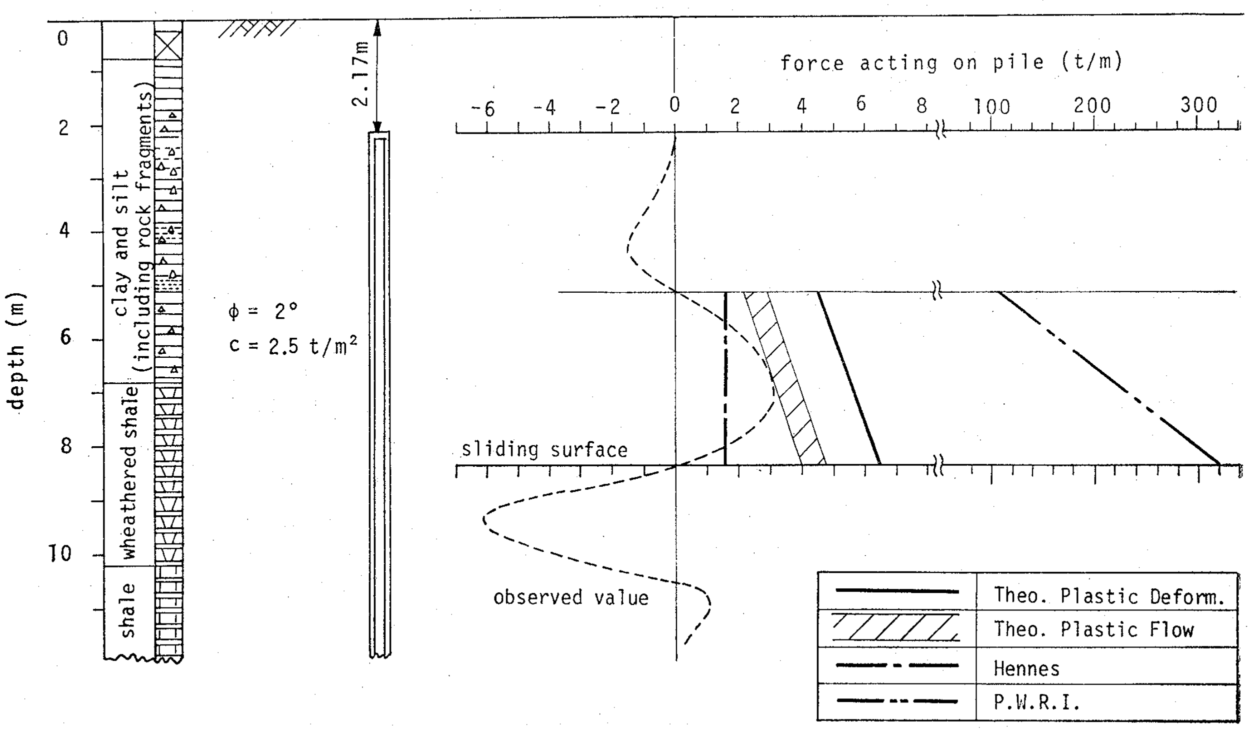
On the other hand, soil properties are shown in Table 1, which are required for obtaining the theoretical values of lateral force. For the theory of plastic deformation, soil constants c and φ are obtained by the shear test or the standard penetration test. For the theory of plastic flow, soil constants τ_y and η_p are estimated by the Komamura's equation (1967) and the measured values by the vane shear test (Nakano, 1963). Because these values of τ_y and η_p are not estimated with accuracy and the theoretical values due to the theory of plastic flow is significantly influenced by the value of $(v_1 \cdot \eta_p)$, it is supposed that the theoretical values due to the theory of plastic flow do not have high reliability. Therefore, the comparison between the observed and the theoretical values due to the theory of plastic flow is restricted to qualitative or rough discussions.

The theoretical lateral forces, obtained by using the soil constants in Table 1, are shown

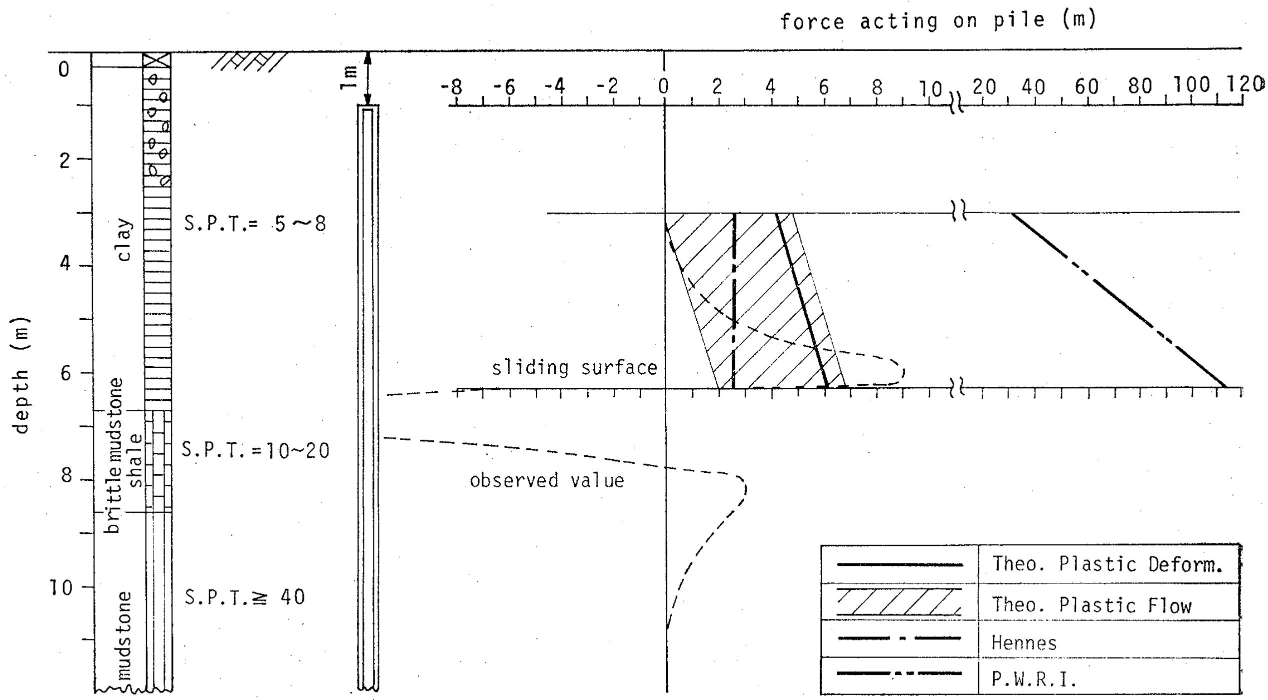
Table 1. Soil properties of plastically deforming ground

soil constant \ pile	Katamachi	Kamiyama		Higashitono	
	B	No. 1	No. 2	No. 2	No. 3
unit weight γ t/m³	1.9	1.9	1.9	1.9	1.9
angle of internal friction φ degree	2	0	0	0	0
cohesion c kg/cm²	0.25	0.41	0.41	0.44	0.51
yield stress τ_y kg/cm²	0.17∼0.29	0.17∼0.41	0.17∼0.41	0.17∼0.44	0.17∼0.51
product of flow velocity and plastic viscosity $(v_1 \cdot \eta_p)$ kg/cm²	3.8×10^{-7}∼ 6.1×10^{-1}	3.8×10^{-7}∼ 2.6×10^{1}	3.8×10^{-7}∼ 2.6×10^{1}	3.8×10^{-7}∼ 6.0×10^{1}	3.8×10^{-7}∼ 5.5×10^{1}

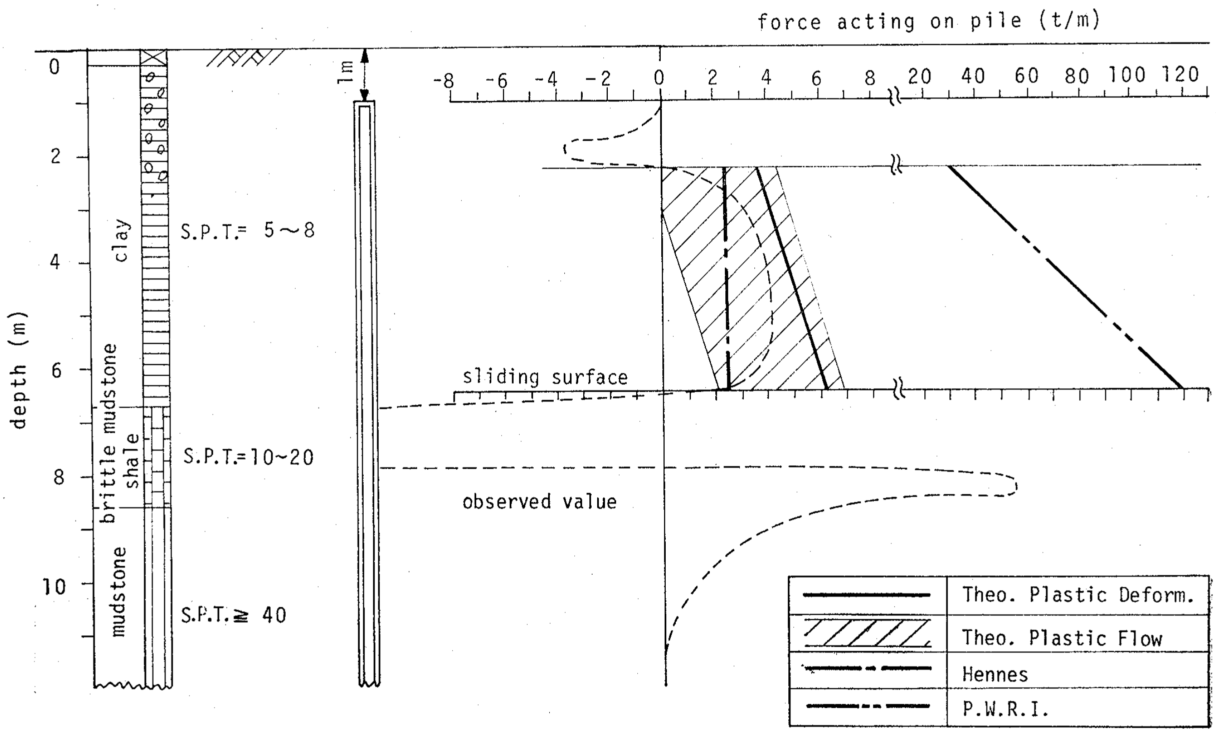
in Fig. 13 (a)~(e). The solid lines and the shadowed portions correspond to the theories of plastic deformation and plastic flow, respectively. And for the purpose of comparison, the theoretical values due to the Hennes' equation and the equation of P. W. R. I. are also shown by chain lines with one dot and two dots, respectively.



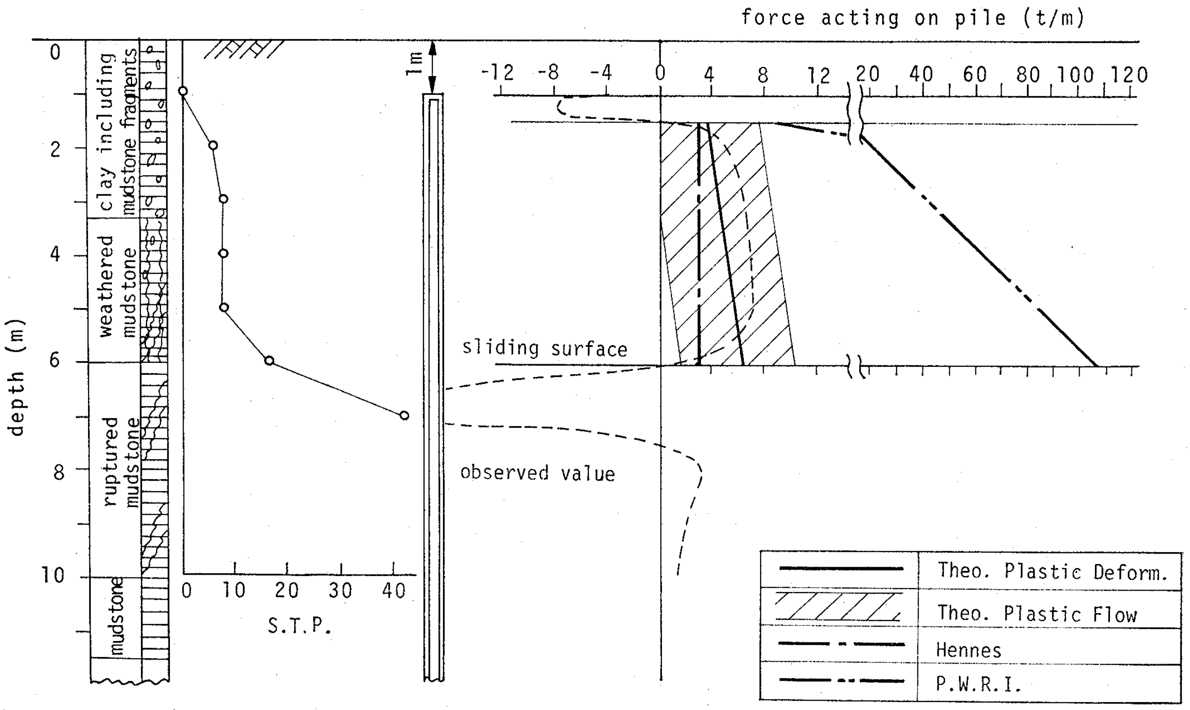
(a) Katamachi B pile



(b) Kamiyama No. 1 pile

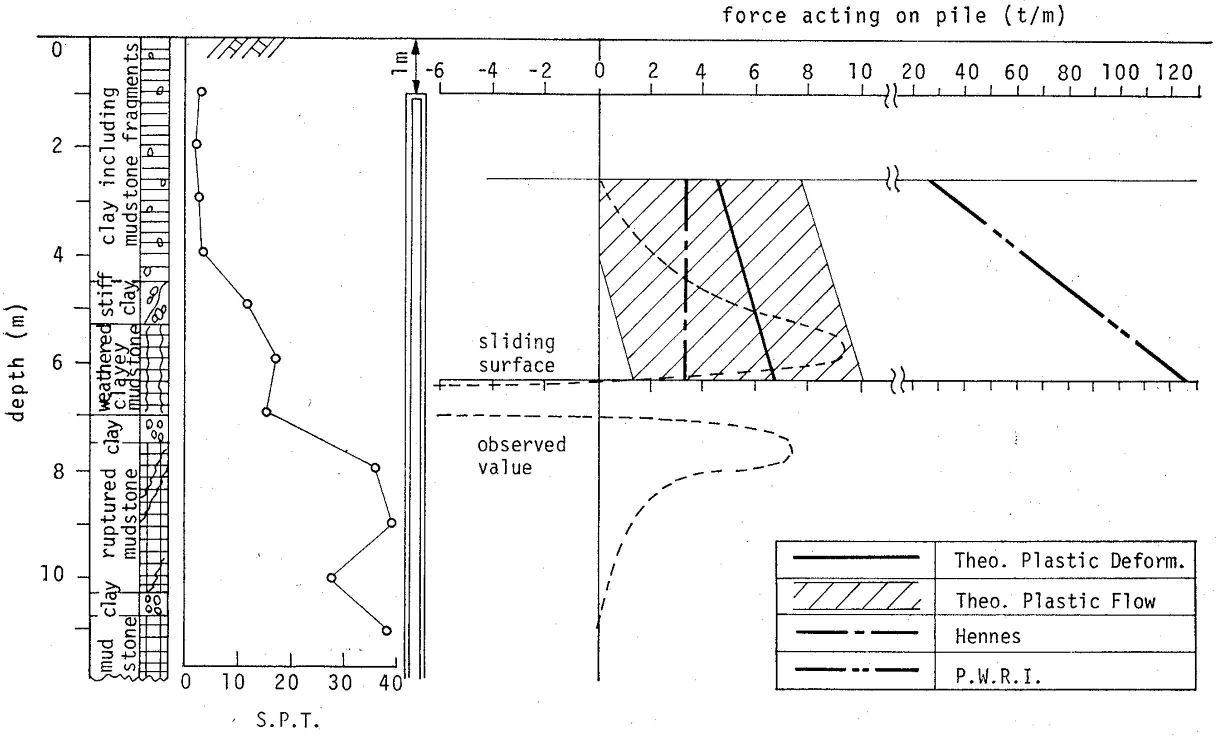


(c) **Kamiyama No. 2 pile**



(d) **Higashitono No. 2 pile**

In Fig. 13 (a)~(e), there are two types of distributions of observed lateral forces. The one is a case where reaction acts at the top of pile, and the other is a case where the lateral force concentrates between 1 and 2 m above the sliding surface and where the distribution of them seems to be a triangle. It is considered that such triangular distributions of lateral force have been obtained as a result of influences of pile deformation and



(e) **Higashitono No. 3 pile**

Fig. 13. Condition of surrounding ground and comparison between the theoretical and the observed values of lateral force acting on stabilizing piles against landslides

non-uniform movements of sliding layer with depth. Whereas, the theoretical distributions of lateral force are trapezoidal, because it is assumed that the piles are perfectly rigid.

The observed and the theoretical lateral forces are compared quantitatively. The theoretical values due to the Hennes' equation are less than the observed ones, and those due to the equation of P. W. R. I. greater than the observed ones. The theoretical values due to the theories of plastic deformation and plastic flow fall between the values due to the Hennes' equation and the equation of P. W. R. I., and they are in approximately the same order of magnitude as the observed values. These facts seem to be a matter of course, because in the Hennes' equation the compressive and the shear deformations of soil between piles is not considered and in the equation of P. W. R. I. the limiting value of lateral force is obtained. As the soil constants used in the theory of plastic flow are comparatively rough, it does not seem fruitful to make more quantitative discussions on the theoretical values due to the theory of plastic flow. However, when soil constants are obtained with a certain degree of accuracy, the reliability of the theory of plastic flow will be discussed in more detail and the possibility of its availability may remain, because the observed values are included in the regions of the theoretical ones.

Next, each pile will be discussed. Fig. 14 shows the comparison between the observed and the theoretical total lateral forces. The theory of plastic deformation, the Hennes' equation and the equation of P. W. R. I. are chosen as the theoretical values. Generally speaking, the theoretical values due to the equation of P. W. R. I. overestimate the observed ones. In the Katamachi B pile, the observed total lateral force is the closest to the theoretical one due to the Hennes' equation. But the largest observed value of lateral force is considerably larger than that of Hennes' equation as seen from Fig. 13 (a). And the theoretical value is obtained under the assumption that piles do not deform, but the observed value is affected by pile deformation; especially it becomes very small near both the surface of sliding and the top of pile. Moreover, as this pile is a reinforced concrete pile and its bending strength is comparatively small (about 1/2.5 of that of steel pipe pile used), it is considered that the bending failure occurs in the pile before the surrounding ground becomes plastic and the lateral force becomes smaller. From the reasons described above, it is supposed that the observed total lateral force acting on the Katamachi B pile apparently resembles the theoretical value due to the Hennes' equation. Therefore, it is considered that the Hennes' equation may fundamentally underestimate the lateral force. In the Higashitono No. 2 pile and the Kamiyama No. 2 pile, the theoretical values due to the theory of plastic deformation approximate the observed ones in both the total lateral force and its distribution, and the theoretical ones due to the Hennes' equation are underestimated, as seen from Figs. 13 (c), (d) and 14. In the piles of Higashitono No. 3 and Kamiyama No. 1, the observed total lateral forces are approximately plotted between the theoretical values due to the theory of plastic deformation and the Hennes' equation, as seen from Fig. 14. But the distributions of observed and the theoretical lateral forces are considerably different, as seen from Fig. 13 (b) and (e). It is supposed that this difference may be caused by the pile deformation because of no reaction near the pile top.

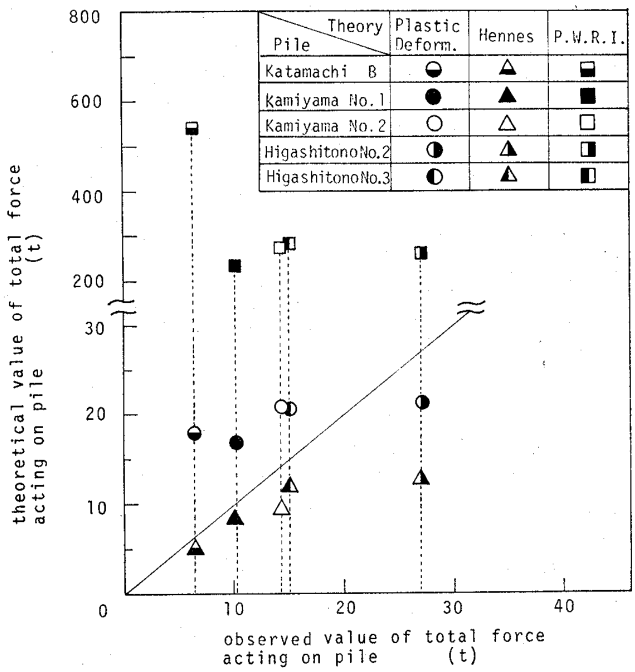


Fig. 14. Comparison between the theoretical and the observed values of total lateral force acting on stabilizing piles against landslides

The above discussions may be summarized as follows: The theory of plastic deformation is based on the assumptions that the piles are rigid and that the ground above the sliding surface reaches a plastic state only just around piles. In such a case that these assumptions do not hold, that is, the effect of pile deformation is large and the bending failure occurs in piles before the assumed plastic state occurs, the observed value of lateral force becomes smaller than the theoretical one due to the theory of plastic deformation, and it happened to approximate the Hennes' equation apparently (as the Katamachi B pile). In the case where no reaction occurs at the pile top behind a pile, the distribution of observed lateral force differs from the theoretical one, because of large deformation of pile (as the piles of Higashitono No. 3 and Kamiyama No. 1). But, in the case where the assumption regarding pile deformation is approximately held by the reaction at the pile top behind a pile, the observed total lateral force and its distribution may approximate the theoretical ones due to the theory of plastic deformation (as the piles of Higashitono No. 2 and Kamiyama No. 2). It may be concluded that the lateral force acting on piles due to the plastically deforming ground may be estimated by the theory of plastic deformation, under the condition of restrained pile top.

CONCLUSIONS

In this paper have been investigated the growth mechanism and the method of estimating the lateral force acting on stabilizing piles due to the surrounding soil undergoing defomation. The following conclusions may be made:

(1) The theoretical solutions of the lateral force are obtained considering the interval between piles in a row and two types of plastic states in the theories of plastic deformation and plastic flow.

(2) In both theories, the lateral force acting on a pile p per unit thickness of layer increases as the ratio D_2/D_1 decreases and it increases repidly as D_2/D_1 decreases still more, in the case of a constant diameter of pile.

(3) In the theory of plastic deformation, the lateral force p increases as the angle of internal friction φ or the cohesion c increases. It increases nonlinearly for φ or linearly for c, in the case where the diameter and the interval of piles are constant. And, it increases linearly as (D_1-D_2) increases when D_2/D_1 is constant.

(4) In the theory of plastic flow, the lateral force p increases as $(v_1 \cdot \eta_p)$ increases, but it does not change very much as τ_y changes. And, it increases linearly as (D_1-D_2) increases when D_2/D_1 is constant.

(5) The theoretical lateral forces due to both the theories of plastic deformation and plastic flow agree, in terms of the order of magnitude, with the observed ones which are obtained in the stabilizing piles against landslides. Whereas, the theoretical values due to the Hennes' equation underestimate and the equation of P.W.R.I. overestimate the observed values.

(6) The lateral force p may be estimated by the theory of plastic deformation under the condition of restrained pile top.

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NOTATION

B = width of a channel

c = cohesion of soils

C_1, C_2, C_3, C_4 = integration constants

d = diameter of pile

D = distance to the direction of piles in a row

D_1 = center-to-center interval between piles in a row

D_2 = clear interval between piles

H = thickness of soil layer

L = length of a channel

$m = 16\ \eta_p v_1 D_1/\pi^2$

N_φ = flow value $(\tan^2(\pi/4+\varphi/2))$

p = lateral force acting on a pile per unit thickness of layer in the direction of x-axis

$p_{BB'}$ = lateral force acting on the plane BB′ per unit thickness of layer in the direction of x-axis

p_1 = viscous force acting on surfaces EB and E′B′ per unit thickness of layer in the direction of x-axis

p_2 = total force due to earth pressure acting on surfaces EB and E′B′ per unit thickness of layer in the direction of x-axis

p_3 = force acting on surfaces AE and A′E′ per unit thickness of layer in the direction of x-axis

$\Delta p'$ = applied pressure in a channel

p_0 = total shearing force acting on side walls of a channel
r = radius of $\widehat{GG'}$
r_1 = radius of $\widehat{EE'}$
r_2 = radius of $\widehat{BB'}$
v_1 = velocity through the surface $\widehat{BB'}$
v_2 = velocity through the surface $\widehat{EE'}$
v_r = velocity through the surface $\widehat{GG'}$
v_0 = velocity of plug in a channel
v_p = velocity of plug in a flow between piles
x = direction of deformation in a soil layer
z = depth
$\alpha = \pi/4 + \varphi/2$
γ = unit weight of soil
η_p = plastic viscosity
σ_x = normal stress on the surface making a right angle with x-axis
σ_α = normal stress on the surface making an angle of α with x-axis
τ_y = yield stress
φ = angle of internal friction of soils

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