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STRESS-DILATANCY RELATIONS OF ANISOTROPIC SANDS IN THREE DIMENSIONAL STRESS CONDITION

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ABSTRACT

The stress-dilatancy equations which relate principal strain increment ratio with principal stress ratio in the previous studies are first reviewed and shown is the necessity to investigate the relationship between the stress-dilatancy equation for axisymmetric deformation and that for general stress condition and to clarify the effect of inherent anisotropy on the stress-dilatancy equations. Then, in summary form, three fundamental postulates are introduced as the bases on which a theory to investigate above mentioned two problems is to be established. The stress-dilatancy performance of sands predicted by the proposed theory were compared with several experimental data obtained by other investigators. These data include those of triaxial compression and extension tests, general stress condition tests ($\sigma_1>\sigma_2>\sigma_3$) and plane strain tests. This comparison disclosed the relevence of the theory in spite of its simplicity.

Key words: anisotropy, dilatancy, drained shear, plasticity, sand
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INTRODUCTION

Among several stress-dilatancy equations which have been proposed, the simplest one is the equation for axisymmetric deformation proposed by Rowe (1962) and Rowe, Barden and Lee (1964) as,

$$R=DK \tag{1}$$

where $R=\sigma_1/\sigma_3$, $K=\tan^2(\pi/4+\phi_\mu/2)$ and $D=-2d\varepsilon_3/d\varepsilon_1$ for the triaxial compression case of $\sigma_2=\sigma_3$ or $D=-d\varepsilon_3/2d\varepsilon_1$ for the triaxial extension case of $\sigma_2=\sigma_1$. Barden and Khayatt (1966) examined Eq. (1) by the precise triaxial and extension tests on sand and showed that Eq. (1) fits the experimental data of triaxial compression and extension tests excellently. Furthermore, Horne (1965) derived the theoretical stress-dilatancy equations for the general stress condition of $\sigma_1>\sigma_2>\sigma_3$ as follows.

$\sigma_1>\sigma_2>\sigma_3\,(d\varepsilon_2<0)$

$$-\frac{\sigma_1 d\varepsilon_1}{\sigma_2 d\varepsilon_2+\sigma_3 d\varepsilon_3}=K \tag{3}$$

$\sigma_1>\sigma_2>\sigma_3\,(d\varepsilon_2=0$; plane strain)

$$-\frac{\sigma_1 d\varepsilon_1}{\sigma_3 d\varepsilon_3}=K \quad \text{or} \quad \frac{\sigma_1}{\sigma_3}=-\frac{d\varepsilon_3}{d\varepsilon_1}K \tag{4}$$

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$$\sigma_1>\sigma_2>\sigma_3\,(d\varepsilon_2>0)$$
$$-\frac{\sigma_1 d\varepsilon_1+\sigma_2 d\varepsilon_2}{\sigma_3 d\varepsilon_3}=K \tag{5}$$

However, Barden, Khayatt and Wightman (1969) showed by conducting triaxial compression and extension tests and plane strain tests on same sand that the value of K in Eq. (4) for the plane strain case is larger than that in Eq. (1) for the triaxial compression and extension cases. This was explained by Barden et. al (1969) as such that the minimum degree of freedom for interparticle movement occurs under the condition of plane strain. However, this explanation would be rather vague and it is be necessary to investigate the stress-dilatancy equations for the general stress condition more rationally based on experimental data.

Furthermore, it may be dointed out that so far the inherent anisotropy has not been considered directly in deriving the stress dilatancy equations. Meanwhile, Arthur and Menzies (1972) prepared the samples with the various inherent anisotropies by pouring air-dried sand into tilted cuboidal mold with various tilting angles and conducted triaxial compression tests where $\sigma_1>\sigma_2=\sigma_3$. They showed the case where $d\varepsilon_2/d\varepsilon_3$ is about 1.5 in spite of $\sigma_2=\sigma_3$. Such inherent anisotropy was also shown by El-Shoby (1969) who conducted isotrodic consolidation tests of cylindrical triaxial samples prepared by pouring sand in a vertically stood mold. In this case, it was shown that $d\varepsilon_1<d\varepsilon_3$ in spite of $\sigma_1=\sigma_3$ in isotropic consolidation process.

It is the first aim of this paper to investigate the relationship between the stress-dilatancy equation for axisymmetric deformation and that for general stress condition where $\sigma_1>\sigma_2>\sigma_3$. The second aim is to clarify the effect of inherent anisotropy on the stress-dilatancy equations. In this paper, only the case is concerned where the axes of principal stresses do not rotate.

FUNDAMENTAL EQUATIONS

Any element in soil mass is under three-dimensional stress condition. As it is rather complicated to investigate stress-dilatancy relations under such three-dimensional stress condition from the beginning, it is better to suppose idealized two-dimensional stress condition as the first step. Now, define two principal stresses in two-dimensional stress system by σ_X and σ_Y, whehe $\sigma_X>\sigma_Y$ and define two principal strain increments by $d\varepsilon_{XXY}$ and $d\varepsilon_{YXY}$ which correspond to σ_X and σ_Y respectively. The suffix XY of $d\varepsilon_{XXY}$ and $d\varepsilon_{YXY}$ means $\sigma_X>\sigma_Y$ stress-system. In this case,

$$\left.\begin{aligned}\sigma_X&=\sigma_1\text{ ; major principal stress}\\ \sigma_Y&=\sigma_3\text{ ; minor principal stress}\end{aligned}\right\} \tag{6}$$

$$\left.\begin{aligned}d\varepsilon_{XXY}&=d\varepsilon_1\text{ ; major principal strain increment}\\ d\varepsilon_{YXY}&=d\varepsilon_3\text{ ; minor principal strain increment}\end{aligned}\right\} \tag{7}$$

and volumetric strain increment is expressed as

$$dv=d\varepsilon_{XXY}+d\varepsilon_{YXY} \tag{8}$$

On the other hand, Eq. (1) may be rewritten in the triaxial compression case as

$$\frac{\sigma_1}{\sigma_3}=K\left(1-\frac{dv}{d\varepsilon_1}\right) \tag{9}$$

and in the triaxial extension case as

$$\frac{\sigma_1}{\sigma_3}=K\frac{1}{1-\dfrac{dv}{d\varepsilon_3}} \tag{10}$$

Stress-dilatancy equation in idealized two-dimensional stress condition may be obtained by substituting Eqs. (6), (7) and (8) into Eq. (9) as

$$\frac{\sigma_X}{\sigma_Y}=K\left(1-\frac{d\varepsilon_{XXY}+d\varepsilon_{YXY}}{d\varepsilon_{XXY}}\right)=-K\frac{d\varepsilon_{YXY}}{d\varepsilon_{XXY}} \qquad (11)$$

The same equation is also obtained by substituting Eqs. (6), (7) and (8) into Eq. (10). The stress-dilatancy equation in the idealized two-dimensional stress condition as expressed by Eq. (11) is the first basic postulate of the theory.

Next, it is necessary to correlate strain increments in the idealized two-dimensional stress condition with strain increments in the ordinary three-dimensional stress condition by using other postulates. Then, define three principal stresses by σ_X, σ_Y and σ_Z and define three principal strain increments by $d\varepsilon_X$, $d\varepsilon_Y$ and $d\varepsilon_Z$ which correspond to σ_X, σ_Y and σ_Z respectively.

There are three combinations of two principal stresses as $\sigma_X-\sigma_Y$, $\sigma_X-\sigma_Z$ and $\sigma_Y-\sigma_Z$. In each combination, there are two cases where relative largeness between two principal stresses are different such as $\sigma_X>\sigma_Y$ and $\sigma_Y>\sigma_X$. Therefore, six idealized two-dimensional slippings in six different idealized two-dimensional stress conditions may be possible for an element under three-dimensional stress condition as shown in Table 1, while all of six two-dimensional slippings are not induced at the same time. Strain increment components in six two-dimensional slippings are summarized in third, fourth and fifth columns in Table 1. Then suppose the case where $\sigma_X>\sigma_Y>\sigma_Z$ for example. In this case, possible idealized two-dimensional stress-systems are $\sigma_X>\sigma_Y$, $\sigma_X>\sigma_Z$ and $\sigma_Y>\sigma_Z$ stress-systems. On the other hand, Matsuoka (1974) proposed such a postulate that each of three principal strain increments in three-dimensional stress condition caused by shear deformation is obtained by linear summation of two components which are produced by two different idealized two-dimensional slippings. According to this postulate, three principal strain increments under $\sigma_X>\sigma_Y>\sigma_Z$ stress-system may be expressed using notations in Table 1 as

$$\left.\begin{aligned} d\varepsilon_X&=d\varepsilon_{XXY}+d\varepsilon_{XXZ}\\ d\varepsilon_Y&=d\varepsilon_{YXY}+d\varepsilon_{YYZ}\\ d\varepsilon_Z&=d\varepsilon_{ZXZ}+d\varepsilon_{ZYZ}\end{aligned}\right\}\ (\sigma_X>\sigma_Y>\sigma_Z) \qquad (12)$$

Eq. (12) means that for example $d\varepsilon_X$ is composed of $d\varepsilon_{XXY}$ and $d\varepsilon_{XXZ}$ which are strain increment components in two idealized two-dimensional slippings of $\sigma_X>\sigma_Y$ stress system

Table 1. Six idealized two-dimensional stress-systems

Stress System (1)	(2)	$d\varepsilon_X$ (3)	$d\varepsilon_Y$ (4)	$d\varepsilon_Z$ (5)	Stress-dilatancy Eq. (6)	Ψ (7)	h (8)	f (9)
$\sigma_X-\sigma_Y$	$\sigma_X>\sigma_Y$	$d\varepsilon_{XXY}$	$d\varepsilon_{YXY}$	—	$\frac{\sigma_X}{\sigma_Y}=-K\frac{d\varepsilon_{YXY}}{d\varepsilon_{XXY}}$	$\Psi_{XY}=\frac{\sigma_X{}^K}{\sigma_Y}$	h_{XY}	f_{XY}
	$\sigma_Y>\sigma_X$	$d\varepsilon_{XYX}$	$d\varepsilon_{YYX}$	—	$\frac{\sigma_Y}{\sigma_X}=-K\frac{d\varepsilon_{XYX}}{d\varepsilon_{YYX}}$	$\Psi_{YX}=\frac{\sigma_Y{}^K}{\sigma_X}$	h_{YX}	f_{YX}
$\sigma_X-\sigma_Z$	$\sigma_X>\sigma_Z$	$d\varepsilon_{XXZ}$	—	$d\varepsilon_{ZXZ}$	$\frac{\sigma_X}{\sigma_Z}=-K\frac{d\varepsilon_{ZXZ}}{d\varepsilon_{XXZ}}$	$\Psi_{XZ}=\frac{\sigma_X{}^K}{\sigma_Z}$	h_{XZ}	f_{XZ}
	$\sigma_Z>\sigma_X$	$d\varepsilon_{XZX}$	—	$d\varepsilon_{ZZX}$	$\frac{\sigma_Z}{\sigma_X}=-K\frac{d\varepsilon_{XZX}}{d\varepsilon_{ZZX}}$	$\Psi_{ZX}=\frac{\sigma_Z{}^K}{\sigma_X}$	h_{ZX}	f_{ZX}
$\sigma_Y-\sigma_Z$	$\sigma_Y>\sigma_Z$	—	$d\varepsilon_{YYZ}$	$d\varepsilon_{ZYZ}$	$\frac{\sigma_Y}{\sigma_Z}=-K\frac{d\varepsilon_{ZYZ}}{d\varepsilon_{YYZ}}$	$\Psi_{YZ}=\frac{\sigma_Y{}^K}{\sigma_Z}$	h_{YZ}	f_{YZ}
	$\sigma_Z>\sigma_Y$	—	$d\varepsilon_{YZY}$	$d\varepsilon_{ZZY}$	$\frac{\sigma_Z}{\sigma_Y}=-K\frac{d\varepsilon_{YZY}}{d\varepsilon_{ZZY}}$	$\Psi_{ZY}=\frac{\sigma_Z{}^K}{\sigma_Y}$	h_{ZY}	f_{ZY}

NOTE ; $d\varepsilon_{i_{ij}}=\frac{\partial\Psi_{ij}}{\partial\sigma_i}d\lambda_{ij},\ d\varepsilon_{i_{ji}}=\frac{\partial\Psi_{ji}}{\partial\sigma_i}d\lambda_{ji}$

and $\sigma_X > \sigma_Z$ stress system, respectively. Note that possible are six different three-dimensional stress-systems where relative largeness among three stresses are different such as $\sigma_X > \sigma_Y > \sigma_Z$, $\sigma_X > \sigma_Z > \sigma_Y$, $\sigma_Y > \sigma_Z > \sigma_X$, $\sigma_Y > \sigma_X > \sigma_Z$, $\sigma_Z > \sigma_X > \sigma_Y$ and $\sigma_Z > \sigma_Y > \sigma_X$. For other stress-systems besides $\sigma_X > \sigma_Y > \sigma_Z$, other equations which correspond to Eq. (12) can be derived using strain increment components listed in columns (3), (4) and (5) of Table 1. The linear summation of two components in idealized two-dimensional slippings as expressed by Eq. (12) is the second basic postulate.

As to deformation properties of sand, Poorooshasb, Holubec and Sherbourne (1966, 1967), Poorooshasb (1971) and Tatsuoka and Ishihara (1974a) showed that sand behaves as a strain hardening elasto-plastic material under some conditions. In general, elastic part of strain is neglegible in comparison with non-reversible part of strain (Barden, Khayatt and Wightman, 1969). Also in this paper, elastic part of strain is neglected. Therefore, strain increment components in idealized $\sigma_X > \sigma_Y$ stress-system for example are correlated with each other by using a plastic potential function as

$$\left.\begin{aligned} d\varepsilon_{XXY} &= \frac{\partial \psi_{XY}}{\partial \sigma_X} d\lambda_{XY} \\ d\varepsilon_{YXY} &= \frac{\partial \psi_{XY}}{\partial \sigma_Y} d\lambda_{XY} \end{aligned}\right\} \tag{13}$$

where $d\varepsilon_{XXY}$ and $d\varepsilon_{YXY}$ are plastic strain increment components and ψ_{XY} is the plastic potential function for $\sigma_X > \sigma_Y$ stress-system and $d\lambda_{XY}$ is a scalar depending on the state of the element of sand denoted by mean void ratio, inherent anisotropy, stress conditions, stress history etc.. Eq. (13) is the third fundamental postulate. The simplest plastic potential function ψ_{XY} which satisfies Eq. (11) is expressed as (Barden and Khayatt, 1966),

$$\psi_{XY} = \frac{\sigma_X{}^K}{\sigma_Y} \Delta_{XY} \quad (\sigma_X > \sigma_Y) \tag{14}$$

where $\Delta_{XY} = 0$ when $\sigma_X = \sigma_Y$ and $\Delta_{XY} = 1$ when $\sigma_X > \sigma_Y$ since shear deformation occurs only when $\sigma_X \neq \sigma_Y$. Similarly in $\sigma_Y > \sigma_X$ stress-system, plastic potential function is denoted by

$$\psi_{YX} = \frac{\sigma_Y{}^K}{\sigma_X} \Delta_{YX} \quad (\sigma_Y > \sigma_X) \tag{15}$$

and strain increment components are expressed as

$$\left.\begin{aligned} d\varepsilon_{XYX} &= \frac{\partial \psi_{YX}}{\partial \sigma_X} d\lambda_{YX} \\ d\varepsilon_{YYX} &= \frac{\partial \psi_{YX}}{\partial \sigma_Y} d\lambda_{YX} \end{aligned}\right\} \tag{16}$$

It is to be noted that $d\lambda_{YX}$ is not identical with $d\lambda_{XY}$ when sand has inherent anisotropy which is in general inevitable when sample is prepared under the effect of gravitation. Similar equations to Eqs. (13) and (16) can be obtained for other four cases and stress-dilatancy equations and plastic potential functions for six two-dimensional slippings are summarized in sixth and seventh columns in Table 1.

Then $d\lambda_{XY}$ for example may be expressed as

$$d\lambda_{XY} = h_{XY}(\sigma_X/\sigma_Y,\ p,\ K_{XY},\ e)\, df_{XY} \tag{17}$$

where h_{XY} is the hardening function in $\sigma_X > \sigma_Y$ stress-system and df_{XY} is the increment of the yield function f_{XY}. K_{XY} is the parameter depending on the state of the element of sand and in general

$$\left.\begin{aligned} K_{XY} \neq K_{YX} \neq K_{XZ} \neq K_{ZX} \neq K_{YZ} \neq K_{ZY} \\ h_{XY} \neq h_{YX} \neq h_{XZ} \neq h_{ZX} \neq h_{YZ} \neq h_{ZY} \end{aligned}\right\} \tag{18}$$

And f_{XY} may be expressed as

$$f_{XY}=f_{XY}(\sigma_X/\sigma_Y,\ p,\ L_{XY},\ e) \tag{19}$$

where L_{XY} is the parameter depending on the state of the element. Note that f_{XY} is the yield function in $\sigma_X>\sigma_Y$ stress-system and in general

$$f_{XY}\neq f_{YX}\neq f_{XZ}\neq f_{ZX}\neq f_{YZ}\neq f_{ZY} \tag{20}$$

Eq. (20) was partially verified in the triaxial compression and extension cases (Tatsuoka and Ishihara, 1974, b). Yielding of sand occurs only when

$$f_{XY}=f_{XYc} \quad \text{and} \quad df_{XY}>0 \tag{21}$$

where f_{XYc} is the highest value of f_{XY} in the loading process.

STRESS-DILATANCY EQUATIONS

By combining three basic postulats:

(1) stress-dilatancy equations in idealized two-dimensional stress-systems as Eq. (11),

(2) linear summation of two strain increment components as expressed by Eq. (12) and

(3) notations using plastic potential functions as expressed by Eq. (13),

stress-dilatancy relations for four cases are derived in the following, triaxial compression case of $\sigma_1>\sigma_2=\sigma_3$, triaxial extension case of $\sigma_1=\sigma_2>\sigma_3$, general stress condition case of $\sigma_1>\sigma_2>\sigma_3$ and plane strane case of $d\varepsilon_2=0$. In the following, Z-axis means vertical direction and X-axis and Y-axis mean horizontal directions. Then, for triaxial compression case for example, there are three different cases such as $\sigma_X>\sigma_Y=\sigma_Z$, $\sigma_Y>\sigma_Z=\sigma_X$ and $\sigma_Z>\sigma_X=\sigma_Y$. When it is necessary to distinguish such three cases as above, suffixes X, Y and Z are used in place of suffixes 1, 2 and 3. When not necessary, suffixes 1, 2 and 3 are used. For other three cases, similarly to triaxial compression case, suffixes X, Y and Z are used only when it is necessary to identify the directions of three principal stresses σ_1, σ_2 and σ_3.

The relationship between K and interparticle friction angle ϕ_u is not discussed because this is not the main aim of this paper. Therefore, for all cases in the following, the values of K are determined from stress-dilatancy plottings of data of conventional triaxial compression tests where σ_1-direction is identical with the vertical direction at the time of sample preparation.

Triaxial Compression ($\sigma_1>\sigma_2=\sigma_3$)

In this case, three principal strain increments $d\varepsilon_1$, $d\varepsilon_2$ and $d\varepsilon_3$ are derived, noting that $\Delta_{23}=0$ due to $\sigma_2=\sigma_3$, as follows.

$$\left.\begin{aligned}
d\varepsilon_1&=d\varepsilon_{112}+d\varepsilon_{113}\\
&=\frac{\partial\Psi_{12}}{\partial\sigma_1}d\lambda_{12}+\frac{\partial\Psi_{13}}{\partial\sigma_1}d\lambda_{13}\\
&=\frac{\partial}{\partial\sigma_1}\left(\frac{\sigma_1{}^K}{\sigma_2}\Delta_{12}\right)d\lambda_{12}+\frac{\partial}{\partial\sigma_1}\left(\frac{\sigma_1{}^K}{\sigma_3}\Delta_{13}\right)d\lambda_{13}\\
&=K\left(\frac{\sigma_1{}^{K-1}}{\sigma_2}d\lambda_{12}+\frac{\sigma_1{}^{K-1}}{\sigma_3}d\lambda_{13}\right),\\
d\varepsilon_2&=d\varepsilon_{212}+d\varepsilon_{223}\\
&=\frac{\partial\Psi_{12}}{\partial\sigma_2}d\lambda_{12}+\frac{\partial\Psi_{23}}{\partial\sigma_2}d\lambda_{23}\\
&=\frac{\partial}{\partial\sigma_2}\left(\frac{\sigma_1{}^K}{\sigma_2}\Delta_{12}\right)d\lambda_{12}+\frac{\partial}{\partial\sigma_2}\left(\frac{\sigma_2{}^K}{\sigma_3}\Delta_{23}\right)d\lambda_{23}\\
&=-\frac{\sigma_1{}^K}{\sigma_2{}^2}d\lambda_{12}
\end{aligned}\right\} \tag{22}$$

and
$$\left.\begin{aligned} d\varepsilon_3 &= d\varepsilon_{313} + d\varepsilon_{323} \\ &= \frac{\partial \Psi_{13}}{\partial \sigma_3} d\lambda_{13} + \frac{\partial \Psi_{23}}{\partial \sigma_3} d\lambda_{23} \\ &= \frac{\partial}{\partial \sigma_3}\left(\frac{\sigma_1{}^K}{\sigma_3}\Delta_{13}\right) d\lambda_{13} + \frac{\partial}{\partial \sigma_3}\left(\frac{\sigma_2{}^K}{\sigma_3}\Delta_{23}\right) d\lambda_{23} \\ &= -\frac{\sigma_1{}^K}{\sigma_3{}^2} d\lambda_{13} \end{aligned}\right|$$

By eliminating $d\lambda_{12}$ and $d\lambda_{13}$ from Eq. (22), the stress-dilatancy equation for triaxial compression is obtained as

$$\frac{\sigma_1}{\sigma_3} = \frac{\sigma_1}{\sigma_2} = -K\frac{d\varepsilon_2 + d\varepsilon_3}{d\varepsilon_1} = K\left(1 - \frac{dv}{d\varepsilon_1}\right) \quad (\sigma_1 > \sigma_2 = \sigma_3) \tag{23}$$

This is the same equation as Eq. (1). However, it is noteworthy that Eq. (23) is not affected by the relative largeness between $d\varepsilon_2$ and $d\varepsilon_3$. In anisotropic sand, it is possible that $d\varepsilon_2$ is not identical with $d\varepsilon_3$ in spite of $\sigma_2 = \sigma_3$. From Eq. (22), the ratio of $d\varepsilon_3$ to $d\varepsilon_2$ is derived as

$$\frac{d\varepsilon_3}{d\varepsilon_2} = \frac{d\lambda_{13}}{d\lambda_{12}} = -\frac{1}{(\sigma_1/\sigma_3)/K(d\varepsilon_3/d\varepsilon_1) + 1} \tag{24}$$

In conventional triaxial tests with cylindrical samples this ratio can not be obtained ordinarily. Meanwhile Arthur and Menzies (1972) conducted triaxial compression tests on cuboidal samples (10cm×10cm×10cm) which were prepared by pouring air-dried rounded Leighton Buzzard sand into a tilted mold. Therefore, σ_1-direction is not necessarily identical with vertical pouring direction. They defined the tilting angle θ as shown in Fig. 1. When $\theta = 90°$, σ_1-direction is identical with the pouring direction. σ_2-direction is always the direction of layering which is horizontal when sample is prepared. However, σ_3-direction is not the direction of layering except when $\theta = 90°$. They conducted drained triaxial compression test of $\sigma_2 = \sigma_3 = \text{const.} = 0.539 \text{kg/cm}^2$ and σ_1 increasing. In Fig. 2, it is shown that the values of $d\varepsilon_3/d\varepsilon_2$ obtained by Eq. (24) using the measured values of σ_1/σ_3 and $d\varepsilon_3/d\varepsilon_1$ fit with measured values of $d\varepsilon_3/d\varepsilon_2$ and that $d\varepsilon_3/d\varepsilon_2$ is not a unit except when $\theta = 90°$ and increases with decrease in θ. And it may be seen from Fig. 3 that Eq. (23) holds even for the case of $d\varepsilon_2 \neq d\varepsilon_3$. Similar triaxial compression tests with cylindrical samples made by pouring saturated Toyoura sand into a tilted mold under water were conducted by Oda (1972 a, b). Test results show that stress-strain relationships depend on the angle θ bet-

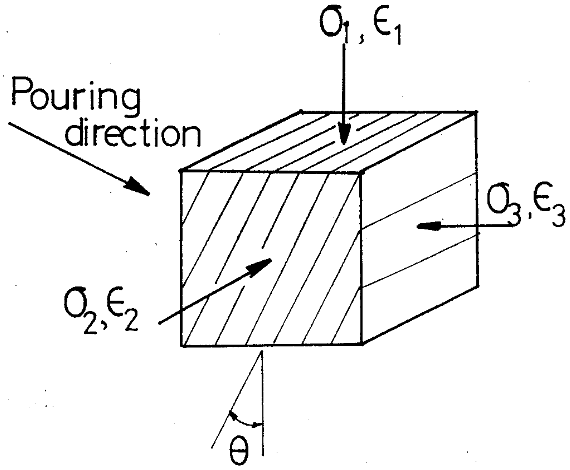

Fig. 1. Reference direction (after Arthur and Menzies, 1972)

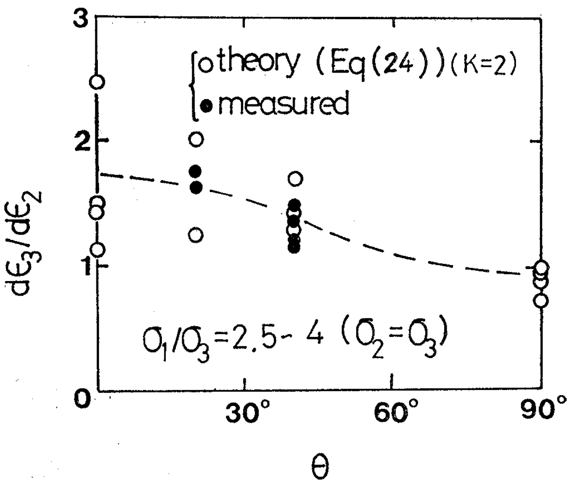

Fig. 2. The ratio $d\varepsilon_3/d\varepsilon_2$ in triaxial compression (after data of Arthur and Menzies, 1972)

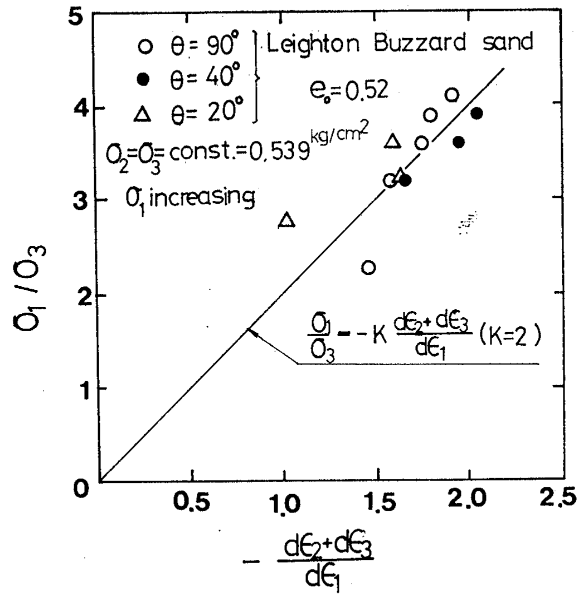

Fig. 3. Stress-dilatancy plots (after data of Arthur and Menzies, 1972)

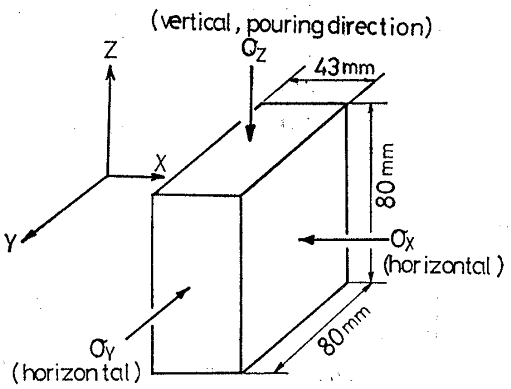

Fig. 4. Specimens used by Ishihara, Yamada and Kitagawa (1975)

ween the σ_3-direction and the pouring direction due to the variation of inherent anisotropy and also show that Eq. (23) holds irrespectively of the variation of anisotropy.

Triaxial Extension $(\sigma_1=\sigma_2>\sigma_3)$

Also in this case, three principal strain increments are obtained by the similar procedure to the case of triaxial compression. In this case, Δ_{12} is zero since $\sigma_1=\sigma_2$. Therefore,

$$\left.\begin{aligned} d\varepsilon_1&=d\varepsilon_{1_{12}}+d\varepsilon_{1_{13}}=0+d\varepsilon_{1_{13}}=K\frac{\sigma_1^{K-1}}{\sigma_3}d\lambda_{13}\\ d\varepsilon_2&=d\varepsilon_{2_{12}}+d\varepsilon_{2_{23}}=0+d\varepsilon_{2_{23}}=K\frac{\sigma_2^{K-1}}{\sigma_3}d\lambda_{23}\\ d\varepsilon_3&=d\varepsilon_{3_{13}}+d\varepsilon_{3_{23}}=-\left(\frac{\sigma_1^K}{\sigma_3^2}d\lambda_{13}+\frac{\sigma_2^K}{\sigma_3^2}d\lambda_{23}\right)\end{aligned}\right\} \qquad (25)$$

By eliminating scalars $d\lambda_{13}$ and $d\lambda_{23}$ from Eq. (25), the stress-dilatancy equation for the triaxial extension case is obtained as

$$\frac{\sigma_1}{\sigma_3}=\frac{\sigma_2}{\sigma_3}=-K\frac{d\varepsilon_3}{d\varepsilon_1+d\varepsilon_2}=K\frac{1}{1-\frac{dv}{d\varepsilon_3}} \quad (\sigma_1=\sigma_2>\sigma_3) \qquad (26)$$

This is same as Eq. (1) but it is to be noted that Eq. (26), similarly as in Eq. (23), is not affected by the ratio $d\varepsilon_1/d\varepsilon_2$ which is not necessarily a unit.

Ishihara, Yamada and Kitagawa (1975) conducted triaxial compression and extension tests using rectangular samples as shown in Fig. 4 under the condition of $p=1/3(\sigma_1+\sigma_2+\sigma_3)=$const. $=1$ kg/cm². Z-direction in Fig. 4 means pouring direction and X, Y-directions are horizontal. Samples were prepared by pouring saturated Fuji River sand into a mold. Ishihara et al. conducted four kinds of tests listed in Table 2. Both of ZCD and YCD tests are triaxial compression tests but $\sigma_1=\sigma_Z$ in the former and $\sigma_1=\sigma_Y$ in the latter. In triaxial extension tests, similarly, $\sigma_3=\sigma_Z$ in ZED-test and $\sigma_3=\sigma_X$ in XED-test. Fig. 5 indicates that triaxial compression tests ZCD and YCD show different volumetric strain $v\sim$stress ratio q/p' relationship and similarly triaxial extension tests ZED and XED show different $v\sim q/p'$ relationship. This would be due to inherent anisotropy caused by sedimentation of sand particles under gravitation. The stress-dilatancy equation for ZCD and

Table 2. Lists of tests conducted by Ishihara, Yamada and Kitagawa (1975)

Test name	Type of test	σ_Z	σ_Y	σ_X	q
ZCD	Triaxial comp.	σ_1	σ_3	σ_3	$\sigma_1-\sigma_3$
ZED	Triaxial ext.	σ_3	σ_1	σ_1	$\sigma_3-\sigma_1$
YCD	Triaxial comp.	σ_3	σ_1	σ_3	$\sigma_1-\sigma_3$
XED	Triaxial ext.	σ_1	σ_1	σ_3	$\sigma_3-\sigma_1$

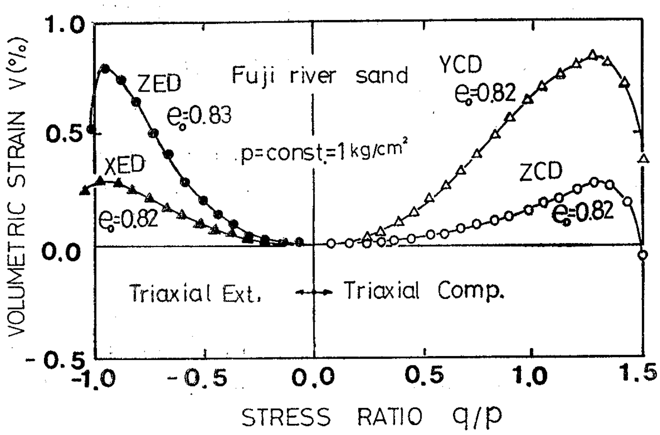

Fig. 5. Volumetric sirain characteristics in triaxial compression and extension tests (after Ishihara, Yamada and Kitagawa, 1975)

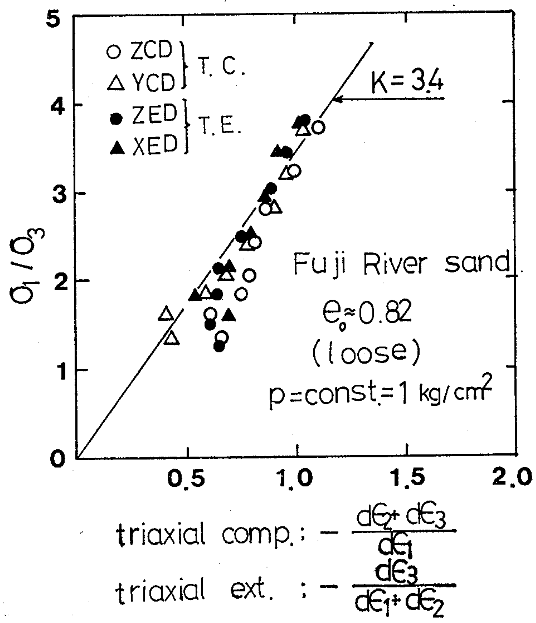

Fig. 6. Stress-dilatancy plots (after data of Ishihara, Yamada and Kitagawa, 1975)

YCD tests is expressed by Eq. (23) and that for ZED and XED tests is expressed by Eq. (26). Fig. 6 shows that these equations expressed by a solid line fit experimental data excellently. But, note that $d\varepsilon_Z/d\varepsilon_Y$ in XED-test ($\sigma_Z=\sigma_Y>\sigma_X$) and $d\varepsilon_X/d\varepsilon_Z$ in YCD-test ($\sigma_Y>\sigma_X=\sigma_Z$) are not units as shown in Fig. 7. The theoretical relationships among principal strain increments in each test are derived from equations in Table 1 as follows.

○ in XED-test ($\sigma_Z=\sigma_Y>\sigma_X$)

$$\left.\begin{aligned} d\varepsilon_Z &= d\varepsilon_{ZZX}=K\frac{{\sigma_Z}^{K-1}}{\sigma_X}d\lambda_{ZX} \\ d\varepsilon_Y &= d\varepsilon_{YYX}=K\frac{{\sigma_Y}^{K-1}}{\sigma_X}d\lambda_{YX} \\ d\varepsilon_X &= d\varepsilon_{XZX}+d\varepsilon_{XYX}=-\frac{{\sigma_Z}^K}{{\sigma_X}^2}d\lambda_{ZX}-\frac{{\sigma_Y}^K}{{\sigma_X}^2}d\lambda_{YX} \end{aligned}\right\} \quad (27)$$

where $d\lambda_{ZX}$ and $d\lambda_{YX}$ are scalars. As X and Y directions are horizontal and exchangeable for each other, it may be assumed in this case that

$$\left.\begin{aligned} d\lambda_{ZX} &= d\lambda_{ZY} \\ d\lambda_{YX} &= d\lambda_{XY} \end{aligned}\right\} \quad (28)$$

From Eqs. (27) and (28), it is found that $d\varepsilon_Z/d\varepsilon_Y$ is $d\lambda_{ZX}/d\lambda_{YX}=d\lambda_{ZY}/d\lambda_{XY}$ and in this case this is about 0.55 as shown in Fig. 7. Then a parameter a which depends upon the degree of anisotropy of sample may be defined by

$$a=\frac{d\lambda_{ZX}}{d\lambda_{YX}}=\frac{d\lambda_{ZY}}{d\lambda_{XY}} \tag{29}$$

It may be anticipated that a is a unit in the case of isotropic sand. From, Eqs. (27) and (29), the stress-dilatancy equation for this case is obtained as

$$\frac{\sigma_1}{\sigma_3}=\frac{\sigma_Z}{\sigma_X}=\frac{\sigma_Y}{\sigma_X}=-K\frac{d\varepsilon_X}{d\varepsilon_Z+d\varepsilon_Y}=-\frac{K}{1+a}\frac{d\varepsilon_X}{d\varepsilon_Y}=-\frac{K}{1+\frac{1}{a}}\frac{d\varepsilon_X}{d\varepsilon_Z} \tag{30}$$

If sample is isotropic, then $a=1.0$ and Eq. (30) becomes

$$\frac{\sigma_1}{\sigma_3}=\frac{\sigma_Z}{\sigma_X}=\frac{\sigma_Y}{\sigma_X}=-\frac{K}{2}\frac{d\varepsilon_X}{d\varepsilon_Y}=-\frac{K}{2}\frac{d\varepsilon_X}{d\varepsilon_Z}=-\frac{K}{2}\frac{d\varepsilon_3}{d\varepsilon_1} \tag{31}$$

In Fig. 8, the values of σ_Y/σ_X and $-d\varepsilon_X/d\varepsilon_Y$ and the values of σ_Z/σ_X and $-d\varepsilon_X/d\varepsilon_Z$ are plotted. It is seen from this figure that Eq. (30) fits experimental data but Eq. (31) which may be applied for isotropic material does not hold.

○ in YCD-test ($\sigma_Y>\sigma_X=\sigma_Z$)

$$\left.\begin{aligned} d\varepsilon_Z&=d\varepsilon_{ZYZ}=-\frac{\sigma_Y{}^K}{\sigma_Z{}^2}d\lambda_{YZ}\\ d\varepsilon_Y&=d\varepsilon_{YYZ}+d\varepsilon_{YYX}=K\frac{\sigma_Y{}^{K-1}}{\sigma_Z}d\lambda_{YZ}+K\frac{\sigma_Y{}^{K-1}}{\sigma_X}d\lambda_{YX}\\ d\varepsilon_X&=d\varepsilon_{XYX}=-\frac{\sigma_Y{}^K}{\sigma_X{}^2}d\lambda_{YX}\end{aligned}\right\} \tag{32}$$

In this case, by defining another parameter a' which depends on the degree of anisotropy

$$a'=\frac{d\lambda_{YX}}{d\lambda_{YZ}}=\frac{d\lambda_{XY}}{d\lambda_{XZ}} \tag{33}$$

of sample by the stress-dilatancy equation is obtained from Eqs. (32) and (33) as

$$\frac{\sigma_1}{\sigma_3}=\frac{\sigma_Y}{\sigma_X}=\frac{\sigma_Y}{\sigma_Z}=-K\frac{d\varepsilon_Z+d\varepsilon_X}{d\varepsilon_Y}=-K(1+a')\frac{d\varepsilon_Z}{d\varepsilon_Y}=-K\left(1+\frac{1}{a'}\right)\frac{d\varepsilon_X}{d\varepsilon_Y} \tag{34}$$

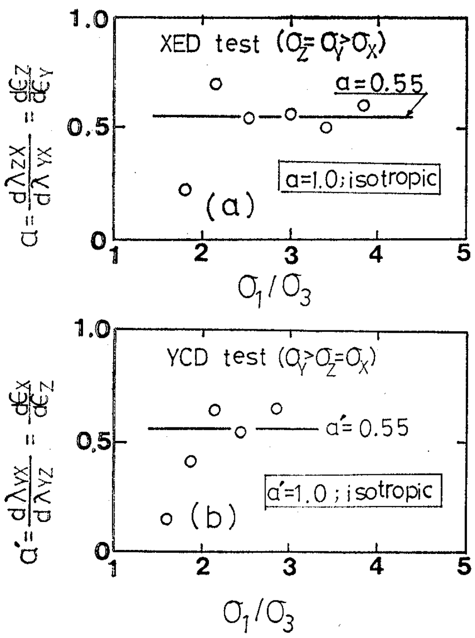

Fig. 7. Ratios of principal strain increments (after data of Ishihara, Yamada and Kitagawa, 1975)

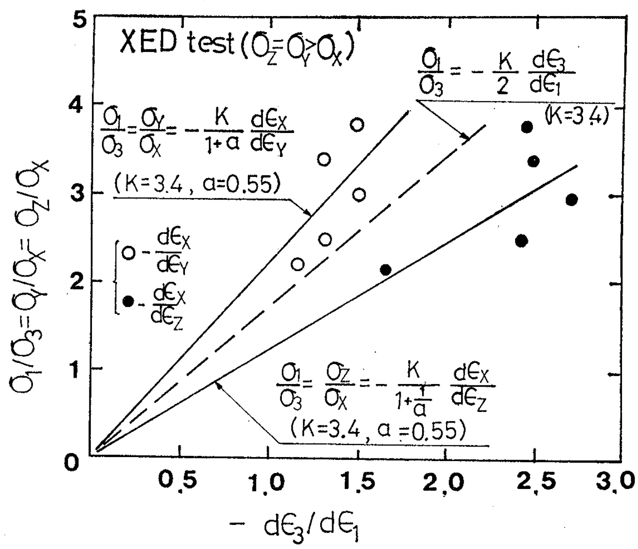

Fig. 8. Stress-dilatancy plots (after data of Ishihara, Yamada and Kitagawa, 1975)

The value of a' is about 0.55 in this case as shown in Fig. 7. The considerable scattering of values of a and a' are observed in Fig. 7 and a and a' may be functions of σ_1/σ_3. However, at present, the function can not be determined based on few data. Therefore a and a' are considered as constants for simplicity in this paper.

○ in ZCD-test $(\sigma_Z>\sigma_Y=\sigma_X)$

$$\left.\begin{aligned} d\varepsilon_Z&=d\varepsilon_{ZZY}+d\varepsilon_{ZZX}=K\frac{\sigma_1^{K-1}}{\sigma_3}2d\lambda_{ZY}\\ d\varepsilon_Y&=d\varepsilon_{YZY}=-\frac{\sigma_1^K}{\sigma_3^2}d\lambda_{ZY}\\ d\varepsilon_X&=d\varepsilon_{XZX}=d\varepsilon_Y \end{aligned}\right\} \tag{35}$$

○ in ZED-test $(\sigma_Z<\sigma_Y=\sigma_X)$

$$\left.\begin{aligned} d\varepsilon_Z&=d\varepsilon_{ZYZ}+d\varepsilon_{ZXZ}=-\frac{\sigma_1^K}{\sigma_3^2}2d\lambda_{YZ}\\ d\varepsilon_Y&=d\varepsilon_{YYZ}=K\frac{\sigma_1^{K-1}}{\sigma_3}d\lambda_{YZ}\\ d\varepsilon_X&=d\varepsilon_{XXZ}=d\varepsilon_Y \end{aligned}\right\} \tag{36}$$

From Eqs. (32) through (36), volumetric strain increments $dv=d\varepsilon_Z+d\varepsilon_Y+d\varepsilon_X$ in each test are obtained as

$$\left.\begin{aligned} &\text{ZCD-test ; } dv_{\mathrm{ZCD}}=\left(K\frac{\sigma_1^{K-1}}{\sigma_3}-\frac{\sigma_1^K}{\sigma_3^2}\right)\cdot 2d\lambda_{ZY}\\ &\text{YCD-test ; } dv_{\mathrm{YCD}}=(\quad {\prime\prime} \quad)\cdot(d\lambda_{YZ}+d\lambda_{YX})\\ &\text{ZED-test ; } dv_{\mathrm{ZED}}=(\quad {\prime\prime} \quad)\cdot 2d\lambda_{YZ}\\ &\text{XED-test ; } dv_{\mathrm{XED}}=(\quad {\prime\prime} \quad)\cdot(d\lambda_{YX}+d\lambda_{ZX}) \end{aligned}\right\} \tag{37}$$

From Eqs. (29), (33) and (37), the ratios of volumetric strain increments are obtained as

$$\begin{aligned} dv_{\mathrm{ZCD}} : dv_{\mathrm{YCD}} : dv_{\mathrm{ZED}} : dv_{\mathrm{XED}}&=2a\cdot a' : (1+a') : 2 : (a\cdot a'+a')\\ &=1 : 2.6 : 3.3 : 1.42 \end{aligned} \tag{38}$$

Therefore it may be supposed that the disagreement in the magnitudes of the volumetric strains of four tests shown in Fig. 5 would be due to the inherent anisotropy which is denoted by the parameters a and a'. Accordingly it may be anticipated that the volumetric strains of four tests shown in Fig. 5 would coincide with each other for the same stress ratio σ_1/σ_3 when sample is isotropic where a and a' are units.

General Stress Condition $(\sigma_1>\sigma_2>\sigma_3)$

In this case, three principal strain increments are obtained as

$$\left.\begin{aligned} d\varepsilon_1&=d\varepsilon_{112}+d\varepsilon_{113}=K\frac{\sigma_1^{K-1}}{\sigma_2}d\lambda_{12}+K\frac{\sigma_1^{K-1}}{\sigma_3}d\lambda_{13}\\ d\varepsilon_2&=d\varepsilon_{212}+d\varepsilon_{223}=-\frac{\sigma_1^K}{\sigma_2^2}d\lambda_{12}+K\frac{\sigma_2^{K-1}}{\sigma_3}d\lambda_{23}\\ d\varepsilon_3&=d\varepsilon_{313}+d\varepsilon_{323}=-\frac{\sigma_1^K}{\sigma_3^2}d\lambda_{13}-\frac{\sigma_2^K}{\sigma_3^2}d\lambda_{23} \end{aligned}\right\} \tag{39}$$

Note that in general $d\lambda_{12}\neq d\lambda_{13}\neq d\lambda_{23}$ due to the anisotropy in sand. However, when $d\lambda_{12}\neq d\lambda_{13}$ and $d\lambda_{12}\neq d\lambda_{23}$, it is impossible to eliminate $d\lambda_{12}$, $d\lambda_{13}$ and $d\lambda_{23}$ from Eq. (39) and also impossible to derive the stress-dilatancy equation which does not include $d\lambda_{12}$, $d\lambda_{13}$ and $d\lambda_{23}$. On the other hand when σ_1-axis coincides with Z-axis which means both of vertical direction and pouring direction at the time of sample preparation, σ_2-axis and σ_3-axis are horizontal axes which are X-axis and Y-axis as shown in Fig. 4. For this case,

X and Y directions are exchangeable for each other. Therefore,

$$\left.\begin{aligned} d\lambda_{12}=d\lambda_{13}=d\lambda_{ZY}=d\lambda_{ZX} \\ d\lambda_{23}=d\lambda_{32}=d\lambda_{YX}=d\lambda_{XY} \end{aligned}\right\} \tag{40}$$

In this case, Eq. (39) becomes

$$\left.\begin{aligned} d\varepsilon_1&=K\left(\frac{\sigma_1^{K-1}}{\sigma_2}+\frac{\sigma_1^{K-1}}{\sigma_3}\right)d\lambda_{ZY} \\ d\varepsilon_2&=-\frac{\sigma_1^K}{\sigma_2^2}d\lambda_{ZY}+K\frac{\sigma_2^{K-1}}{\sigma_3}d\lambda_{YX} \\ d\varepsilon_3&=-\left(\frac{\sigma_1^K}{\sigma_3^2}d\lambda_{ZY}+\frac{\sigma_2^K}{\sigma_3^2}d\lambda_{YX}\right) \end{aligned}\right\} \tag{41}$$

From Eq. (41), the ratios $d\varepsilon_2/d\varepsilon_1$ and $d\varepsilon_3/d\varepsilon_1$ are obtained as

$$\frac{d\varepsilon_2}{d\varepsilon_1}=-\frac{\sigma_1}{\sigma_3}\frac{-\frac{\sigma_3}{\sigma_2}\cdot a+K\left(\frac{\sigma_2}{\sigma_1}\right)^K}{K\left(1+\frac{\sigma_2}{\sigma_3}\right)\cdot a} \tag{42}$$

$$\frac{d\varepsilon_3}{d\varepsilon_1}=-\frac{\sigma_1}{\sigma_3}\frac{a+\left(\frac{\sigma_2}{\sigma_1}\right)^K}{K\left(1+\frac{\sigma_3}{\sigma_2}\right)a} \tag{43}$$

where $a=d\lambda_{ZY}/d\lambda_{YX}$. Furthermore, by eliminating a from Eqs. (42) and (43), the stress-dilatancy equation for $\sigma_1>\sigma_2>\sigma_3$ stress condition is obtained as

$$\frac{\sigma_1}{\sigma_3}\frac{\frac{\sigma_2}{\sigma_3}+\frac{1}{K}}{\frac{\sigma_2}{\sigma_3}+1}=-K\frac{d\varepsilon_3+\frac{1}{K}\frac{\sigma_2}{\sigma_3}d\varepsilon_2}{d\varepsilon_1}\quad(\sigma_1>\sigma_2>\sigma_3\ ;\ \sigma_1=\sigma_Z) \tag{44}$$

Note that Eq. (44) holds only when two among three scalars, $d\lambda_{12}$, $d\lambda_{13}$ and $d\lambda_{23}$ in Eq. (39) are identical. Eq. (44) corresponds to Eqs. (3), (4) and (5) which were proposed by Horne. But, Eq. (44) differs from Eqs. (3), (4) and (5) in some aspects.

Ko and Scott (1967 and 1968) conducted p=const.=20 psi (1.41 kg/cm²) shear test under general stress condition. Stress paths they employed are shown in Fig. 9 and they are denoted by RS 90 (triaxial comp.), RS 75, RS 60, RS 45 and RS 30 (triaxial ext.) tests respectively. Specimens were cuboidal (9.4×9.4×9.4 cm) and air-dried Ottawa sand was used. Since in this case σ_1-axis coincides with vertical axis, Eqs. (41) through (44) can be examined by Ko and Scott's data. Fig. 10 shows the relationship between $d\varepsilon_2/d\varepsilon_1$ and σ_1/σ_3 of these tests. To RS 30 (TE)-test which is identical with the XED test of Ishihara et al., the following equations which are derived from Eqs. (27) and (29) may be applied.

$$\left.\begin{aligned} d\varepsilon_1&=K\frac{\sigma_1^K}{\sigma_3}d\lambda_{ZX} \\ d\varepsilon_2&=K\frac{\sigma_2^K}{\sigma_3}d\lambda_{YX} \\ \frac{d\varepsilon_2}{d\varepsilon_1}&=\frac{1}{d\lambda_{ZX}/d\lambda_{YX}}=\frac{1}{a} \end{aligned}\right\} \tag{45}$$

It may be seen from data of RS 30-test shown in Fig. 10 with theoretical lines that in this case the value of a is between 0.8 and 1.0. This indicates that samples prepared by Ko and Scott have slight anisotropy. Theoretical curves for RS 45, RS 60 and RS 75 tests which are expressed by solid curves in Fig. 10 are Eq. (42) and that for RS 90 (TC)-test is Eq. (23) which is independent of a. The value of K was obtained from stress-dilatancy

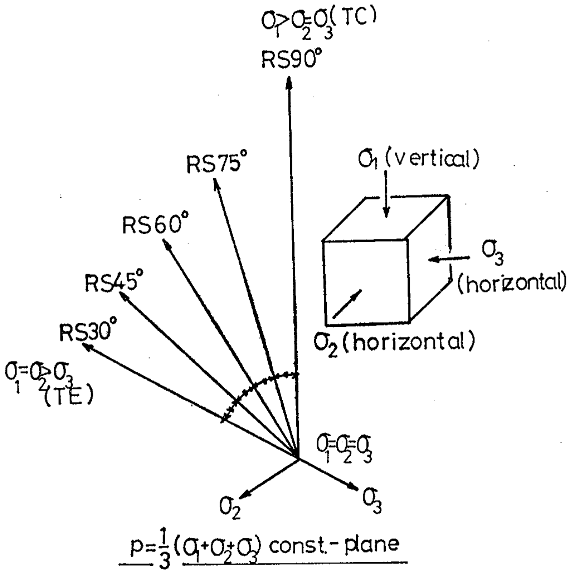

Fig. 9. Stress paths and sample employed by Ko and Scott (1967 and 1968)

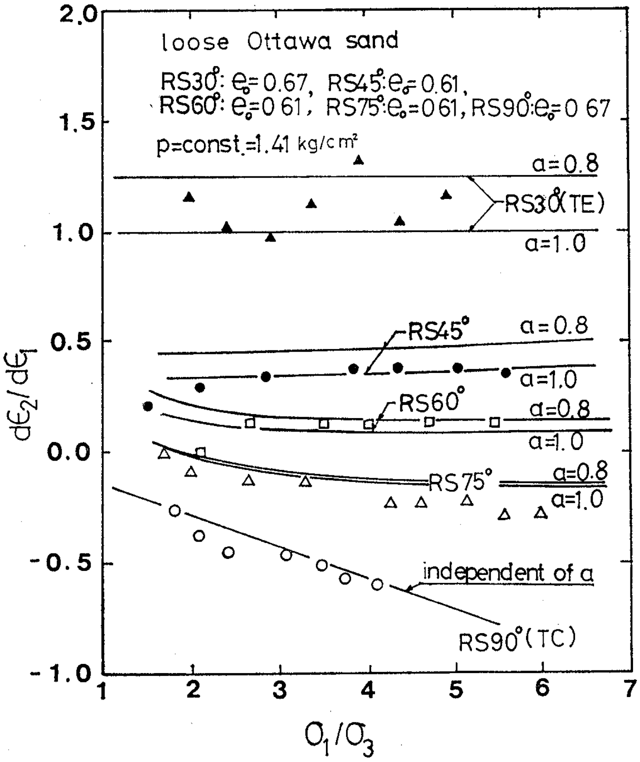

Fig. 10. $d\varepsilon_2/d\varepsilon_1 \sim \sigma_1/\sigma_3$ **relationships (after data of Ko and Scott, 1967 and 1968) (Solid curves are theoretical curves)**

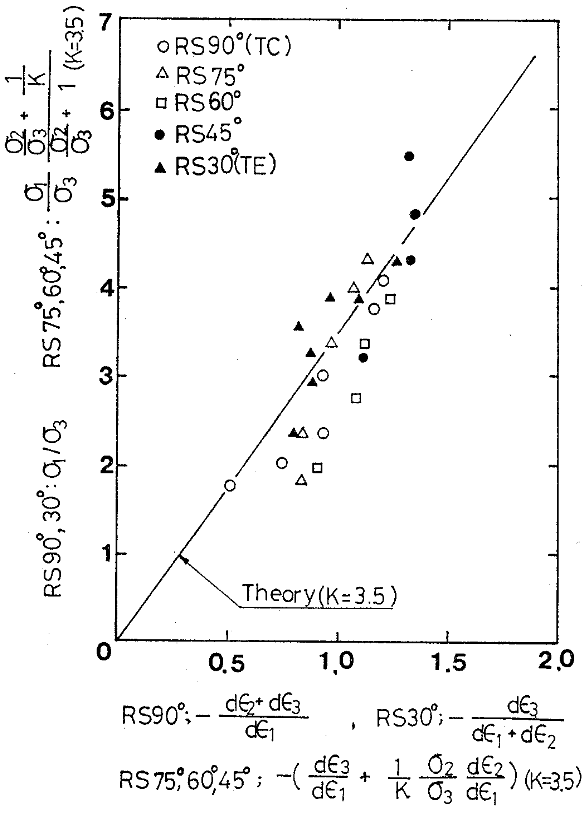

Fig. 11. Stress-dilatancy plots (after data of Ko and Scott, 1967 and 1868)

plotting of RS 90 test. The stress-dilatancy plottings of various test results are shown in Fig. 11 where the theoretical curves for RS 90 and RS 30 tests are Eqs. (23) and (26), respectively and those for RS 75. RS 60 and RS 45 tests are Eq. (44). It may be seen from Figs. 10 and 11 that theoretical curves fit experimental data. It is noteworthy that stress-dilatancy plottings in Fig. 11 are not affected by the parameter *a*. Therefore it is to be noted that the degree of inherent anisotropy in sample do not affect such stress-dilatancy equations as Eqs. (23), (26) and (44), but strain increment ratios are affected by the degree of inherent anisotropy as shown by Eqs. (24), (42) and (43) or as shown in Figs. 2, 7, 8 and 10.

Plane Strain Condition ($\sigma_1 > \sigma_2 > \sigma_3$ *and* $d\varepsilon_2 = 0$)

Plane strain test is one of the general stress condition tests under the condition of $d\varepsilon_2=$

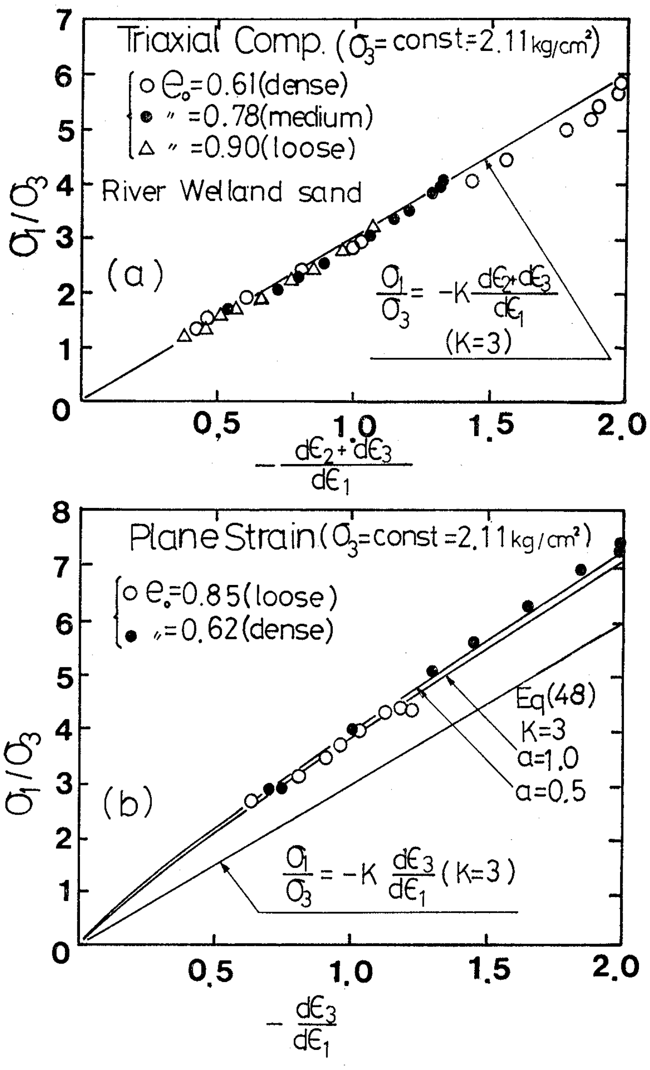

Fig. 12. Stress-dilatancy plots (after Barden et al., 1969)

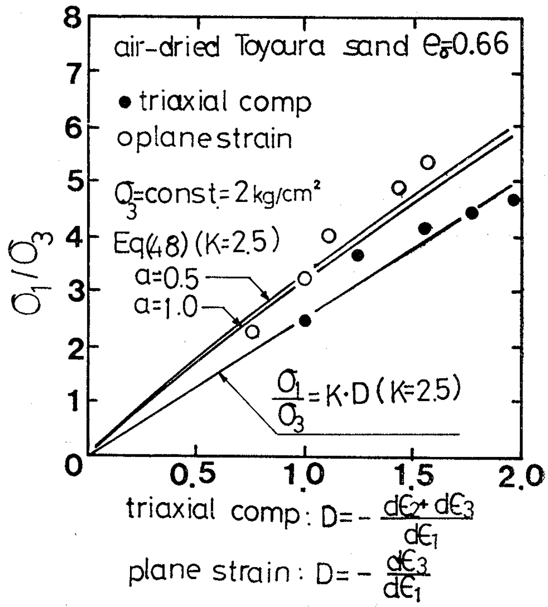

Fig. 13. Stress-dilatancy plots (after data of Wade, 1963)

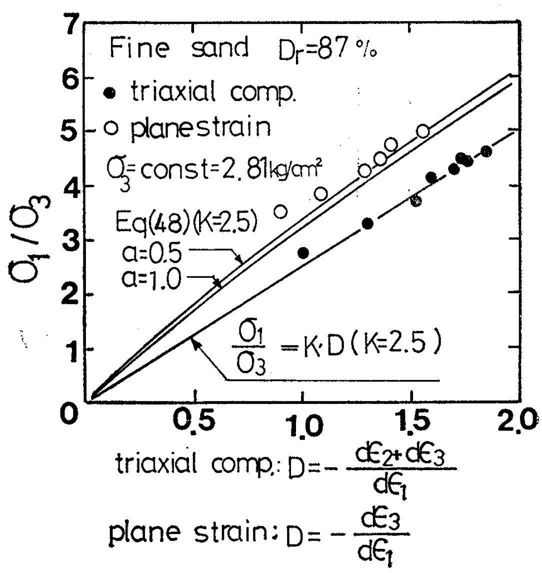

Fig. 14. Stress-dilatancy plots (after data of Ichihara and Matsuzawa, 1970)

0. Since many plane strain tests have been conducted, it is valuable to examine the theory by data of plane strain tests. Since σ_1-direction coincides with the vertical direction in all of the plane strain tests quoted in the following, Eqs. (40) through (44) are applicable to this case under the condition of $d\varepsilon_2=0$. From Eq. (41) and $d\varepsilon_2=0$-condition, σ_2 is correlated with σ_1, σ_3 and a as follows.

$$d\varepsilon_2=\left\{-\frac{\sigma_1{}^K}{\sigma_2{}^2}\cdot a+K\frac{\sigma_2{}^{K-1}}{\sigma_3}\right\}d\lambda_{YX}=0 \tag{46}$$

Eq. (46) becomes

$$\frac{\sigma_2}{\sigma_3}=K^{-\frac{1}{K+1}}\left(\frac{\sigma_1}{\sigma_3}\right)^{\frac{K}{K+1}}a^{\frac{1}{K+1}} \tag{47}$$

where $a=d\lambda_{ZY}/d\lambda_{YX}$. The stress-dilatancy equation for the plane strain condition is obtained by substituting $d\varepsilon_2=0$ into Eq. (44) as

$$\frac{\sigma_1}{\sigma_3}\frac{\dfrac{\sigma_2}{\sigma_3}+\dfrac{1}{K}}{\dfrac{\sigma_2}{\sigma_3}+1}=-K\frac{d\varepsilon_3}{d\varepsilon_1}\quad(\sigma_1>\sigma_2>\sigma_3\ ;\ \sigma_1=\sigma_Z,\ \ d\varepsilon_2=0) \tag{48}$$

where σ_2/σ_3 is obtained from Eq. (47). Note that Eq. (48) is not identical with Eq. (4) which is stress-dilatancy equation for plane strain test proposed by Horne. Both of conventional triaxial compression tests and σ_3=constant plane strain compression tests on same sands were conducted by Barden, Khayatt and Wightman (1969), Wade (1963), Ichihara and Matsuzawa (1970), Green (1971) and Cornforth (1964) test results of which are shown

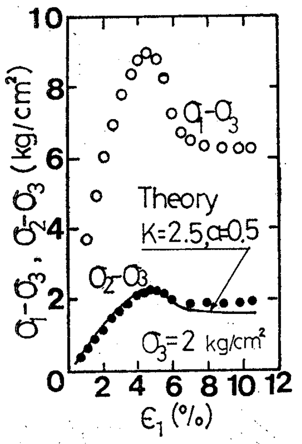

Fig. 15. Stress-strain relationsips of a plane strain test (after Ichihara and Matsuzawa, 1970)

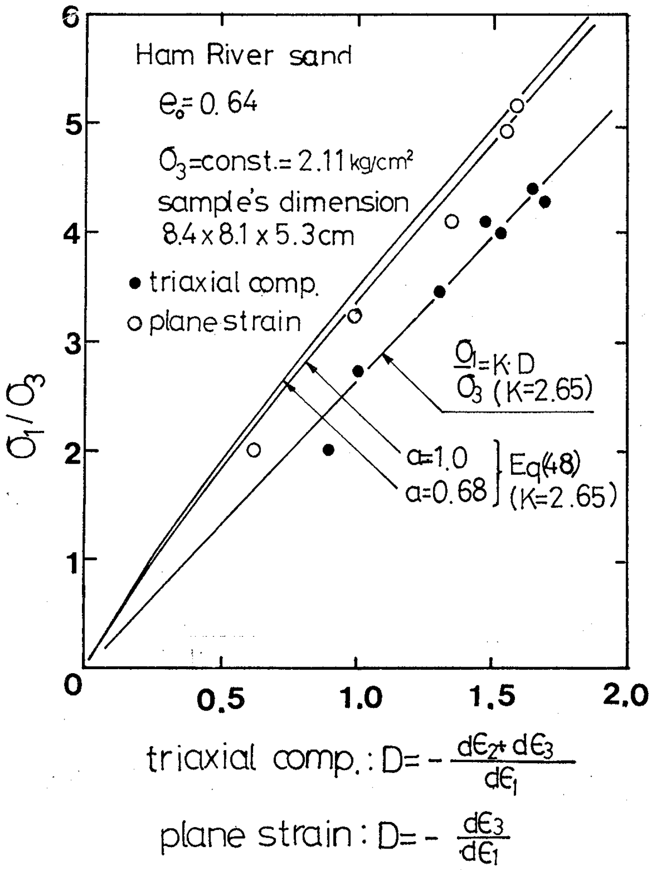

Fig. 16. Stress-dilatancy plots (after data of Green, 1972)

in Figs. 12 through 19. Fig. 12(a) shows stress-dilatancy plot for pre-peak parts of triaxial compression tests on specimens of $H=10$cm and $D=10$cm which were conducted by Barden et al. (1969). River Welland sand was employed. It is seen from Fig. 12(a) that K is about 3 for this case. In Fig. 12(b), stress-dilatancy plot for pre-peak parts of plane strain tests on specimen of 10cm×10cm×20cm and theoretical curves are presented. The straight line represents Eq. (4) proposed by Horne and it may be seen that when the same value of K as in the triaxial compression case is used, the value of $-d\varepsilon_3/d\varepsilon_1$ predicted by Eq. (4) is over-estimated for the plane strain case. On the other hand, the curves represent Eq. (48) where σ_2/σ_3 is obtained by substituting $K=3.0$, the refered value of σ_1/σ_3 and $a=1.0$ or 0.5 into Eq. (47). It may be seen from Fig. 12(b) that the effect of the value of a on the theoretical curves is relatively small and the theoretical curves fit experimental data excellently. The effectiveness of Eq. (48) with the same value of K as in the triaxial compression case are also shown in Figs. 13, 14 and 16 where data are quoted from Wade (1963), Ichihara and Matsuzawa (1970) and Green (1972), respectively. In these figures, the straight lines represent both of the stress-dilatancy equations for the triaxial compression case expressed by Eq. (23) and Horne's plane strain stress-dilatancy equation expressed by Eq. (4) and the curves are the plane strain stress-dilatancy relations represented by Eq. (48).

Furthermore, it is to be noted that while the variation of the intermediate principal stress σ_2 during shear can not be predicted by Eq. (4), Eq. (47) offers the variation of σ_2/σ_3 as a function of σ_1/σ_3. Note that in Eq. (47) the value of $a=d\lambda_{ZY}/d\lambda_{YX}$ have considerable effects on σ_2/σ_3. This is illustrated in Fig. 18 where data after Cornforth (1964) are also plotted. It may be seen from Fig. 18 that the value of a for this case is between 0.3 and 0.4. In Fig. 19 the theoretical curve of σ_2 by eq. (47) is compared with the measured one where the value of K is obtained from the triaxial compression test conducted by Cornforth (1964) on the same sand as in plane strain tests and a of 0.4 is employed. It may be found from this figure that the predicted value of σ_2 by *Eq.* (47) fit the experimental data for

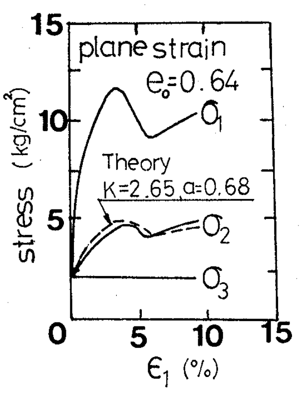

Fig. 17. Stress-strain relationships of a plane strain test (after Green, 1972)

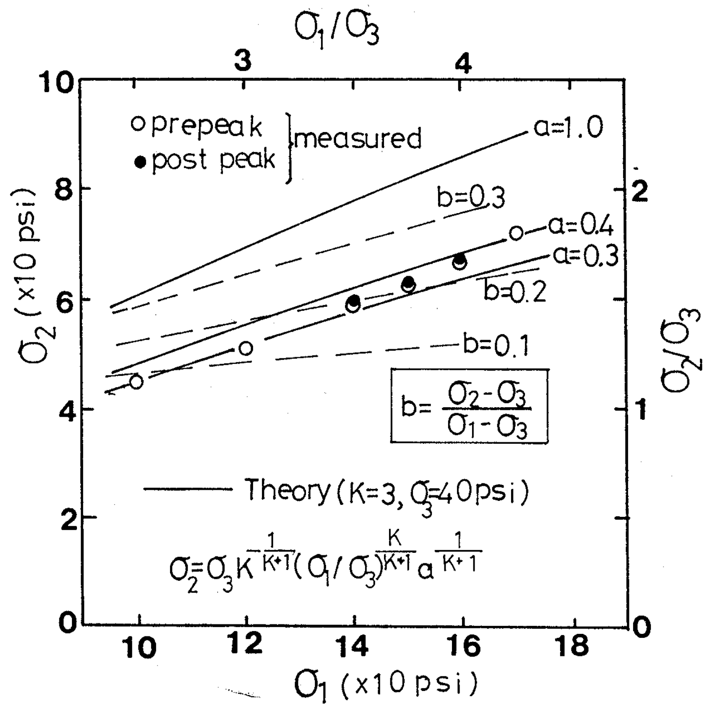

Fig. 18. Variation of σ_2 in plane strain test (after data of Cornforth, 1964)

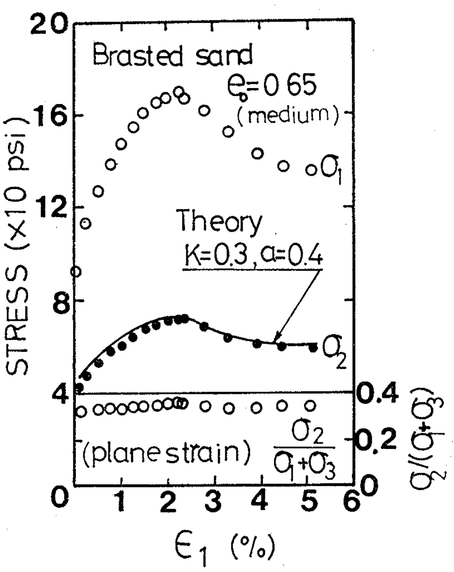

Fig. 19. Stress-strain relationships of a plane strain test (after Cornforth, 1964)

both of pre-peak and post-peak. Similar result is shown in Fig. 15 where the value of K is obtained from Fig. 14 and the value of a is employed for the theoretical curve to fit the experimental curve. However, it is noteworthy that these values of a of around 0.5 are similar to the value of a shown in Fig. 7(a) where data of XED test are plotted. On the other hand, Green (1972) conducted also the socalled XED test as shown in Fig. 4 and the value of a of 0.68 was obtained by the same procedure as shown in Fig. 7(a). Fig. 17 shows that the theoretical curve with the measured value of a of 0.68 fits the measured curve.

The ratio between the deviator stresses $\sigma_1-\sigma_3$ and $\sigma_2-\sigma_3$ is denoted by a parameter b as

$$b=\frac{\sigma_2-\sigma_3}{\sigma_1-\sigma_3} \tag{49}$$

It is shown in Fig. 18 that the value of b in the plane strain condition is not necessarily constant but varies in accordance with σ_1/σ_3. The theoretical relationships between b and σ_1/σ_3 which is obtained from Eq. (47) are illustrated in Fig. 20. This shows that b increases up to $\sigma_1/\sigma_3=4$ but have almost constant values for the larger value of σ_1/σ_3. Note that the theoretical relationship between b and σ_1/σ_3 shown in Fig. 20 cannot be applicable to the case of $\sigma_1=\sigma_3$, because this relationship is derived from Eq. (39) which holds only when $\sigma_1>\sigma_2>\sigma_3$.

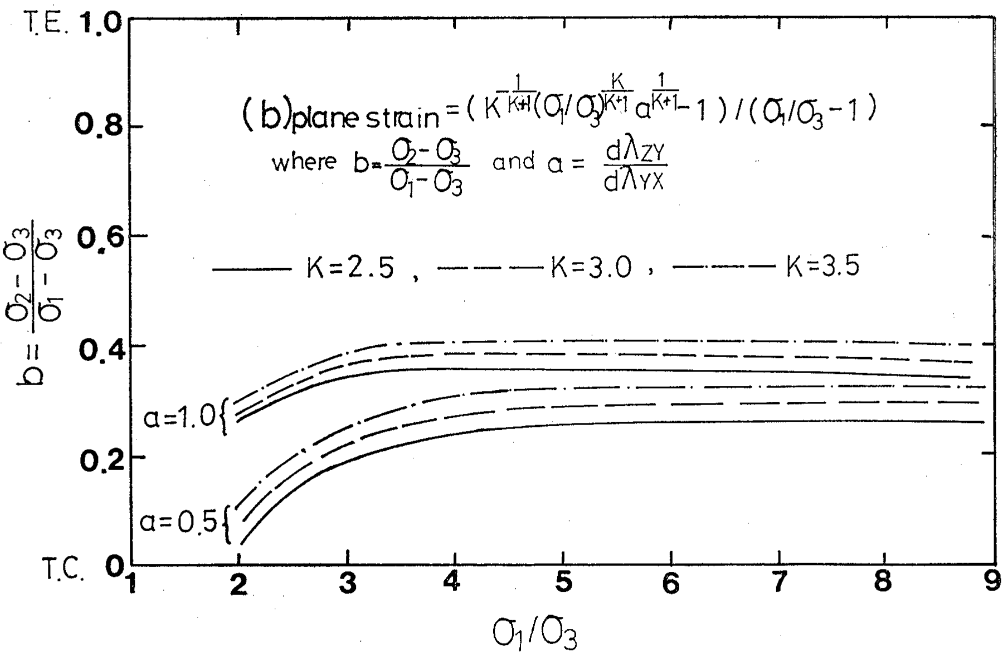

Fig. 20. Theoretical variation of *b*-value in plane strain tests

CONCLUSIONS

On the basis of three fundamental postulates, a theory was proposed by which it is possible to correlate the stress-dilatancy equation for axisymmetric deformation with those for general stress condition in the rather simple form and to assess the effect of inherent anisotropy in sands. It is also shown that the stress-dilatancy equations denoted by principal stress ratios and principal strain increment ratios are not affected by the inherent anisotropy in any case. Furthermore, the comparison of the theory with experimental data was performed and it disclosed the relevance of the theory in spite of its simplicity.

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NOTATION

a, a' = parameters related with inherent anisotropy where

$$a=\frac{d\lambda_{ZX}}{d\lambda_{YX}} \quad \text{and} \quad a'=\frac{d\lambda_{YX}}{d\lambda_{YZ}}$$

$b=(\sigma_2-\sigma_3)/(\sigma_1-\sigma_3)$

D = dilatancy rate

e_0 = void ratio at the start of shear

f_{XY}, f_{YX} = yield functions in $\sigma_X>\sigma_Y$ stress-system and $\sigma_Y>\sigma_X$ stress-system, respectively

f_{XYc} = current yield functions of f_{XY}

df_{XY} = yield function increment

h_{XY}, h_{YX} = hardening functions in $\sigma_X>\sigma_Y$ stress-system and $\sigma_Y>\sigma_X$ stress-system, respectively

$K=\tan^2(\pi/4+\phi_\mu/2)$

K_{XY}, K_{YX} = parameters depending on the state of the element

L_{XY}, L_{YX} = parameters depending on the state of the element

p = mean principal stress $=(\sigma_1+\sigma_2+\sigma_3)/3$

q = shear stress $=\sigma_1-\sigma_3$ in triaxial compression and $\sigma_3-\sigma_1$ in triaxial extension

R = principal stress ratio $=\sigma_1/\sigma_3$

v = volumetric strain

dv = volumetric strain increment

Δ_{XY}, Δ_{YX} = 1.0 when $\sigma_X \neq \sigma_Y$ and zero when $\sigma_X=\sigma_Y$

ε_1, ε_2, ε_3 = major, intermediate and minor principal strains

$d\varepsilon_1$, $d\varepsilon_2$, $d\varepsilon_3$ = major, intermediate and minor principal strain increments

$d\varepsilon_X$, $d\varepsilon_Y$, $d\varepsilon_Z$ = three principal strain increments

$d\varepsilon_{XXY}$, $d\varepsilon_{YXY}$ = principal strain increments in ideal two-dimensional $\sigma_X>\sigma_Y$ stress-system

θ = angle between pouring direction at sample preparation and σ_3-direction

σ_1, σ_2, σ_3 = major, intermediate and minor principal stresses

σ_X, σ_Y, σ_Z = three principal stresses

ϕ_μ = mean angle of interparticle friction

ψ_{XY}, ψ_{YX} = plastic potential functions in $\sigma_X>\sigma_Y$ stress-system and in $\sigma_Y>\sigma_X$ stress-system, respectively

$d\lambda_{XY}$, $d\lambda_{YX}$, $\cdots$ = scalars

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