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INFLUENCE OF DEGREE OF SHEAR STRESS REVERSAL ON THE LIQUEFACTION POTENTIAL OF SATURATED SAND*

Discussion by KENNETH L. LEE**

The authors have presented a valuable study to the profession. Several aspects of this study were of particular interest to the writer and he is in general agreement with the conclusions and implications from the material presented. The following comments are presented by way of augmenting and amplifying the data and concepts set forth in the original paper.

First, the authors recognize that the initial consolidation stresses below a spread footing or mat foundation are not the same as assumed in the ideal free field liquefaction analysis with zero shear stress ($\tau_0=0$) on the horizontal and assumed failure plane. Rather, as shown in their Fig. 1, τ_0 varies from element to element within the foundation soil, as in an earth dam, and a more correct laboratory simulation requires anisotropic consolidation of test specimens. To the writer's knowledge, this extra sophistication has not yet been done for footing problems, although it is standard practice in the seismic stability analysis of earth dams. Nevertheless, the need to use anisotropic consolidation for seismic bearing capacity studies has been recently recognized, especially with respect to foundation soil liquefaction potential studies for offshore gravity structures (Young et al., 1975). More recently, Yoshimi and Tokimatsu (1975) have presented experimental data confirming that the liquefaction potential in soil foundations below footings varies with the position (and hence the initial consolidation stress) in the soil.

Second, the effect of anisotropic consolidation on the response behavior of a saturated sand has been a topic of considerable importance with respect to the seismic stability analysis of earth dams (Seed et al., 1969 and Seed et al., 1975). Seed and Lee (1969) have presented data from anisotropically consolidated cyclic triaxial tests which are qualitatively similar to the anisotropically loaded torsion simple shear data presented by the authors.

Complete symmetric reversed cyclic loading produced the classical liquefaction response. Nonreversing cyclic loading produced progressive strains but the excess pore water pressure did not build up high enough to produce true liquefaction in the sense that the effective stress was reduced to zero. Intermediate nonsymmetric, but partially reversed cyclic stresses led to high pore pressures, but requiring higher cyclic stresses for liquefaction than for complete symmetric reversing stress conditions.

These early observations led Seed and Lee (1969) to suggest that for anisotropically consolidated samples, as required for earth dam seismic stability analyses, excess pore pressure was not a useful factor on which to base a failure criteria. As an alternative,

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they proposed that a more appropriate criteria should be based on cyclic strains. A value of accumulative amplitude cyclic strain of 5% in a cyclic triaxial test has since been used in many seismic stability analyses for earth dams (cf. Seed et al 1975).

This strain definition of failure overcomes the difficulties associated with silty or clayey soils where accurate pore pressures cannot be measured under rapid cyclic loading associated with simulated earthquake testing. It also overcomes the difficulties associated with the observations that the pore pressure response of anisotropically consolidated specimens (with non or partially reversed cyclic loading) will always be different from the pore pressure response of isotropically consolidated (symmetrically loaded) test specimens.

The selection of 5% single amplitude strain as a failure criterion is admittedly only an arbitrary decision. Some designers use other criteria. Clearly more research is needed to relate the laboratory test results and field behavior more closely than can be done at present. Nevertheless, while awaiting this needed additional research, it should be noted that the 5% cyclic strain criteria has given realistic and useful results in a number of post event analyses and designs for future structures.

Third, the observation by the authors that the liquefaction strength of this clean sand was independent of cyclic frequency within the range of 1 to 12Hz, is in almost total agreement with many other studies. The author has summarized data from several other independent investigations which show the liquefaction strength of clean sand under cyclic loading to be independent of cyclic frequency (at least within the range investigated, 1/12 to 28Hz.). On the other hand, there is evidence that this uniqueness does not apply to clay soils. Thiers and Seed (1969) present data which show that cyclic strength decreases somewhat with decreasing cyclic frequency.

To the writer, these observations concerning frequency effects on cyclic strength are quite logical. The writer believes that cyclic strength deteriorations are strain dependent phenomena, and not a stress dependent result. Cyclic strains wear away the interparticle contacts allowing the particles to move closer together. This tendency results in an increase in net pore pressure which in turn leads to a weaker soil. It therefore follows that the larger the cyclic strains, the weaker will be the sample under a given number of cyclic stress applications. If a soil creeps under load, then it will strain more per cycle than a soil which does not creep. Other things being equal, fewer cycles will be required to fail samples which creep than samples which do not creep. Thus, since clays generally creep it should be expected that the longer the load is applied per cycle (i.e. the longer the cyclic period) the weaker will be the clay soil under cyclic loading. On the other hand, sands generally do not creep much under sustained load, so it should not be expected that sands would show enough cyclic frequency effect to be discernible within the general range of scatter of the data.

Fourth, and finally, the authors suggest from their Fig. 13 that the cyclic strength data from cyclic triaxial and cyclic torsion simple shear are approximately the same, provided the strength data are all plotted versus the mean normal consolidation stress. This is a matter which the writer would like to discuss in some detail, because this question of cyclic strength for various consolidation stress conditions has been raised by a number of authors over the past several years. Since this question is one of continuing interest, perhaps a brief background review would be in order to put the various suggestions in a common perspective.

Seed and Lee (1966) described the reversing stress triaxial test as a possible tool for laboratory studies of seismic loading on a soil element below a level surface in the field. It was hypothesized that the three key stress features were needed to simulate in the laboratory were (1) the normal and (2) the shear stress on the potential failure plane during consolidation, and (3) the cyclic shear stress on the potential failure plane during seismic loading. For level ground surface conditions the potential failure plane was assumed to be horizontal. The ideal laboratory test to simulate these conditions was suggested to be an ideal cyclic simple shear test (without practical boundary effects). The torsion simple shear apparatus ideally provides better boundary conditions than the practical simple shear apparatus. An alternative to these types of equipment was the cyclic triaxial test. Isotropic consolidation provided zero shear stress on the failure plane (in fact, all planes) before cyclic loading, and symmetric cyclic axial stress pulses provided cyclic shear stresses on planes at 45° to the axis of the sample. Data for cyclic triaxial tests were used successfully by Seed and Idriss (1968) in a case history study to explain the observed liquefaction that developed at Niigata during the 1964 earthquake. The seismic stresses used in that early study were computed on the assumption that the soil responded as an ideal elastic system.

Meanwhile, Peacock and Seed (1968) developed a cyclic simple shear apparatus and found that cyclic strengths with this equipment were only about half of the strengths from cyclic triaxial tests, when compared on the basis of equal normal consolidation stress on the potential failure plane (σ_{3c} for triaxial and σ_{vc} for simple shear).

Meanwhile, developments in the seismic response analysis techniques had advanced to the point where nonlinear strain dependent soil properties could be used in the calculations to more correctly simulated true field conditions. Combining non-linear seismic response calculated seismic stresses with cyclic strengths from simple shear tests for the Niigata case history, both of which were lower than used in the earlier study, again led to a satisfactory agreement between observed and calculated liquefaction (Seed and Idriss, 1968).

An additional study by Lee and Seed (1967) used cyclic loading on anisotropically consolidated triaxial samples to simulate seismic effects conditions within earth slopes where the shear stress on the potential failure plane was greater than zero before the earthquake. Comparing the cyclic strength data on the basis of normal consolidation, stress on the potential failure plane generally shows higher strengths for anisotropically consolidated soil than for isotropically consolidated soil.

Further concerning earth dams, Seed et al. (1969) used non-linear response analyses with cyclic triaxial tests corrected for simple shear and field conditions to back figure the observed case history of liquefaction failure of the Sheffield dam in 1925. Because sloping surfaces were involved, anisotropic consolidation was required involving non-zero shear stress on the potential failure planes prior to the cyclic loading. Again, a reasonably good comparison was obtained between predicted and observed liquefaction of this field case.

This original recommendation concerning correlations between cyclic triaxial and cyclic field conditions (Peacock and Seed, 1968) was later modified to include the effect of overconsolidation which produced high K_0 conditions (Seed and Peacock, 1971). For normally consolidated soils the cyclic simple shear strengths were only about half the values obtained by cyclic triaxial tests. However, for overconsolidated soils such that $K_0 \approx 1.0$, it was found that cyclic simple shear and cyclic triaxial tests gave about the same results. A similar conclusion may be drawn from the results presented by Ishihara and Yasuda (1975) for normally consolidated soil prepared in a hollow cylinder to $K_0=1.0$ conditions and tested in cyclic torsion simple shear. The cyclic strengths are identical to those obtained on the same sand in an isotropic consolidation cyclic triaxial test. More recently De Albe, Chan and Seed (1975) have presented data from large shaking table tests on normally consolidated sand which quantify the differences between cyclic triaxial, cyclic simple shear and field cyclic shear liquefaction strengths.

A convenient method of expressing the cyclic strength data for laboratory or field purposes is in a normalized form of a ratio of cyclic shear stress causing failure in N cycles to normal consolidation stress prior to the cyclic disturbance. For cyclic triaxial tests the convenient stresses are: cyclic shear stress $\sigma_{dp}/2\sigma_{3c}$ where σ_{dp} is the maximum amplitude of cyclic axial or deviator stress, and σ_{3c}, is the minor principal consolidation stress. For the field, or for laboratory simple shear or torsion shear conditions, the most convenient stress components are: τ_p, the maximum amplitude cyclic shear stress and τ_{vc}, the vertical normal stress on a horizontal plane.

Conversion from the cyclic triaxial to the laboratory simple shear or the field conditions is then conveniently done as follows

$$\left(\frac{\tau_p}{\sigma_{vc}}\right)=C_r\left(\frac{\sigma_{dp}}{2\sigma_{3c}}\right) \tag{8}$$

The term C_r is an empirical conversion factor. The latest recommendations by De Albe (1975) for C_r, for clean normally consolidated sands is as follows: Laboratory simple shear, $C_r \approx 0.6$ to 0.66; field, $C_r=0.55$ to 0.59 (ave. 0.57). These values are independent of soil density. For clay soils, $C_r \approx 1.0$ (Thiers and Seed, 1969) and for anisotropically consolidated soils with $K_c \geqslant 1.5$ $(\alpha=\tau_c/\sigma_{vc} \geqslant 0.2)$, $C_r \approx 1.0$ (Seed and Idriss, 1969).

Eq. (8) uses normal consolidation stress on the failure (horizontal) plane as a basis for reducing the cyclic strength data, which is different from using the mean normal consolidation stress as done by the authors and by Finn (1972) and Finn et al. (1971). An advantage of using the mean normal stress as a basis of data reduction is that it is a stress invariant which appeals to the more rigorous tastes. Finn found that on the mean normal stress basis there was no difference between cyclic simple shear and cyclic triaxial data for one normally consolidated sand. The author's Fig. 13 appears to confirm this for another sand, although the data are too scattered and cover too many conditions to clarify this conclusion. The disadvantage of using mean normal stress is that one needs to estimate K_0, and also the results are not particularly adaptible in that form for use in seismic stability analyses.

The advantage of using the normal stress on the failure plane σ_{vc} as a basis of data reduction is that σ_{vc} is readily calculated and carries a practical meaning to the design engineer. Also the data are readily compared with the results of seismic response calculations for seismic stability analyses.

It is of interest to note what value of C_r would be implied in a comparative set of cyclic triaxial and simple shear data that gave the same results when compared on the basis of mean normal stress σ_m. Finn reasoned that the simple shear test was a plane strain test so that only two principal stresses should be used in the calculation.

$$\sigma_m=\sigma_{vc}/2(1+K_0)=K_2\sigma_{vc} \tag{9}$$

The authors use all three principal stresses:

$$\sigma_m=\sigma_{vc}/3(1+2K_0)=K_3\sigma_{vc} \tag{10}$$

Assuming $K_0=0.4$, which is reasonably typical for sands, leads to $K_2=0.7$ and $K_3=0.6$.

For the special case where the cyclic triaxial and cyclic simple shear strengths are equal when normalized on a mean normal stress basis, Eq. (8) may be rewritten as follows

$$\left(\frac{\tau_p}{K\sigma_{vc}}\right)_s=\left(\frac{\sigma_{dp}}{2\sigma_{3c}}\right)_t \tag{11}$$

Comparison of Eq. (8) and Eq. (11) indicates that for this special case $C_r=K$ and depending on the definition of σ_m, then $C_r=0.6$ to 0.7 which is approximately what has been suggested by Seed and his coworkers for normally consolidated sands with zero shear stress on the failure plane prior to the cyclic loading.

It is of interest to compare the cyclic torsion simple shear data with the cyclic triaxial data presented by the authors from their Fig. 13 with the above stated recommendations. For this purpose, the first named author kindly supplied the writer with original data from which Fig. 24 has been prepared. Only those data for tests at approximately the same density are shown in Fig. 24. No conversions are made to extrapolate from one density to another.

The range of data scatter for each series of tests is typical of many studies with which the writer is familiar. As first suggested by Seed and Lee (1966), but never verified experimentally in published form, the results of cycling the axial stress only are the same as the results from cycling both the axial stress and the cell pressure (Fig. 24(a)). For the low K_c ratio tests there is no clear distinction between isotropic and anisotropic consolidated torsion simple shear data (Fig. 24(b)). However, this is not a significant conclusion because the value of $K_c=1.2$ is too low and the data too scattered to distinguish clearly whether or not there is any strength gain with anisotropic consolidation as first noted by Seed, Lee and Idriss (1969). Finally, Fig. 24(c) shows that the value of C_r required to convert cyclic triaxial to cyclic torsion simple shear is about the same as suggested by De Albe, Chan and Seed (1975) for clean normally consolidated sand.

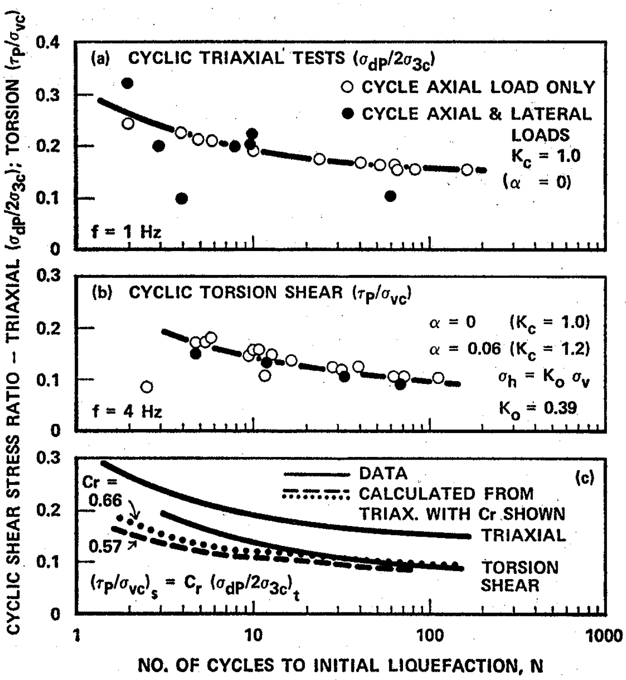

Fig. 24. Compilation and comparison of cyclic triaxial and cyclic torsion shear initial liquefaction data on loose saturated Bandaijima sand. Density$\simeq$1.45 gms/cm^3 (Yoshimi and Oh-oka, 1975)

The aforementioned studies have provided the basis for many seismic stability analyses which have been performed for both research and design objectives. The justification for using the techniques, including the several empirical assumptions involved, lies in the past observed cases of seismic instability. Since there are several inherent assumptions involved in using these procedures, engineers must use caution in research leading to improvements with any of the assumptions or parameters involved. Large changes in one parameter without consideration of the overall problem may lead to erroneous conclusions. The authors are encouraged in their pioneering work to include the effect of anisotropic consolidation stress below foundations of buildings in seismic liquefaction analyses. It is hoped that they will continue with these studies and include case history comparisons and design recommendations in future papers.

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ONE-DIMENSIONAL VOLUME CHANGE CHARACTERISTICS OF SANDS UNDER VERY LOW CONFINING STRESSES*

Discussion by KENNETH L. LEE**

The authors have presented an excellent study which vividly quantifies the volume change behavior of saturated sand before and after liquefaction. The writer wishes to comment on only one aspect; the large increase in compressibility that occurs as a result of liquefaction (Fig. 8) for soil at the same void ratio. This type of behavior has been noted by others in different situations. Lee and Albaisa (1974) observed a major increase in volumetric strain of granular soil after liquefaction as compared with the compressibility before liquefaction. Finn et al (1970) remarked on the relative ease of samples to liquefy in laboratory cyclic triaxial tests after having once been liquefied and then reconsolidated. The writer and others have also noticed that soil consolidated after liquefaction is relatively easy to reliquefy, but have not written about it. At first the writer attributed this soil weakness to severe change in shape of the sample which developed a neck or other discontinuity when first liquefied so that any reloading would produce stress concentration

* By Yoshiaki Yoshimi, Fumio Kuwabara and Kohji Tokimatsu, Vol. 15, No. 3, Sept. 1975, pp. 51-60.

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