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PLANE STRAIN CONSOLIDATION OF A CLAY LAYER WITH FINITE THICKNESS

HAKUJU YAMAGUCHI* and YUKITOSHI MURAKAMI**

ABSTRACT

The present study is concerned with the two- and three-dimensional (multi-dimensional) consolidation of a clay layer with finite thickness, on the basis of Biot's theory.

As the results of analysis, the degree of settlement is presented, and a peculiar phenomenon that the settlement is more retarded than that of Terzaghi's theory is predicted. Since it seems the matter relevant to the Mandel-Cryer effect, its physical cause is examined.

The excess pore-water pressure and its time history are discussed quantitatively, by taking account of the dilatancy characteristics of a clay. It is revealed from experimental results that the behavior of the pore-water pressure can be satisfactorily explained by Biot's theory, if the dilatancy is taken into account.

Key words: clay, cohesive soil, consolidation, dilatancy, model test, pore pressure, saturation, settlement

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INTRODUCTION

Among two- and three-dimensional (multi-dimensional) consolidation theories, Rendulic's and Biot's theories are well known in general. Some problems have been solved on the basis of these theories. In these days, superiority of Biot's theory is being appreciated. However, its mathematical treatment is much more complicated and difficult than that of Rendulic's theory. It has been, therefore, considered that the use of Biot's theory is rather limited in practical problems.

A few studies of multi-dimensional consolidation based on Biot's theory have been published. Among these, the work by Gibson et al. (1970) or Yoshikuni et al. (1975) may be the only one, in which the analytical results are given in a form applicable to practical problems in soil mechanics, as far as the authors know. The former study is concerned in the degree of settlement of a clay layer with finite thickness subjected to a strip or a circular load on its surface, while the latter in the consolidation of a clay cylinder with external radial drainage. In the former study, however, the authors are afraid that there is a mistake in its mathematical treatment, viz. an incorrect application of Jordan's lemma. By reason of the above, the authors (1972) have published a study of the same multi-dimensional consolidation problem by making use of a different analytical method. In the analysis, however, it is obliged to impose a restriction on the boundary condition that the

* Department of Civil Engineering, Tokyo Inst. of Tech., Oh-okayama, Meguro-ku, Tokyo.
** Department of Civil Engineering, Yamanashi University, Takeda, Kofu, Yamanashi.
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horizontal displacement of a layer surface is restricted, for the sake of avoiding a mathematical complication. Therefore, the analysis is unsatisfactory itself and the applicability of its result is limited.

The present study treats the multi-dimensional consolidation problem under the more reasonable condition that a layer surface is free from displacement. The objectives of this study are to overcome the difficulty in Biot's theory, to prepare numerical results applicable to actual problems and to examine the peculiar consolidation characteristics obtained herein. Besides, recognising an appreciable influence of shear deformation on the multi-dimensional consolidation, the present study includes a discussion on the dilatancy effect for which Biot's theory has no control.

THEORY AND ANALYSIS

Biot (1941) has formulated a theory for two- and three-dimensional consolidation. According to this theory, the fundamental equations of consolidation of a fully saturated clay in a two-dimensional case are expressed as follows:

$$\left.\begin{aligned} &G\left\{\nabla^2 u_i-\frac{1}{1-2\nu'}\varepsilon_{v,i}\right\}=\sigma_{,i} \qquad (i=x,\ z)\\ &\frac{\partial\varepsilon_v}{\partial t}=c_v\nabla^2\varepsilon_v\\ &\frac{\gamma_w c_v}{k}\nabla^2\varepsilon_v=-\nabla^2\sigma \end{aligned}\right\} \tag{1}$$

where ε_v, u_i and σ stand for a volumetric strain, displacement in the direction i and excess pore-water pressure respectively. Besides, c_v, k and γ_w are the coefficients of consolidation and of permeability and weight of water in unit volume respectively.

Two-dimensional consolidation problems can be solved on the basis of Eqs. (1) by using the method of a separation of variables and the general solutions are given in the following forms:

$$\begin{aligned}\varepsilon_v=\frac{2}{\pi}\int_0^\infty &\{\sum_\alpha(A_{\xi\alpha}\cos(\alpha z)+B_{\xi\alpha}\sin(\alpha z))\exp(-St)\\ &-(C_\xi\cosh(\xi z)+D_\xi\sinh(\xi z))\}\cos(\xi x)d\xi\end{aligned} \tag{2}$$

$$\begin{aligned}\sigma=\frac{2}{\pi}\int_0^\infty &\left\{\sum_\alpha\left[\frac{\gamma_w c_v}{k}(A_{\xi\alpha}\cos(\alpha z)+B_{\xi\alpha}\sin(\alpha z))\right.\right.\\ &\left.\left.+E_{\xi\alpha}\cosh(\xi z)+F_{\xi\alpha}\sinh(\xi z)\right]\exp(-St)\right\}\cos(\xi x)d\xi\end{aligned} \tag{3}$$

$$w=\frac{2}{\pi}\int_0^\infty\{\Psi_1(\xi,\ z)\exp(-St)+\Psi_2(\xi,\ z)\}\cos(\xi x)d\xi \tag{4}$$

$$\sigma_z=\frac{2}{\pi}\frac{\gamma_w c_v}{\mu\cdot k}\int_0^\infty\{\Psi_3(\xi,\ z)\exp(-St)+\Psi_4(\xi,\ z)\}\cos(\xi x)d\xi \tag{5}$$

$$\tau_{xz}=\frac{2}{\pi}\frac{\gamma_w c_v}{\mu\cdot k}\int_0^\infty\{\Psi_5(\xi,\ z)\exp(-St)+\Psi_6(\xi,\ z)\}\sin(\xi x)d\xi \tag{6}$$

where

$$\begin{aligned}\Psi_1(\xi,\ z)=\sum_\alpha&\left\{\frac{\alpha}{\xi^2+\alpha^2}(A_{\xi\alpha}\sin(\alpha z)-B_{\xi\alpha}\cos(\alpha z))\right.\\ &+\frac{1}{G}\left[\left(\frac{z}{2}\cosh(\xi z)-\frac{1}{4\xi}\sinh(\xi z)\right)E_{\xi\alpha}-\left(\frac{1}{4\xi}\cosh(\xi z)\right.\right.\\ &\left.\left.\left.-\frac{z}{2}\sinh(\xi z)\right)F_{\xi\alpha}\right]+(I_{\xi\alpha}\sinh(\xi z)+J_{\xi\alpha}\cosh(\xi z))\right\}\end{aligned}$$

$$\Psi_2(\xi,\ z)=-\frac{1}{1-2\nu'}\left\{\left(\frac{z}{2}\cosh(\xi z)-\frac{3-4\nu'}{4\xi}\sinh(\xi z)\right)C_\xi\right.$$
$$\left.-\left(\frac{3-4\nu'}{4\xi}\cosh(\xi z)-\frac{z}{2}\sinh(\xi z)\right)D_\xi\right\}$$
$$-(H_\xi\cosh(\xi z)+G_\xi\sinh(\xi z))$$

$$\Psi_3(\xi,\ z)=\sum_\alpha\left\{\frac{\xi^2}{\xi^2+\alpha^2}(A_{\xi\alpha}\cos(\alpha z)+B_{\xi\alpha}\sin(\alpha z))\right.$$
$$+\frac{\xi}{G}\left[\left(\frac{1}{4\xi}\cosh(\xi z)-\frac{z}{2}\sinh(\xi z)\right)E_{\xi\alpha}-\left(\frac{z}{2}\cosh(\xi z)\right.\right.$$
$$\left.\left.\left.-\frac{1}{4\xi}\sinh(\xi z)\right)F_{\xi\alpha}\right]-\xi(I_{\xi\alpha}\cosh(\xi z)+J_{\xi\alpha}\sinh(\xi z))\right\}$$

$$\Psi_4(\xi,\ z)=\frac{\xi}{1-2\nu'}\left\{\left(-\frac{1}{4\xi}\cosh(\xi z)+\frac{z}{2}\sinh(\xi z)\right)C_\xi\right.$$
$$\left.+\left(\frac{z}{2}\cosh(\xi z)-\frac{1}{4\xi}\sinh(\xi z)\right)D_\xi\right\}$$
$$+\xi(G_\xi\cosh(\xi z)+H_\xi\sinh(\xi z))$$

$$\Psi_5(\xi,\ z)=\sum_\alpha\left\{\frac{\xi\alpha}{\xi^2+\alpha^2}(A_{\xi\alpha}\sin(\alpha z)-B_{\xi\alpha}\cos(\alpha z))\right.$$
$$+\frac{\xi}{G}\left[\left(\frac{z}{2}\cosh(\xi z)+\frac{1}{4\xi}\sinh(\xi z)\right)E_{\xi\alpha}+\left(\frac{1}{4\xi}\cosh(\xi z)\right.\right.$$
$$\left.\left.\left.+\frac{z}{2}\sinh(\xi z)\right)F_{\xi\alpha}\right]+\xi(I_{\xi\alpha}\sinh(\xi z)+J_{\xi\alpha}\cosh(\xi z))\right\}$$

$$\Psi_6(\xi,\ z)=-\frac{\xi}{1-2\nu'}\left\{\left(\frac{z}{2}\cosh(\xi z)+\frac{1}{4\xi}\sinh(\xi z)\right)C_\xi\right.$$
$$\left.+\left(\frac{1}{4\xi}\cosh(\xi z)+\frac{z}{2}\sinh(\xi z)\right)D_\xi\right\}$$
$$-\xi(G_\xi\sinh(\xi z)+H_\xi\cosh(\xi z))$$
$$S=c_v(\alpha^2+\xi^2)$$

and

$$\mu=\frac{1-\nu'}{1-2\nu'}$$

in which $A_{\xi\alpha}$, $B_{\xi\alpha}$, $\cdots$, $J_{\xi\alpha}$ are integral constants.

With the initial and the specified boundary conditions as shown in Fig. 1, the integral constants can be determined. However, it should be noted here that the drainage condition is restricted on account of the limit of mathematical treatment. That is, nothing but the solution under the condition that the surface of the layer is pervious but the bottom face impervious can be obtained.

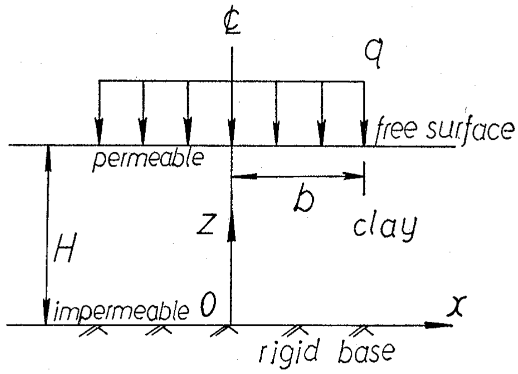

Fig. 1. Flexible strip loading

First of all, from both the condition that the bottom is impervious and the initial condition of $\varepsilon_v=0$ at $t=0$, Eqs. (2) and (3) give:

$$\sum_i A_{\xi\alpha_i}\cos(\alpha_i z)-C_\xi\cosh(\xi z)=0 \tag{7}$$

$$B_{\xi\alpha_i}=0 \tag{8}$$

On the other hand, considering the condition that the surface is pervious, we have:

$$A_{\xi\alpha_i}\cos(\alpha_i H)+\frac{k}{\gamma_w c_v}E_{\xi\alpha_i}\cosh(\xi H)=0 \tag{9}$$

Noting the condition that no vertical displacement should occur and no shear stress is assumed to act on the bottom, Eqs. (4) and (6) conclude that:

$$J_{\xi\alpha_i}=F_{\xi\alpha_i}=0 \tag{10}$$

Since the vertical stress on the surface is specified by a constant surface load, we obtain:

$$\frac{\xi^2}{\xi^2+\alpha_i^2}A_{\xi\alpha_i}\cos(\alpha_i H)+\frac{\xi}{G}\left\{\frac{1}{4\xi}\cosh(\xi H)-\frac{H}{2}\sinh(\xi H)\right\}E_{\xi\alpha_i}-\xi I_{\xi\alpha_i}\cosh(\xi H)=0 \tag{11}$$

And by the condition of the surface being free from shear stress:

$$\frac{\xi\alpha_i}{\xi^2+\alpha_i^2}A_{\xi\alpha_i}\sin(\alpha_i H)+\frac{\xi}{G}\left\{\frac{H}{2}\cosh(\xi H)+\frac{1}{4\xi}\sinh(\xi H)\right\}E_{\xi\alpha_i}+\xi I_{\xi\alpha_i}\sinh(\xi H)=0 \tag{12}$$

Eqs. (9), (11) and (12) together with the condition of $E_{\xi\alpha_i}\neq 0$ and $k/\gamma_w c_v=(1-2\nu')/2G(1-\nu')$ lead the following characteristic equation:

$$\bar{a}(\xi)\alpha^2+\bar{b}(\xi)\alpha\cdot\tan(\alpha H)+\bar{c}(\xi)=0 \tag{13}$$

in which

$$\bar{a}(\xi)=\mu\{\xi H\operatorname{sech}(\xi H)+\sinh(\xi H)\}$$
$$\bar{b}(\xi)=-\xi\cosh(\xi H)$$
$$\bar{c}(\xi)=\xi^2\{\bar{a}(\xi)-\sinh(\xi H)\}$$

and μ denotes an auxiliary constant defined before: $\mu=(1-\nu')/(1-2\nu')$. These eigenvalues α_i ($i=1, 2,\cdots$) can be obtained by making use of the Newton-Raphson method on an electronic computer as follows. Referring to Eq. (7), $\bar{A}_{\xi\alpha_i}(=A_{\xi\alpha_i}/C_\xi)$ may be approximately expressed as the solutions of a set of the linear equations:

$$\sum_{i=1}^{n}\bar{A}_{\xi\alpha_i}\int_0^H g_i(z)\,g_1(z)\,dz=\int_0^H f(z)g_1(z)\,dz$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\sum_{i=1}^{n}\bar{A}_{\xi\alpha_i}\int_0^H g_i(z)\,g_n(z)\,dz=\int_0^H f(z)g_n(z)\,dz$$

where

$$g_i(z)=\cos(\alpha_i z)$$
$$f(z)=\cosh(\xi z)$$

And n is a large integer properly specified. If $A_{\xi\alpha_i}$ is obtained, $E_{\xi\alpha_i}$ is settled from Eq. (8) as that:

$$E_{\xi\alpha_i}=-\frac{\gamma_w c_v}{k}\frac{\cos(\alpha_i H)}{\cosh(\xi H)}A_{\xi\alpha_i}$$

$I_{\xi\alpha_i}$ is, in consequence, specified by Eqs. (11) and (12).

Finally, the vertical displacement w and the excess pore-water pressure σ are expressed in terms of the dimensionless quantities as follows:

$$w-w_{T=\infty}=\frac{2}{\pi}q\cdot\frac{H}{2G}\int_0^\infty\frac{C_\xi}{\xi}\tanh(\xi)\sum_{i=1}^{n}\bar{A}_{\xi\alpha_i}\cos(\bar{\alpha}_i)$$
$$\cdot\exp\{-(\xi^2+\bar{\alpha}_i^2)T\}\cos(\xi X)d\xi \tag{14}$$
(at the surface; $Z=1$)

$$\sigma=\frac{2}{\pi}q\int_0^\infty C_\xi\sum_{i=1}^{n}\left\{\cos(\bar{\alpha}_i Z)-\frac{\cos(\bar{\alpha}_i)}{\cosh(\xi)}\cosh(\xi Z)\right\}$$
$$\cdot\bar{A}_{\xi\alpha_i}\exp\{-(\xi^2+\bar{\alpha}_i^2)T\}\cos(\xi X)d\xi \tag{15}$$

where

$$\bar{\alpha}_i=\alpha_i H,\ \ X=x/H,\ \ Z=z/H,\ \ T=c_v t/H^2,\ \ \xi=\xi H.$$

And

$$C_\xi=\frac{2(1-\nu')\sin(\beta\xi)}{\xi\{\cosh(\xi)+\xi\,\mathrm{cosech}(\xi)\}},\ \ \beta=b/H$$

in which C_ξ is such a constant as leading Eq. (2) under the condition of $t\to\infty$ to the elastic solution of volumetric strain.

Consequently, the excess pore-water pressure and the degree of settlement in the progress of consolidation can be easily obtained from the above solutions. It should be noticed that these solutions are influenced by Poisson's ratio, since a set of eigenvalues is dependent upon it.

As to the case of the three-dimensional (axi-symmetrical) problem that a circular load acts on the surface, the solutions would be obtained, if $\alpha J_1(\alpha\xi)J_0(R\xi)$ takes the place of $2\sin(\beta\xi)\cos(X\xi)/\pi\xi$ in the solutions of the two-dimensional problem, in which $\alpha=a/H$ and $R=r/H$. A constant a is a radius of the circular load and r is a radial coordinate. J_0 and J_1 denote the cylindrical functions of order zero and one respectively.

DEGREE OF SETTLEMENT

In practice, it is important to give the theoretical expression for the time characteristics of consolidation settlement of a clay layer with finite thickness. Since the consolidation settlement makes progress as time elapses, the degree of settlement can be accepted reasonably for it, which is conventionally defined as follows:

$$U_s(T)=\frac{w(T=T)-w(T=0)}{w(T=\infty)-w(T=0)} \tag{16}$$

where w is the vertical displacement which is given by Eq. (14). It can be said from the above equation that the degree of settlement expresses the ratio of the consolidation settlement at an arbitrary time to the total consolidation. From an engineering point of view, this is an important parameter in consolidation problem and is, in general, defined in terms of the surface settlement of a clay layer.

The theoretical relationships of time and the degree of settlement at $x=0$ are shown in Fig. 2. It has been already shown in the preceding study (Yamaguchi and Murakami, 1973) that, in the case of the horizontal displacement on a surface restricted, the degree of settlement is never smaller than that of the one-dimensional consolidation for any loading or drainage condition. When regarding Fig. 2, however, it would be found that a peculiar phenomenon comes out. That is to say, with the exception of the case that the value of Poisson's ratio is 0.5, there arises the phenomenon that the rate of settlement is later than that of the one-dimensional consolidation, though depending on the ratio of the load breadth to the layer thickness. In particular, this tendency becomes more remarkable as the value of Poisson's ratio approaches zero. Besides, it is seen from this result of numerical calculation that fixing Poisson's ratio at a certain value the delay is

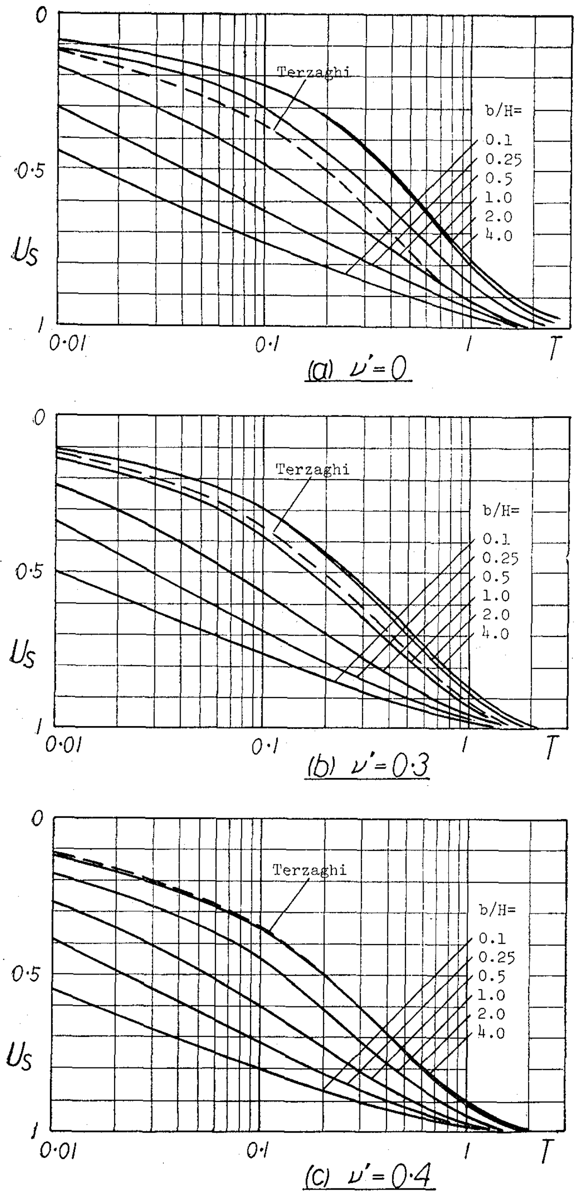

Fig. 2. Relationships between time factor and degree of settlement (under strip load)

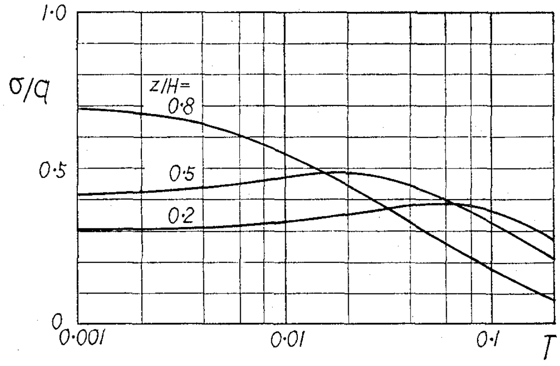

Fig. 3. Time history of excess pore-water pressure ($b/H=0.5$)

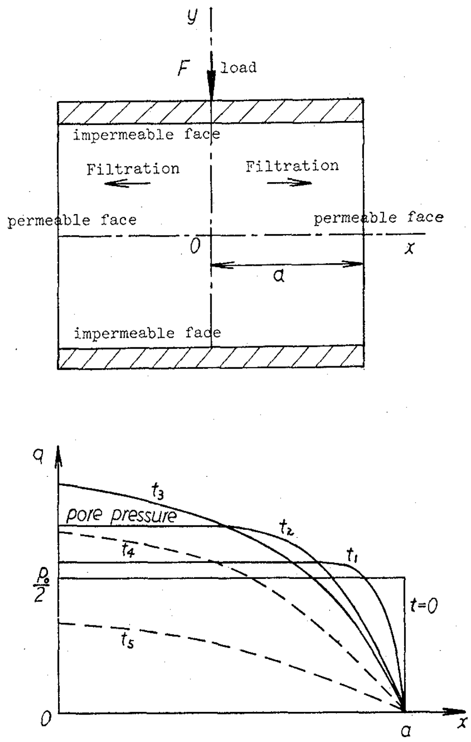

Fig. 4. The Mandel-Cryer effect (after Mandel (1953))

most striking when the ratio b/H is about 4.

It is supposed that this phenomenon is exceedingly related to the Mandel-Cryer effect which would be mentioned later in detail, in view of the quantitative characteristics of it. Considering the fact that the excess pore-water pressure and the deformation of soil skeleton are mutually interrelated in the process of consolidation, it can be expected that the progress of settlement and the dissipation of pore-water pressure have a strong correlativity, even though there may not be a unique relationship between them. In the cause of the temporary increase in the pore-water pressure, therefore, the phenomenon that the rate of settlement during the multi-dimensional consolidation is later than that during the one-dimensional consolidation may happen.

On the other hand, as shown before, in the case that the horizontal displacement on the layer surface is restricted, the predicted pore-water pressure decreases monotonously as time passes and the Mandel-Cryer effect has nothing to do with it. In consequence, there exists no factor that the rate of settlement is retarded.

It is theoretically shown that the characteristics of time-degree of settlement relationships are approximately equivalent to those of the case that the horizontal displacement of a layer surface is restricted, when the value of Poisson's ratio is about 0.5. Therefore it can be said without waiting for an experimental evidence that the present theoretical relationship is valid when the value of a clay is nearly equal to 0.5. Because, it has been concluded in the preceding study that a comparison with theoretical and experimental results is satisfactory in this case. When the value of Poisson's ratio is far smaller than 0.5, however, it is necessary to evidence experimentally whether the present theoretical relationships are applicable.

MANDEL-CRYER EFFECT

The excess pore-water pressure for the present problem can be calculated by Eq. (15). The numerical results are shown in Fig. 3, which illustrate the time histories of the pressure at the three specified points on the center-line of a strip load. It should be noticed that the pressure increases temporarily during the early stage of consolidation. Moreover, it can be appreciated that the time required till the pressure reaches maximum is different at each point and that it is earlier as the distance between the point and a permeable surface is shorter.

This phenomenon is called the Mandel-Cryer effect after the investigators who have predicted it theoretically. However, as in one-dimensional consolidation this phenomenon does not take place, it is supposed that a special condition is necessary for the occurrence of this phenomenon.

According to Yoshikuni (1973), Biot's equation can be reformed to:

$$\frac{\partial \sigma}{\partial t}=c_v \nabla^2 \sigma+\frac{\partial \Phi}{\partial t} \tag{17}$$

where Φ is a potential function of consolidation defined by Yoshikuni himself. It is stressed by him that the characteristics of the pore-water pressure and the Mandel-Cryer effect depend exclusively upon Φ. In his paper, however, he touches no physical cause why this phenomenon occurs. Then, herein the authors' consideration would be described.

Except for all-round consolidation, the sum of total stresses may change in the process of consolidation. It may be caused by the fact that the resultant deformation takes place when the pore-water pressure dissipates and this implies the change in the state of the structure of soil particles which is in equilibrium under the anisotropic condition of stress or strain. Accordingly, the change in the stress cannot but happen on account of such a force as keeps the equilibrium of the structure up. The stress change is considered to be most remarkable when Poisson's ratio is zero because the drainage volume of pore-water or the volume change of soil is maximum. It may be presumed that the change in the sum of total stresses during the initial stage of consolidation puts a burden not on the effective stress but on the pore-water pressure. Because, under undrained condition, the immediately reactable stress is not the effective stress but the pore-water pressure. So, it is not unaccountable that there exists the phenomenon that the temporary increase in the pore-water pressure breaks out. It is supposed that the Mandel-Cryer effect occurs when the increase in the pore-water pressure due to the stress change is greater than the absolute quantity of the decrease in the pressure due to the drainage. Considering as above, it can be explained that the Mandel-Cryer effect appears at earlier time as the distance between the point and a permeable face is shorter. At the point which is in the neighborhood of the permeable face, a hydraulic gradient is excessive at the initial stage of consolidation so that the dissipation of pore-water pressure is promoted rapidly. In consequence, the

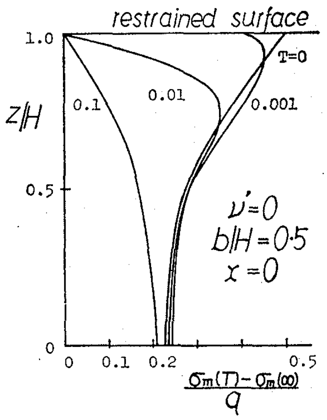

Fig. 5. Time history of average total stress

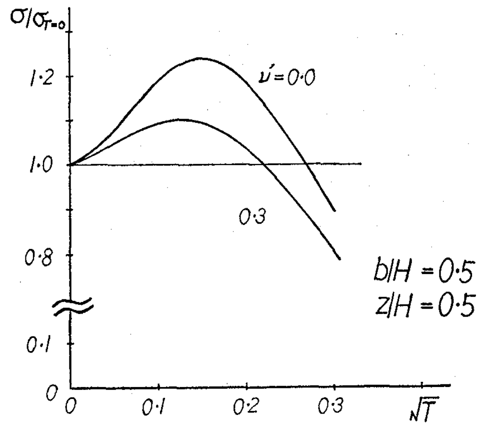

Fig. 6. Influence of Poisson's ratio on the Mandel-Cryer effect

temporary increase in the pore-water pressure is confined to occur within a very short time after the consolidation has set in. On the other hand, at the point which is far distant from the permeable face, the drainage is dull so that the pore-water pressure tends to be affected directly by the change in the sum of total stresses. The time that the Mandel-Cryer effect appears is retarded and the phenomenon itself becomes relatively conspicuous.

However, it should be recalled herein that this effect results from the quantitative correlation between the change in the average of total stresses and the decrease in the pore-water pressure due to the drainage. This matter can be understood if examining the result obtained under the condition that the horizontal displacement of a clay surface is restricted. Under this condition, it has been shown in the authors' preceding paper (1972) that the pore-water pressure σ satisfies the differential equation of a parabolic type $\dot{\sigma}=c_v\nabla^2\sigma$. Hence, generally, σ decreases monotonously with the lapse of time so that the Mandel-Cryer effect does not occur. It is, however, apparent that the average of total stresses increases temporarily even under this condition, as shown in Fig. 5. Consequently, it can not be concluded that the change in the sum of total stresses induces necessarily the transient increase of pore-water pressure.

Now, as can be seen from Eq. (15), the pore-water pressure is influenced by Poisson's ratio ν' since eigenvalues $\alpha_i(i=1, 2,\cdots)$ are functions of ν'. In Fig. 6, the influence of ν' on the pressure is shown. It is shown that the Mandel-Cryer effect appears most remarkably when $\nu'=0$ and that it is enfeebled as ν' approaches 0.5. This result is identical with that analysed by Cryer (1963) for the spherical consolidation.

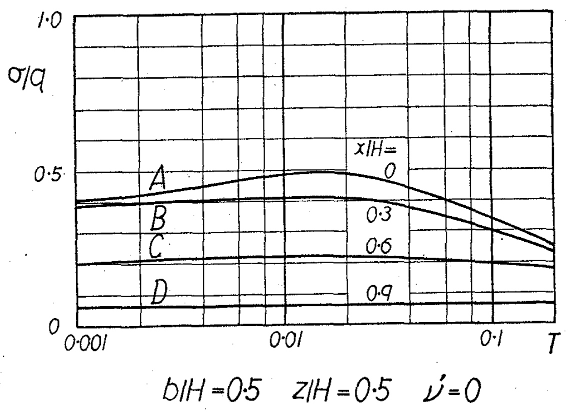

Fig. 7. The Mandel-Cryer effect along horizontal axis

Fig. 7 shows the appearance of the Mandel-Cryer effect along the horizontal axis. It appears markedly at point A on the center-line of a load, and it grows weak at such points as B and C. Comparing with the analytical result by Mandel (1953) (see Fig. 4), there can be seen the existence of a similarity between them. In addition, the result may support the propriety of the above consideration that the Mandel-Cryer effect results from the relative and quantitative relationship between the change in the pore-water pressure and in the sum of total stresses.

Aboshi (1955) has carried out a test of

measuring the pore-water pressure within a clay mass under a three-dimensional (axial symmetrical) condition. The Kanaura-wan clay was placed into a vessel which had dimensions of 100cm×90cm×90cm. When it was subjected to a circular load of 50cm in diameter under the drainage condition that only the upper face of the mass was permeable, the pore-water pressure within the mass was measured at thirty different points by using piezometers. In particular, the time history of the measured pressure along the center-line of the load is quoted in Fig. 8. It may be seen that the two characteristics of pore-water pressure predicted from the theoretical analysis appear in the experimental result. That is, one is the appearance of the Mandel-Cryer effect and the other is its character that the time required till the pressure reaches a peak value is longer as the distance between the measurement point and the permeable face is greater. At this stage, the theoretical and the experimental results cannot be quantitatively compared because the boundary conditions are different. It can be, however, concluded at least that the movements of the pore-water pressure in the process of the multi-dimensional consolidation are more fully explained by Biot's theory than by Rendulic's theory.

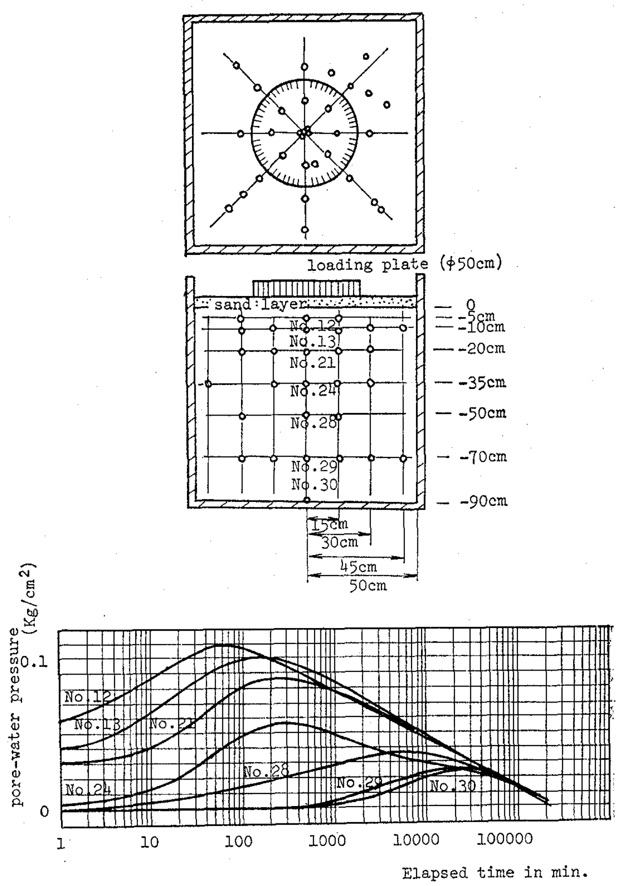

Fig. 8. Experimental result by Aboshi (1955)

MEASUREMENT OF PORE-WATER PRESSURE

A series of tests of measuring the excess pore-water pressure was conducted to evidence the theoretical result by using two kinds of clays. The aims of this experiment are to examine the adaptability of the theoretical prediction of the pore-water pressure derived from Biot's theory, to investigate the dilatancy characteristics of a clay and to establish the estimation method for the pore-water pressure prediction on the basis of the above investigation.

Table 1. Soil properties

	Sakuragicho clay	Ohgishima clay
G_S	2.74	2.72
ϕ'	31.0 (degrees)	33.2 (degrees)
LL	69.9 (%)	—
PL	30.9 (%)	—
c_v	1.5×10^{-2} (cm²/min)	7.2×10^{-2} (cm²/min)
ν'	0.4 ($p_c\leq 2$ kg/cm²)	0.4 ($p_c\leq 2$ kg/cm²)
w	72.4% ($p_c=0.1$ kg/cm²)	74.0% ($p_c=0.1$ kg/cm²)

The clays used in the test were the Sakuragicho alluvial clay and the Ohgishima sandy clay, and their soil properties are shown in Table 1. Test apparatus were composed of a vessel, a load plate, a pore-water pressure device with semi-conductor, a X-Y recorder and a DC-amplifier.

The internal dimensions of the vessel were 28cm×56cm×56cm. The clay layer of 15cm in thickness was evenly set inside the vessel. As for the load plate, its form was a strip of 15cm in width which was made of vinyl chloride and lead bullets so as to hold flexibility. Then, to keep the drainage condition that only the surface of clay layer was permeable, a thin sand layer was placed upon the clay layer. Under such a condition, the pore-water pressure device was set up at the measurement point in advance. The calibration curve of this device had been determined and provided previously. The range of the load intensity was settled by taking into account the accuracy of the load intensity, the device and consolidation pressure in triaxial tests, viz. 20(g/cm^2) to 100(g/cm^2). The preconsolidation pressure or surcharge had been settled equal to half of the strip-load intensity for each test. The points of measuring the pore-water pressure were fixed along the center-line of the load. Because, the domain that the characteristics of the pore-water pressure appears most remarkably is along this line, as has been seen from the theoretical result.

As a result of this test, the phenomenon that the pore-water pressure increases temporarily at the initial stage of consolidation, which has been theoretically predicted under the present boundary conditions, was confirmed on both clays. In addition, by using the value of Poisson's ratio ν' and the coefficient of consolidation c_v which have been determined by preliminary tests, the predicted and the measured results are compared. Concerning the Mandel-Cryer effect, it can be seen that the time at which the peak of pore-water pressure appears agrees within the error of 80% and that the quantity is fairly agreeable within the error of 20%. So, it should be emphasized that Biot's theory is very excellent for predicting the Mandel-Cryer effect. As for the quantity of pore-water pressure measured in the tests, it is found from Fig. 9 that the observed value is greater than the predicted one, and that the difference is about 20% at the measurement point O. This reason may be ascribed to the dilatancy effect of a clay. So, here should be developed the estimation of the pore-water pressure due to the dilatancy.

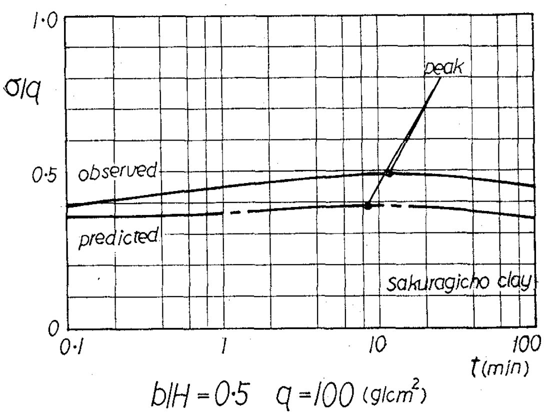

Fig. 9. Observed excess pore-water pressure at point O

First of all, it is necessary to obtain experimentally the dilatancy characteristics of the clay used in the tests, on the assumption that the characteristics follow a unique rule. For this purpose, a series of undrained shear tests was conducted by making use of a triaxial apparatus. Although it has been concluded from the work by Karube et al. (1966) that the dilatancy characteristics of a clay are not influenced by consolidation pressure, it was settled within the range of 0.5(kg/cm^2) to 3.0 (kg/cm^2) in these tests for the sake of checking this conclusion.

In Fig. 10, the dilatancy characteristics of the clays are shown. Herein, the bulk modulus of a clay K_s was measured by all-round consolidation tests and the results are given in Fig. 11. It can be noticed from the experimental result that the dilatancy characteristics are not affected by consolidation pressure. Hence, if the ratio of octahedral shear stress to octahedral normal stress τ_{oct}/σ'_{oct} at the points of measuring the pore-water pressure is estimated from the theory of elasticity or Biot's theory, the dilatancy d can be analysed and the pore-water pressure due to the dilatancy would be eventually obtained from d and K_s, by means of using the following equation:

$$\sigma_d = -\frac{d}{K_s}$$

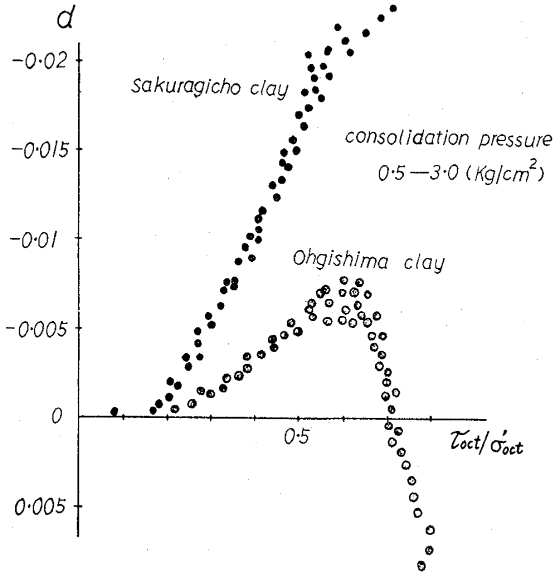

Fig. 10. Dilatancy characteristics

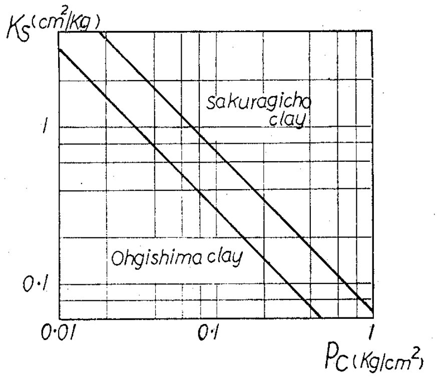

Fig. 11. Relationship between p_c and K_s

Strictly speaking, as the pressure is a function of time, it would change. The change may be, however, negligible within an hour in this case after consolidation has set out, though it is naturally due to the value of the coefficient of consolidation and the drainage length.

Consequently, the predicted value of the pore-water pressure taking account of the dilatancy correction, i.e., the sum of the pore-water pressure derived from Biot's theory and that due to the dilatancy would be compared with the experimental result and the comparison is shown in Fig. 12.

It is seen from the figure that the agreement is fairly satisfactory within the error of 5%. Since Biot's theory is formulated on the assumption that a clay is elastic, the effect of shear deformation on volumetric strain is not taken into account. However, it can be concluded here that the behavior of the pore-water pressure can be fairly predicted by it if one makes up for this effect in the theory. This conclusion may be applicable in the prediction of the final consolidation settlement also.

The pore-water pressure was, moreover, measured at such some points along the center-line of the load as shown in Fig. 3, for the sake of evidencing the theoretical result. The

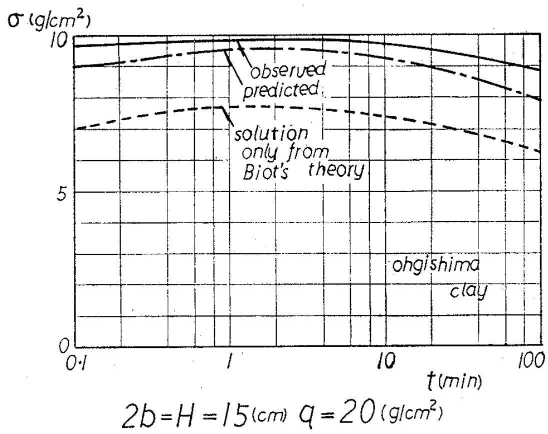

Fig. 12. Comparison of experimental and theoretical results by taking account of dilatancy effect (at point O)

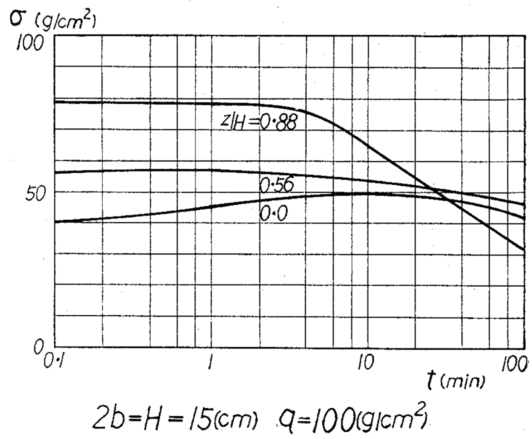

Fig. 13. Measured pore-water pressure (along $x=0$)

measured pressure is illustrated in Fig. 13. It can be seen that the actual characteristics of pore-water pressure is much alike to that predicted by the theory and that the pore-water pressure measured at the points except point O agrees quantitatively with the solution obtained from Biot's theory alone. It is suggested that, at these points, the dilatancy effect is relatively small compared with that at point O. This fact can be also proved by the analytical result that the total stresses are nearly equal and the factor τ_{oct}/σ'_{oct} is small at these points.

CONCLUDING REMARKS

In the present study, a multi-dimensional consolidation problem of a clay layer with finite thickness has been discussed on the basis of Biot's theory. As a result, the following conclusions may be made:

(1) Concerning the time-degree of settlement characteristics it can happen that the consolidation is later than the one-dimensional one for a smaller value of Poisson's ratio ν' and a larger value of b/H. This phenomenon is relevant to the Mandel-Cryer effect. However, the above tendency becomes weak when Poisson's ratio ν' is nearly 0.5.

(2) The time-degree of settlement relationships derived from the present and the preceding studies is identical when Poisson's ratio is approximately 0.5.

(3) The Mandel-Cryer effect which has been known in the problems of a spherical consolidation etc. can be acknowledged in the present problem also. The behavior that the time at which the peak of pore-water pressure appears tends to be earlier as the distance between the measurement point and a permeable face is shorter can be evidenced by the experiment. Hence, the superiority of Biot's theory should be appreciated in comparison with Rendulic's theory by which this phenomenon cannot be explained entirely.

(4) The quantitative estimation of the pore-water pressure can be carried out by Biot's theory, with taking account of the effect of dilatancy of a clay. An expression of the dilatancy characteristics required in the estimation has been experimentally formulated to some degree, though there is still a room for investigating it further.

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NOTATION

a = radius of a circular load
b = half width of a strip load
c_v = coefficient of consolidation
d = volumetric strain due to dilatancy
G = shear modulus
G_s = specific gravity of soil particles
H = thickness of a clay layer
k = coefficient of permeability
K_s = bulk modulus of soil skeleton
LL = liquid limit
p_c = consolidation pressure
PL = plastic limit
q = load intensity
t = time

T = time factor
u_i = displacement of soil in the direction i
ϕ' = angle of effective internal friction
ε_v = volumetric strain
ν' = Poisson's ratio of soil skeleton
σ = excess pore-water pressure
σ'_{oct}, τ_{oct} = octahedral stresses
σ_z = vertical stress
τ_{xz} = shear stress
γ_w = weight of water in unit volume

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