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OBSERVATIONAL PROCEDURE OF SETTLEMENT PREDICTION

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ABSTRACT

The use of analytical solution of consolidation equation for settlement prediction is not always effective since such conditions as an initial distribution of excess pore water pressure, drain length, final vertical strain of soils and the coefficient of consolidation are sometimes quite uncertain in practical engineering problems. For this reason, an observational settlement prediction is freshly presented. First, the linear ordinary differential equation derived is demonstrated to give a settlement-time relationship. After that, by using the difference form of the equation (an autoregressive equation), observational procedure of settlement prediction is proposed. Two kinds of practical methods are presented. One is a graphical method, the advantage of which is its simplicity. The other is the method based on the Bayesian inference of the non stationary stochastic process, which can give a predictive probability distribution of future settlement and then also provide a preliminary theory for reliability-based design of settlement problems.

The proposed methodology is demonstrated to be also applicable for some other special problems including settlement due to drainage from sand piles.

Key words: cohesive soil, computer application, consolidation, graphical analysis, measurement, settlement, statistical analysis

IGC: E2/D5

INTRODUCTION

The one-dimensional consolidation theory is an important contribution of soil mechanics to practical foundation engineering. Settlement resulting from one-dimensional compression and one-dimensional drainage has been widely recognized to be well explained by this theory. However, for the theory to be also effective in the case of future settlement prediction, some other engineering judgement has been additionally indispensable.

It is known that a consolidation equation, a partial differential equation of parabolic type, produces a unique solution when a coefficient of the equation and the initial and boundary conditions have been prior determined. This is common knowledge so that the practical applications of this equation have exhibited a tendency to utilize only this characteristic of the solution. In fact, according to conventional settlement analysis, conditions such as an initial distribution of excess pore water pressure, drain length, final vertical strain of soils and the coefficient of consolidation were naturally considered to be given in advance of the analysis. It is, however, also commonly accepted that the estimation of these conditions usually goes with a high degree of uncertainty. Therefore, the

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engineer working on settlement prediction is usually expected to have had some previous experience.

In this paper, a new idea of settlement prediction is presented, the philosophy of which is based on "Observational Procedure." The trend equation of time series data of settlement is derived first from the one-dimensional consolidation equation, after which the future settlement is predicted by using the past observations. Two kinds of practical methods are proposed. One is a graphical method, the advantage of which is its simplicity. The other is the method based on the Bayesian inference of a non stationary stochastic process, which can give a predictive probability distribution of future settlement and then also provide a preliminary theory for reliability-based design of settlement problems.

The proposed methodology is demonstrated to be also applicable for some special problems including settlement due to drainage from sand piles.

MASTER EQUATION OF SETTLEMENT-TIME RELATION

In this section, the ordinary differential equation derived gives a settlement-time relationship.

As a fundamental equation, Mikasa's equation is adopted, that is

$$\dot{\varepsilon}=c_v\varepsilon_{zz} \tag{1}$$

in which

$\varepsilon(t, z)$: vertical strain (volumetric strain),

$t(\geqq 0)$: time,

z : depth from the top of clay stratum and

c_v : coefficient of consolidation.

In Eq. (1) the upper dot, $\cdot$ represents time-differentiation and the lower script, z the differentiation with respect to depth, z. Even if the coefficients of both permiability and volume compressibility vary from time to time, the Eq. (1) is still effective when c_v remains constant (Mikasa, 1963). In addition, Eq. (1) makes it easy to express settlement. For these reasons Eq. (1) is adopted instead of Terzaghi's equation.

The solution of Eq. (1) is expressed here by introducing two unknown functions of time, T and F, as follows:

$$\begin{aligned}\varepsilon(t, z)=T&+\frac{1}{2!}\left(\frac{z^2}{c_v}\dot{T}\right)+\frac{1}{4!}\left(\frac{z^4}{c_v^2}\ddot{T}\right)+\cdots\\&+zF+\frac{1}{3!}\left(\frac{z^3}{c_v}\dot{F}\right)+\frac{1}{5!}\left(\frac{z^5}{c_v^2}\ddot{F}\right)+\cdots\end{aligned} \tag{2}$$

Eq. (2) shows that

$$T=\varepsilon(t, z=0)$$

and

$$F=\varepsilon_z(t, z=0)$$

The vertical strain ε in Eq. (1) is originally defined in an Eulerian sense, but, in this paper, ε is approximately regarded as a Lagrangean strain for simplicity. The formula of settlement prediction by regrading ε as an Eulerian strain has already been established by the author. The results, however, become a complex nonlinear problem and therefore they are beyond the scope of practical applicability.

The typical two boundary conditions are considered next.

(1) Drainage from both top and bottom boundaries.

Fig. 1 shows this boundary condition, which is formulated as

$$\varepsilon(t, z=0)=\bar{\varepsilon} : \text{constant} \tag{3}$$

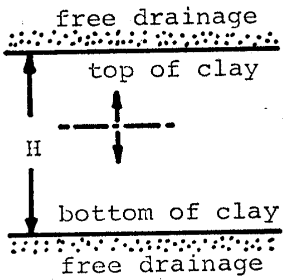

Fig. 1. Drainage from top and bottom boundaries

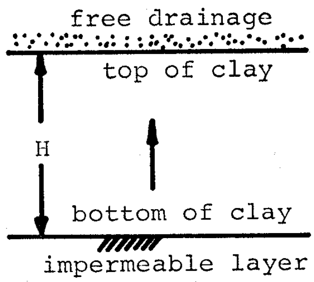

Fig. 2. Upward drainage

$$\varepsilon(t,\ z=H)=\underline{\varepsilon}\ :\ \text{constant} \tag{4}$$

where H represents the thickness of clay stratum and both $\bar{\varepsilon}$ and $\underline{\varepsilon}$, the unknown boundary values respectively.

Substitution of Eq. (3) into Eq. (2) gives

$$T=\bar{\varepsilon}\ :\ \text{constant} \tag{5}$$

which shows that upper half of the solution (2) becomes constant. Then, from Eq. (4), we have the linear ordinary differential equation of unknown F with constant coefficients,

$$F+\frac{1}{3!}\left(\frac{H^2}{c_v}\dot{F}\right)+\frac{1}{5!}\left(\frac{H^4}{c_v^2}\ddot{F}\right)+\cdots=\frac{\underline{\varepsilon}-\bar{\varepsilon}}{H} \tag{6}$$

(2) Upward drainage (Fig. 2)

When the bottom of the clay layer is impermiable, boundary conditions are given as follows:

$$\varepsilon(t,\ z=0)=\bar{\varepsilon}\ :\ \text{constant} \tag{7}$$

$$\varepsilon_z(t,\ z=H)=0 \tag{8}$$

Since Eq. (7) is the same as Eq. (3), it also follows that

$$T=\bar{\varepsilon}\ :\ \text{constant} \tag{9}$$

Then, calculating Eq. (8) from the solution (2), we get

$$F+\frac{1}{2!}\left(\frac{H^2}{c_v}\dot{F}\right)+\frac{1}{4!}\left(\frac{H^4}{c_v^2}\ddot{F}\right)+\cdots=0 \tag{10}$$

which is also a linear ordinary differential equation of F with constant coefficient.

Now, for any boundary condition, the settlement of clay stratum can be expressed as

$$\rho(t)=\int_0^H \varepsilon(t,\ z)\,dz \tag{11}$$

in which $\rho(t)$ represents the settlement at time, t. Substituting the solution (2) into Eq. (11), since $T=\bar{\varepsilon}$: constant, we have

$$\rho(t)=\bar{\varepsilon}H+\frac{1}{2!}(H^2F)+\frac{1}{4!}\left(\frac{H^4}{c_v}\dot{F}\right)+\frac{1}{6!}\left(\frac{H^6}{c_v^2}\ddot{F}\right)+\cdots \tag{12}$$

By the sequential differentiation of Eq. (12) with respect to t, we get the following set of equations:

$$\left.\begin{aligned}
\dot{\rho}&=\frac{1}{2!}(H^2\dot{F})+\frac{1}{4!}\left(\frac{H^4}{c_v}\ddot{F}\right)+\frac{1}{6!}\left(\frac{H^6}{c_v^2}\dddot{F}\right)+\cdots\\
&\ \ \vdots \qquad\qquad \vdots \qquad\qquad \vdots\\
\overset{(n)}{\rho}&=\frac{1}{2!}(H^2\overset{(n)}{F})+\frac{1}{4!}\left(\frac{H^4}{c_v}\overset{(n+1)}{F}\right)+\frac{1}{6!}\left(\frac{H^6}{c_v^2}\overset{(n+2)}{F}\right)+\cdots\\
&\ \ \vdots \qquad\qquad \vdots \qquad\qquad \vdots
\end{aligned}\right\} \tag{13}$$

Comparing the set of equations, Eq. (12) and Eqs. (13), with Eq. (6) or Eq. (10), the unknown function F can be eliminated. When Eq. (6) is the case, it follows that

$$\rho+\frac{1}{3!}\left(\frac{H^2}{c_v}\dot{\rho}\right)+\frac{1}{5!}\left(\frac{H^4}{c_v^2}\ddot{\rho}\right)+\cdots=\frac{H}{2}(\bar{\varepsilon}+\underline{\varepsilon}) \tag{14 a}$$

Similarly, from Eq. (10), we get

$$\rho+\frac{1}{2!}\left(\frac{H^2}{c_v}\dot{\rho}\right)+\frac{1}{4!}\left(\frac{H^4}{c_v^2}\ddot{\rho}\right)+\cdots=H\bar{\varepsilon} \tag{14 b}$$

Eqs. (14) give a settlement-time relationship under the condition in which an external consolidation load is constant, that is, Eqs. (14) are the results derived from the stationary boundary conditions, Eq. (3) and Eq. (7).

It is easily ascertained that Eqs. (14) are absolutely stable for $t\rightarrow\infty$, that is, all the eigenvalues of these equations are negative real numbers which are different from each other. In succeeding sections, however, we discuss the practical problems in which c_v, H and boundary conditions of drainage and load are entirely uncertain. Therefore, the expressions of the solutions of Eqs. (14) by the use of prominent eigenvalues and boundary values, this is common in conventional analysis, can not be efficient for settlement prediction.

It should be noted that the higher order differential terms of Eqs. (14) are negligibly small. Then, the following n-th order approximation equation is adopted as a master equation of settlement-time relationship:

$$\rho+c_1\dot{\rho}+c_2\ddot{\rho}+\cdots+c_n\overset{(n)}{\rho}=C \tag{15}$$

where c_1, c_2, $\cdots$, c_n and C are regarded as unknown constants. It will be shown later that Eq. (15) is also applicable for some other settlement prediction problems. Introducing discrete time as

$$\left.\begin{array}{l} t_j=\Delta t\cdot j, \quad j=0,\ 1,\ 2,\ \cdots \\ \Delta t:\text{constant}, \end{array}\right\} \tag{16}$$

Eq. (15) can be reduced to a difference form,

$$\rho_j=\beta_0+\sum_{s=1}^{n}\beta_s\rho_{j-s} \tag{17}$$

in which ρ_j denotes $\rho(t_j)$, the settlement at the time $t=t_j$ and the coefficients β_0 and β_s ($s=1$, 2, $\cdots$, n) are unkown plant parameters. Eq. (17) will give us an idea of observational settlement prediction.

For the convenience sake of succeeding discussions, the first order approximation equation,

$$\rho+c_1\dot{\rho}=C \tag{18}$$

is examined here. Let initial condition be

$$\rho(t=0)=\rho_0 \tag{19}$$

in which the time, $t=0$, should be taken at the time after loading works since Eqs. (14) have been derived from the stationary boundary conditions which do not vary from time to time. When this is the case, Eq. (18) can be easily solved as

$$\rho(t)=\rho_f-(\rho_f-\rho_0)\exp\left(-\frac{t}{c_1}\right) \tag{20}$$

where

$$\rho_f=C,$$

the final settlement, sometimes referred to as the stable state of ρ. On the other hand, the first order difference equation is expressed as

$$\rho_j=\beta_0+\beta_1\rho_{j-1} \tag{21}$$

The coefficients, β_0 and β_1 have the following values. Substituting the stable state,

$$\rho_j=\rho_{j-1}=\rho_f, \tag{22}$$

into Eq. (21), we obtain

$$\rho_f=\frac{\beta_0}{1-\beta_1}. \tag{23}$$

Furthermore, by the recursive operation with respect to j, Eq. (21) yields

$$\rho_j=\frac{\beta_0}{1-\beta_1}-\left\{\frac{\beta_0}{1-\beta_1}-\rho_0\right\}(\beta_1)^j \tag{24}$$

which can be compared with Eq. (20). From this, it follows that

$$l_n\beta_1=-\frac{\Delta t}{c_1}\begin{cases}=-\dfrac{6c_v}{H^2}\Delta t : \text{the case of both top and bottom drainage} & (25)\\ =-\dfrac{2c_v}{H^2}\Delta t : \text{the case of upward drainage} & (26)\end{cases}$$

which suggests that β_1 does not depend on the boundary values, $\bar{\varepsilon}$ and $\underline{\varepsilon}$; β_1 is independent of load.

GRAPHICAL SETTLEMENT PREDICTION

We first examine the effectiveness of the first order approximation equation. Assume that we have had $n+1$ settlement observations, $(\rho_0, \rho_1, \cdots, \rho_n)$ generated by a constant external load. Using these observations we can plot n points, (ρ_k, ρ_{k-1}) for $k=1, 2, \cdots, n$, on the (ρ_j, ρ_{j-1}) co-ordinate. From this, we can visually see whether all these points lie on a straight line as Eq. (21) suggests or not. Figs. 3 and Fig. 4 show the actual results obtained from the settlement observations of extensive reclaimed lands in Japan. The data for Figs. 3 are from the reclaimed land at Kobe Port. Fig. 4 is the rearranged result from the data published by Aboshi (1969). In every case of these figures, settlement was considered to be resulting from one-dimensional consolidation. Through these figures,

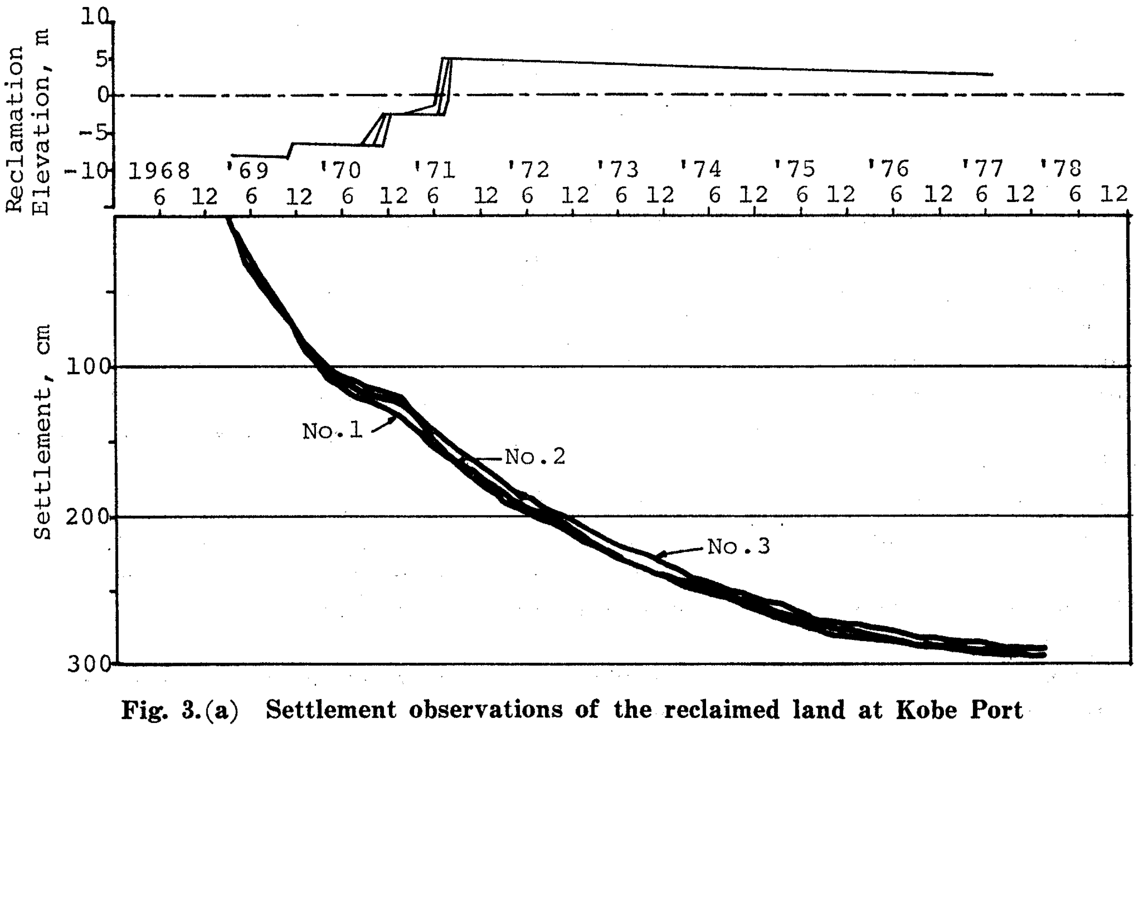

Fig. 3.(a) Settlement observations of the reclaimed land at Kobe Port

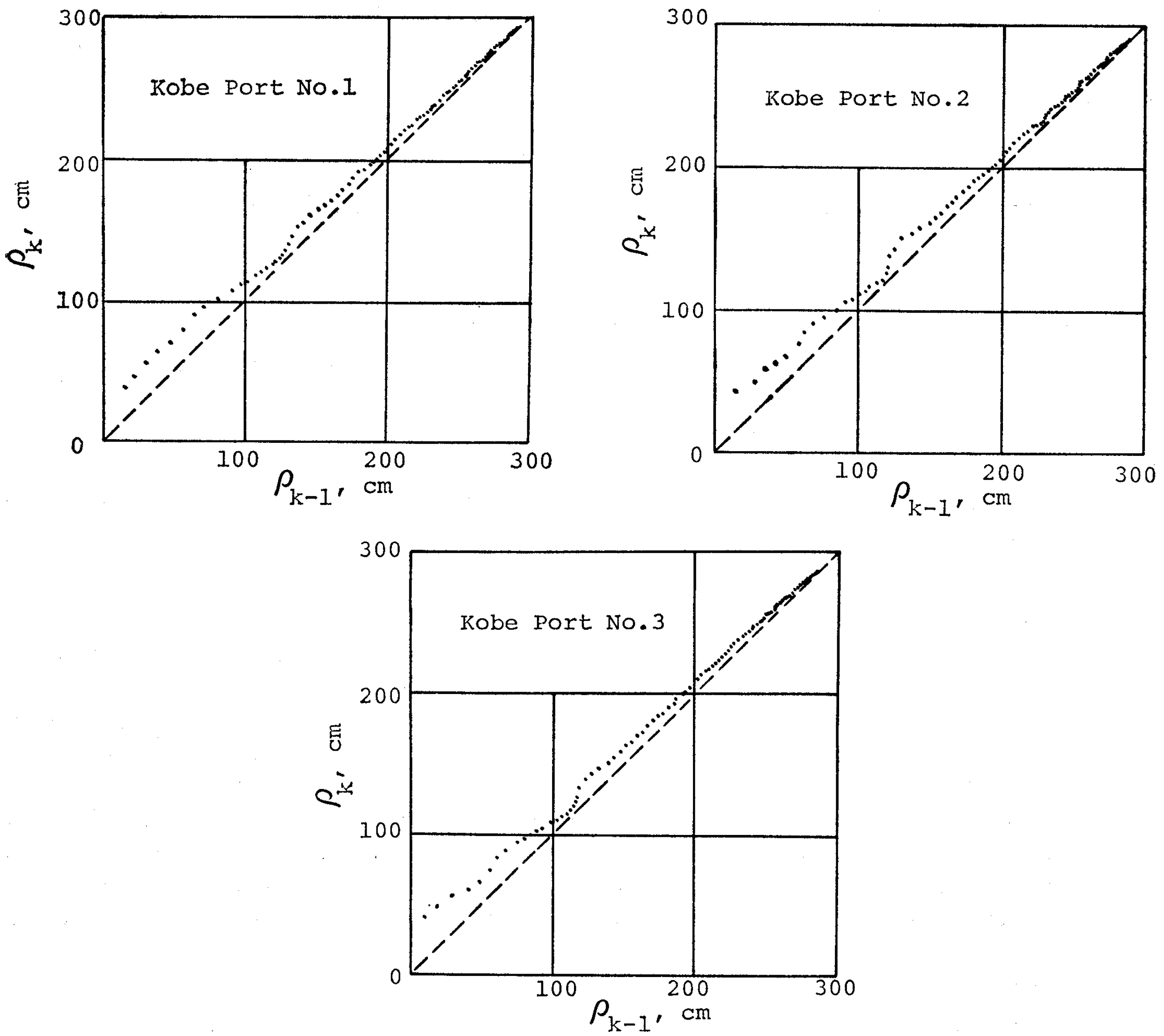

Fig. 3.(b) $\rho_j-\rho_{j-1}$ **relationship**

a time interval, Δt was taken to be 3 months (91-92 days).

As shown in the figures, the first order difference equation, Eq. (21), gives a good fitting to the observations, which enables us to predict future settlement by graphical method. This procedure is illustrated in Fig. 5. Since the rough estimates of β_0 and β_1 are given by the intercept and the slope of a fitted straight line, it is possible to predict settlement of any future time, j, by using Eq. (24). Moreover, the intercepting point of this line with the 45°-line represents the final settlement, because ρ_f is given by $\rho_j=\rho_{j-1}$. If necessary, the unknown coefficient of time factor, c_v'/H^2, can be easily calculated backward by Eq. (25) or (26).

In the case of multi-staged loading, the straight line, $\rho_j=\beta_0+\beta_1\rho_{j-1}$, will be moved up as is also shown in Fig. 5. When the settlement is relatively small compared to the thickness of clay layer, the shifted line becomes almost parallel to the initial line because β_1 is determined not from the external load but from the thickness, H and the coefficient of consolidation, c_v.

This graphical method is quite useful in situations in which simplicity is more valuable than strictness, but followings are noteworthy.

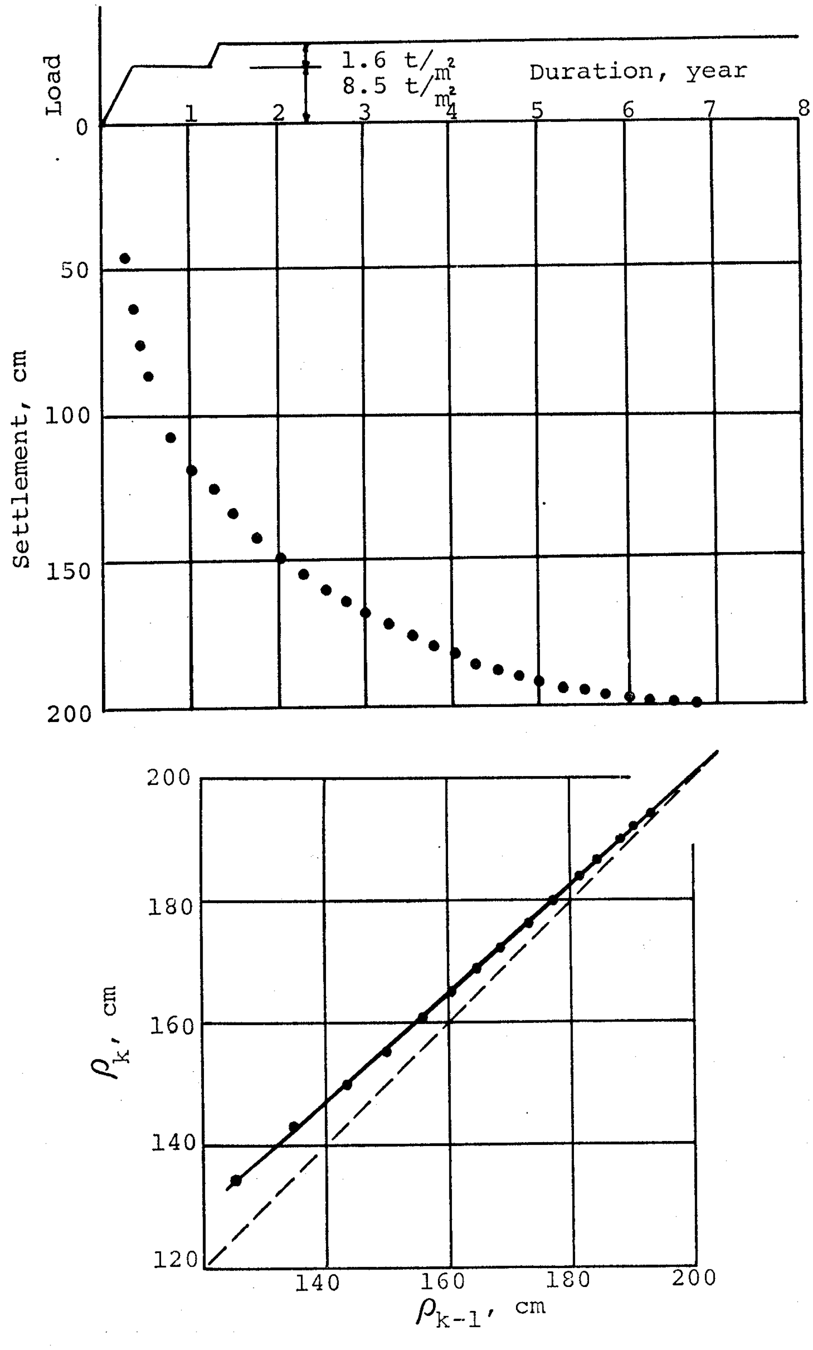

Fig. 4. Settlement observations at Shin-Ube thermoelectric power station (after Aboshi (1969))

(1) To obtain β_0, we are generally compelled to extrapolate the fitted line to the ρ_{k-1} $=0$ axis. So, if the banking period is so long and then ρ_0 becomes so large, the extrapolation should be performed very carefully.

(2) The accuracy of the graphical method depends mainly on a time interval, Δt. The longer Δt becomes, the higher the accuracy is. For this purpose, the rearrangement of observations is sometimes recommended. If, as an example, we observe settlement every week and Δt is taken to be 10 weeks, then, letting $\rho[k]$ denote the settlement of the k-th week, we have the following 10 time-series of observations, that is;

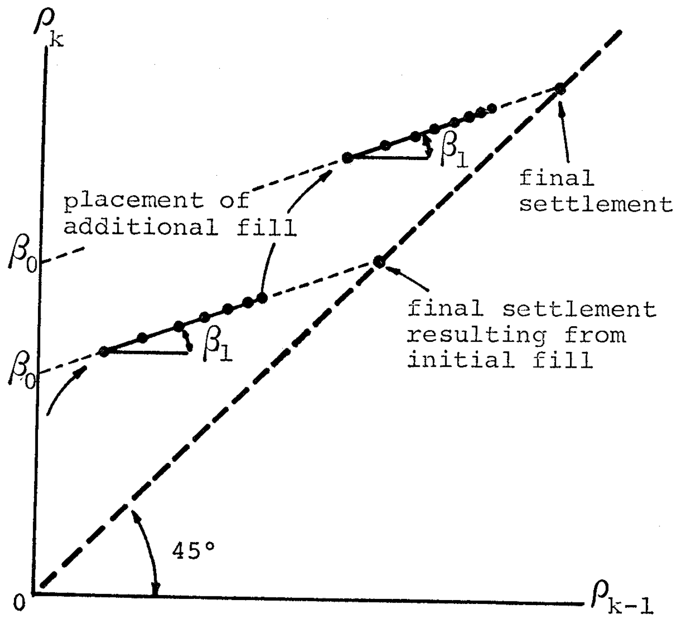

Fig. 5. Graphical method of settlement prediction

$$\begin{array}{llll} (1) & \rho[0], & \rho[10], & \rho[20], \cdots \\ (2) & \rho[1], & \rho[11], & \rho[21], \cdots \\ \vdots & \vdots & \vdots & \vdots \\ (10) & \rho[9], & \rho[19], & \rho[29], \cdots \end{array}$$

Since β_0 and β_1 are independent of an absolute time, all the time-series data will be plotted on the same straight line.

RELIABILITY ANALYSIS

The solution of Eq. (18), the first order approximation equation, is predicted here by using Bayesian techniques under the conditions in which c_1 and C are unknown at the time $t=0$. One definite advantage that the probabilistic approach has over the graphical method is that the predicted value is given with its reliability.

As is implied by the previous discussions, the first order antoregressive model,

$$\rho_j=\beta_0+\beta_1\rho_{j-1}+\sigma\eta_j \tag{27}$$

is chosen here as being a suitable one for settlement prediction, in which η_j is the random variable with zero mean and unity deviation. In order to make successive calculations simple, the followings must be assumed: one is that η_j is a Gaussian random variable and the other is

$$E[\eta_j\eta_k]=\begin{cases} 1: j=k \\ 0: j\neq k \end{cases} \tag{28}$$

where $E[\ \ ]$ denotes an expectation operation. It may be unnecessary to state that Eq. (28) does not suggest the statistical independency between ρ_j and ρ_k. Three unknown parameters, β_0, β_1 and σ are sometimes written as θ for simplicity, that is,

$$\theta=(\beta_0,\ \beta_1,\ \sigma)' : \text{unknown parameter vector.}$$

Now suppose that the set of observations,

$$\rho^t=(\rho_0,\ \rho_1,\ \cdots,\ \rho_t) \tag{29}$$

has been already obtained. We, then, have the posterior Probability Distribution Function (pdf) of unknown θ as follows:

$$\xi(\theta|\rho^t)\propto\xi(\theta)\prod_{j=1}^{t}p(\rho_j|\theta,\ \rho_{j-1}) \tag{30}$$

where

$\xi(\theta)$: a-priori pdf of θ

$p(\rho_j|\theta,\ \rho_{j-1})$: Gaussian pdf with mean, $(\beta_0+\beta_1\rho_{j-1})$, and variance σ^2.

When we have no information at the time, $t=0$, $\xi(\theta)$ can be given by a diffuse prior pdf,

$$\xi(\theta)\propto\frac{1}{\sigma} \tag{31}$$

If we have some information at that time, $\xi(\theta)$ can be expressed by the more informative one than Eq. (31). When we take $\xi(\theta)$ as a Gauss-gamma joint pdf, the posterior pdf, $\xi(\theta|\rho^t)$, also becomes a Gauss-gamma joint pdf.

Next derived is the pdf of ρ_j conditioned by θ and ρ_0. This pdf is calculated from

$$p(\rho_j|\theta,\ \rho_0)=\underbrace{\int\cdots\int}_{(j-1)}\prod_{k=1}^{j}p(\rho_k|\theta,\ \rho_{k-1})\cdot d\rho_1 d\rho_2\cdots d\rho_{j-1} \tag{32}$$

By using the mathematical inductive method, the $p(\rho_j|\theta,\ \rho_0)$ is demonstrated to be a Gaussian pdf, the mean, M_j, and the variance, Σ_j^2, of which are given by

$$\left.\begin{aligned} M_j&=\frac{\beta_0}{1-\beta_1}-\left(\frac{\beta_0}{1-\beta_1}-\rho_0\right)(\beta_1)^j \\ \text{and}\qquad \Sigma_j&=\sigma\sqrt{\frac{1-\beta_1^{2j}}{1-\beta_1^2}} \end{aligned}\right\} \tag{33}$$

respectively. Although both M_j and Σ_j are monotone increasing functions of j, since $l_n\beta_1<0$, they are also bounded from above:

$$\left.\begin{aligned} \lim_{j\to\infty}M_j&=\frac{\beta_0}{1-\beta_1} \\ \lim_{j\to\infty}\Sigma_j&=\sigma/\sqrt{1-\beta_1^2} \end{aligned}\right\} \tag{34}$$

Now, what we should examine here is the meaning of σ. If we take a time series observation ρ^t at only one spot on a clay stratum, Σ_j represents the random fluctuation of ρ_j from the trend M_j. However, we can generally observe settlement at various spots located within a certain area. Let ${}^l\rho^t$ be a time series observation of settlement at spot "l", and let

$$\boldsymbol{\rho}^t=({}^1\rho^t,\ {}^2\rho^t,\ \cdots,\ {}^m\rho^t) \tag{35}$$

in which m is the number of observation spots. In this case, if a settlement process can be assumed to be spatially stationary, the σ estimated by the posterior pdf,

$$\xi(\theta|\boldsymbol{\rho}^t)\propto\xi(\theta)\prod_{l=1}^{m}\prod_{j=1}^{t}p({}^l\rho_j|\theta,\ {}^l\rho_{j-1}) \tag{36}$$

can be considered to include the property of unequal settlement characterized by the spatial interval between observation spots, $l=1,\ 2,\ \cdots,\ m$. When this is the case, introducing the concept of an initial distribution of ρ_0, $p(\rho_0)$, we obtain

$$p(\rho_j|\theta)=\underbrace{\int\cdots\int}_{(j)}p(\rho_0)\prod_{k=1}^{j}p(\rho_k|\theta,\ \rho_{k-1})\cdot d\rho_0 d\rho_1\cdots d\rho_{j-1} \tag{37}$$

which is different from Eq. (32). An initial distribution of ρ, $p(\rho_0)$ will represent the distribution of unequal settlement just after loading. Provided that $p(\rho_0)$ can be expressed as a Gaussian pdf, $p(\rho_j|\theta)$ becomes also a Gaussian pdf just as $p(\rho_j|\theta,\ \rho_0)$.

Now back to our prediction problem. We have already been able to derive the predictive pdf of ρ_j conditioned only by the set of observations, ρ^i, as follows:

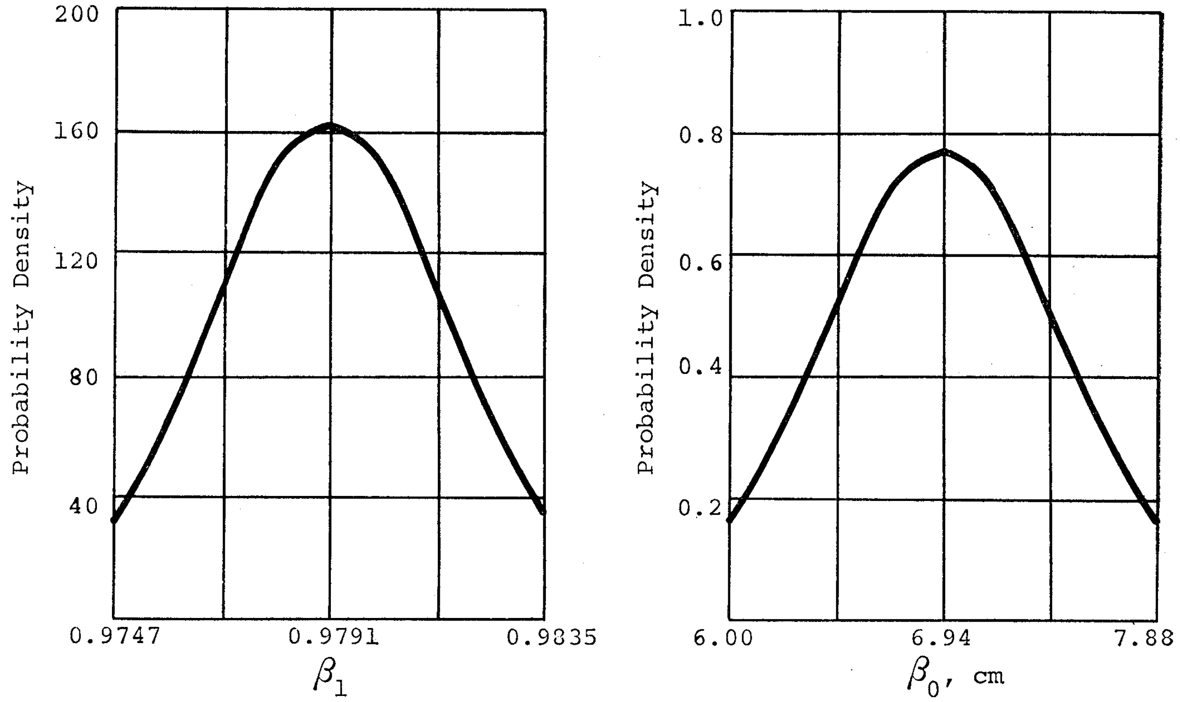

Fig. 6. Posterior marginal pdf's of β_0 and β_1 conditioned by the data observed untill December, 1972 at Kobe Port, No. 3

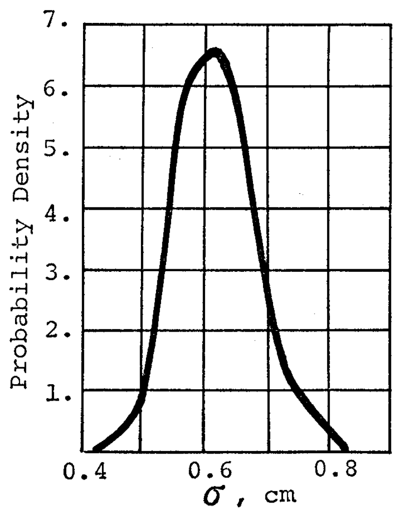

Fig. 7. Posterior marginal of σ by the same data of Fig. 6

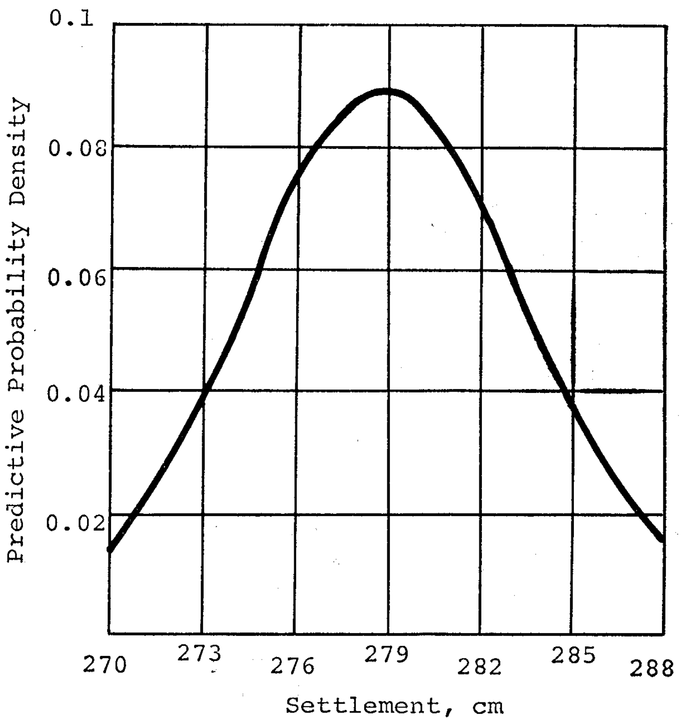

Fig. 8. Predictive pdf of the settlement of December, 1976 from the data of December, 1972

$$p(\rho_j|\rho^t)=\int p(\rho_j|\theta,\ \rho_0)\xi(\theta|\rho^t)d\theta, \tag{38}$$

where

$$d\theta=d\beta_0 d\beta_1 d\sigma.$$

Reliability of settlement prediction will be defined from Eq. (38). For example, the probability of taking ρ_j between ρ_a and ρ_b at time j can be given by

$$\text{Prob.}\ (\rho_a<\rho_j<\rho_b|\rho^t)=\int_{\rho_a}^{\rho_b} p(\rho_j|\rho^t)d\rho_j \tag{39}$$

The calculation of Eq. (38) or Eq. (39) can not be performed without an electronic computer because of the multiple inegrations in these equations.

Figs. 6, 7 and 8 are the numerical results of the applications of Eq. (30) and Eq. (38) to the observations of Kobe Port No. 3, which are illustrated in Figs. 3. To obtain the results, both a diffuse prior pdf and the data observed until December, 1972 were employed. The prediction was conducted towards the settlement of December, 1976, the realized value of which was 281 cm. Fig. 8 is the predictive pdf of the settlement. Figs. 6 and 7 are the marginal pdf's of β_0, β_1 and σ of Eq. (30), respectively. For these calculations Δt was taken to be 30~31 days.

SOME SPECIAL PROBLEMS

In this section we derive a general prediction formula of future settlement through investigations of three special problems.

Secondary Compression due to Creep

If in the simplest case, the secondary compression of clay stratum can be approximately described by a single Voigt model subject to a constant external load. Then in this case the secondary settlement, ρ_{II}, is considered to satisfy

$$\rho_{\text{II}}+c\dot{\rho}_{\text{II}}=P, \tag{40}$$

the first order linear differential equation with constant coefficients. Since the solution of Eq. (40) is given as

$$\rho_{\text{II}}=\rho_{f\text{II}}-(\rho_{f\text{II}}-\rho_{0\text{II}})\exp\left(-\frac{t}{c}\right) \tag{41}$$

in which $\rho_{0\text{II}}$ is the initial condition of Eq. (40) and $\rho_{f\text{II}}=P$, then, the total settlement can be expressed as follows:

$$\rho=(\rho_{f\text{I}}+\rho_{f\text{II}})-(\rho_{f\text{I}}-\rho_{0\text{I}})\exp\left(-\frac{t}{c_1}\right)$$
$$-(\rho_{f\text{II}}-\rho_{0\text{II}})\exp\left(-\frac{t}{c}\right) \tag{42}$$

where the settlement resulting from Terzaghi's consolidation theory is denoted by the lower subscript, $_{\text{I}}$, and is expressed also by the solution of the first order approximation equation, Eq. (20). It may be easily noticed that ρ of Eq. (42) is the formal solution of a stable 2nd order linear ordinary differential equation with constant coefficients. Then, by introducing a difference form, the next 2nd order autoregressive equation,

$$\rho_j=\beta_0+\sum_{s=1}^{2}\beta_s\rho_{j-s} \tag{43}$$

is suggested as being suitable for settlement prediction including creep factor.

Settlement of Clay Stratum with Sand Seams

In an ordinary soil exploration, a thin sand seam sometimes goes unnoticed. However, if the sand seam allows free drainage, this overlooking causes a great error in settlement-time prediction. Fig. 9 shows the simple case of such a situation, in which the total settlement of both stratum I and II is given by

$$\rho(t)=\rho_{\mathrm{I}}(t)+\rho_{\mathrm{II}}(t) \tag{44}$$

where $\rho_{\mathrm{I}}(t)$ and $\rho_{\mathrm{II}}(t)$ represent settlements of stratum I and stratum II respectively. Suppose that the sand seam of Fig. 9 allows free drainage. Furthermore, for simplicity, let $\rho_{\mathrm{I}}(t)$ and $\rho_{\mathrm{II}}(t)$ be expressed by the form of Eq. (20). When this is the case, Eq. (44) suggests that the total settlement $\rho(t)$ can also be predicted in the same way as is given in the discussion of the secondary compression due to creep, that is, by the 2nd order autoregressive equation.

Sand Drains

In order to reduce the time for consolidation, vertical sand drains are sometimes used. As is well known, if the equal vertical strain hypothesis is approximately satisfied, the degree of consolidation can be expressed by a single mode, that is,

$$U=1-\exp(\lambda t) \tag{45}$$

where an eigenvalue λ is determined from both a horizontal consolidation coefficient and geometrical dimensions of sand piles. According to the tradition in soil engineering field, settlement can be treated as being proportional to the degree of consolidation. Eq. (45) shows, therefore, that the settlement by sand drains can be predicted by the first order autoregressive equation.

General Prediction Formula

The above discussions imply that a general settlement prediction model is given by

$$\rho_j=\beta_0+\sum_{s=1}^{n}\beta_s\rho_{j-s}. \tag{46}$$

An actual settlement may be governed by many different factors. However, if those factors can be mathematically formulated as an eigenvalue problem, prediction formula of Eq. (46) is quite general.

It should be noted that Eq. (46) is the same form as that of Eq. (17), although the physical meanings of the coefficients are quite different.

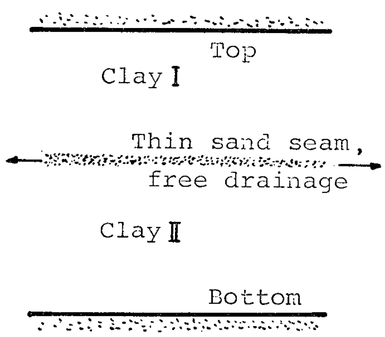

Fig. 9. Drainage from thin sand seam

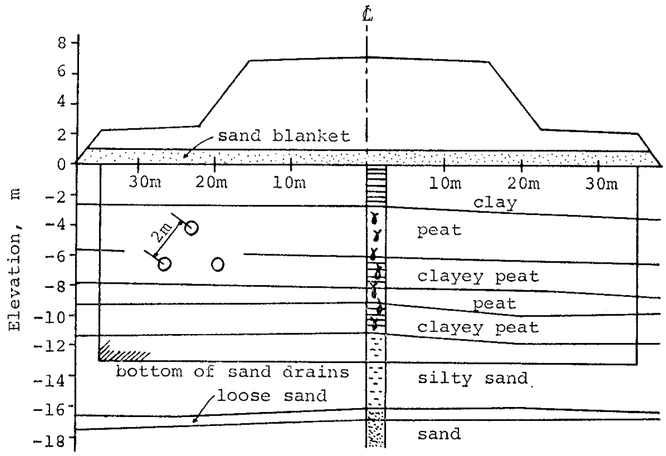

Fig. 10. Soil profile and embankment cross section of Iwamizawa test fill

The statistical parameter identification of Eq. (46) for $n \geq 2$ from past observations is, however, hopeless at present, since the settlement process is not stationary, that is, $E[\rho_j] \neq$ const. An analytical expression for a predictive pdf of ρ_j by Bayesian statistics is formally possible in the same way as is presented in the case of $n=1$ in the previous section. The results, however, include super multiple integrations. Therefore, the author recommends the least square estimates of β_0 and β_s ($s=1, 2, \cdots$) if the long term observation is available and the considerably large time interval, $\Delta t = t_j - t_{j-1}$, can be adopted.

Practically speaking, the first order prediction model, Eq. (21) or Eq. (27) should be taken first. When there exists only one prominent eigenvalue, the first order appoxima-

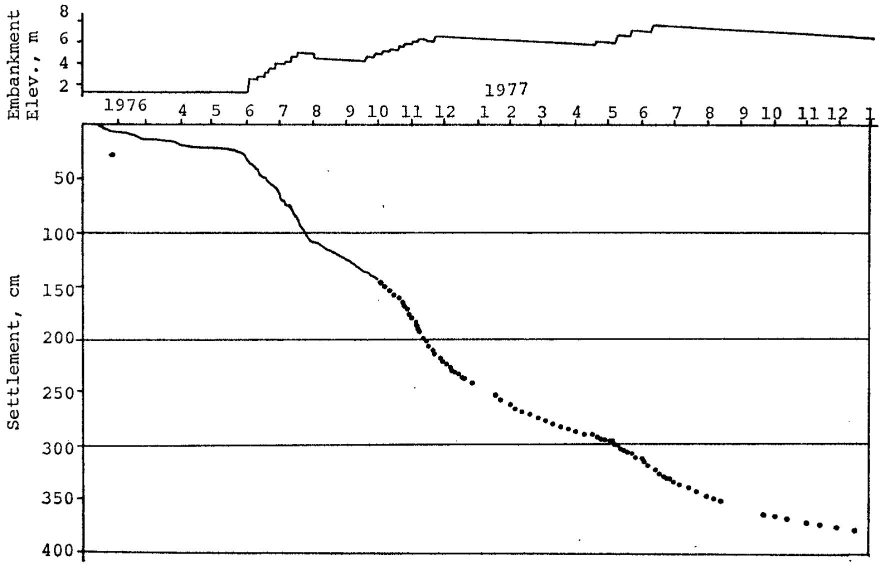

Fig. 11. Settlement observations at sand-drain test section of Iwamizawa test fill

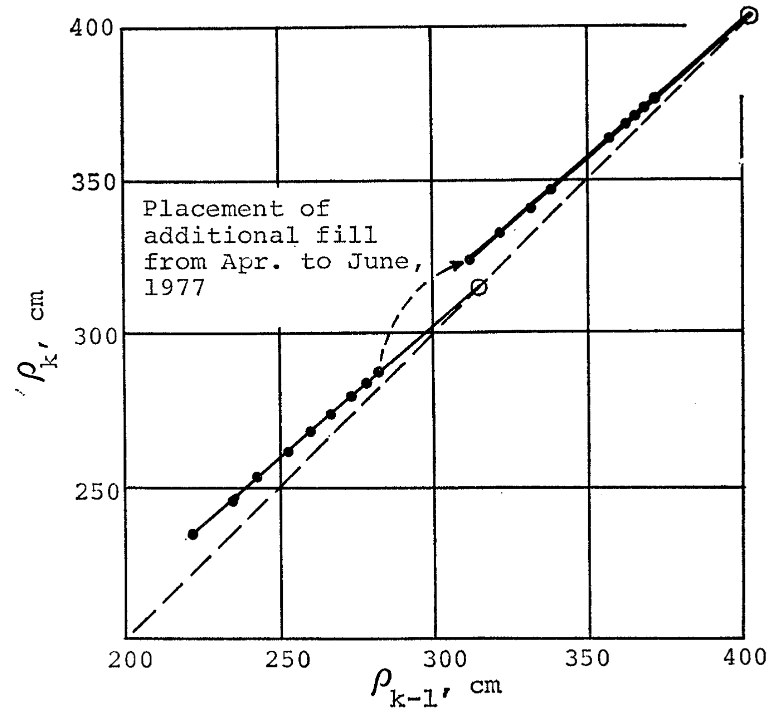

Fig. 12. Application of proposed graphical method to observations of Iwamizawa test fill

tion method will give satisfactory results. If the model does not give a good approximation to observations, the second order prediction model, Eq. (46) with $n=2$, is to be tried next.

A Practical Example of A Complex Boundary Value Problem

The applicability of the proposed settlement prediction method to a complex multilayered clay deposit was examined by using actual data provided from the test fill which was constructed during 1975-1978 at Iwamizawa in Hokkaido as a portion of the Do-o expressway by Japan Highway Public Corporation. The soil profile and the embankment cross section at the test section is shown in Fig. 10. As shown in the figure, the clay deposit consists of 7 subsoil layers including clay, peat and silty clay stratum. Sand piles were driven till the elevation of -13 m for the purpose of vertical drain through the piles. Measured settlements under center of embankment are given in Fig. 11. The embankment was left alone two times with no additional fill, the first time is from the later part of November, 1976 to the midst of April, 1977 and the other time began in the last of May, 1977. The proposed graphical settlement prediction method was applied to the observations obtained during these two periods. The results is illustrated in Fig. 12 which shows the remarkable applicability of the first order autoregressive prediction model. For the illustration of Fig. 12, the time interval, Δt, is taken to be 15 days.

CONCLUSIONS

The observational method for settlement prediction is freshly proposed in this paper. For one-dimensional consolidation problems and sand-drain problems, the first order autoregressive equation, Eq. (21) or Eq. (27) is demonstrated to be a suitable one for settlement prediction. For this prediction formula, both the graphical and reliabilistic approaches were presented.

The proposed method is considered to have a great advantage to predict settlement of an arbitrary future time under situations in which a coefficient of consolidation and both initial and boundary conditions are quite uncertain. Such situations are considered to be so common in engineering practice that the proposed method will have a certain availability. As for the unknown conditions, they can be guessed backward, if necessary, from settlement observations by using the estimated coefficients of the first order autoregressive equation.

The higher order autoregressive equation, Eq. (46), suggests to provide more precise settlement prediction and to be applicable to some special problems, that is, one is concerned with secondary compression and the other is the settlement of multilayered stratum with sand seams.

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