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DISCUSSIONS

CO-ORDINATION NUMBER AND ITS RELATION TO SHEAR STRENGTH OF GRANULAR MATERIAL*

Discussion by NOBUCHIKA MOROTO**

Every investigation on behavior of granular systems used to start with the examination of ideal packings. The individual elements of grain assemblies are mostly taken as spheres. This is an idealization of nature, of course, an abstraction, and it is understood that the results of such an investigation are not quite ready for direct application in practice. However, the outcome of the mental work on idealized systems may be a better understanding of nature, the realization of some important facts and the establishment of some, at least qualitative statements.

The term "coordination number" gives the number of spheres which are in direct contact with any given sphere. We get an interesting relation if we plot the coordination number N as a function of the void ratio e or the porosity n. The coordination number of regular packings of uniform sphere is given in Table 4 after Filep (1936) (from Kezdi (1964)). From this table, the relationship between N and n can be given as

$$N \cdot n \doteqdot 3 \tag{6}$$

This relationship can be easily transformed to

$$N=3\left(1+\frac{1}{e}\right) \tag{7}$$

Smith et al. (1929) gives a semi-empirical equations

$$N=26.5-\frac{10.7}{(1-n)} \tag{8}$$

Table 4. Porosity of different regular pakings

Coordination number N	Porosity n
4	0.718
5	0.558
6	0.476
7	0.439
8	0.395
9	0.352
10	0.302
11	0.281
12	0.259

Filep (1936) (after Kézdi's paper (1964))

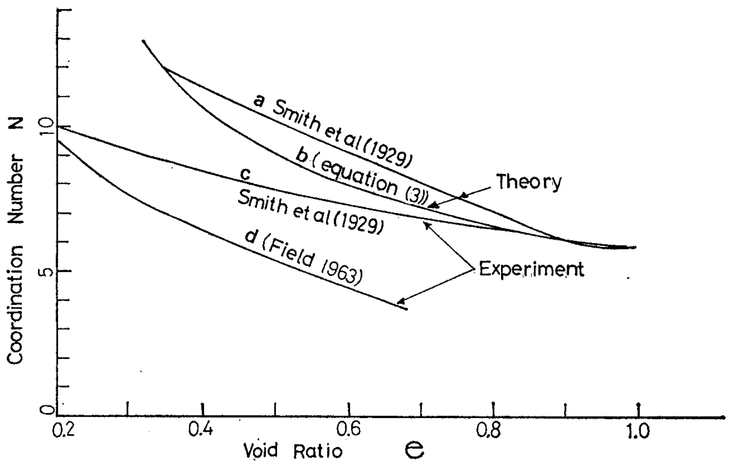

Fig. 20.

* By Masanobu Oda, Vol. 17, No. 2, June 1977, pp. 29–42.
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Field (1963) experimentally obtained the following equation

$$N=\frac{12}{1+e} \tag{9}$$

The relationship of coordination number N and void ratio e are plotted in Fig. 20.

The relationship between void ratio and shearing strength proposed by Mogami (1965) is written by

$$\sin\phi=\frac{k}{1+e} \tag{10}$$

where ϕ is the internal friction angle and k is constant. Using Eqs. (9) and (10), we have

$$\sin\phi\fallingdotseq\frac{k}{1+e_0}\fallingdotseq\beta N \tag{11}$$

where β is constant and e_0 is the initial void ratio.

In order to study void distribution pattern, the writer has proposed an entropy in relation to distribution of local void ratio in a particulate material. The entropy H is written as

$$H=-\int p(e)\ln p(e)de \tag{12}$$

where $p(e)$ is a distribution function. The values of H which can be calculated from the data obtained experimentaly by investigators are plotted in Fig. 21, where Smith et al. used lead shots and Marsal (1972) ran his tests using gravels.

It is interesting to note that (1) On Fig. 20, we can see that the curves of loose and medium dense state (curves of a, b and c) are getting together. It is remarkable that in the loose state the curves of a, b and c meet each other, and if we shift the curve a to some extent we can have a unique relationship between the experimentally obtained figure and theoretically introduced one (curve d). (2) On Fig. 21, the relations are linear and decrease with increasing void ratio. If we shift the curves to some extent, we can have a unique relation. The relation between entropy H and void ratio e is written as

$$H=A\cdot e+B \tag{13}$$

where A and B are constants.

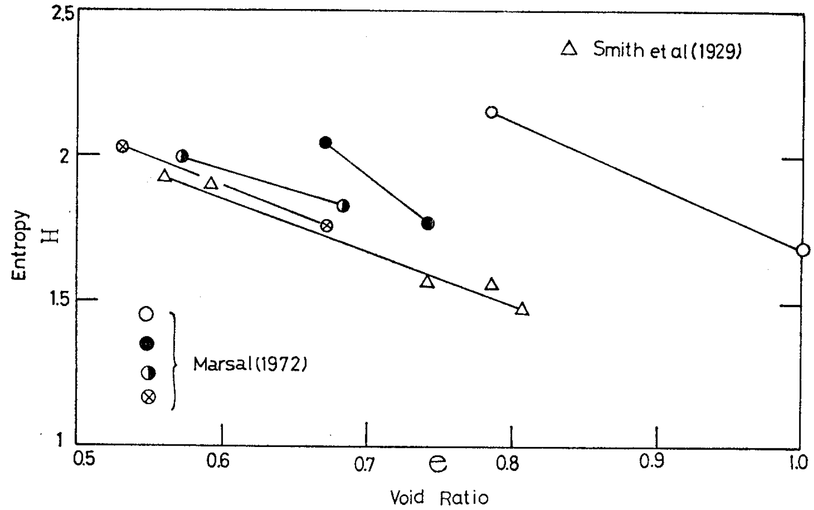

Fig. 21.

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EXPERIMENTAL STUDY OF ANISOTROPIC SHEAR STRENGTH OF SAND BY PLANE STRAIN TEST*

Discussion by J. R. F. Arthur** and T. Dunstan**

Oda, Koishikawa and Higuchi have made a valuable contribution to our knowledge of inherent anisotropy. Arthur and Assadi (1977) have published some data for Leighton Buzzard sand sheared in plane strain which corroborates the data of Oda et al. for a low stress level ($\sigma_3' \approx 0.5$ kg/cm^2). Fig. 2 of Arthur and Assadi (1977) is reproduced here (Fig. 19). The top curve labelled H refers to homogeneous dense samples deposited slowly through air and sheared in plane strain ; θ is equivalent to δ in Fig. 12 of Oda et al.. Leighton Buzzard sand shows the same increase in inherent strength anisotropy as Toyoura sand when sheared in plane strain as opposed to triaxial compression.

The minimum stress ratios recorded at higher stress levels ($\sigma_3'=2.0$, 4.0 kg/cm^2) in Fig. 12 for $\delta=24°-30°$ are especially interesting. Again, there is some parallel in Curve R of Fig. 19 (Fig. 2 of Arthur and Assadi (1977)) where a minimum is clearly implied by the data, but not fully defined. However, the cause of this minimum is the inhomogeneity and induced anisotropy of a pre-existing rupture layer at a sensitive orientation $(\pi/2-\phi'/2)$ to the major principal stress direction. The minima in Fig. 12 are clearly due to inherent anisotropy and occur when the plane of particle deposition coincides with a plane of maximum stress obliquity oriented at $(\pi/2-\phi'/2)$ to the major principal stress direction.

The absence of these minima at low stress levels is a stimulant to speculation. If the useful fiction of the concept of stress in granular media is accepted it is at least reasonable to propose that, in a so-called uniformly stressed granular material, there will be variations in magnitudes of principal stresses for small spaces within the soil. Stress distributions of this kind were represented in Fig. 20 (Fig. 11 of Arthur, Dunstan, Al-Ani and Assadi (1977)) for both a low stress level and a higher stress level. According to the means of these distributions of the principal stresses the stress ratios σ_1'/σ_3' are the same for both stress levels, but only at the low level are the independent variations of the stresses sufficient to cause a wide distribution of the stress ratio σ_1'/σ_3' in local regions of the material.

This wide distribution results in some local spaces within the material which fail whilst others are not straining at all ; there is then an inhomogeneity of stress which causes an inhomogeneous distribution of strain rate. The effect of these inhomogeneities depends on the geometric scale over which they occur ; if these variations occur over a much smaller linear scale than the original directional fabric inhomogeneities (that is, those related to the plane of deposition) then the size of the fabric inhomogeneities will be reduced by

* By Masanobu Oda, Isao Koishikawa and Toshio Higuchi, Vol. 18, No. 1, Mar. 1978, pp. 25-38.
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