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STRESS CONDITIONS AND STRESS HISTORIES AFFECTING SHEAR MODULUS AND DAMPING OF SAND UNDER CYCLIC LOADING

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ABSTRACT

To evaluate the effects of various static stress conditions and stress histories on shear modulus and damping of sand under cyclic loadings, a comprehensive series of cyclic torsional shear tests on hollow cylindrical specimens of a clean sand was performed. All the tests were performed and the constant value of mean principal stress $=1.0\,\mathrm{kgf/cm^2}$ (about 100 $\mathrm{kN/m^2}$) under fully drained condition. From the test results, the followings were found. For a wide range of shear strain amplitude ($\gamma=5\times10^{-5}\sim5\times10^{-3}$), the effect of stress ratio on shear modulus is minor in the triaxial compression case but considerable in the triaxial extension case. Initial shear stress decreases shear modulus especially for the triaxial compression case. Although shear modulus is affected by various static stress conditions, its strain-dependency is insensitive to those static stress conditions. The effects of stress ratio and initial shear stress on damping ratio are less important than shear strain amplitude γ and mean principal stress p. It was found that the effects of overconsolidation history and previous larger cyclic shear stress history on G and η are also less important than γ and p.

Key words: dynamic, repeated load, sand, special shear test, stress path, torsion
IGC: D 6/D 7

INTRODUCTION

Laboratory methods currently used for evaluating shear moduli and damping capacities of soils under cyclic loadings involve various tests ; triaxial tests, simple shear tests, resonant-column tests or torsional shear tests. Most of these tests are performed on normally consolidated virgin specimens having no shear stress histories with use of stational sinusoidal wave forms. In situ soil elements, however, can be subjected to more complicated static and dynamic stress conditions than in ordinary laboratory tests. Therefore, to evaluate shear moduli and dampings of in situ soil elements under general stress conditions when random earthquake motions are applied, several factors affecting shear moduli and dampings

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of soils should be evaluated such as stress ratio, initial shear stress, cyclic shear stress history, overconsolidation history etc.

Simple shear tests can be considered to simulate most closely the stress condition to which in situ soil elements are subjected in horizontally layered sites. This is because in situ K_0-stress condition can be naturally simulated in simple shear tests. Youd and Craven (1975) have found that the lateral stress in cyclic simple shear test on an air-dry sand increases with the increase in the number of cyclic loading especially for larger strain amplitude. It is, however, difficult to evaluate lateral stress in ordinary simple shear tests. On the other hand, in resonant-column tests and in torsional shear tests using solid or hollow cylindrical specimens, lateral stress can be easily controlled and determined, while it is, in general, difficult to simulate the K_0-stress condition. From those reasons, the effects of stress ratio on shear modulus and damping has been evaluated by resonant-column tests or torsional shear tests.

Hardin and Black (1966) have shown by performing torsional resonant-column tests and static cyclic torsional shear tests on solid cylindrical specimens of dry clean sand that the shear modulus at $\gamma=2.5\times10^{-5}$ are almost independent of the stress ratio σ_a/σ_r (axial stress/radial stress) ranging from 1.0 to around 2.0, with the mean principal stress $p=1/3(\sigma_a+2\sigma_r)$ being kept constant. This finding was confirmed later by several investigators (Kuribayashi, Iwasaki and Tatsuoka, 1973; Shibata and Tai, 1976; Yanagisawa and Yan, 1977).

Hardin and Drnevich (1972a) have demonstrated by performing static cyclic torsional shear test on hollow cylindrical specimens of dry Ottawa Sand that the effects of initial shear stress on shear modulus and damping at $\sigma_a/\sigma_r=1.0$ and for $\gamma<10^{-3}$ are much less important than mean principal stress.

Hardin and Black (1966) and Afifi and Richart (1973) have confirmed that the increases in the shear modulus of clean sands for $\gamma=2.5\times10^{-5}$ or less due to overconsolidation histories are quite small, although those effects on G and η of finer soils are more significant.

The effect of the previous shear stress history on G and η have been examined by Park and Silver (1975) by performing cyclic triaxial tests on a clean sand. They have shown that for the axial strain amplitude larger than 10^{-4} the shear modulus and the damping ratio are insensitive to the previous stress or strain histories with the previous strain amplitude being less than the present strain amplitude at which G and η were measured.

Although a larger number of investigations have been performed in this field as described above, the effects of the static stress conditions and the stress or strain histories on shear moduli and damping ratios have not been clarified for a wide range of strain amplitude and for a more general stress condition.

Herein reported are the results of a comprehensive series of static cyclic torsional shear tests on hollow cylindrical saturated specimens of a clean sand with the shear strain amplitude ranging from around 5×10^{-5} to around 10^{-2}. Examined in this investigations were; (1) the effects of stress ratio both in the triaxial compression case and in the triaxial extention case, (2) the effects of initial shear stress, (3) the combined effects of stress ratio and initial shear stress, (4) the effects of overconsolidation and (5) the effects of previous larger cyclic shear stress history.

DEVICE AND TEST PROCEDURES

The device used in this investigation is identical to the one used in the previous studies (Iwasaki, Tatsuoka and Takagi, 1978; Tatsuoka, Iwasaki and Takagi, 1978; Tatsuoka, Iwasaki, Yoshida, Fukushima and Sudo, 1979).

Hollow cylindrical specimens being 10 cm high, 6 cm in innerdiameter and 10 cm in outerdiameter were enclosed in a triaxial cell and cyclic torsional loads were applied on the top of a specimen. In any test in this investigation, a saturated densely packed specimen of fresh Toyoura Sand was sheared under the fully drained condition. Physical properties of the sand are $G_s=2.64$, $e_{max}=0.96$, $e_{min}=0.64$, $D_{10}=0.12$ mm, $D_{60}=0.175$ mm and the grain shape is angular. Specimens were prepared by pouring fresh de-aired Toyoura Sand into a mold fulfilled with de-aired water with a spoon. Densification was achieved by tapping the mold with a wooden hammer. Fig. 1 illustrates the static stress condition and the way of appling cyclic shear stress τ_d. Static stresses are axial stress σ_a, radial stress σ_r and initial shear stress τ_0. In this test, the confining pressure both in the hollow and in the outside is same. Therefore, the static tangential stress σ_θ can be considered identical to the static radial stress σ_r which is applied by air pressure. In this sense, G and η under more generalized static stress condition such as $\sigma_a \neq \sigma_\theta \neq \sigma_r$ with τ_0 can not be evaluated with this device. All the tests were conducted under $p=1.0$ kgf/cm² (about 100 kN/m²).

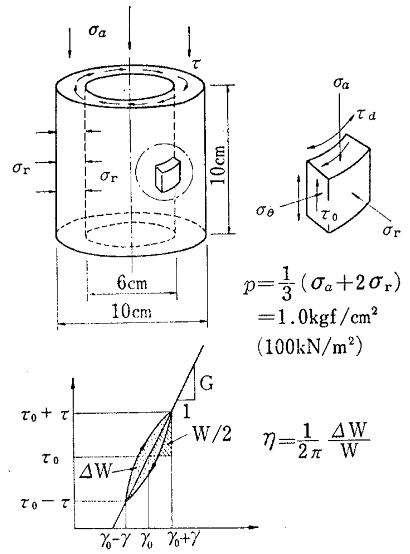

Fig. 1. Stress condition in torsional shear hollow cylindrical specimens and definitions of shear modulus G and damping ratio η

Five different series of tests were carried out as (1) SR-series, (2) τ_0-series, (3) τ_0+SR-series, (4) OC-series, (5) τ_d-history series. The test conditions are listed in Table 1.

The SR-series is to evaluate the effect of stress ratio on G and η. In these tests, the stress ratio SR is defined as σ_1/σ_3. SR was varied as $SR=\sigma_1/\sigma_3=\sigma_a/\sigma_r=1.5$, 2.0, 3.0, 4.0 and 5.0 in the triaxial compression case and $SR=\sigma_1/\sigma_3=\sigma_r/\sigma_a=2.0$, 3.0, 4.0 and 5.0 in the triaxial extension case. In Table 1, TC 2 for example, means the test where the sample is consolidated in the triaxial compression stress condition of $\sigma_a/\sigma_r=2.0$ and TE 2 means the test where the sample is consolidated in the triaxial extension stress condition of $\sigma_r/\sigma_a=2.0$. For each stress condition, two tests were performed except TE 4. Each consolidation stress condition was achieved as follows. In the test TC 2, for example, firstly a sample was consolidated isotropically at $\sigma_a=\sigma_r=0.75$ kgf/cm² (about 75 kN/m²) and then, keeping σ_r constant, σ_a was increased up to 1.5 kgf/cm² (about 150 kN/m²). After consolidated about one hour at that stress condition, a cyclic torsional shear test was started. On the other hand in test TE 2, firstly a sample was consolidated isotropically at $\sigma_a=\sigma_r=0.6$ kgf/cm² (about 60 kN/m²) and then, keeping σ_a constant, σ_r was increased up to 1.2 kgf/cm² (about 120 kN/m²) and that stress condition was kept constant for an hour before cyclic tests.

In the τ_0-series, an initial shear stress τ_0 was applied to an isotropically consolidated specimen. Therefore, the static stress condition in this test is $p=1.0$ kgf/cm² (about 100 kN/m²), $\sigma_a/\sigma_r=1.0$ and $\tau_0 \neq 0.0$. As the values of τ_0, 0.05, 0.10 and 0.20 kgf/cm² (about 5, 10 and 20 kN/m²) were employed.

In the τ_0+SR-series, an initial static shear stress τ_0 was applied to an anisotropically consolidated specimen ($\sigma_a/\sigma_r \neq 1.0$) and then a cyclic shear stress τ_d was applied. Employed static stress conditions in this series are listed in Table 1.

Except the τ_d-history series which will be described later, so called the stage test was adopted (Park and Silver, 1975). A specimen was firstly torsionally sheared ten times at 0.1 cycle per second under constant amplitude of τ_d with the shear strain amplitude being around 5×10^{-5}. Then, with increasing shear stress amplitude, the cyclic loadings of ten

Table 1. List of data

Test No.	T.C. or T.E.	σ_a/σ_r or σ_r/σ_a	τ_0/p	e_0
1*	T.C.	1.5	0	0.626
2		1.5		0.630
3*		2.0		0.662
4		2.0		0.636
5*		3.0		0.594
6		3.0		0.614
7*		4.0		0.621
8		4.0		0.644
9*		5.0		0.619
10		5.0		0.622
11*	T.E.	2.0	0	0.611
12		2.0		0.602
13*		3.0		0.626
14		3.0		0.643
15*		4.0		0.656
16*		5.0		0.644
17		5.0		0.733

Test No.	T.C. or T.E.	σ_a/σ_r or σ_r/σ_a	τ_0/p	e_0
18		1.0	0.05	0.659
19+				0.618
20			0.10	0.634
21+				0.639
22			0.20	0.615
23+				0.693
24	T.C.	2.0	0.05	0.627
25			0.10	0.635
26			0.20	0.612
27		3.0	0.05	0.631
28			0.10	0.652
29			0.20	0.576
30	T.E.	2.0	0.05	0.585
31			0.10	0.640
32			0.20	0.573
33		3.0	0.05	0.658
34			0.10	0.605
35			0.20	0.618

1) T.C. means the triaxial compression stress condition of $\sigma_a/\sigma_r>1.0$ and T.E. means the triaxial extension stress condition of $\sigma_r/\sigma_a>1.0$.
2) Void ratio at the beginning of shearing.
* These data are shown in Figs. 2(a) and 7(a) and represented by the mark + in Figs. 3 and 4.
+ These data are represented by solid marks in Fig. 2(b).

times at each shear stress amplitude were repeated until the specimen failed.

In the *OC*-series, samples were firstly consolidated by the isotropic confining pressure larger than 1.0 kgf/cm^2 for 12 hours. Then, the confining pressure was decreased to 1 kgf/cm^2 and the stage test was performed. The overconsolidation ratios employed were 2.0, 3.0 and 4.0.

Lastly, in the τ_d-history series, the amplitude of τ_d or γ was varied both increasingly and decreasingly among stages. For a some amplitude of cyclic shear stress τ_d, ten cycle loadings were repeated. The purpose of this series is to evaluate the effects of previous loading of τ_d which is larger than the present value at which G and η were measured.

EFFECTS OF STATIC STRESS CONDITIONS

Shear Modulus

The shear moduli at tenth cycle obtained from the *SR*-series, the τ_0-series and the τ_0+

SR-series were divided by G^* defined by Eq. (1).

$$G^*=700\frac{(2.17-e)^2}{1+e}p^{0.5} \quad (1)$$

in which G^* is shear modulus in kgf/cm^2, e is the void ratio and p is the mean principal stress in kgf/cm^2, both being the values at the time of measuring G. The relationships between G/G^* and γ which were obtained through the procedure described above are shown in Fig. 2. Since it has been found that the shear modulus of Toyoura Sand is proportional to $(2.17-e)^2/(1+e)$ for the shear strain amplitude ranging from 10^{-6} to 10^{-2} (Iwasaki, Tatsuoka and Takagi, 1978), it can be considered that the variations of G/G^* in Fig. 2 are due to various static stress conditions. The solid curves and the hatched zones are the average curve and the range of scattering of the data for $SR=\sigma_a/\sigma_r=1.0$ and $\tau_0=0.0$, respectively. These were obtained by the previous study (Iwasaki, Tatsuoka and Takagi, 1978). It can be seen from Fig. 2 that shear modulus varies considerably by the variation of static stress condition. To examine how shear modulus is affected by the values of *SR* and τ_0, the ratio R of the shear modulus for $SR\fallingdotseq 1.0$ and/or $\tau_0\fallingdotseq 0.0$ to that for $SR=1.0$ and $\tau_0=0.0$ was defined as

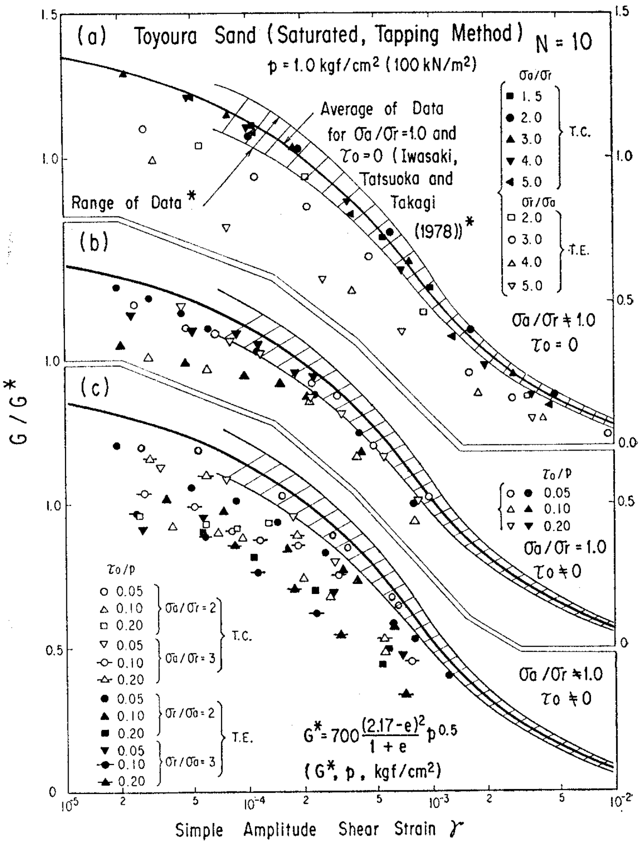

Fig. 2. $G/G^*\sim\gamma$ relationships in various static stress conditions ($N=10$)

$$R=\frac{G \text{ for } SR\fallingdotseq 1.0 \text{ and/or } \tau_0\fallingdotseq 0.0}{G_r=G \text{ for } SR=1.0 \text{ and } \tau_0=0.0} \text{ for the same values of } e,\ p \text{ and } \gamma \quad (2)$$

The values of R were obtained by dividing G/G^* for $SR\fallingdotseq 1.0$ and/or $\tau_0\fallingdotseq 0.0$ by G/G^* for $SR=1.0$ and $\tau_0=0.0$. Those values were read off for $\gamma=5\times10^{-5}$, 10^{-4}, 2×10^{-4}, 5×10^{-4}, 10^{-3} and 3×10^{-3} from the $G/G^*\sim\gamma$ relationships as shown in Fig. 2. The relationships among R, *SR* and τ_0/p for $N=2$ and 10 are shown in Figs. 3 and 4, repectively. The followings can be seen from both figures :

(1) In the triaxial compression case of $\sigma_a/\sigma_r>1.0$ with $\tau_0=0.0$, the effect of stress ratio $SR=\sigma_a/\sigma_r$ on G is not important for $SR<4.0$ for the shear strain amplitude ranging from 5×10^{-5} to 3×10^{-3} under the constant value of p. Shear modulus for $SR=5.0$ is somewhat smaller than that for $SR\leqq 4.0$, but its reduction is not so large. The comparison of the data by this investigation with those by other investigations at $\gamma=10^{-4}$ or less is shown in Fig. 5. The data by this investigation are those at $\gamma=10^{-4}$. The other data in Fig. 5 were obtained by resonant-column tests. It can be seen from Fig. 5 that the data by this investigation are well consistent with other data. The test results shown above indicate that when $\sigma_a/\sigma_r>1.0$ and $\tau_0=0$ such as in horizontally layered deposits, shear modulus can be properly estimated from mean principal stress irrespectively of *SR* for *SR* less than 4.0 for a wide range of γ, say from 10^{-6} to 10^{-2}.

(2) Differently from the triaxial compression case, shear moduli in the triaxial extension case of $\sigma_r/\sigma_a>1.0$ with $\tau_0=0.0$ under the constant value of p decrease almost linearly with

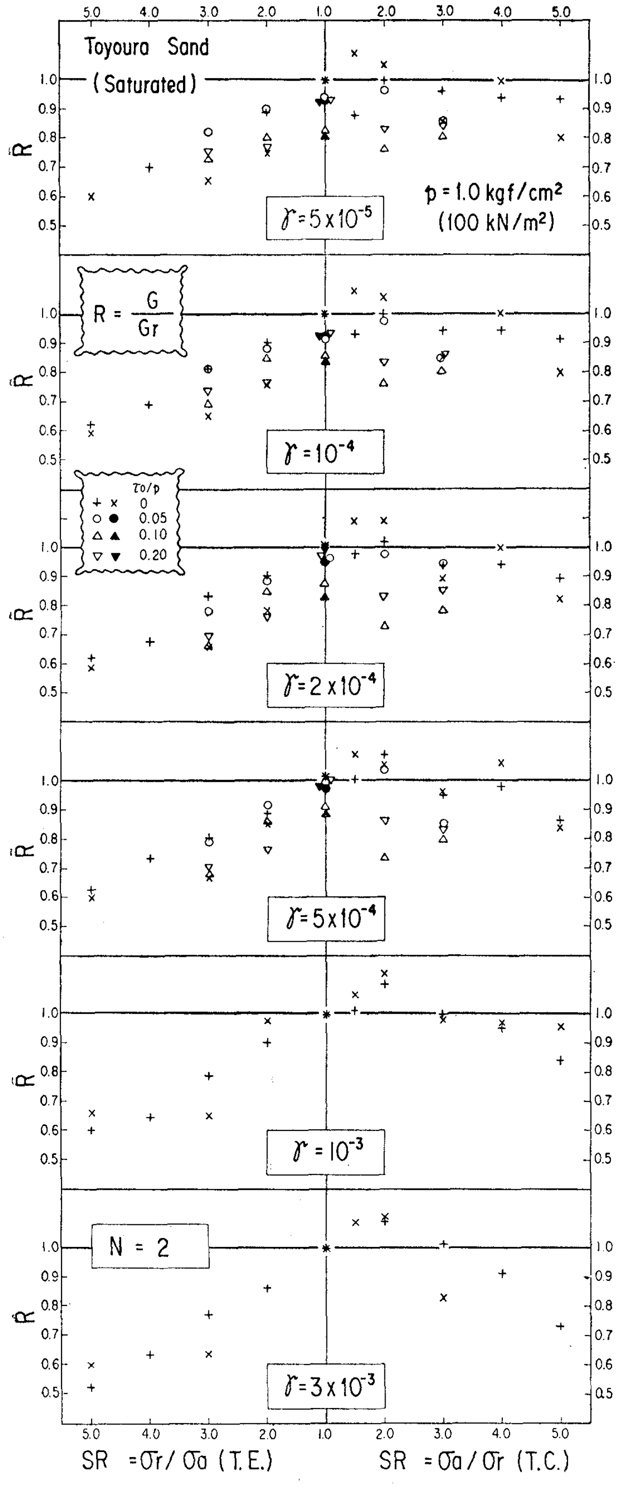

Fig. 3. Relationship among shear modulus ratio, stress ratio and initial shear stress (N=2)

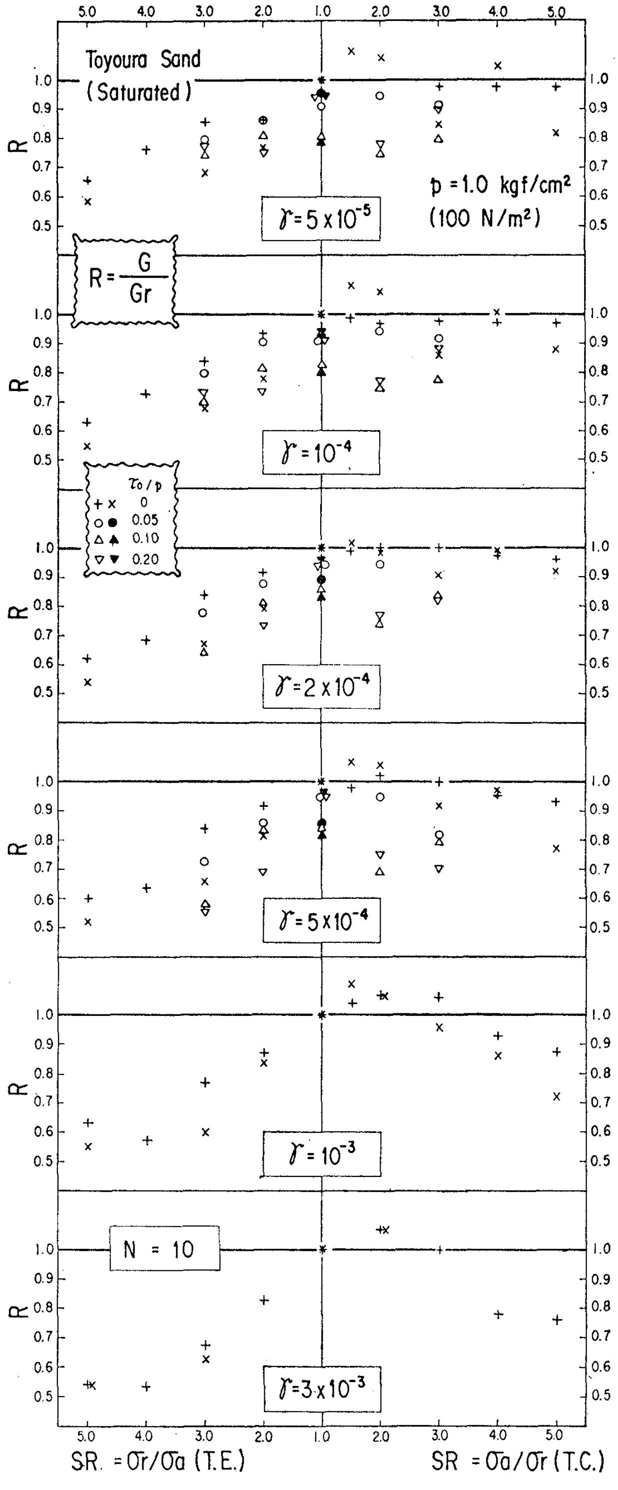

Fig. 4. Relationship among shear modulus ratio, stress ratio and initial shear stress (N=10)

increasing SR from $SR=1.0$. This shows that shear modulus in the triaxial extension case should be evaluated taking into account stress ratio besides mean principal stress p.

(3) Shear modulus decreases with increasing initial shear stress τ_0 especially both for the isotropical consolidation case of $\sigma_a/\sigma_r=1.0$ and for the triaxial compression case of $\sigma_a/$

Table 2. Legend for Fig. 5

Simbols	Sands	e	Devices	p (kgf/cm^2)	γ	References
○	Ottawa	unknown	R.C. (Hardin)	1.0	2.5×10^{-5}	Hardin and Black (1966)
●				3.45		
▷	Toyoura	0.69	R.C. (Drnevich)	2.0	5×10^{-5}	Kuribayashi, Iwasaki and Tatsuoka (1976)
▶					10^{-4}	
△	Toyoura	0.62	R.C. (Drnevich)	1.0	10^{-4}	Shibata and Tai (1976)
▽	Toyoura	0.68	R.C. (Solid Sample)	1.0	unknown	Yanagisawa and Yan (1977)
▼		0.65		3.5		
□		0.66		4.5		

$\sigma_r>1.0$. The rate of decrease in shear modulus due to τ_0 is 20% for $\tau_0/p=0.2$. In the triaxial extension case of $\sigma_r/\sigma_a>1.0$, however, the effect of τ_0 on G is not important.

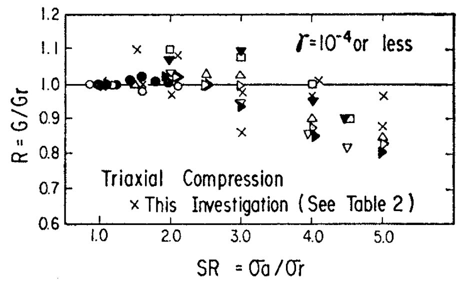

Fig. 5. Comparison of the relationships between shear modulus ratio and stress ratio by various investigations

The fashion of the decrease in shear modulus with increasing shear strain amplitude for $SR\neq 1.0$ and/or $\tau_0=0.0$ was compared with that for $SR=1.0$ and $\tau_0=0.0$. For this purpose, the values of $G/\{G\}_{\gamma=10^{-4}}$ at $N=10$ were obtained by dividing G/G^* in Fig. 2 by G/G^* at $\gamma=10^{-4}$. By this procedure, the effects of shear strain amplitude on G in various stress condition can be isolated from the effects of variation of void ratio during shearing. Fig. 6 shows the relationship between $G/\{G\}_{\gamma=10^{-4}}$ and γ for three cases. The solid curves in Fig. 6 are those for $\sigma_a/\sigma_r=1.0$ and $\tau_0=0.0$. It is seen from Fig. 6(a) that the strain-dependency of G in the case of $SR\neq 1.0$ and $\tau_0=0.0$ is not affected by stress ratio. Also in the case of $\sigma_a/\sigma_r=1.0$ and $\tau_0\neq 0$ (Fig. 6(b)), it is seen that the strain-dependency of G is not affected by the value of τ_0/p. It is seen, however, from Fig. 6(c) that the strain-dependency of G for $\tau_0/p=0.2$ both for $\sigma_a/\sigma_r=3.0$ and for $\sigma_r/\sigma_a=3.0$ is somewhat larger than that for $\sigma_a/\sigma_r=1.0$ and $\tau_0=0.0$. The differences from the case of $\sigma_a/\sigma_r=1.0$ and $\tau_0=0$ are within 30%. On the other hand, it is also a fact that the estimation of in situ shear modulus can include such a large error as 30% in some cases. Therefore, judged from the test results shown in Fig. 6, it can be said that the strain-dependency of shear modulus for soil elements under various static stress conditions is not strongly affected by the static stress condition and can be determined from cyclic tests on isotropically consolidated specimens ($\sigma_a/\sigma_r=1.0$ and $\tau_0=0.0$). Then, the dynamic response analyses of horizontally or inclined layered deposits or soil structures can be somewhat simplified utilizing the strain-dependency of shear modulus which is not affected by static stress conditions.

By the theory of Hardin and Drnevich (1972 b), the strain-dependency of shear modulus for a particular soil can be represented by a hyperbolic curve :

$$G/G_{\max}=\frac{1}{1+\gamma/\gamma_r} \qquad (3)$$

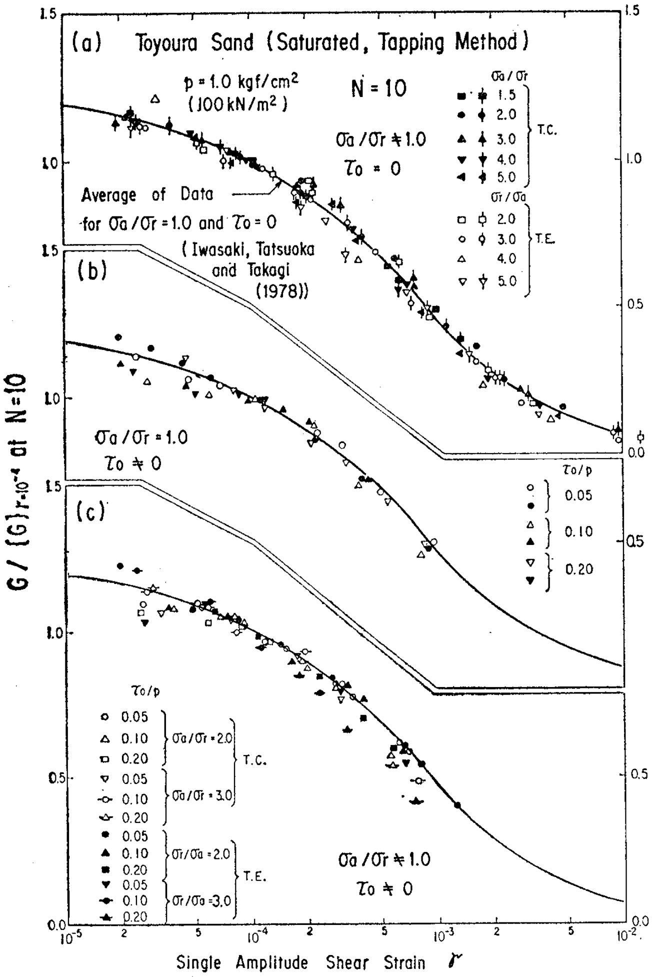

Fig. 6. Strain-dependencies of shear modulus in various static stress conditions (N=10)

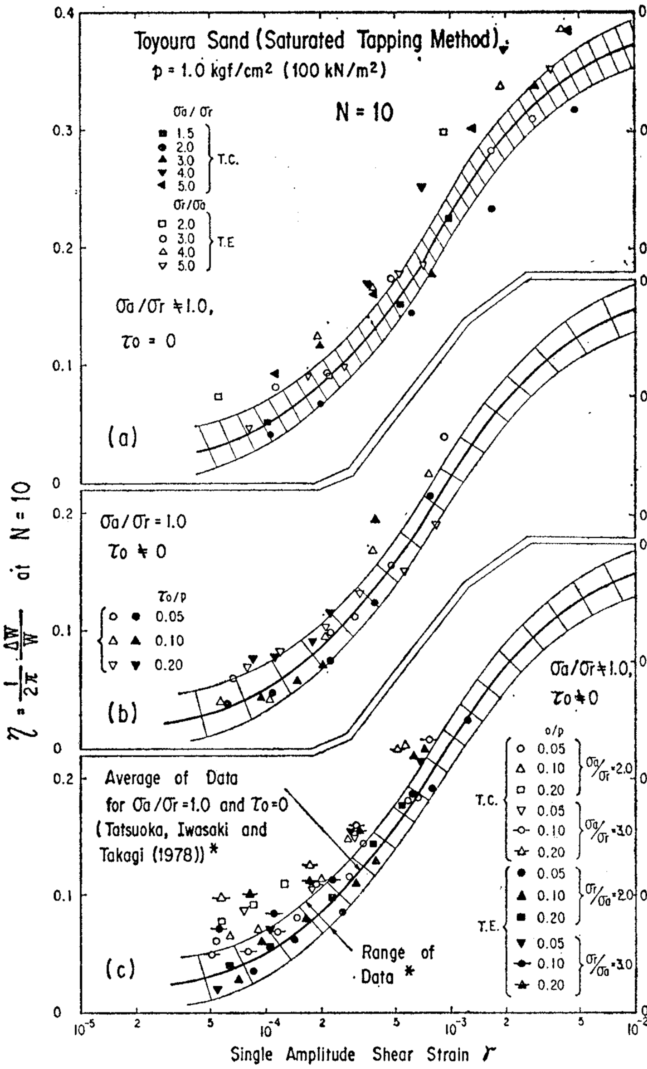

Fig. 7. $\eta \sim \gamma$ relationships in various static stress conditions (N=10)

$$\gamma_r : \text{reference strain} = \frac{\tau_{\max}}{G_{\max}} \tag{4}$$

For sand in the case of $\sigma_a/\sigma_r \fallingdotseq 1.0$ and $\tau_0 = 0.0$, $\tau_{\max}$ is obtained from

$$\tau_{\max} = \left[\left\{\left(\frac{1+(\sigma_r/\sigma_a)}{2}\right)\cdot\sin\phi\right\}^2 - \left(\frac{1-(\sigma_r/\sigma_a)}{2}\right)^2\right]^{1/2}\cdot\sigma_a \tag{5}$$

$G_{\max}$ means G at $\gamma=0.0$ or very small value. As seen from Eq. (3), γ_r means the shear strain amplitude γ at which $G/G_{\max}=0.5$. Therefore, strain-dependency can be represented only by γ_r. The test results shown in Fig. 6(a) mean that the value of γ_r is rather independent of σ_r/σ_a and τ_0/p when the strain-dependency of the test results are represented by Eq. (3). Now, let us examine whether γ_r calculated from Eqs. (4) and (5) is affected by σ_r/σ_a or not. First, for example, it can be considered from Fig. 4 that $G_{\max}$ for $SR=\sigma_a/\sigma_r=5.0$ is around $0.9\times G_{\max}$ for $SR=1.0$. Then, with $\phi=45°$, which is relevant for dense Toyoura Sand, the values of γ_r both for $SR=1.0$ and 5.0 can be calculated from Eqs. (4) and (5). Then, the ratio of γ_r for $SR=5.0$ to that for $SR=1.0$ can be obtained. This value is 0.48. This means that following Eqs. (3), (4) and (5), the value of γ_r is considerably affected by SR. This is not consistent with the test results shown in Fig. 6

(a). Such an inconsistency was also found for the triaxial extension case with $\tau_0=0.0$. For the case of $SR\neq1.0$ and $\tau_0\neq0.0$, how to derive τ_{max} is not clear in the paper by Hardin and Drnevich (1972 b). It is likely at present that it is rather complicated to use the stress condition-dependent strain-dependency of shear modulus expressed by Eqs. (3), (4) and (5) and that for ordinary earthquake response analyses it is sufficient to use a stress condition-independent strain-dependency of shear modulus.

Damping Ratio

Fig. 7 shows the relationships between damping ratio η and shear strain amplitude γ for $N=10$ obtained from three different test series. Figs. 8 and 9 show the relationships among

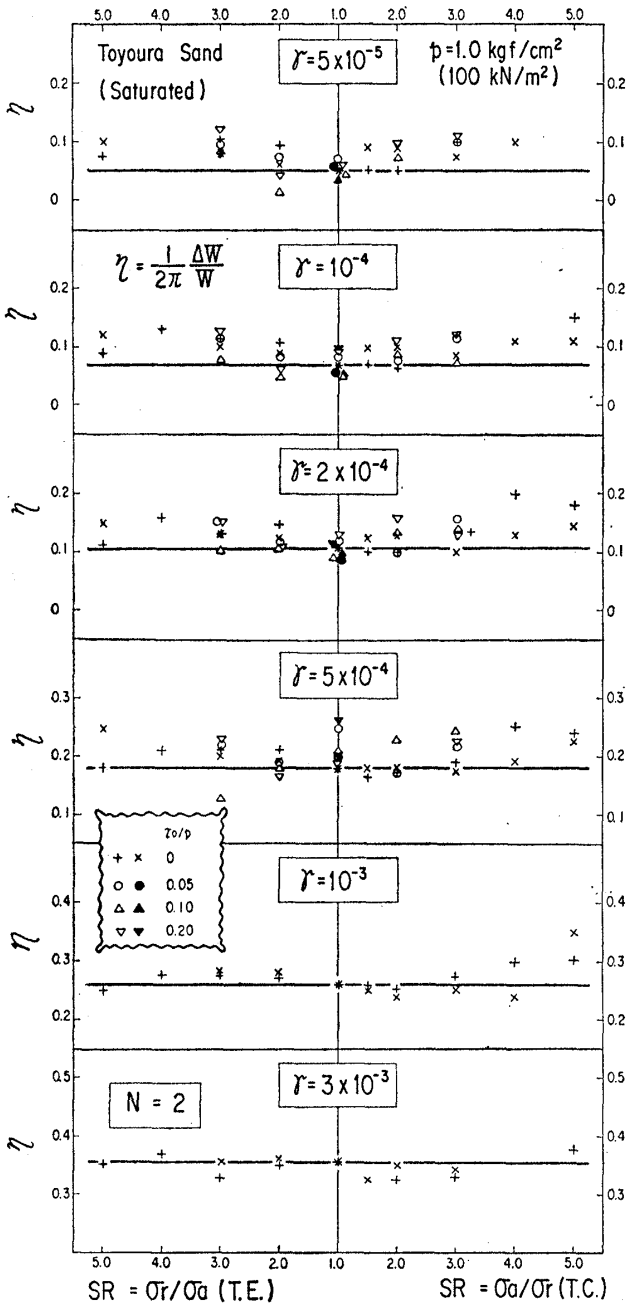

Fig. 8. Relationship among damping ratio, stress ratio and initial shear stress (N=2)

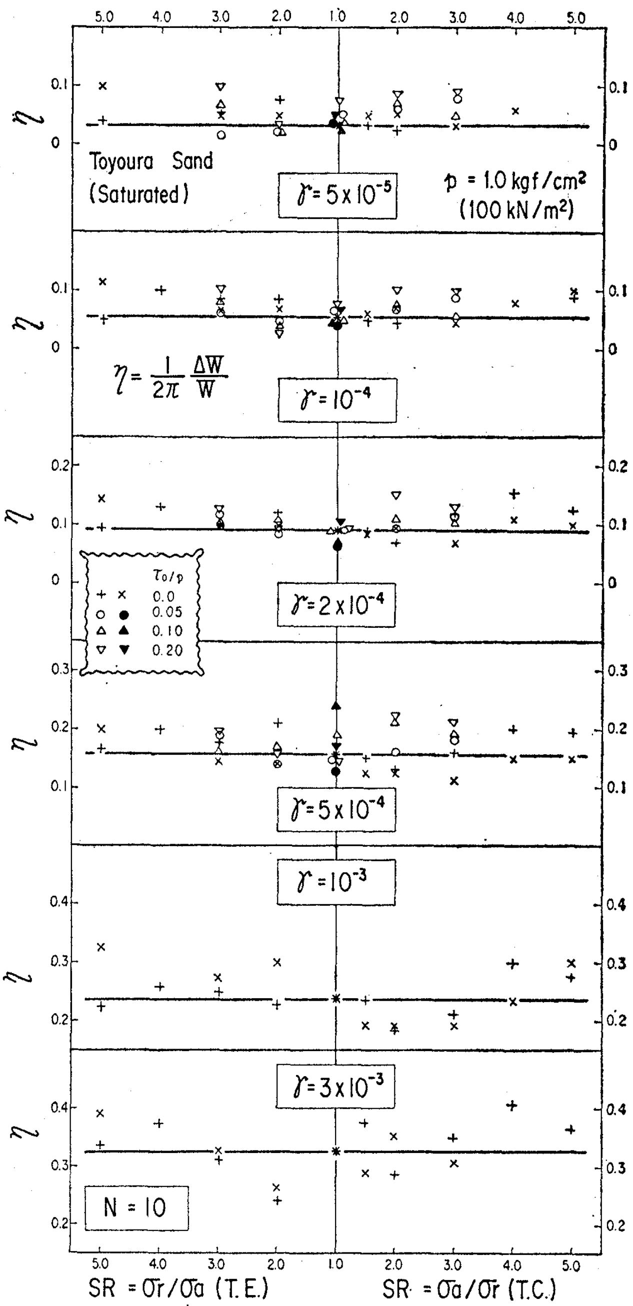

Fig. 9. Relationship among damping ratio, stress ratio and initial shear stress (N=10)

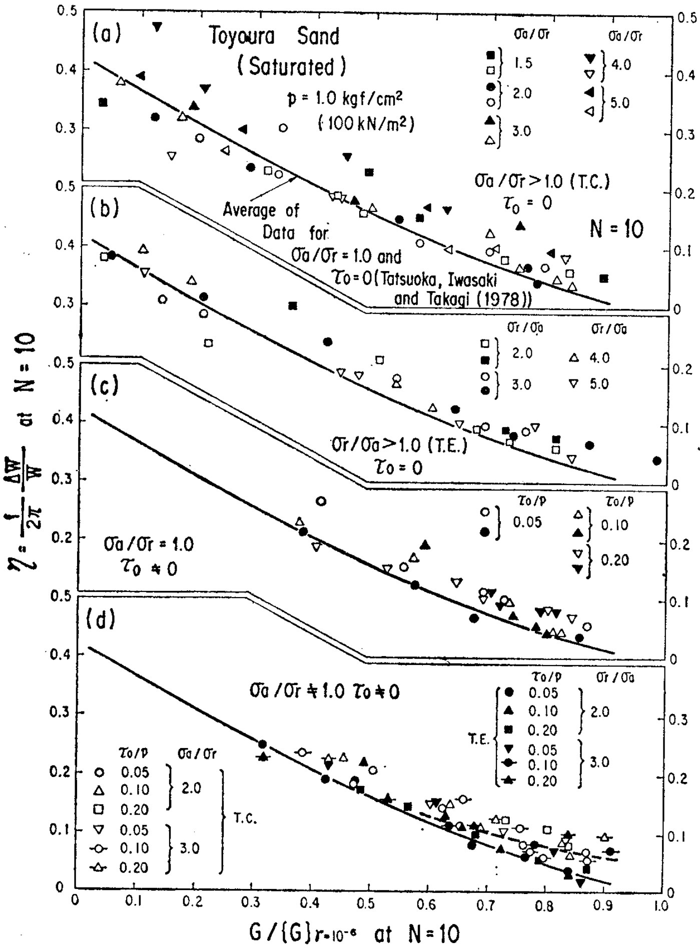

Fig. 10. $\eta \sim G/\{G\}_{\gamma=10^{-6}}$ **relationships in various stress conditions** ($N=10$)

η, SR and τ_0/p at various values of γ for $N=2$ and 10, respectively. The value of η in those figures were read off from the $\eta \sim \gamma$ relationships shown in Fig. 7. The horizontal solid lines in Figs. 8 and 9 mean the average values of η for $SR=1.0$ and $\tau_0=0.0$ obtained by Tatsuoka, Iwasaki and Takagi (1978). From those figures, a trend showing that η increases slightly both with the increase in SR and with the increase in τ_0/p for γ less than 5×10^{-4} can be seen. For larger values of γ, that trend is not so clear. Due to the scatter of the data, however, a determinate relationships among η, SR and τ_0/p could not be established. As a whole, it is obvious that the effects of SR and τ_0/p on η are less important than shear strain amplitude and mean principal stress.

On the other hand, it has been clarified that the η is closely related to the ratio of G to $G_{max}=G$ at $\gamma=0.0$ (Hardin and Drnevich, 1972 b) or the ratio of G to $\{G\}_{\gamma=10^{-6}}=G$ at $\gamma=10^{-6}$ (Tatsuoka, Iwasaki and Takagi, 1978). To obtain the relationship between η and $G/\{G\}_{\gamma=10^{-6}}$, it was postulated that the strain-dependency of G between $\gamma=10^{-6}$ and 10^{-4} is independent of SR and τ_0/p, as suggested in Fig. 6. In the case of $SR=1.0$ and $\tau_0=0.0$, the ratio of G at $\gamma=10^{-6}$ to that at 10^{-4} of Toyoura Sand was estimated equal to 1.24 (Iwasaki, Tatsuoka and Takagi, 1978). The value of $G/\{G\}_{\gamma=10^{-6}}$ were obtained by dividing the value of $G/\{G\}_{\gamma=10^{-4}}$ of each data shown in Fig. 6 by 1.24. Fig. 10 shows the relationships between η and $G/\{G\}_{\gamma=10^{-6}}$. The solid curves in Fig. 10 are the average curves for the isotropically consolidated specimens ($SR=1.0$ and $\tau_0=0.0$). It can be seen from Figs. 10 (a) and 10(b) that the effects of SR on the relationship between η and $G/\{G\}_{\gamma=10^{-6}}$ for $\tau_0=0.0$ are not important. This suggests that the value of η for horizontally layered deposits having any value of K_0 can be estimated adequately from the value of $G/\{G\}_{\gamma=10^{-6}}$ utilizing the relationship between η and $G/\{G\}_{\gamma=10^{-6}}$ which is established for isotropically consolidated specimens. Note that the relationship between $G/\{G\}_{\gamma=10^{-6}}$ and γ is also independent of SR (The relationship is, however, affected by p). It can be seen from Figs. 10(c) and 10(d) that the effects of τ_0 on the $\eta \sim G/\{G\}_{\gamma=10^{-6}}$ are minor but the combined effects of SR and τ_0 are not negligible for larger value of $G/\{G\}_{\gamma=10^{-6}}$ than around 0.5. For example, for $SR=3.0$ and $\tau_0/p=0.1\sim0.2$, the broken line in Fig. 10(d) instead of the solid line can be proposed.

EFFECTS OF STRESS HISTORIES

Overconsolidation

The shear moduli of isotropically overconsolidated specimens ($OCR=2$, 3 and 4) for $N=$

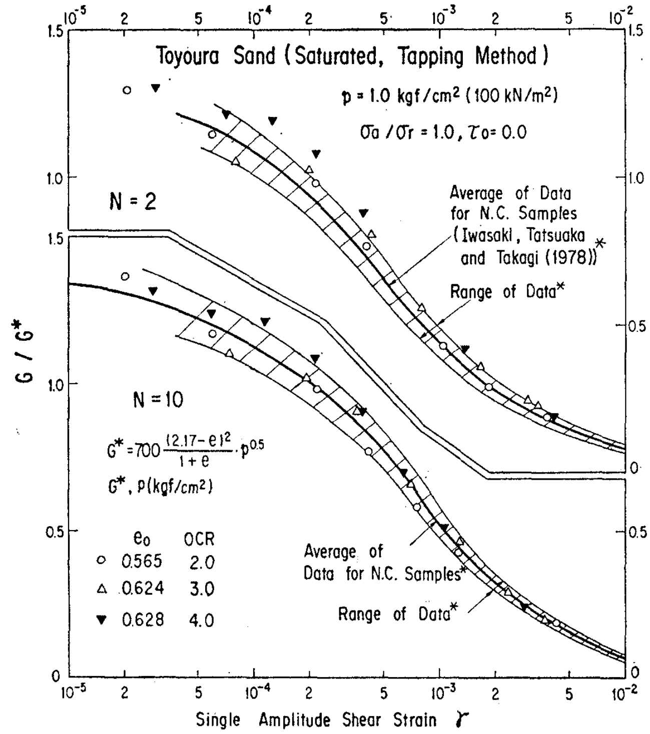

Fig. 11. ***G/G*∼γ* relationships of overconsolidated specimens**

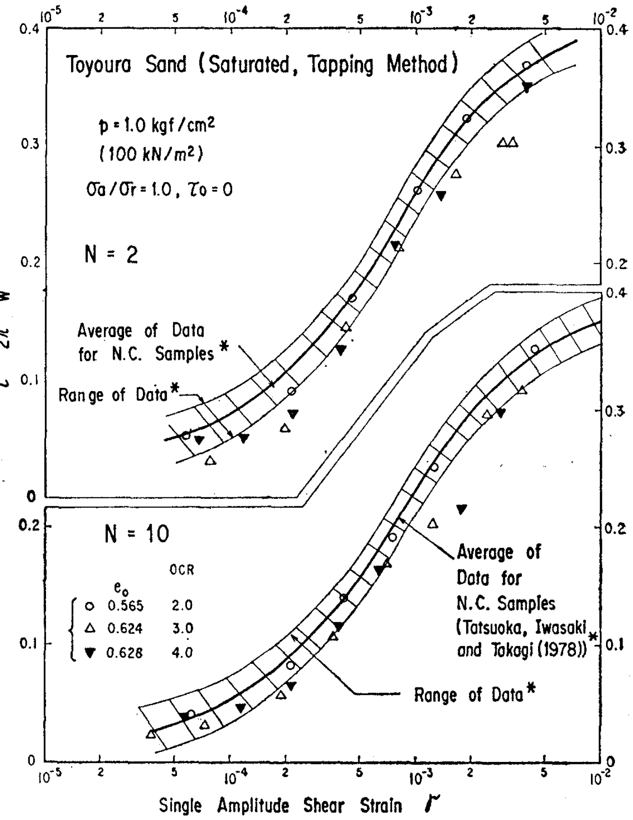

Fig. 12. ***η∼γ* relationships of overconsolidated specimens**

2 and 10 divided by G^* are shown in Fig. 11. The solid curves represent G/G^* of normally consolidated specimens. Note that the increase in shear modulus due to the decrease in void ratio during overconsolidation histories disappears by having been divided by G^*. Therefore, it can be considered that the discrepancy of G/G^* of overconsolidated specimens from those of normally consolidated specimens are only due to the overconsolidation history itself. It can be seen from Fig. 11 that although G/G^* increases slightly by overconsolidation for $\gamma=5\times10^{-5}\sim5\times10^{-3}$ especially for smaller number of cyclic loading, the effects of overconsolidation on shear modulus are less important than shear strain amplitude and mean principal stress. Fig. 12 shows the relationship between the damping ratios of overconsolidated specimens and those of normally consolidated specimens for $N=2$ and 10. The latters are represented by solid curves. It can be noticed that the damping ratio of overconsolidated specimens are slightly smaller than those of normally consolidated ones for $\gamma=5\times10^{-5}\sim5\times10^{-3}$ especially for $N=2$. It can also be said, however, that the effects of overconsolidation on damping ratio are not important. Note that such conclusions as above have been obtained by the previous studies (Hardin and Black, 1966 etc) only for $\gamma=2.5\times10^{-5}$ or less.

Shear Stress Histories

All the test results described so far were obtained by so called "stage" tests. The effects of cyclic shear stress or strain histories on G and η at the shear stress or strain amplitudes which are smaller than those during the cyclic shear stress histories were examined by the τ_d-history series. The test results are shown in Figs. 13 through 16. In Figs. 13 and 14, both hollow circles between A and B and solid circles between C and D mean the data which were obtained during the stages among which the amplitude of τ_d is always increasing.

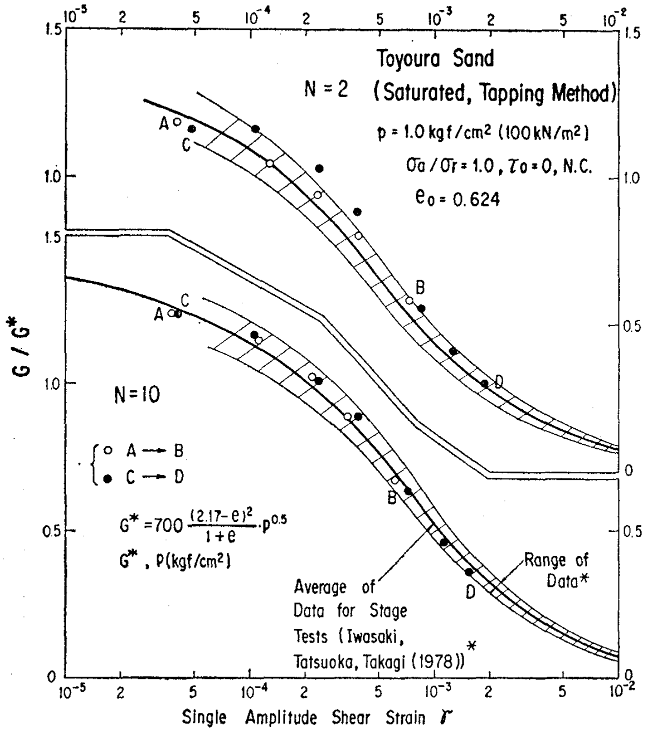

Fig. 13. $G/G^{*}\sim\gamma$ **relationships by** τ_d**-reversing test**

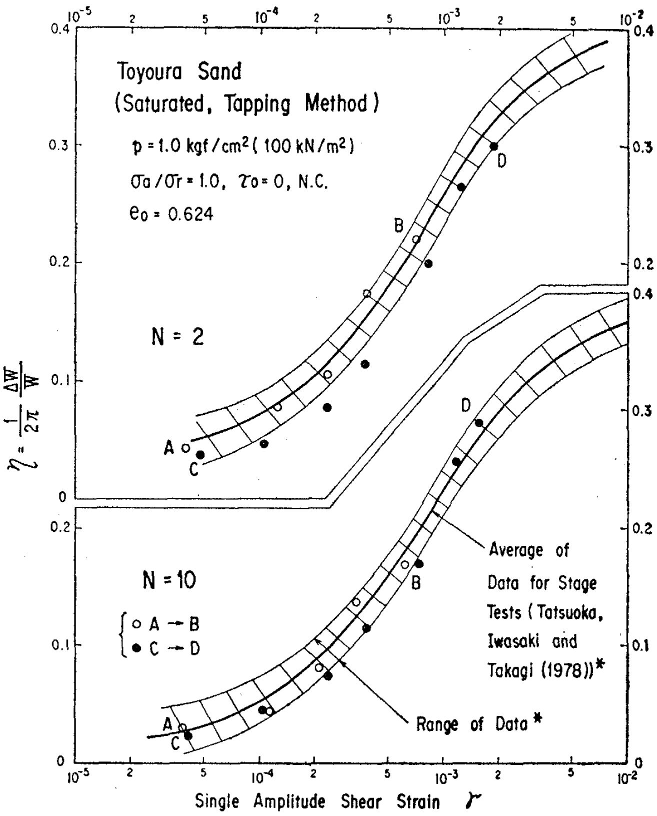

Fig. 14. $\eta\sim\gamma$ **relationships by** τ_d**-reversing test**

In each stage, ten times of cyclic loading at almost constant amplitude of τ_d were repeated. The data between C and D are, however, obtained after having applied the cyclic shear stress history AB to the specimen. It can be seen from Figs. 13 and 14 that the cyclic shear stress or strain history of AB increased slightly G/G^* and decreased slightly η for $N=2$ for the shear strain amplitude which were smaller than the maximum shear strain amplitude at B during the stress history of AB. The effects of the stress history of AB on G/G^* and η for $N=10$ can be considered negligible. Note that the differences of G/G^* between hollow circles and solid circles for the same value of γ do not include the contribution of the decrease in void ratio during the stress history of AB. Figs. 15 and 16 show the results of the other test. In that case, the hollow circles between A and B represent the results during the stage test, but solid circles between B and C are those obtained during the stages among which the amplitude of τ_d was always decreasing. It can be seen from these figures that the effects of previous cyclic shear stress history on G/G^* and η are slightly larger than in the cases of Figs. 13 and 14, especially for smaller shear strain amplitudes. This may be due to the fact that for smaller shear strain amplitudes the number of previous cyclic loading at larger shear strain amplitude is larger in the case of Figs. 15 and 16 than in the case of Figs. 13 and 14. It can also be seen from Figs. 15 and 16 that the effects of the stress history with larger cyclic shear strain amplitudes on G/G^* and η are larger for $N=2$ than for $N=10$.

During random earthquake motions, the amplitude of τ_d increases and decreases repeatedly. The variations of shear modulus and damping ratio during radom earthquake motions can be considered to be caused by several factors : (1) the variation of shear strain amplitude, (2) the variation of effective mean principal pressure, (3) the variation of stress ratio

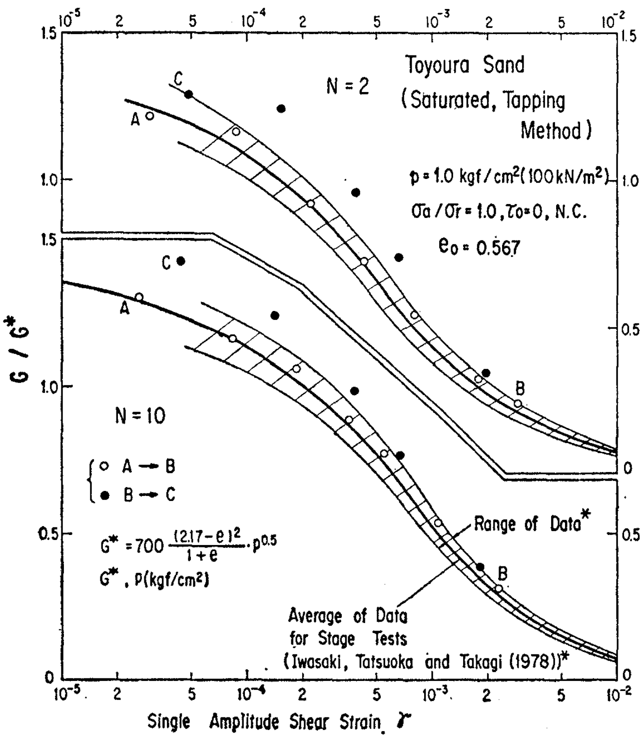

Fig. 15. $G/G^{*}\sim\gamma$ **relationships by** τ_d**-reversing test**

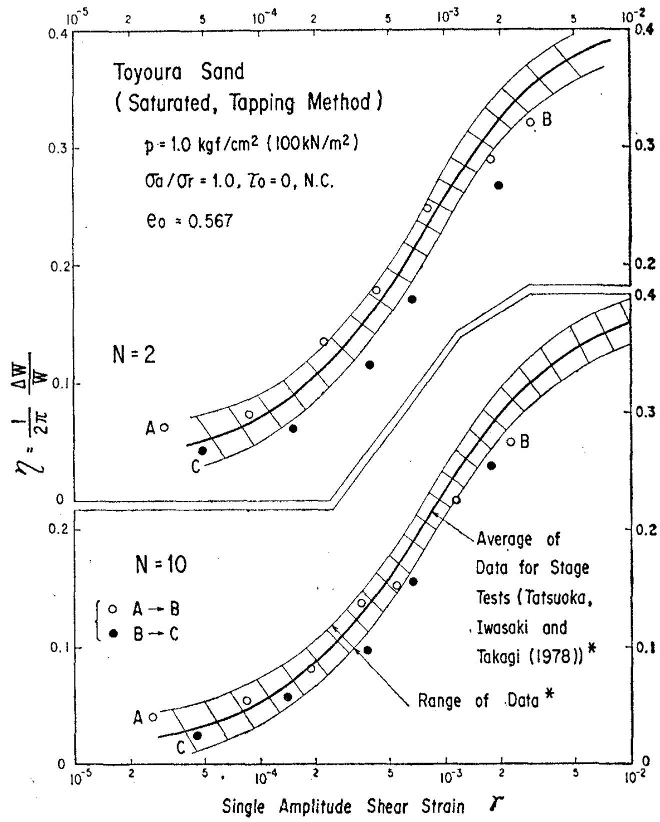

Fig. 16. $\eta\sim\gamma$ **relationships by** τ_d**-reversing test**

and initial shear stress, (4) the variation of void ratio and (5) the previous random cyclic shear stress histories. In the case of saturated undrained loose sand deposits, the first and second factors are most important. As to the last factor, judging from the test results shown in Figs. 13 through 16, its effects can be considered to be less important than those of shear strain amplitude, void ratio (only for shear modulus) and effective mean principal stress.

CONCLUSIONS

To evaluate the effects of static stress conditions and stress histories on equivalent shear modulus and damping ratio of a clean sand for a wide range of shear strain amplitude, a comprehensive series of cyclic torsional shear tests was performed. From the test results and the analyses presented, the followings can be derived :

(1) Under a constant mean principal stress, the effects of stress ratio on shear modulus are more remarkable in the triaxial extension case than in the triaxial compression case. In the latter case, shear modulus can be considered almost independent of stress ratio less than about 4.0 for $\gamma=5\times10^{-5}\sim3\times10^{-3}$.

(2) By applying initial shear stress, shear modulus decreases, especially for the case of $\sigma_a/\sigma_r\geqq1.0$.

(3) Although shear modulus are affected by both stress ratio and initial shear stress, its strain-dependency is almost independent of those factors.

(4) The effects of stress ratio and initial shear stress on damping ratio are less important than shear strain amplitude and mean principal stress for a wide range of shear strain amplitude.

(5) The relationship between η and $G/\{G\}_{\gamma=10^{-6}}$ can be considered almost independent of stress ratio and initial shear stress, except for $G/\{G\}_{\gamma=10^{-6}}$ larger than 0.6 in the case where both τ_0/p larger than 0.1 and σ_a/σ_r larger than 2.0 are applied.

(6) The effects of overconsolidation and cyclic shear stress history with larger shear strain amplitudes on both shear modulus and damping ratio of the clean sand tested are less important than those of effective mean principal stress, void ratio and shear strain amplitude for $\gamma=5\times10^{-5}$ to 5×10^{-3}.

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NOTATION

e_0 = initial void ratio at the beginning of cyclic shearing

$p=1/3(\sigma_a+2\sigma_r)$: mean principal stress

G = shear modulus

G_r = shear modulus of isotropically consolidated specimen

$\{G\}_{\gamma=10^{-4}}$, $\{G\}_{\gamma=10^{-6}}$ = G at $\gamma=10^{-4}$ and G at $\gamma=10^{-6}$, respectively

$G_{\max}$ = G at $\gamma=0$

$G^*=700\dfrac{(2.17-e)^2}{1+e}p^{0.5}$ (G^* and p in kgf/cm^2)

N = number of cyclic loading

OCR = overconsolidation ratio

$SR=\sigma_1/\sigma_3$

R = the ratio of the shear modulus at $SR\neq1.0$ and/or $\tau_0\neq0.0$ to G_r = that at $SR=1.0$ and $\tau_0=0$

τ_0 = initial static shear stress

τ_d = cyclic shear stress

σ_1, σ_3 = major and minor principal stresses

σ_a, σ_r = axial and radial stresses

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