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TECHNICAL NOTES※

COMPARISON OF TRUE AND RESIDUAL FRICTION ANGLES

P. PURUSHOTHAMA RAJ*

ABSTRACT

Both original and shear-induced fabrics have a significant influence on the deformation and strength properties of clays. The shear-induced structure appears to be the prime cause for the formation of constant residual angle irrespective of the stress history or the initial structure. Initiation of shear-induced fabric is controlled by the mobilization of true angle of friction. A comparison of true and residual friction angles from field data is made. The correlations of Bjerrum for residual friction angle and Skempton-Gibson-Bjerrum for true friction angle with plasticity index have been used to get additional data for the comparison. It is found that the residual friction angle is nearly equal to true friction angle for soils with clay content greater than about 40%. Residual friction angle approaches true friction angle at low normal pressure in soils with high percent of montmonillonite.

Key words: residual strength, shear strength, progressive failure, soil structure, cohesive soil, clay minerals
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INTRODUCTION

The works of Skempton (1964) and Morgenstern and Tchalenko (1967 a, b) on laboratory samples and samples from shear zones of landslides have revealed the presence of shear-induced structure at residual state. The influence of known original fabric (or initial structure) has been studied at length upto residual by Morgenstern and Tchalenko (1967 a) and Ramiah and Raj (1971). Further it has been drawn to attention that the shear induced structure seemed to be the primary cause for the formation of constant residual angle, ϕ'_r, irrespective of the stress history or the initial structure.

It is often cited that at or near failure slip planes develop marking the initiation of shear induced fabric and according to Gibson (1953) the appearance of these planes is controlled by the mobilization of Hvorslev's angle of friction. It was thought (Gibson, 1953; Gould, 1960) that the ultimate strength which might control stability of slope was equal to the true angle of friction, ϕ_e, without cohesion. A preliminary study by Ramiah and Raj (1972) on synthetic soils (obtained by mixing bentonite, kaolinite and fine sand in different proportions by weight) showed a favourable comparison (Table 1) of residual friction angles (from reversal shear box on normally consolidated intact specimens) and Hvorslev's true friction angles being determined by adopting the procedure of Crawford

* Associate Professor of Civil Engineering, Bangalore University, Bangalore-560 056, India.
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Table 1. Comparison of angles of friction (Ramiah & Raj, 1972)

Sl. No.	Soil	Percent clay content	Residual Friction Angle, ϕ_r' (degree) Normal pressure-kN/m² 35.16	70.30	105.47	Average+	Hvorslev's True friction angle, ϕ_e (degree)
1	100B : 0K : 0S	71.00	12.40	8.74	6.82	7.00	7.30
2	75B : 25K : 0S	56.00	10.62	8.40	8.25	8.50	11.94
3	50B : 50K : 0S	41.00	11.20	10.50	10.00	10.50	11.68
4	25B : 75K : 0S	26.00	17.80	17.20	14.20	16.60	12.54
5	0B : 100K : 0S	11.00	24.23	23.69	23.23	23.00	22.45
6	0B : 75K : 25S	8.25	23.75	23.30	20.60	22.50	21.25
7	0B : 50K : 50S	5.50	26.80	28.80	26.60	28.50	23.00
8	0B : 25K : 75S	2.75	32.22	33.42	33.42	33.00	33.69
9	0B : 25K : 100S	0.00	42.20	36.65	34.53	35.00	—
10	25B : 0K : 75S	17.75	26.50	25.20	23.50	24.00	15.43
11	50B : 0K : 50S	35.50	18.62	15.47	16.40	17.50	14.31
12	75B : 0K : 25S	53.25	14.60	10.62	8.62	10.50	11.31
13	25B : 25K : 50S	20.50	18.02	17.20	16.01	17.00	12.00
14	25B : 50K : 25S	23.25	16.30	15.40	16.28	16.00	13.05
15	50B : 25K : 25S	38.25	10.80	9.87	9.57	9.50	11.06

Note : +Obtained by plotting shear and normal stresses considering $c_r'=0$
B : Bentonite (LL=400% ; PL=45.75%)
K : Kaolinite (LL=66% ; PL=43.40%)
S : Fine Sand

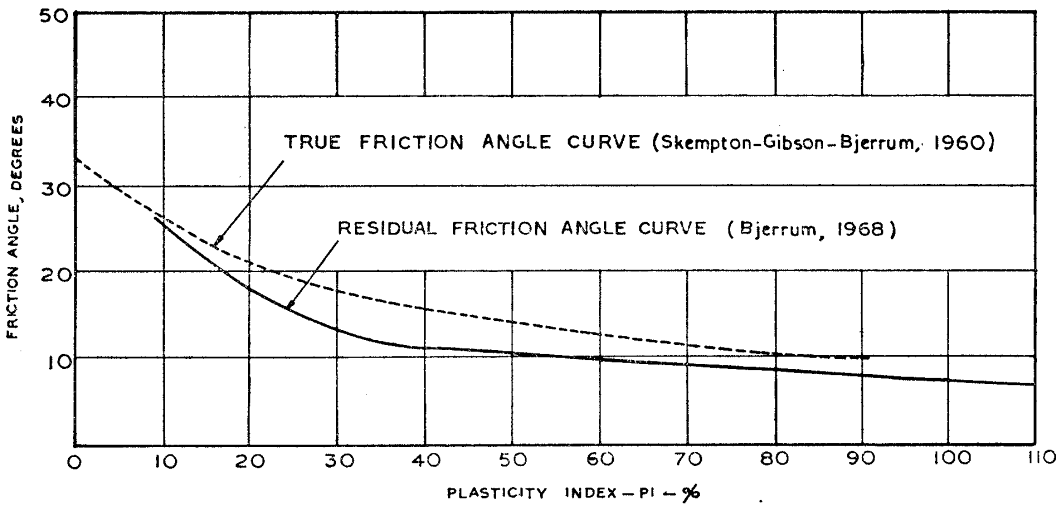

Fig. 1. Variation of friction angle with plasticity index

(1961) on consolidated undrained triaxial tests. Thus, it appears to be appropriate to compare the true angle of friction with residual angle of friction.

COMPARISON OF ANGLES OF FRICTION

The results of Ramiah and Raj (1972) of synthetic soils and the correlations presented by Bjerrum (1968) for residual angle of friction and Skempton-Gibson-Bjerrum for true friction angle with plasticity index, (reported by Bjerrum and Simons, 1960), Fig. 1, showed the scope for the present study.

Thus a carefully made literature survey yielded Table 2 for natural clays with clay content greater than 40% in majority of the cases. In cases where ϕ_e values are not available values are obtained from Skempton-Gibson-Bjerrum correlation. The data from

Table 2. Field data on true and residual friction angles

Sl. No.	Soil type and Source	P. I. %	L. I. %	% Clay con-tent	Type of shear test	Normal Pressure range (kN/m²)	Residual strength range (degree)	True friction range (degree)	Remarks
1	Vienna Clay Austria (Hvorslev, 1960)	25.0		23.0	RS	—	17.50	17.50	
2	Little Belt Clay Denmark (Hvorslev, 1960)	91.0	—	77.0	RS	—	9.00[1]	10.00	1. Kenney (1967)
3	Modelo Clay California (Gould, 1960)	31~37	−0.20 to −0.27	31~34	—	—	12.5[a]	11.50	
4	Seven Sisters Clay Canada (O'Neil, 1962)	76.0	—	—	R_0	—	13.0	11.00[b]	
5	Walton's Wood England Carboniferous mudstone-(slip) (Skempton and Petley, 1967)	31.0	0.10	69	DS	30.76~246.97			
6	Sevenoaks- a) Atherfield Clay-(Slip)	40.0	0.03	51	DS	61.52~246.97 246.97~494.82	15.0 10.0	15.5[b]	
	b) Reworked Atherfield Clay (Slip) (Skempton and Petley, 1967)	40.0	0.08	58	DS	—	16.0	15.5[b]	
7	Jari-Upper Siwalik Clay-(Slip)-W. Pakistan (Skempton and Petley, 1967)	34	−0.23	45	DS	61.79~741.79	12.0	17.0[b]	
8	Sukian-Upper Siwalik Clay (Minor shear)-W. Pakistan (Skempton and Petley, 1967)	32	−0.38	52	DS	61.79~307.62	14.0	17.5[b]	
9	Boom Clay-Belgium (DeBeer, 1969)	52.2	0.00	49	RS	43.95~219.73	19~24	21.0[2]	2. Marivoet (1948)
10	Oxford Clay, England (James, 1971)	25.50	—	40~60	DS	49.22~196.87	16	14.5~19.0[b] 24.1[3]	3. Skempton (1948) from test
11	Brown London Clay England a) Guildford-(Slip)	51	0.04	57	DS DS	30.76~123.05 123.05~215.33	13.8) 11.6)	12.5[4] 14.0[5]	4. Gibson (1953)-modified parameter
	b) Hendon	49		60	DS DS	24.61~190.72 190.72~276.85	14.6) 13.1)	14.5[b]	5. Henkel (1960)
	c) Walthsmetow (Slip) (Bishop et al., 1971)	42		68	DS DS RS RS	55.37~ 81.74 81.74~166.11 6.15~ 61.79 61.79~309.37	15.5) 12.8) 10~14) 8~10)		
12	Blue London Clay (Bishop et al, 1971)	41	0.00	58	DS RS	49.22~553.71 30.76~246.97	13.7) 9.4)	12.5[4]) 14.0[5])	
13	Weald Clay from Arlington (Bishop et al., 1971)	33	−0.06	52	RS RS	30.76~246.97 246.97~650.39	10~16.0) 9~10.0)	17.0[b]	
14	Varved silt-Cod Beck (Morgenstern and Tchalenko, 1967b)	34	—	20~50	DS	—	20	17.0[b]	
15	Silty Clay-Fiddler's Ferry (Morgenstern and Tchalenko, 1967b)	21	—	30	DS	—	24	21.0[b]	
16	Kepur Marl a) Kings Norton	10	0.0	—	TS	—	32~23	26[b]	
	b) Erdington	10	−0.0	—	TS	—	29~22	26[b]	
	c) Bells Lane (Chandler, 1969)	18	0.0	—	TS	—	24~18	22[b]	
17	Lias Clay (Chandler, 1970)	32	0.125	52	TS	—	18.5	17[b]	
18	Gault Clay (Hutchinson, 1969)	31 51	— —	58 50	DS DS	— —	19.0 12.0	17.5[b] 14.0[b]	

Sl. No.	Soil type and Source	P. I. %	L. I. %	% Clay content	Type of shear test	Normal Pressure range (kN/m²)	Residual strength range (degree)	True friction range (degree)	Remarks
19	Mula Clay-India (Datye et al., 1967)	35~57	0.10	—	DS	—	16~18	13.5~17.0[b]	
20	Muda dam Mudstone (James, 1970)	19	0.42	—	DS	274.22~766.40	19.5	21.5[b]	

NOTE : a-Residual angle obtained from Bjerrum's (1968) Chart (Fig. 1)
b-True friction angle obtained from Skempton-Gibson-Bjerrum (1960) Chart (Fig. 1)
DS-Reversal direct shear test
RS-Ring or torsion shear apparatus
R_0-Special Direct shear apparatus of O'Neil (1962)
TS-Triaxial shear with precut specimen

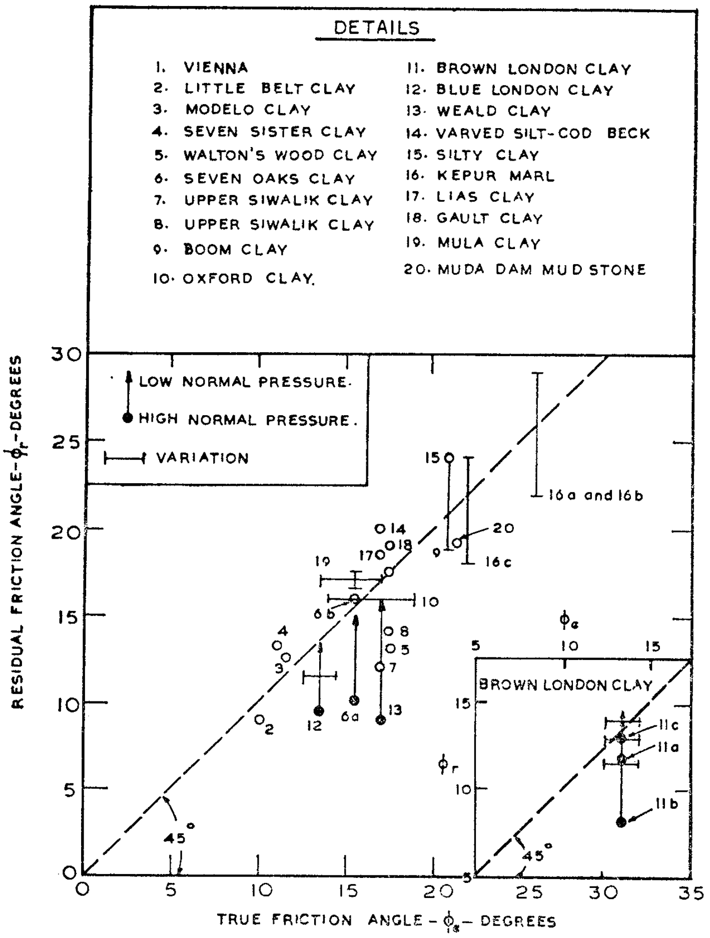

Fig. 2. Comparison of true and residual friction angles

Table 2 is plotted in Fig. 2. The range of true and residual friction angles are also plotted in a few cases. In general, it is found that the true friction angle is almost equal to the residual friction angle. But in clays, probably with high montmorillonite mineral there is a variation in residual angle with normal pressure (e. g. Brown London Clay). It appears that in such cases the residual friction angle at low normal pressures corresponds to true friction angle.

CONCLUSION

The residual friction angle appears to be equal to true friction angle for soils with clay content greater than 40%. In soils with high montmorillonite mineral content the residual friction angle approaches true friction angle at low normal pressure.

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