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BEARING CAPACITY OF RECTANGULAR FOOTINGS ON CLAYS OF STRENGTH INCREASING LINEARLY WITH DEPTH

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ABSTRACT

A complete set of bearing capacity factor for rectangular footings on clays of undrained shear strength ($\phi=0$) increasing linearly with depth was compiled by combining the exact plasticity solution for continuous footings and the slip circle solution for rectangular footings. Effect of increasing strength with depth was expressed in term of the ratio kB/c_0, where k is the rate of increase in strength with depth, c_0 is the strength at the clay surface and B is footing width. It is examined that the value of kB/c_0 likely to be encountered in practice is within the range from 0.2 to 30.

As compared with results of the present analysis, three other kinds of bearing capacity formulae proposed for the case of strength increasing with depth, were found to overestimate the bearing capacity. The maximum extent of overestimation expected in practical problem is almost 300%.

A set of approximate expressions for bearing capacity factor was proposed for convenience of practical use, which is function of the ratio kB/c_0 and the footing shape B/L. The maximum extent of underestimation by these approximate expressions is 5%.

Key words : bearing capacity, cohesive soil, continuous footing, footing, fully saturated soil, shallow foundation, slip surface
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INTRODUCTION

It is well known that the undrained shear strength c_u ($\phi=0$) of clays increases with depth. In the case of normally consolidated and lightly overconsolidated marine clays without dried crust, the strength increases linearly with depth. The bearing capacity problem for these soils, therefore, should be approached by taking into account this strength distribution. Davis and Booker (1973) presented the exact plasticity solution for the bearing capacity of continuous footings on the surface of clays of strength increasing linearly with depth. No exact plasticity solution, however, seems to have been obtained for rectangular footings on clays of such strength distribution.

The slip circle method, on the other hand, has been often used in practice for working out the bearing capacity of clays of inhomogeneous strength. The Author (1966) presented the bearing capacity solution for rectangular footings on clays of strength increasing

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linearly with depth, by the use of slip circle method. As has been pointed out by Davis and Booker, the slip circle method is likely to give an upper bound to the correct solution. In the absence of exact solution for rectangular footings, however, no other method than the slip circle method seems to be available for assessing the influence of footing shape on the bearing capacity.

The purpose of the present paper is to combine two different kinds of solutions, the exact plasticity solution for continuous footings and the solution for rectangular footings by the slip circle method, in order to take into consideration the change in the shape factor for rectangular footings with various linear strength distribution of clays. Until the complete set of plasticity solution becomes available, this sort of combination would be of some value in engineering practice.

PLASTICITY SOLUTION

The case considered in the analysis is shown in Fig. 1. The clay stratum is assumed to be saturated with water table equal to or above its surface. Undrained shear strength c_u is assumed to increase linearly with depth and expressed $c_u=c_0+kz$, $(\phi=0)$. c_0 is the strength at the clay surface and k is the rate of increase in strength with depth, which is equal to the product of the submerged unit weight and the (c/p) value of particular clay. The exact plasticity solution was obtained for continuous footings $(B/L=0)$, with rough and smooth bases respectively. And the bearing capacity q was expressed in the form

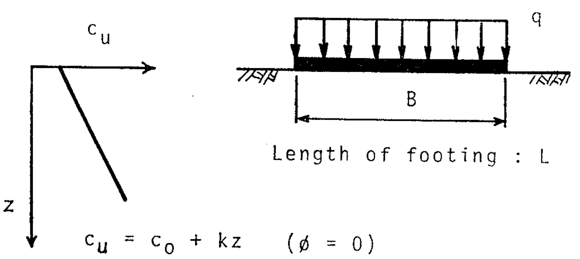

Fig. 1. Condition for the analysis

$$q=F\left[(2+\pi)c_0+\frac{kB}{4}\right] \quad (1)$$

where F is a dimensionless factor depending only in the ratio kB/c_0 (Davis and Booker,

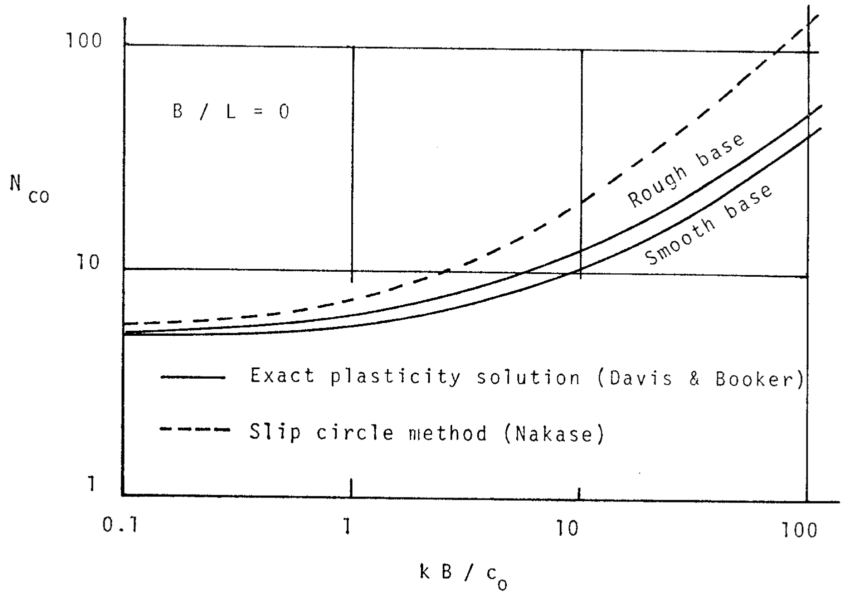

Fig. 2. Bearing capacity factor for continuous footings

1973).

The result of analysis by Davis and Booker was presented in graphs showing relationship between the factor F and the ratio kB/c_0. It is found that in the extreme cases of either $c_0=0$ or $k=0$, the factor F becomes unity irrespective of roughness of the footing base, which gives the bearing capacities

$$\left.\begin{array}{l} q=(2+\pi)c_0 \cdots k=0 \\ q=kB/4 \cdots\cdots\cdots c_0=0 \end{array}\right\} \quad (2)$$

Eq. (1) can be written in the form with the bearing capacity factor

$$q=c_0\left[(2+\pi)F+\frac{F}{4}\frac{kB}{c_0}\right]=c_0 N_{c_0} \quad (3)$$

where N_{c_0} is the bearing capacity factor for continuous footings ($B/L=0$). Solid lines in Fig. 2 show relationship between the bearing capacity factor N_{c_0} and the ratio kB/c_0, which are worked out by the F vs kB/c_0 relationship by Davis and Booker.

SOLUTION BY SLIP CIRCLE METHOD

The slip circle method was employed for obtaining the bearing capacity of rectangular footings on the surface of clays with strength increasing linearly with depth (Nakase, 1966). In this analysis, the slip surface was assumed to be a part of horizontal cylinder emerging from an edge of the footing as shown in Fig. 3(a). In evaluation of the resistance along the two side surfaces, a sort of degree of mobilization was taken into consideration by introducing the reduction factor of r/R as shown in Fig. 3(b). This concept of resistance mobilization was also employed by Tschebotarioff (1951) for the case of rectangular footings on clays of homogeneous strength.

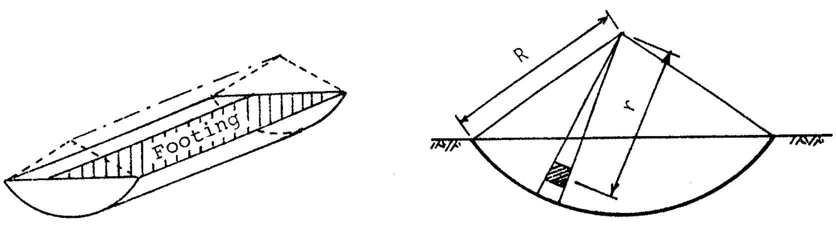

Fig. 3. Key sketch to the slip circle method

The bearing capacity factor for continuous footings ($B/L=0$) obtained by the slip circle method is shown by dashed line in Fig. 2. In the extreme cases of either $c_0=0$ or $k=0$, the bearing capacity of continuous footings by the slip circle method becomes

$$\left.\begin{array}{l} q=5.52\,c_0 \cdots\cdots k=0 \\ q=1.125\,kB \cdots c_0=0 \end{array}\right\} \quad (4)$$

Eq. (4) implies that the slip circle method overestimates the bearing capacity for continuous footings by 7.4% for the cace of $k=0$, and 450% for $c_0=0$.

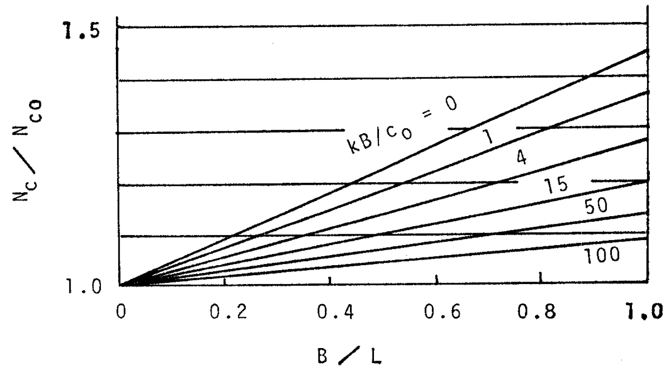

Fig. 4. Change in bearing capacity factor with footing shape (Slip circle method)

Change in the bearing capacity

Table 1. Bearing capacity factor and shape factor by slip circle method

kB/c_0	Bearing capacity factor		Shape factor n
	$B/L=0$	$B/L=1$	
0	5.52	7.95	0.440
0.1	5.77	8.24	0.428
0.2	6.01	8.51	0.416
0.4	6.43	8.99	0.398
0.6	6.82	9.44	0.384
0.8	7.19	9.87	0.373
1.0	7.55	10.28	0.362
2	9.31	12.30	0.321
4	12.52	15.94	0.273
5	14.01	17.64	0.259
6	15.46	19.30	0.248
8	18.30	22.56	0.233
10	21.07	25.71	0.220
15	27.78	33.28	0.198
20	34.25	40.50	0.183
25	40.7	47.6	0.170
30	46.7	54.2	0.161
35	53.0	61.1	0.153
40	59.2	67.9	0.147
50	71.6	81.2	0.134
60	83.8	94.1	0.123
70	96.0	106.7	0.112
80	107.8	118.8	0.102
90	119.4	130.7	0.095
100	131.0	142.4	0.087

factor N_c with the footing shape, B/L, is shown in Fig. 4, where the ratio N_c/N_{c_0} is plotted against B/L for various values of the ratio kB/c_0. From this figure it is found that the bearing capacity factor for rectangular footings may be expressed in the form

$$N_c = N_{c_0}\left[1+n\frac{B}{L}\right] \qquad (5)$$

where n is a dimensionless shape factor depending only in the ratio kB/c_0. In the case of homogeneous strength ($k=0$), the bearing capacity of rectangular footings is

$$q=5.52\,c_0\left[1+0.44\frac{B}{L}\right] \qquad (6)$$

which coincides to the Tschebotarioff's expression. Bearing capacity factor N_{c_0} and shape factor n by the slip circle method are shown in Table 1.

COMBINATION OF TWO SOLUTIONS

The point of this combination is to determine an appropriate value of the shape factor n for the case of homogeneous strength, since this particular case gives rise to the link between the exact solution and that by the slip circle method. Available information of the shape factor n in connection with the plasticity solution is given in Table 2. The shape factor of 0.3, recommended in the first edition of the Terzaghi and Peck's text book, has been widely used in engineering practice. Judging from the information in Table 2, however, let it be assumed that $n=0.2$ in the present analysis.

Table 2. Shape factor n ($c_u=c_0$, $k=0$)

Reference	n	Notes
Terzaghi-Peck (1948)	0.3	Semiempirical
Skempton (1951)	0.22	
Meyerhof (1951)	0.10 0.22	Smooth, circular or square Rough, circular or square
Terzaghi-Peck (1967)	0.2	Semiempirical
Yamaguchi (1969)	0.10	Smooth, circular

Let the bearing capacity factor N_{c_0} for continuous footings be taken that by the exact plasticity solution in Fig. 2. And above mentioned assumption leads to an expression of $N_c=N_{c_0}\left(1+0.2\frac{B}{L}\right)$ for rectangular footings on clays of homogeneous strength. Considering that the shape factor by the slip circle method for the case of homogeneous strength is 0.44 and it is reduced to 0.2, the shape factor obtained by the slip circle method for all other cases of $kB/c_0>0$ shall be reduced to 0.2/0.44.

Thus obtained shape factor n is shown in Fig. 5. In the slip circle method, no distinction is made between the rough base and smooth base. The shape factor in Fig. 5, therefore may be coupled to either of N_{c_0} values by the exact plasticity solution in Fig. 2. Table 3 shows some numericals for the combined solution for bearing capacity factor for rectan-

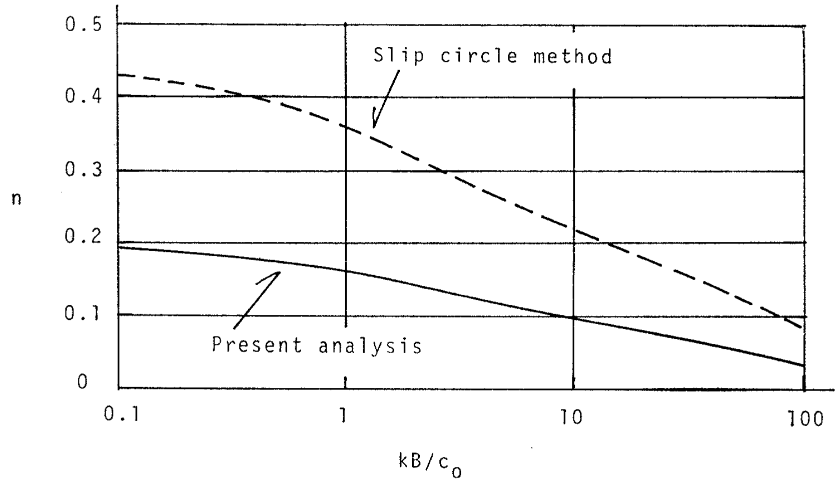

Fig. 5. Shape factor *n*

Table 3. Bearing capacity factor and shape factor

kB/c_0	Bearing capacity factor N_c				Shape factor n
	Rough base		Smooth base		
	$B/L=0$	$B/L=1$	$B/L=0$	$B/L=1$	
0	5.14	6.17	5.14	6.17	0.200
0.1	5.27	6.30	5.21	6.23	0.195
0.2	5.43	6.46	5.28	6.28	0.189
0.4	5.77	6.81	5.43	6.41	0.181
0.6	6.01	7.06	5.57	6.54	0.175
0.8	6.28	7.35	5.72	6.69	0.170
1.0	6.55	7.63	5.84	6.80	0.165
2	7.65	8.77	6.58	7.54	0.146
4	9.21	10.35	7.86	8.83	0.124
5	9.80	10.96	8.35	9.34	0.118
6	10.49	11.68	8.95	9.96	0.113
8	11.71	12.95	9.86	10.91	0.106
10	12.88	14.17	10.72	11.79	0.100
15	15.43	16.82	12.84	14.00	0.090
20	17.85	19.33	14.84	16.07	0.083
25	20.16	21.71	16.75	18.04	0.077
30	22.31	23.94	18.56	19.91	0.073
35	24.38	26.09	20.28	21.70	0.070
40	26.50	28.28	22.02	23.50	0.067
50	30.52	32.38	25.40	26.95	0.061
60	34.74	36.69	28.80	30.41	0.056
70	38.49	40.41	32.15	33.76	0.050
80	42.49	44.44	35.45	37.08	0.046
90	46.30	48.29	38.70	40.36	0.043
100	50.04	52.04	41.90	43.58	0.040

gular footings on clay with strength increasing linearly with depth.

VALUE OF kB/c_0

In the precedings, the effect of increasing strength is expressed in term of the ratio kB/c_0. In normally consolidated or lightly overconsolidated marine clays, the k value is in the range from 0.8 kN/m^3 to 2.0 kN/m^3, which corresponds to the submerged unit weight of clays of 4 kN/m^3 to 5 kN/m^3 and the (c/p) value of 0.2 to 0.4. As for the c_0 value, 2 kN/m^2 to 5 kN/m^2 will be commonly found. If the footing width B of 1 m to

30 m is considered, therefore, the value of the ratio kB/c_0 most likely encountered in practice will be in the range from 0.2 to 30.

COMPARISON WITH OTHER BEARING CAPACITY FORMULAE

In his paper on the bearing capacity of clays, Skempton (1951) recommended that, in the case of inhomogeneous strength with depth, the average value of strength in the range from footing base down to the depth of $(2/3)B$ should be taken as an equivalent strength. In the case of linearly increasing strength, this leads to the equivalent strength of $\bar{c}_u = c_0 + k(B/3)$. In this recommendation Skempton made a remark on the limitation that the use of this equivalent strength should be confined to the case where the strength within the above mentioned range is within ±50% of the average strength. In the case of linearly increasing strength, this limitation corresponds to the range of $kB/c_0 \leq 3$. Applying this equivalent strength, Skempton's expression for bearing capacity factor for rectangular footings becomes

$$N_c = 5.0\left(1+0.2\frac{B}{L}\right)\left(1+\frac{1}{3}\frac{kB}{c_0}\right) \tag{7}$$

In the use of well known Terzaghi's formula, Peck et al. (1953) also referred to the equivalent strength. Their equivalent strength is the average value of strength in the range from footing base down to the depth of B. In the case of linearly increasing strength, this leads to $\bar{c}_u = c_0 + k(B/2)$. They did not mention about the limitation of the use of this equivalent strength. This recommendation was made in connection with the Terzaghi's formula for rectangular footings with rough base and the shape factor of $n=0.3$. In the present comparison, however, in order to be conservative for the present analysis, let it be assumed that the bearing capacity factor for continuous footing is that for smooth base, i. e. $N_{c_0} = 5.14$, and the shape factor is 0.2. The use of $n=0.2$ is based on the second edition of the Terzaghi and Peck's text book (1967). Then new version of the bearing capacity factor by Peck et al becomes

$$N_c = 5.14\left(1+0.2\frac{B}{L}\right)\left(1+\frac{1}{2}\frac{kB}{c_0}\right) \tag{8}$$

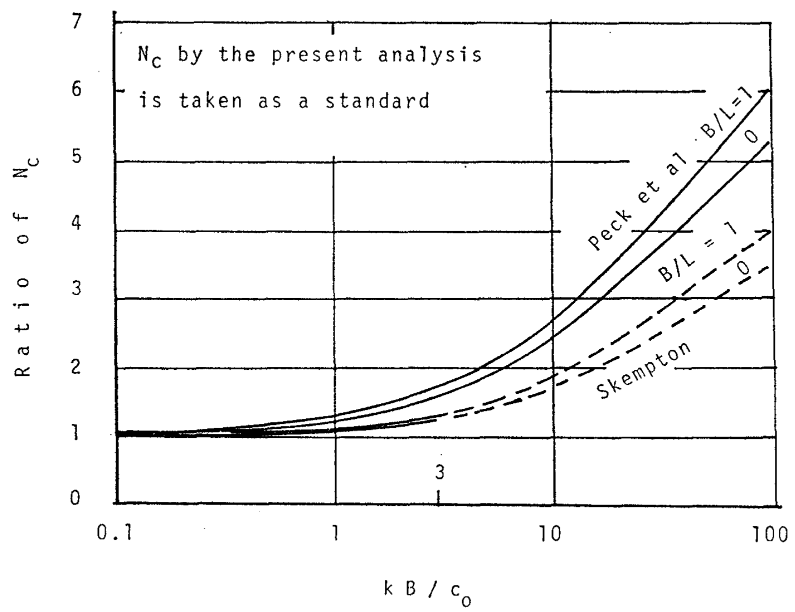

Fig. 6. Comparison of bearing capacity factors

The bearing capacity factors obtained by Eqs. (7) and (8) are compared with that by the present combined analysis in Fig. 6. It is found in the figure that the N_c values by these equations are, in general, larger than that by the present analysis. In the case of Skempton's recommendation, the comparison should be confined in the range of $kB/c_0 \leq 3$. Then it may be said that the Skempton's formula overestimates the bearing capacity by 20% at maximum. On the other hand, the recommendation by Peck et al is found to seriously overestimate the bearing capacity. The extent of overestimation is found to increase with the ratio kB/c_0, and it becomes almost 300% at kB/c_0 of 30, which is considered the upper limit of the ratio encountered in practice.

In their paper on the plasticity solution of bearing capacity of continuous footings on clays of strength increasing linearly with depth, Livneh and Greenstein (1973) recommended to use the strength at the depth of $0.4B$ as an equivalent strength. This equivalent strength is in between those recommended by Skempton and Peck et al.

APPROXIMATE EXPRESSIONS FOR BEARING CAPACITY FACTOR

Values of the bearing capacity factor N_c can be obtained either from Figs. 2 and 5 or from Table 3. In engineering practice, however, the approximate expression for the N_c value would be useful. For this purpose the N_{c_0} vs kB/c_0 curve and n vs kB/c_0 curve are approximated by three straight lines for ranges of kB/c_0 of 0 to 4, 4 to 30 and 30 to 100 respectively. By this approximation, the bearing capacity factor N_c for rectangular footings can be expressed as a function of the ratio kB/c_0 and the footing shape B/L as shown in Table 4. In the Table, the maximum extent of underestimation by these expressions is also shown. Considering the nature of stability problem in clays, underestimation up to 5% may be permissible in engineering practice.

Table 4. Approximate expressions for N_c

Rough base

Range of $\frac{kB}{c_0}$	Approximate expressions	Max. extent of under-estimation $\frac{B}{L}=0$	$\frac{B}{L}=1$
0— 4	$N_c=\left[5.14+1.018\left(\frac{kB}{c_0}\right)\right]\left[1+\left\{0.200-0.019\left(\frac{kB}{c_0}\right)\right\}\frac{B}{L}\right]$	4%	5%
4— 30	$N_c=\left[7.19+0.504\left(\frac{kB}{c_0}\right)\right]\left[1+\left\{0.132-0.002\left(\frac{kB}{c_0}\right)\right\}\frac{B}{L}\right]$	4%	3%
30—100	$N_c=\left[10.43+0.396\left(\frac{kB}{c_0}\right)\right]\left[1+0.057\frac{B}{L}\right]$	2%	1%

Smooth base

Range of $\frac{kB}{c_0}$	Approximate expressions	Max. extent of under-estimation $\frac{B}{L}=0$	$\frac{B}{L}=1$
0— 4	$N_c=\left[5.14+0.680\left(\frac{kB}{c_0}\right)\right]\left[1+\left\{0.200-0.019\left(\frac{kB}{c_0}\right)\right\}\frac{B}{L}\right]$	1%	0%
4— 30	$N_c=\left[6.20+0.412\left(\frac{kB}{c_0}\right)\right]\left[1+\left\{0.132-0.002\left(\frac{kB}{c_0}\right)\right\}\frac{B}{L}\right]$	4%	3%
30—100	$N_c=\left[8.57+0.333\left(\frac{kB}{c_0}\right)\right]\left[1+0.057\frac{B}{L}\right]$	1%	1%

CONCLUDING REMARKS

A set of bearing capacity factor for rectangular footings on clays of strength increasing linearly with depth was compiled by combining the exact plasticity solution for continuous footings and the slip circle solution for rectangular footings. Until the complete set of exact plasticity solution becomes available, this combined solution could be used in actual clays, where the undrained shear strength increases with depth.

As compared with result of the present analysis, three kinds of bearing capacity formulae, so far proposed for the case of clays of strength increasing with depth, were found to overestimate the bearing capacity. The extent of overestimation increases with the kB/c_0 and it becomes almost 300% at $kB/c_0=30$, which is considered the upper limit of this ratio encountered in practical problems.

Approximate expressions for the bearing capacity factor were proposed. These expressions underestimate the bearing capacity by up to 5%. Considering the nature of stability problem in clays, however, this extent of underestimation may be permissible in engineering practice.

Footings on the surface of clays were considered in the present analysis. For the case of footings with embedment depth, results of the present analysis could be readily used with concept of the net loading on the footing, provided that the embedment depth is within the extent for the shallow foundation.

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NOTATION

B=width of footing
c_0=undrained shear strength at surface of clay stratum
$\bar{c}_u$=undrained shear strength
c_u=average value of c_u
F=dimensionaless factor depending only in kB/c_0
k=rate of increase in undrained shear strength with depth
L=length of footing
n=shape factor depending only in kB/c_0
N_c=bearing capacity factor for rectangular footing
N_{c0}=bearing capacity factor for continuous footing
q=ultimate bearing capacity
R=radius of slip circle
r=radius vector in side faces of cylindrical slip surface
z=depth below ground surface
ϕ=angle of shear resistance

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