by Eq. (17). In the case of which it becomes positive, we can not virtually find the value of $(t/r^2)_i$ since $u_i$ in Eq. (20) is defined as the value of which $g$ changes from negative to positive.

# ESTIMATION OF SOIL PARAMETERS BASED ON MONITORED MOVEMENT OF SUBSOIL UNDER CONSOLIDATION*

Discussion by ANNAMARIA CIVIDINI**

The authors have to be congratulated for their very interesting study on the the back analysis of the average elastic constants and permeability coefficients of soft soil deposits. The study contains applications to hypothetical cases (uniform and layered deposits, considering several location of the measurement points) and to actual cases. The problem is solved in the framework of the so-called deterministic approach.

In the analysis of the actual embankment projects, the consolidation process is divided in various period of time and the soil parameters are assumed constant in each period. As the authors themselves suggest, the procedure can provide an insight in the variation of the average elastic parameters during soil consolidation.

The authors note also that "…In actual cases one may not be able to get the measured quantities at all the time steps. However it is not difficult to assess the missing quantities by adeguate interpolation." This may suggest a fictitious increase in the number of data, leading also to the possible propagation of the errors that can affect some of the measured quantities.

In general, the authors note that :

- the results of the back-analysis depend on the number of in-situ measurements and on the location of the measurement points ;
- the pore pressure data may little contribute to improvement of the accuracy of the estimated parameters ;
- it is necessary to define the minimum number of input data needed for reasonable estimates of soil parameters.

In the present discussion some aspects of an alternative, probabilistic approach for back-analysis are briefly recalled. Its applicability to the "design" of in-situ tests is outlined and the results obtained in the interpretation of the field measurements of an actual case are summarized.

The so-called Bayesian approach, belonging to the class of probabilistic procedure, (see e. g. Beliveau, 1974 ; Asaoka, 1979 ; Cividini et al, 1983) is applicable to problems involving a sparsity of data and a relatively large number of unknowns. In addition it allows to consider the following points that are of relevant interest in practice (see e. g. Brown and Burland, 1983 ; Shimizu and Sakurai, 1983 ; Cividini et al, 1981) :

- the different accuracy of the field instruments ;
- the initial estimate (or "a priori" information) of the soil mass properites derived from previous experience, the engineering judgment or preliminary in-situ and laboratory tests ;
- the improvement of the parameter accuracy with respect to the accuracy provided by the initial estimate, that can be seen as a measure of the effectiveness of the measurement program.

In this approach the final parameter estimate (and their reliability) not only depends on the experimental data $\boldsymbol{x}^M$, and on the "a priori" information $\boldsymbol{P}_0$, but it is also related to the "uncertainties" of the two sets of data, represented by the covariance matrices $\boldsymbol{C}_u$ and $\boldsymbol{C}_p{}^0$.

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** Associate Professor, Department of Structural Engineering, Technical University (Politecnico) of Milan, Milan, Italy.

Since the dependence of the response vector $\boldsymbol{x}^m(\boldsymbol{P})$ on the parameter $\boldsymbol{P}$ is in general non linear an iterative procedure based on the linearization of such relation can be adopted for the numerical solution. When the final parameter vector $\boldsymbol{P}_f$ has been determined, it is possible to calculated the "a posteriori" covariance matrix $\boldsymbol{C}_p{}^f$. This requires the evaluation of the "information" matrix (Beliveau, 1974 ; Cividini et al, 1983),

$$\boldsymbol{Q}=(\boldsymbol{L}^T(\boldsymbol{P}_f)\boldsymbol{C}_u{}^{-1}\boldsymbol{L}(\boldsymbol{P}_f))$$

which in turn depends on the uncertainties of the experimental data (matrix $\boldsymbol{C}_u$) and on the sensitivity of the responce vector $\boldsymbol{x}^m(\boldsymbol{P}_f)$ to parameter variations, i. e. on matrix $\boldsymbol{L}$ $(\boldsymbol{P}_f)$. If the entries of the "information" matrix are, in the average, negligible with respect to those of the inverse of the initial covariance matrix, this implies that the measurements do not provide an useful information for the back-analysis. In addition the inverse of the "information" matrix permits to assess the effectiveness of the experimental program with respect to :

a) the instrument accuracy,
b) the number of measurement points, and
c) the location of measurement points.

The "information" matrix can be used in planning, or "designing" the in-situ measurement program (Cividini et al, 1983), according to the following procedure :

1) a suitable set of parameters $\boldsymbol{P}_t$ is adopted in the analysis of the geotechnical problem ;
2) the results of this analysis are used to generate sets of fictitious experimental data. Each set corresponds to a possible measurement program. Some of these sets contain the same number of measurements, but refer to different locations, while others have a different number of data ;
3) for each set of data several back-analyses are carried out by varying the instrument accuracy, i. e. matrix $\boldsymbol{C}_u$ ;
4) the comparison between the back-analyzed parameters and the "true" parameters $\boldsymbol{P}_t$ provides a first measure of the effectiveness of each possible measurement program ;
5) as previously observed, the comparison between the information matrix and the inverse of the initial covariance matrix permits to assess the effectiveness of each measurement program. Specifically, one can assess whether each experimental program provides a significant information on the values of the sought parameters;
6) if large (co)-variances are assumed in the initial estimates for the fictitious back-analyses, the diagonal elements in the inverse of the information matrix coincide with the variances of the estimated parameters. Then, they can be used to set up diagrams relating the scattering of the estimated parameters to the aforementioned factors (a), (b), (c), and consequently, to assess the effectiveness of the possible experimental programs.

The mentioned Bayesian approach has been used in the numerical calibration of a geotechnical system (soft deposit with sand drains under a railway embankment) on the basis of a series of in-situ measurements, (Cancelli and Cividini, 1984). The back-analysis was performed by subdividing the soil mass into zones having different mechnical properties, but considering as unknowns only the permeability coefficients (in the vertical and horizontal directions) and the elastic mo-

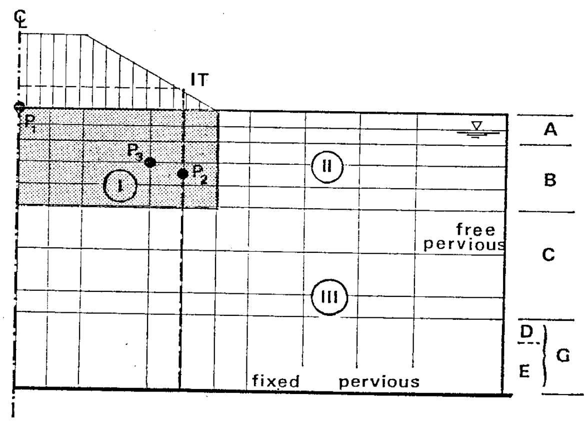


**Fig. 21. Finite element mesh and boundary conditions (a dark area represents the zone with vertical sand drains)**

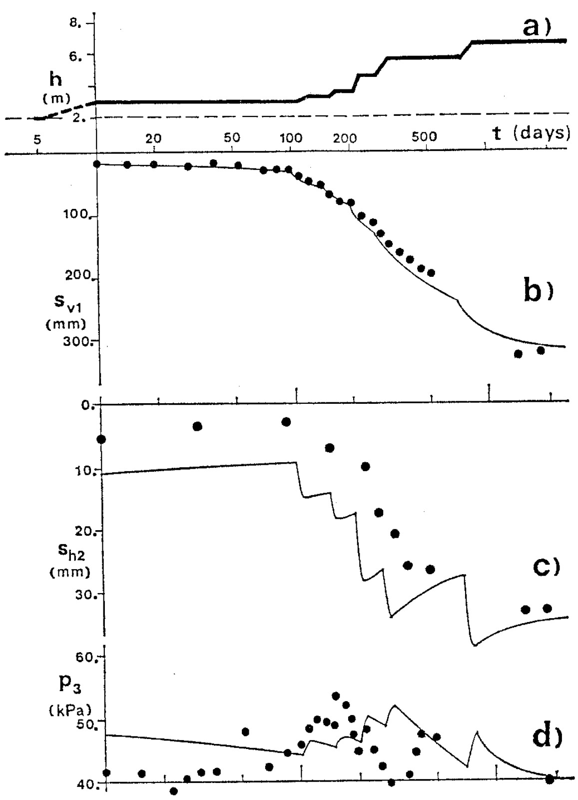


**Fig. 22. Consolidation analysis :**
**a) embankment height ;**
**b) surface settlement (at point $P_1$ in Fig. 21) ;**
**c) lateral movement (at point $P_2$ in Fig. 21) ;**
**d) pore water pressure (at point $P_3$ in Fig. 21) ;**
**(black dots : experimental data)**

dulus for each zone, (cfr. Fig. 21).

Two calibration analyses were carried out separately, both of them based on the finite element discretization of the geotechnical system. The first one led to the optimal values of the elastic moduli of the various zones in which the soft deposit was subdivided (under the assumption of linear elastic and isotropic material behaviour for each zone), while the second one concerned the determination of the permeability coefficients of zones I, II and III (cfr. Fig. 21), both in the isotropic and in the anisotropic cases.

The covariance matrix of the measurement errors $\boldsymbol{C}_u$ was assumed to be diagonal, and its non vanishing terms were defined taking into account : the instrument accuracy ; the lack of information on the seasonal variation of the water table level ; the approximation introduced by assuming the horizontal measurements to be uncorrelated. Note that the initial values of the permeability coefficients were assessed on the basis of the laboratory results and of the presence of sand drains, and that the corresponding initial variances accounted for the large uncertainties in the initial choice.

The results of the analyses show that :

1) the uncertainties of the elastic moduli of layers *B* and *G* (expressed as non dimensional ratios of the variances to the square of the corresponding parameters) are lower, and hence their estimated values are more reliable, than those of layers *A* and *C* ;
2) the displacement boundary conditions on the bottom and right sides of the mesh have a non negligible influence on the final value of the elastic modulus of layer *G* and on its variance ;
3) the attempt to back-analyze separately the elastic coefficients of zones I and II of layers *A* and *B* did not lead to meaningful results. In fact, the difference between the initial and final values of the elastic parameter variance for the zone without sand drains turned out to be very small, thus denoting a limited (negligible) influence of this parameters on the response vector $\boldsymbol{x}^m$ ;
4) the permeability of zone II cannot be evaluated with sufficient accuracy on the basis of the available data (in fact the in-situ measurements are mainly influenced by the vertical permeability of zone I) ;
5) the determination of the vertical permeabilities is more accurate than that of the horizontal ones.

In order to assess the behaviour of the back-calibrated numerical model and of the real geotechnical system, the estimated parameters were adopted in a coupled (two phase) analysis simulating the construction process of the embankment and the results are reported in Fig. 22. A satisfactory agreement was obtained between the numerical results and the corresponding in-situ measurements, but for the pore pressures. In fact, the data from the installed piezometers were affected by the seasonal variation of the water table level the value of which is not known with sufficient precision.

In conclusion the Bayesian approach can be adopted in the estimation of the soil parameters from in-situ measurements and allows to include in the numerical analysis the following factors :

- the different accuracies of the various instruments used in the field ;
- the "a priori" information of the parameters to be estimated ;
- the degree of uncertainties of "a priori" estimate ;
- the degree of uncertainties of the optimal values of the parameters, (in fact this information can be taken into account in the analysis of further engineering works in the same area, on the basis of the same mathematical model) ;

and in general, the Bayesian procedure can be efficiently adopted in order to "design" and to optimize the program of in-situ measurements with respect to the type of parameters to be evaluated and to the accuracy of the chosen instruments.

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# ESTIMATION OF SOIL PARAMETERS BASED ON MONITORED MOVEMENT OF SUBSOIL UNDER CONSOLIDATION*

Discussion by TAKASHI MATSUMOTO** and KIICHI SUZUKI***

It is very important for us, construction contractors to predict the consolidation deformation accurately in order to construct structures safely and exactly. However, so far, computed deformation using the parameters obtained from laboratory test and in-situ test often differ substantially from observations as the authors pointed out.

With the outmost interest the writers have read the contribution of the authors concerning back analysis of consolidation parameters. Especially, the writers have been interested in the followings : 1) The horizontal permeability is not important for consolidation problem. 2) The measured displacements are useful in estamating the soil parameters, whereas measured data only of the pore water pressure do not provide reasonable estamates of soil parameters. 3) The yielding of clay with the increase in shear stress is more significant than the strengthening due to consolidation.

One of the writers, Matsumoto, has ob-

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** Chief, Information Processing Center, Kajima Corporation, Minato-ku, Tokyo.
*** Research Engineer, Information Processing Center, Kajima Corporation, Minato-ku, Tokyo.