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AN ELASTIC ANALYSIS FOR PILED RAFT FOUNDATIONS IN A HOMOGENEOUS SOIL

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ABSTRACT

A boundary element analysis based on an elastic theory is performed to analyse the behaviour of piled raft foundations subjected to vertical load. Characteristics of settlement and load transfer for piled raft foundations whose raft rests on a homogeneous isotropic elastic half-space soil are contrasted with free-standing pile groups and single piles. The critical length of pile groups for a vertical loading (the length beyond which settlement shows little further decrease) is longer than that of single piles whose dimensions are the same as the piles in the groups. The reduction of the settlement caused by the presence of the raft is very small, although the raft transmits 20-40% of the applied load direct to the soil. A significant difference of pile load is recognized at the upper part of the shafts between two kinds of pile group with and without raft, and only small difference is found at the remaining part of the shafts. The ratio of long-term settlement to total final settlement of pile groups is greater than that of single piles. The contact earth pressure on the raft is relatively uniform in the inside area surrounded by piles.

Key words : elasticity, pile, pile group, raft foundation, settlement, vertical load (**IGC** : E 2)

INTRODUCTION

In a conservative method of design for pile foundations, once the decision to introduce piles to a raft foundation has been made because the differential settlement of the raft alone was large, the applied load is considered to be supported by piles and the capacity of the raft is ignored. A contribution of the raft resting on a soil surface to the capacity of the pile group or against the group settlement could be taken into account in design of piled raft foundations. Valuable measurements of load distribution for piles and rafts were made by Hooper (1973), Cooke et al. (1981) and Kakurai et al. (1987). Settlement reducing piles introduced by Burland et al. (1977) can be a means to reduce the differential settlement of shallow foundations using friction piles whose shaft capacity may be fully mobilized under working conditions. However, both the manner of the load transfer from the foundation to the soil and the settlement characteristics of piled raft foundations are not well understood due to the complex system consisting of the piles, the

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raft and the surrounding soil.

A few analytical efforts based on an elastic solution by Mindlin (1936) have been reported to analyse the behaviour of piled raft systems. A boundary element procedure to predict the settlement of piles and pile groups has been developed by Poulos and Davis (1968) and Poulos (1968), and approximate solutions for piled rafts were obtained by Davis and Poulos (1972) based on the interaction between single piles with rigid circular pile caps. Raft flexibility was considered by Hain and Lee (1978) combining the finite element method for the raft with the interaction factor procedure for the pile group. These two approximate methods, involving the use of interaction factors, cannot give more detailed information ; i. e., the stress distribution along a pile shaft and beneath a raft cannot be obtained. A rigorous boundary element procedure for pile groups with rigid rafts was reported by Butterfield and Banerjee (1971). Some case studies indicated that a large amount of the applied load is taken by the raft, although the settlement of the group is little affected by the presence of the raft on the soil surface.

The purpose of this paper is to investigate the settlement behaviour of piled raft foundations and the mechanism of load transfer from the foundation to the soil under working load conditions. The procedure employed is a boundary element technique developed by Poulos, but the present method enables one to determine the stresses acting on all elements of the piles and the raft.

METHOD OF ANALYSIS

The method employed in this paper to analyse the behaviour of pile groups and piled rafts is an extension of the procedure for single piles by Mattes and Poulos (1969). The influence factors for a pile group should be evaluated considering the effects of the other piles or the raft ; therefore, the number of equations to be solved may be large. For this analysis, a special technique such as an interaction factor need not be introduced. Then, the basic procedure will be outlined briefly here.

The pile group consists of N identical elastic piles in which each pile has a length, L, diameter, d, and Young's modulus, E_p, and is divided into n_s shaft elements and n_b base elements. The piles are fixed to a rigid raft which is divided into n_c rectangular elements. The surrounding soil is assumed to be a homogeneous isotropic half-space, having Young's modulus, E_s and Poisson's ratio, ν_s. In the following, a pile group in which the raft directly touches the surface of soil is referred to as "the piled raft (foundation)" and a pile group in which the raft does not touch the soil surface is referred to as "the free-standing (pile) group."

The vertical displacement of the soil adjacent to the piles and the raft due to the stress $\{p\}$ distributed on the pile shaft, the base and the raft is expressed as :

$$\{_s\rho\} = \frac{d}{E_s}[I_s]\{p\} \qquad (1)$$

where $\{_s\rho\}$=soil displacement vector, $\{p\}$= vertical stress vector on pile/raft-soil interface, $[I_s]$=vertical displacement influence factor matrix. The pile displacement is described as the sum of the pile tip displacement and the compression of the pile between the point considered and the tip due to the vertical stress on the pile. The displacement of the foundation and the adjacent soil are equal when slip or local yield on the interface does not occur. The addition of a vertical equilibrium equation for the total system allows all stresses on the interface and the vertical displacement of the raft to be solved.

The characteristics of settlement and load transfer of pile groups are evaluated with several parameters ; e. g., number of piles, N, pile length to diameter ratio, L/d, pile spacing to diameter ratio, s/d, relative stiffness of pile to soil, $K=(E_p/E_s)R_A$ where R_A is the ratio of area of pile section to area bounded by outer circumference, and soil Poisson's ratio, ν_s. Any consistent set of units can be used in the computer program, so that calculation was made without using

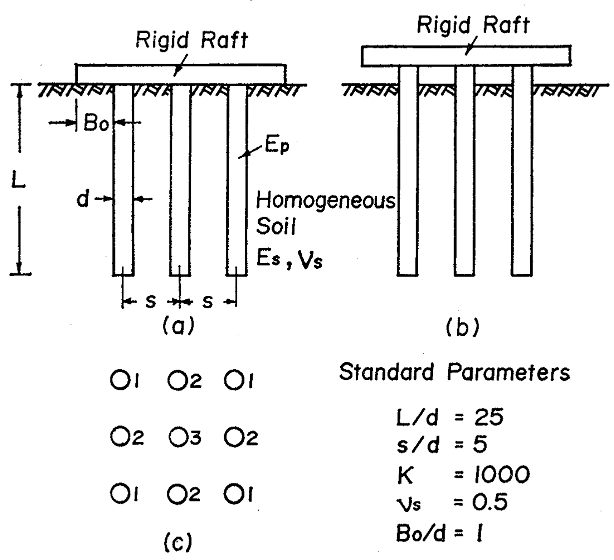

Fig. 1. Standard pile groups analysed. (a) piled raft, (b) free-standing group, (c) key piles

a specified unit. That is equivalent to the calculation using non-dimensional parameters, L/d, s/d etc. Only a square configuration of pile group is examined, using symmetry to reduce the number of equations. The standard values adopted were $N=3\times3$, $L/d=25$, $s/d=5$, $K=1000$, $\nu_s=0.5$ and $B_0/d=1$ where B_0 indicates the width of the overhang area of the raft extended from the surface of the outer piles. In this paper, these values are used unless otherwise specified. The standard pile groups analysed in this paper are illustrated in Fig. 1. While there are many combinations of the above factors, the effects of those are examined by varying one factor and keeping the others constant.

In the following analysis, each pile shaft is divided into ten equal elements and a single element is used for a base. The size of the raft elements in the area enclosing the adjacent piles is double the diameter of the pile, while the elements of the overhang are one half the diameter of the pile, as shown in Fig. 10. The effects of pile-soil slip, non-homogeneity of soil and end-bearing or underreamed piles are not considered in this paper.

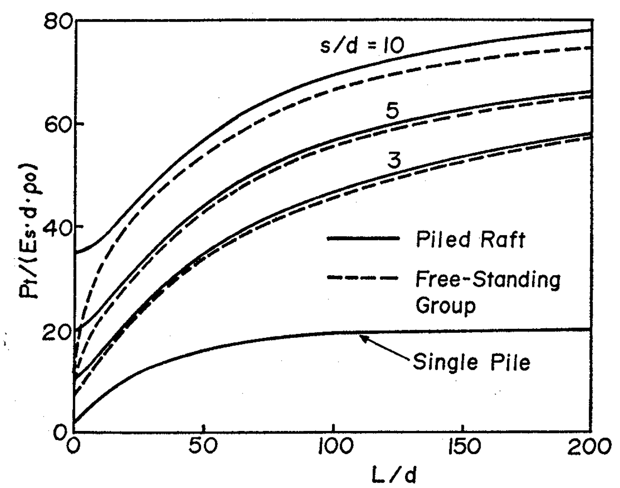

Fig. 2. Dimensionless load-settlement ratio of pile groups

SETTLEMENT OF PILE GROUPS

Load-Settlement Ratio

Fig. 2 shows the dimensionless load-settlement ratio (or pile group stiffness) of piled rafts and free-standing groups consisting of 3×3 piles with $K=1000$ for a wide range of pile length-diameter ratios. The pile group stiffness is represented as $P_t/(E_s d\rho_0)$, where P_t is total applied load on a raft and ρ_0 is vertical displacement of the raft. It can be seen that the load settlement ratio is little affected by the presence of a raft resting on a soil surface. For example, the stiffness of a piled raft with $L/d=25$ and $s/d=5$ is only 4% greater than that of a free-standing group. The small influence of the raft on the group settlement has been pointed out by Whitaker (1960) in the early study of pile groups and has been supported also experimentally by Wiesner and Brown (1978) and Banerjee and Butterfield (1981), and theoretically by Butterfield and Banerjee (1971) and Davis and Poulos (1972).

The load-settlement ratio is slightly greater in piled rafts than in free-standing groups due to the resistance of the soil beneath the raft. The difference of the stiffness between those two pile groups becomes larger for widely-spaced piles where the raft can be more competent against settlement and for short pile groups whose behaviour is similar to that of a raft without piles. However, this slight difference can be negligible for the commonly used spacing of piles in practice.

The effect of the overhang area of the raft on pile group stiffness is negligibly small. Some solutions for piled rafts with $L/d=25$ and $s/d=5$ indicate that the settlement of a piled raft with $B_0/d=1$ is only 2% less than that of the piled raft without overhang.

As L/d increases, the increment ratio of stiffness decreases ; i. e., a critical pile length – the increase of pile length has no or little effect on the settlement beyond this length – can be recognized. Fig. 2 shows that the critical length in a pile group is much greater than that of a single pile whose dimensions are the same as a pile in the group. The critical length may be affected by the spacing of piles, but the influence is not significant as long as the spacing is less than ten pile diameters. The presence of the raft has only a small effect on the critical length as compared with a free-standing group.

Immediate and Long-Term Settlement

The immediate settlement is calculated by using the undrained value of soil Young's modulus and Poisson's ratio of 0.5. The final settlement is calculated by using the drained modulus and drained Poisson's ratio, ν'. Fig. 3 shows the ratio of the immediate settlement, ρ_i, to the total final settlement, ρ_{TF}, for pile groups and single piles. It can be shown that the immediate settlement of the single pile whose length-diameter ratio is greater than 10 is more than 90% of the total final settlement for drained Poisson's ratio of 0.3 and more than 80% for the Poisson's ratio of 0.0. Consolidation settlement can be ignored for single piles because the settlement is predominantly caused by shear deformation of the surrounding soil.

For pile groups, however, the consolidation settlement is more important than for single piles because the soil under the bases

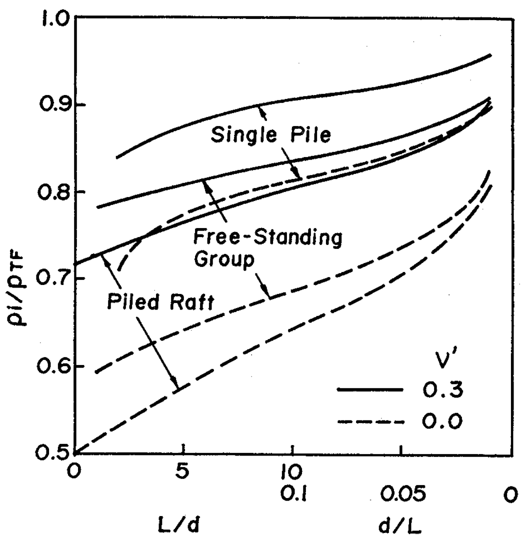

Fig. 3. Proportion of Immediate settlement to total final settlement

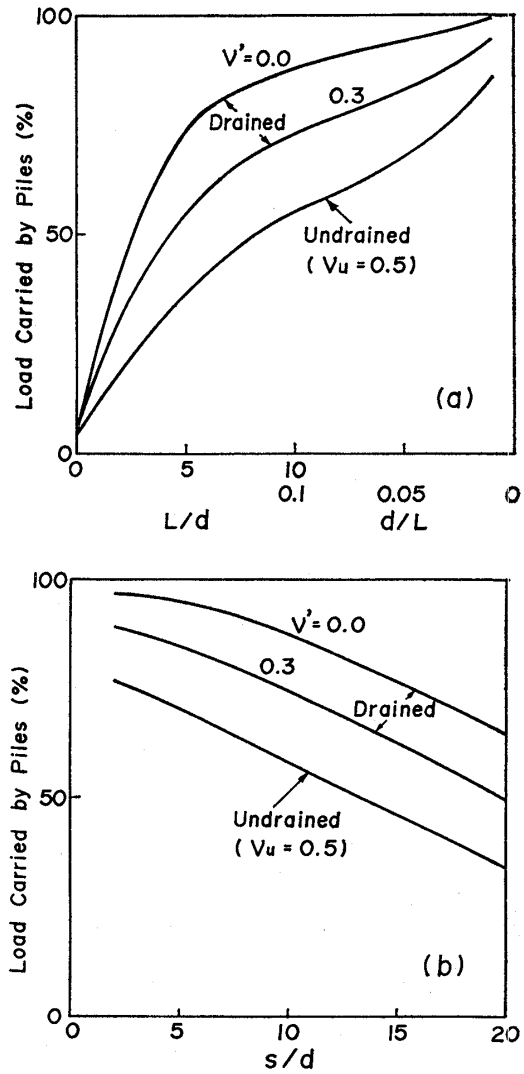

Fig. 4. Load carried by piles under drained condition. (a) influence of pile length, (b) influence of pile spacing

of the pile group is subjected to the compressive stress over a wide and deep zone. Fig. 3 shows the ratio of consolidation settlement to total final settlement for pile groups is about twice that for single piles. The difference of the ratio between piled rafts and free-standing groups is relatively small except for short pile groups. The proportion of the immediate settlement to the total final settlement may be affected by the number of piles, pile length-diameter ratio, pile spacing and other factors, but a limiting value of 0.5 can be given at a raft foundation without piles for $\nu'=0.0$ and 0.72 for $\nu'=0.3$.

LOAD TRANSFER FROM FOUNDATION TO SOIL

Load Carried by Piles and Rafts

Load carried by piles in piled rafts for both drained and undrained conditions is shown in Fig. 4 (a) and (b) against L/d and s/d. It can be seen that a considerable amount of the applied load is carried by the raft under the undrained condition especially for short widely-spaced piles. A high contact pressure on the raft can be induced by incompressible soil which resists the downward movement of the rigid raft. Consequently, the pile load in the piled raft is considerably smaller than that in the free-standing pile

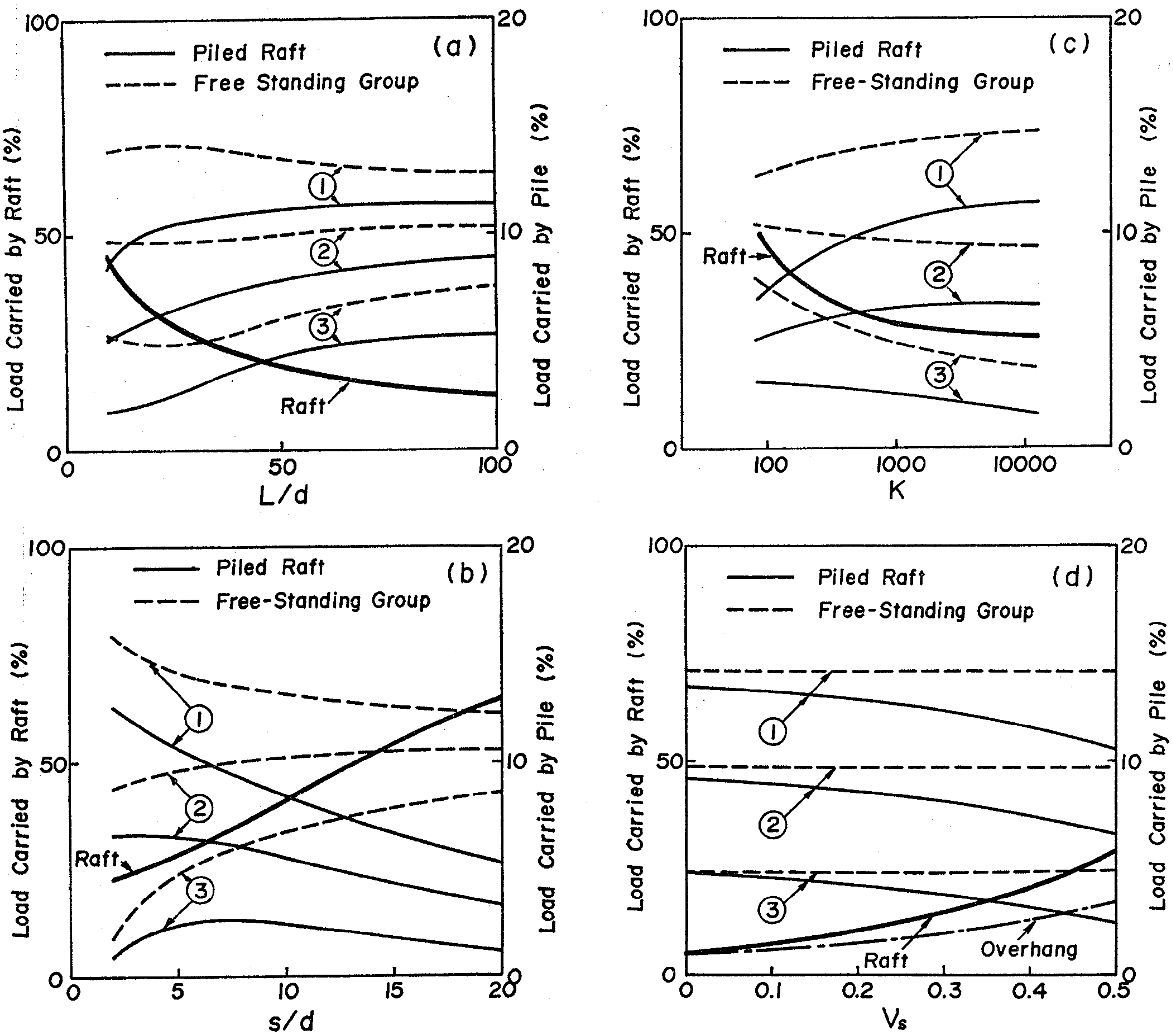

Fig. 5. Load carried by raft and three reference piles. (a) influence of pile length, (b) influence of pile spacing, (c) influence of pile stiffness, (d) influence of soil Poisson's ratio

group. This difference is noteworthy in spite of a small difference of settlement between these two types of pile groups as shown in Fig. 2.

For the drained condition, on the other hand, the proportion of load carried by the raft is reduced and is affected by the drained Poisson's ratio. The reduction is about 20–30% of the total applied load in the case of the drained Poisson's ratio of 0.0.

Fig. 5 shows the percentage of the total applied load carried by the raft and the individual piles in a group. The corner, edge and centre piles in a 3×3 pile group are numbered 1, 2 and 3 as shown in Fig. 1 (c). The difference of the pile head load between piles is remarkable for both types of pile groups. The thick line in Fig. 5 (a) shows the effect of pile length on the load supported by the raft in piled rafts. The raft load changes with the pile length ; the longer the piles, the more load is taken by the piles. Fig. 5 (a) also shows that the effect of pile length on the individual pile load in free-standing groups is basically small, while the difference of pile head load between the piles decreases as the pile length increases.

Fig. 5 (b) shows the effect of the pile spacing of piled rafts on the load carried by the raft. It is apparent that a large load is taken by the raft for a widely-spaced pile groups ; more than 50% of the total applied load is carried by the raft for $s/d>15$. But for the spacing commonly used - s/d less than 10 -, the raft load is 20–40% of the total applied load. This value is less than that in a previous work by Butterfield and Banerjee (1971) whose results suggest that 40–60% of the total load is taken by the base of the raft, although good agreement is obtained in estimating the settlement between these two methods. Further analysis for pile groups consisting of a large number of piles up to 8×8 piles with $s/d=5$, indicates that the load carried by the raft does not exceed 50% of the total applied load.

The individual pile load is largely affected by pile spacing as shown in Fig. 5 (b). A small load is carried by the centre pile of closely-spaced group even if in a free-standing group. As the pile spacing increases, the pile head loads tend to be uniform and are reduced markedly due to the increase of the raft load in piled rafts.

When the piles are highly compressible, a large load is taken by the raft as shown in Fig. 5 (c). Highly compressible piles can prevent the load transfer to the lower part of the piles ; i. e. shortening the critical pile length. For a value of K greater than 1000, however, the load carried by the raft becomes fairly uniform. High compressibility of piles also evens the loads in each pile within a group.

The effect of Poisson's ratio of the soil on the pile head load is significant in piled rafts while nothing is affected in freestanding groups as shown in Fig. 5 (d). For piled rafts, the incompressible soil beneath the raft can support a considerable part of the load from the raft, while the effect of the raft is small at $\nu_s=0.0$ where the contact pressure on the raft is less than 5% of the total applied load.

An elastic analysis indicates that a corner pile in a group takes the maximum load and a centre pile the minimum load. Fig. 6

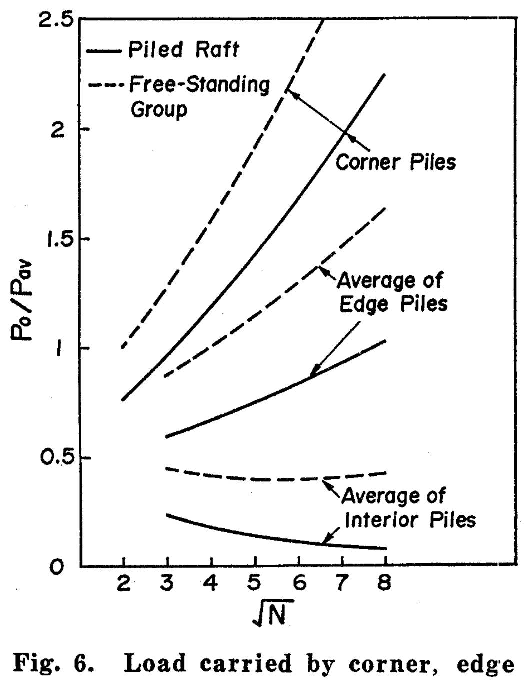

Fig. 6. Load carried by corner, edge and interior piles

shows the average pile head loads among three groups in a pile group (corner, edge and interior) for up to an 8×8 pile group. The pile head load, P_0 is normalized by P_{av} that is the total applied load divided by the number of piles in the group (P_t/N). It can be seen that as the number of piles increases, the difference in pile head loads becomes greater. The interior piles in a large piled raft are subjected to extremely small loads. In an 8×8 pile group, for example, 24% of the total applied load is carried by the interior piles in the free-standing group but only 8% of the total *pile* load is taken by the interior piles in the piled raft, even though the number of the interior piles consititutes 56% of the total.

Previously published test results of instrumented pile groups showed a more even distribution of pile head loads than given by the theory. The model test of a 9 pile group of $s/d=4$ reported by Whitaker (1957) showed that the ratios of loads carried by the corner, side and centre pile were 2.2 : 1.5 : 1. In situ measurements of a piled raft consisting of a large number of piles by Cooke et al. (1981) showed these ratios of 2.2 : 1.7 : 1. The present analysis for a typical example of pile group ($N=3\times3$, $L/d=25$, $s/d=5$, $K=1000$ and $\nu_s=0.5$) shows that the ratios of loads carried by three reference piles are about 3 : 2 : 1 for both piled rafts and free-standing groups. The even distribution of the pile load in the experiment and the in situ measurement can be explained as follows: The local yield in a soil or slip on foundation-soil interface, the relatively high flexibility of the large raft and also the non-homogeneous soil profile whose Young's modulus increase with depth may reduce the high elastic stress concentration on the corner.

Three-dimensional finite element analyses for 3×3 piled rafts were performed by Ottaviani (1975) in elastic conditions. The percentage of the loads carried by the three reference piles and the raft are presented in Table 1 together with the results of the present analysis. Good agreement can be seen between the two results obtained by different methods.

Table 1. Percentage of load carried by piles and raft ($N=3\times3$, $L/d=40$, $s/d=4$, $K=800$, $\nu_s=0.45$, $B_0/d=0.5$)

		Ottaviani (1975)	present analysis
raft		11	11
pile	①	11	12
	②	9	9
	③	7	5

Vertical Distribution of Pile Load

Fig. 7 shows a vertical distribution of the pile load in a 3×3 group. The pile head load in a piled raft is small as compared with that in the free-standing group being discussed in Fig. 5. In the middle and lower parts of the shaft, however, the difference in the load is very small between the two. The load in the piled raft is only 9% less than that in the free-standing group at the depth of a quarter pile length, while the difference reaches 25–50% at the pile head.

Fig. 8 indicates the influence of pile spacing on the vertical distribution of the total pile load as a fraction of the total applied load in the 3×3 pile groups. The solid and broken curves come closer with an increase in the depth. Even in the case of $s/d=10$ in which the curves are widely apart near the surface, they are close at the lower third of the shaft.

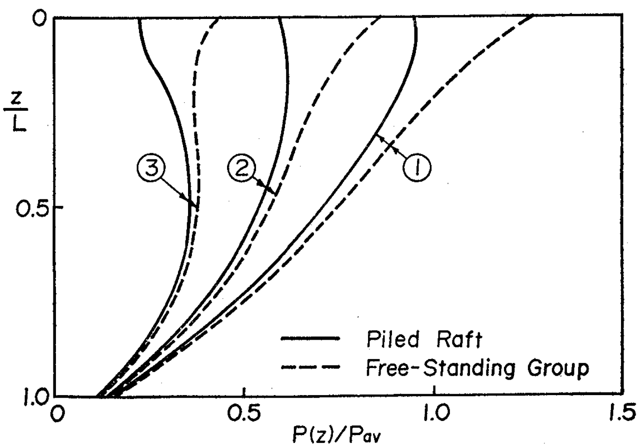

Fig. 7. Vertical distribution of pile load (P_{av} indicates an average pile head load in free-standing group. $P_{av}=P_t/N$)

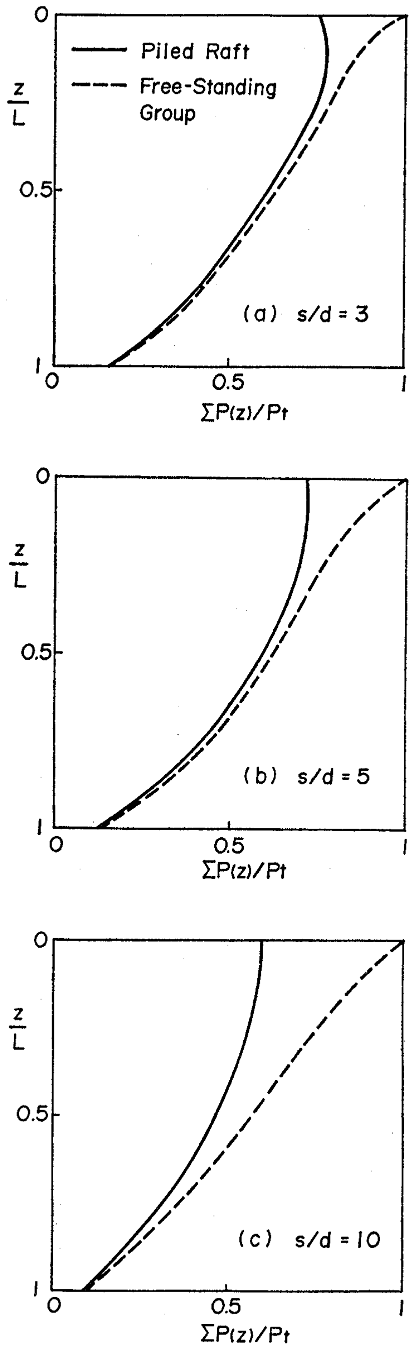

Fig. 8. Vertical distribution of total pile load

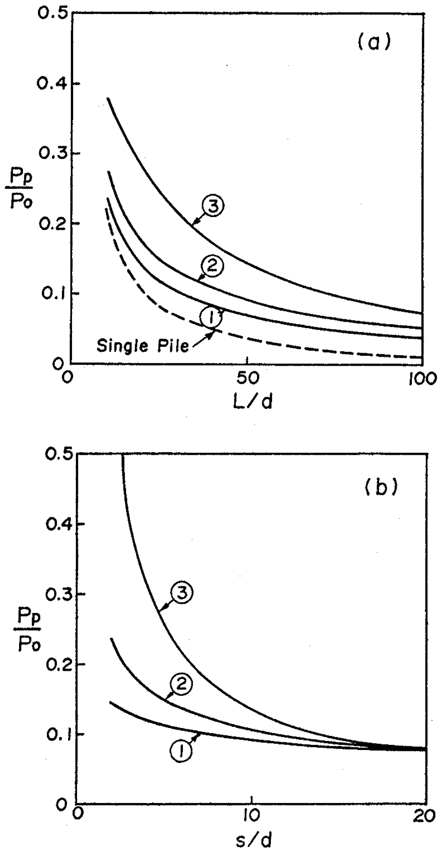

Fig. 9. Load transfer to pile tip. (a) influence of pile length, (b) influence of pile spacing

An interpretation can be drawn : The load transmitted to the soil beneath the raft directly from their interface may be transferred again into the piles, but at a shallow depth. Therefore, a downdrag (negative) friction can occur along the upper part of the pile (or the upward shear stress on the shaft is reduced). This phenomenon occurs in a shallow and interior part under the raft, and any marked difference of pile load due to the presence of the raft cannot be recognized over the major remaining part of the shaft. The manner of the load transfer described above can be a cause for the discrepancy in which the settlement of a pile group is little affected by the presence of the raft, while the load carried by the piles is reduced remarkably by the presence of the raft.

The proportion of the load transferred to a pile tip is small in a long, compressive, single floating pile. Fig. 9 shows the ratio of pile tip load, P_p to the head load, P_0 of an individual pile in free-standing groups. It can be seen that the proportion of the tip load to the head load is greater in pile groups than in single piles. This is marked in the centre pile where both the head load and the shear stress on the shaft are small.

The difference of the tip loads between the individual piles in a group is small compared with the pile head loads as shown in Fig. 7. A higher tip load may appear in a corner pile than in the interior piles, unless the pile stiffness is very low. The effect of pile spacing on the load transferred to the pile tip is also much greater in the interior piles than in the corner piles as shown in Fig. 9 (b). The tip load in piled rafts is omitted because the effect of the presence of the raft is very small as shown in Fig. 7.

Distribution of Contact Pressure on Raft

The dashed line in Fig. 5 (d) indicates the load taken by the part of the overhang in the raft. About 60% of the total raft load is supported by the overhang area, which occupies only less than 30% of the total raft area in the standard case of $B_0/d=1$. When the pile spacing is less than 5 diameters, the inside raft area surrounded by four piles is subjected to small contact pressure, even if the soil is imcompressible.

Fig. 10 shows an example of the contact pressure distribution on a quarter of a 3×3 piled raft. The figures in the rectangular raft elements indicate the percentage of the contact pressure on the element, p_c to the average contact pressure on the raft, p_{cav}. The values of pressure are omitted in symmetrical elements. It can be seen that the pressure on the inside area enclosed by 4 piles is small and uniformly distributed. On the other hand, the pressure on the overhang area is very large (infinite pressure may occur for an infinitely small element division in elastic analysis) and changes rapidly.

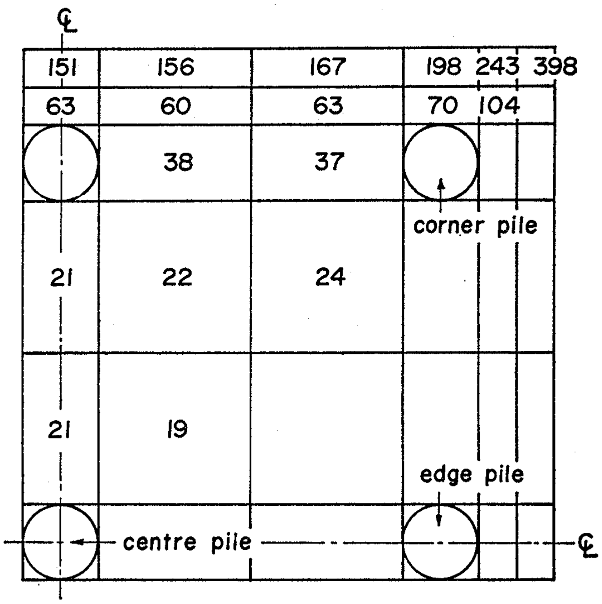

Fig. 10. Distribution of contact pressure on raft, p_c/p_{cav} (%). (p_c=contact pressure on a raft element. p_{cav}= average contact pressure on raft)

For a large pile group, the pressure on the inside raft area is distributed more uniformly. For example, among 7×7 divided elements (enclosed by 4 piles) in a piled raft consisting of 8×8 piles, the minimum pressure at the center of the raft is 76% of the maximum pressure on the element just inside of the corner pile.

CONCLUSIONS

A boundary element procedure is described to analyse the behaviour of piled raft foundations whose raft rests on a homogeneous isotropic elastic half-space soil. When applied to a pile group consisting of a number of identical piles arranged in a square configuration, the following conclusions are drawn :

1. The critical length of pile groups for a vertical loading (the length beyond which settlement shows little further decrease) is longer than that of single piles whose dimensions are the same as the piles in the groups.
2. The percentage of the long-term settlement to the total final settlement of pile groups is greater than that of single piles. In piled rafts, this proportion is smaller than in free-standing groups, but the effect of the raft is small except for short pile groups.
3. The vertical load carried by the raft is about 20–40% of the total applied load in the case of $L/d<50$, $s/d<10$ in an undrained condition.
4. The vertical distribution of the load in a pile within a group is little affected by the presence of the raft except in the upper part of the shaft where the load is reduced by the raft. The load transferred to the pile tip in pile groups is greater than in single piles.
5. The contact pressure on the raft is

relatively uniform in the inside area surrounded by piles. This pressure is much smaller compared with that in the overhang area of the raft.

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NOTATION

A_p=area of pile section
B_0=width of overhang area of raft from surface of outer piles
d=pile diameter
E_p=Young's modulus of pile
E_s=Young's modulus of soil
$[I_s]$=matrix of soil displacement influence factors
K=pile stiffness factor $(=R_A E_P/E_S)$
L=pile length
N=number of piles in a group
n_b=number of base elements of a pile
n_c=number of raft elements
n_s=number of shaft elements of a pile
$P(z)$=axial load in a pile
P_0=load at pile head
P_p=load at pile tip
P_t=total applied load on a raft
$\{p\}$=stress vector on foundation-soil interface
P_{av}=total applied load divided by number of plies in a group (P_t/N)
p_c=Contact pressure on raft
p_{cav}=average contact pressure on raft
R_A=area ratio of pile $(=4\,A_P/\pi d^2)$
s=pile spacing
z=distance below ground surface
ν'=drained Poisson's ratio
ν_s=Poisson's ratio of soil
ν_u=undrained Poisson's ratio of soil
ρ_0=displacement of raft
ρ_t=immediate settlement of raft
ρ_{TP}=total final settlement of raft
$\{_s\rho\}$=soil displacement vector at adjacent points to element of foundation

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