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SHEAR MODULUS AND CYCLIC UNDRAINED BEHAVIOR OF SANDS

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ABSTRACT

The results of laboratory experiments performed to investigate the effects that factors such as fabric, anisotropic consolidation and stress-strain history have on the shear modulus at very small strains (G_0), on the drained modulus degradation curve, and on the cyclic undrained shear strength of sands are summarized. The results presented show that G_0 is relatively insensitive to sand fabric, initial stress ratio, monotonic prestressing, and cyclic strain history. Thus, specimens with the same G_0 can exhibit very different behaviors in cyclic undrained shear, depending upon their initial fabric, initial stress ratio, or stress-strain history. To a lesser degree, this is also true for the shear modulus (G) at larger strains, as interpreted in the conventional manner from drained cyclic shear tests (unload/reload modulus). Based on these findings, the limitations involved in the use of G_0 and G to interpret behavior in cyclic undrained shear are discussed.

Key words : deformation, dynamic, laboratory test, sand, shear modulus (**IGC** : D 6/D 7)

INTRODUCTION

Most experimental investigation of cyclic stress-strain properties of sands have been performed under drained conditions, whence it has been concluded that the shear modulus-shear strain relationship, and the corresponding damping characteristics, are mainly affected by void ratio and mean confining stress (Hardin and Drnevich, 1972 ; Richart, 1977). Other factors have been found to have a modest effect. For example, the drained shear modulus (G) at shear strain amplitudes (γ) ranging between 5×10^{-3} and 3×10^{-1}% was found to be relatively insensitive to drastic changes in the methods of specimen preparation (Iwasaki et al., 1978 ; Tatsuoka et al., 1979 a) and consequently independent of initial fabric ; changes in fabric induced by preshearing of glass spheres resulted in only small variations (less than 8%) in maximum shear modulus (G_0), and reduction in G of the order of 10 to 20% for shear strains of 0.30 to 0.60% (Chen et al., 1988) ; at ordinary principal stress ratios encountered in situ, the effects of stress ratio are rela-

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tively unimportant (Hardin and Drnevich, 1972 ; Tatsuoka et al., 1979 b ; Yu and Richart, 1984, 1985), and probably even negligible considering the uncertainty in evaluating in situ lateral stresses (Schmertmann, 1985) ; monotonic prestressing has been shown to increase or decrease the shear modulus of sands depending upon the stress path (Tatsuoka et al., 1979 b ; Yu and Richart, 1985 ; Tokimatsu et al., 1986), however the changes reported were either negligible or small, the largest effect (15%) being reported by Tokimatsu et al. (1986) for isotropic overconsolidation with an OCR of 4.0 ; strain repetitions reduce the potential for densification under subsequent loading (Finn et al., 1970 ; Silver and Park, 1975 ; Seed, 1979), yet do not affect significantly the modulus degradation curve in drained shear (Silver and Park, 1975 ; Tatsuoka et al., 1979 b). Increases in maximum shear modulus of 10% to 65% due to cyclic strain repetitions have been reported (Shen et al., 1985 ; Tokimatsu et al., 1986 ; Ray and Woods, 1988) ; however, the larger percentages are associated with specimens subjected to as many as 10,000 cycles of significant shear strains (from 0.08 to 0.2%), which is not typical of seismic loading.

Thus, based upon existing data, the conventional interpretation of secant shear modulus in drained cyclic shear appears relatively insensitive to important factors that influence the modulus obtained from either drained monotonic (Lambrechts and Leonards, 1978 ; Baldi et al., 1985 ; Clayton et al., 1985 ; Jamiolkowski et al., 1985 ; Leonards et al., 1986) or cyclic undrained loadings (Ishihara et al., 1979 ; Seed, 1979 ; Susuki and Toki, 1984 ; Vaid and Chern, 1985). The difference is important when attempts are made to predict cyclic undrained behavior using the drained secant modulus or the shear modulus at very small strains, G_0.

This paper examines the factors controlling G_0 and G of sands over a wide range of strain levels and strain histories, along with the potential relationship to their behavior in cyclic undrained shear. In this connection, the paper summarizes the results of laboratory studies carried out to investigate and compare the effects of factors such as anisotropic consolidation and stress-strain history on G_0, on the degradation of G with increasing γ in drained shear, and on the cyclic undrained shear strength. The results of these experiments, and their relationship to previously published data, are used to elucidate fundamental behavior characteristics of sands, as well as to clear up some of the confusion regarding the factors affecting this behavior that arises from the manner in which cyclic shear tests on sands have been interpreted in the past.

LABORATORY EQUIPMENT

The tests summarized in this paper were performed using an apparatus that combines conventional triaxial compression features with resonant column and torsional shear capabilities, as described elsewhere in detail (Alarcon-Guzman et al., 1986). The resonant column and quasi-static torsional loadings are controlled independently and applied to the top of the soil specimen, which is rigidly fixed at its base. The apparatus permits the determination of soil properties on a single solid or hollow cylinder specimen, consolidated either isotropically or anisotropically, over the range of shear strains of engineering interest, that is from 10^{-4} to 10%. Tests were performed on reconstituted specimens of Ottawa 20-30 and 50-70 sands.

Table 1. Characteristics of sands tested

	Ottawa 20-30 (Batch 1)	Ottawa 20-30 (Batch 2)	Ottawa 50-70
Predominant Mineral	Quartz	Quartz	Quartz
Grain Shape Angularity	bulky	bulky	bulky
Specific Gravity	2.65	2.65	2.65
D_{10}(mm)	0.60	0.60	0.23
D_{50}(mm)	0.70	0.72	0.26
D_{60}(mm)	0.72	0.75	0.27
D_{90}(mm)	0.82	0.83	0.30
$C_u=D_{60}/D_{10}$	1.2	1.25	1.2
e_{max}	0.731	0.738	0.852
e_{min}	0.490	0.501	0.585
$e_{max}-e_{min}$	0.241	0.237	0.267

The main characteristics of these materials are given in Table 1. It is noted that two batches of Ottawa 20–30 sand were used; they exhibit minor differences in grain size distribution and minimum and maximum void ratios, and will hereafter be referred to as "Batch 1" and "Batch 2". Solid and hollow specimens were prepared by pluviating the sand in air. When required, saturation was insured by CO_2 purging, percolation of deaired water and backpressuring.

Stage testing of dry specimens was generally carried out as follows. The specimen was first subjected to dynamic loading in the resonant column mode, and the shear modulus was determined from the resonant frequency of the oscillator/soil system for oscillations as small as possible. The power applied to the oscillator was incrementally doubled and the corresponding moduli measured. This operation was repeated until the maximum capacity of the oscillator was reached. These experiments provided shear moduli at single amplitude shear strains ranging from 10^{-4} to about $1.4\times10^{-2}\%$. Following the resonant column tests, the same specimen was then subjected to strain controlled torsional cyclic loading tests performed in stages, with the amplitude of rotation being doubled between successive loading stages. During each stage, several cycles (10 in average) were applied to a predetermined amplitude of the twist angle. For the first loading stage, a minimum single amplitude shear strain between 1.2×10^{-2} to $2.0\times10^{-2}\%$ could be applied, depending upon the size of the specimen, thereby overlapping the strain range for the resonant column tests. The maximum single amplitude shear strain applied during the last stage ranged between 3 and 10%. The test conditions for experiments that did not involve stage testing varied as a function of the factors being investigated. The specific characteristics of each test, e.g. initial conditions and loading sequence, are given in the following sections.

TEST RESULTS AND INTERPRETATION

Shear Modulus under Drained Conditions

1. *Relative density and mean confining stress*–In order to have a reference for comparing the effects of other factors, the influence of relative density and mean confining stress on the shear modulus is illustrated first for Ottawa 20–30 sand (Batch 1). A change in relative density from 40% to 90% is reflected by a relatively modest change in the modulus degradation curves obtained using stage testing techniques on dry hollow specimens (Fig. 1). At shear strains greater than about $5\times10^{-2}\%$, G is almost indepen-

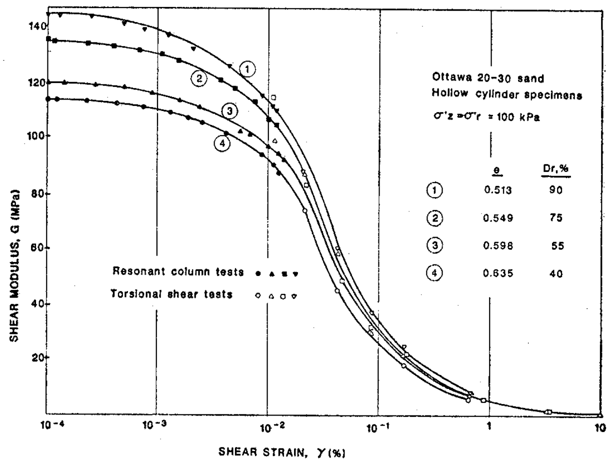

Fig. 1. Effect of relative density on shear modulus-shear strain relationship

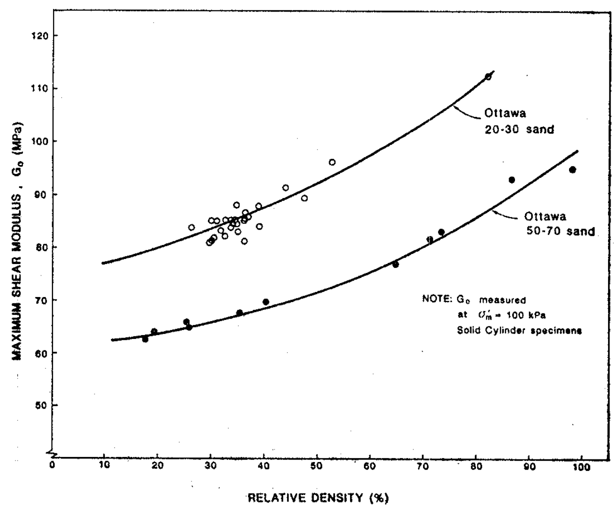

Fig. 2. Variation of maxiumum shear modulus with relative density

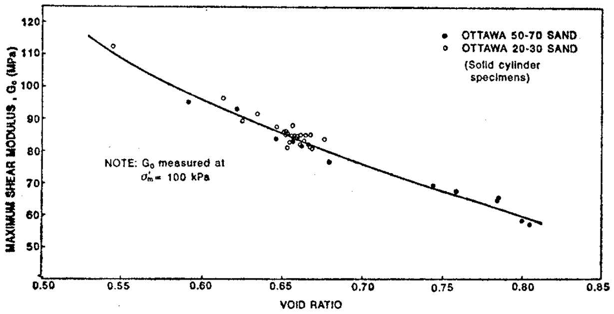

Fig. 3. Variation of maximum shear modulus with void ratio

dent of whether the sand is moderately loose or relatively dense.

The changes in G_0 as a function of relative density were further investigated by tests performed on saturated solid specimens of Ottawa 20-30 (Batch 2) and 50-70 sands for relative densities ranging between 18% and 95%. The results (Fig. 2) illustrate the modest influence that relative density has on G_0 of these sands. It is noted that for sands with differences between e_{max} and e_{min} larger than that corresponding to Ottawa 20-30 or 50-70 sand (Table 1), the variation of shear modulus with relative density can be more significant than the variations observed in Figs. 1 and 2 (Seed and Idriss, 1970 ; Seed et al., 1984). Fig. 2 clearly shows that two relatively similar sands at the same relative density can have markedly different values of G_0. The data in Fig. 2

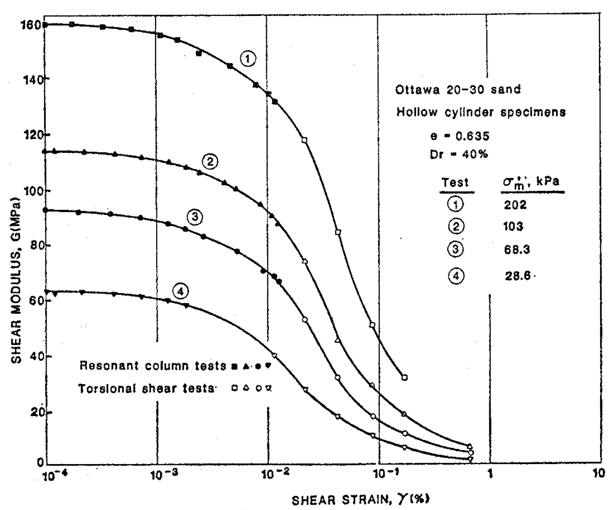

Fig. 4. Effect of confining pressure on shear modulus-shear strain relationship

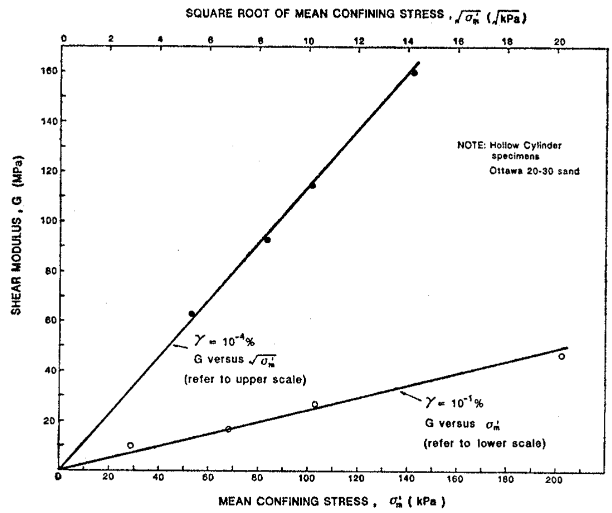

Fig. 5. Relationship between shear modulus and mean confining stress

are replotted vs. void ratio in Fig. 3, which shows that G_0 is related to void ratio rather than to relative density. Thus, the shear wave velocity of granular soils is also related more to density than to relative density, which implies that inferences regarding the state of compaction of granular soils from geophysical measurements alone may be subject to considerable error.

The large influence of the mean confining stress (σ'_m) on the modulus reduction curve is illustrated in Fig. 4 for dry hollow specimens of Ottawa 20-30 sand (Batch 1) with a relative density of about 40%. When the modulus values corresponding to a particular shear strain level are plotted versus the mean confining stress, it is observed that at small shear strains the modulus values are proportional to the square root of σ'_m, whereas at large shear strains the shear moduli become a linear function of σ'_m (Fig. 5). This agrees with previous test results on a variety of sands (e. g. Hardin and Drnevich, 1972 ; Iwasaki et al., 1978 ; Kokusho, 1980 ; Leonards et al., 1986).

In Figs. 1 and 4, the shear moduli at a shear strain amplitude of about 10^{-2}% obtained from the resonant column (high frequency tests, $f>100$ Hz) and torsional shear (low frequency tests, $f \simeq 0.1$ to 0.2 Hz) tests agree well in spite of the great disparity in frequency. Such agreement has also been

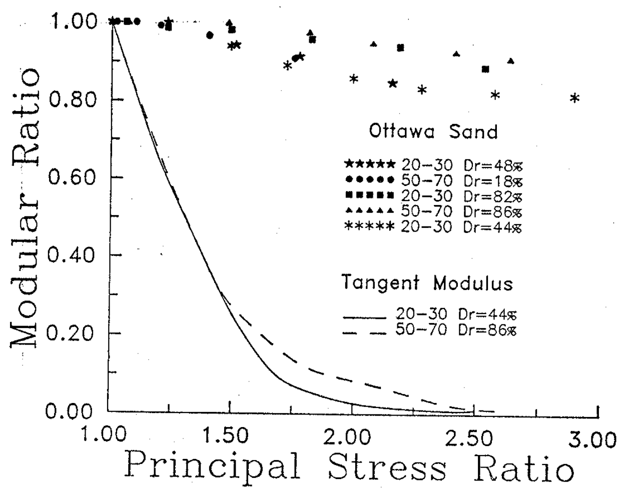

Fig. 6. Effect of principal stress ratio on modular ratio

reported by previous investigators (Richart, 1977 ; Iwasaki et al., 1978). However, it is too early to conclude that frequency has no effect on the stress-strain response of sands, especially under undrained conditions. Besides frequency, the number of loading cycles is also very different between the two portions of the test. The relative contribution of each one of these factors to the response of sands in resonant column tests is not yet fully established. Furthermore, it will be shown later that when stage testing procedures are not used, the shear modulus values from resonant column and torsional shear tests can differ significantly.

2. *Stress ratio*-Specimens of Ottawa 20-30 (Batch 2) and 50-70 sands with relative densities ranging between 18% and 86% were isotropically consolidated to a mean confining stress of 100 kPa and G_0 was measured at values of γ of about 10^{-4}% in the resonant column mode. Thereafter, values of G_0 for these specimens were obtained after subjecting them monotonically to increasing levels of torsional shear stress, $\tau_{z\theta}$, until failure developed. The results from these tests are shown by the data plotted in Fig. 6. The upper part of the plot shows the effect of principal stress ratio on modular ratio (i. e. G_0 at a given stress ratio divided by G_0 at a stress ratio of 1.0). The largest reduction in G_0 was on the order of 18%, with most values being less than 10%. The data agree with those obtained by Yu and Richart (1984). The insensitivity of G_0 to stress ratio is an additional indication that it is not sensitive to changes in fabric since rearrangement of particles certainly occurs in a drastic manner as the stress ratio increases to failure (Oda, 1972 ; Tobita, 1983).

The tangent modulus degradation curves for two of the tests computed from the shear stress-shear strain data during monotonic loading are also shown in Fig. 6. The effects of stress ratio on G_0 (small strain, unloading/reloading parameter) and on the tangent modulus (large strain, initial loading parameter) are drastically different. For example, at a stress ratio of 2, the reduction in G_0 is about 10% while the reduction in tangent modulus is more than 90%. Thus, the modulus in the first quarter of the first loading cycle in a drained cyclic shear test (equivalent to monotonic loading), and hence the behavior in undrained cyclic shear, are very strongly affected by stress ratio but G_0 is not. This important fact is discussed further below.

When drained cyclic shear tests are reported in terms of a shear modulus degradation curve (G/G_0 vs γ), there is a tendency to conclude that stress ratio effects are not significant. This is illustrated in Fig. 7, which shows essentially identical modulus degradation curves obtained using stage testing on two replicate dry hollow specimens of Ottawa 20-30 (Batch 1) sand with the same D_r and σ'_m, although the specimens had been consolidated under stress paths with stress ratios $\sigma'_z/\sigma'_r \simeq 1.0$ and 2.0, respectively. Similar results were obtained by Tatsuoka et al. (1979 b). The effects that the initial state of stress may have on sand behavior are not reflected in the drained modulus degradation curves partly because of the measurement and interpretation techniques conventionally used. The conventional interpretation of G at any given level of γ is a secant unloading-reloading modulus that can be much greater than the secant modulus from the first one-quarter of the loading cycle. It is

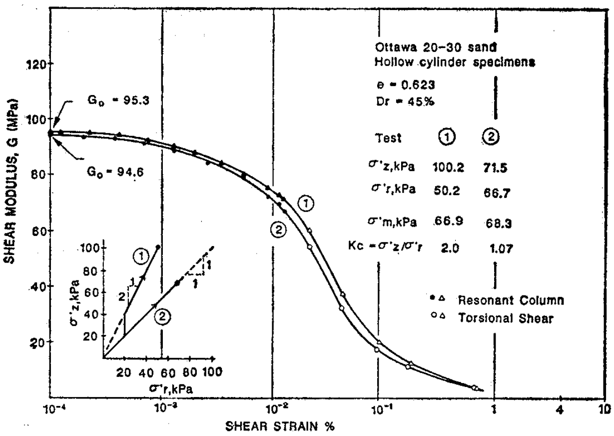

Fig. 7. Effect of consolidation stress ratio on modulus reduction curve

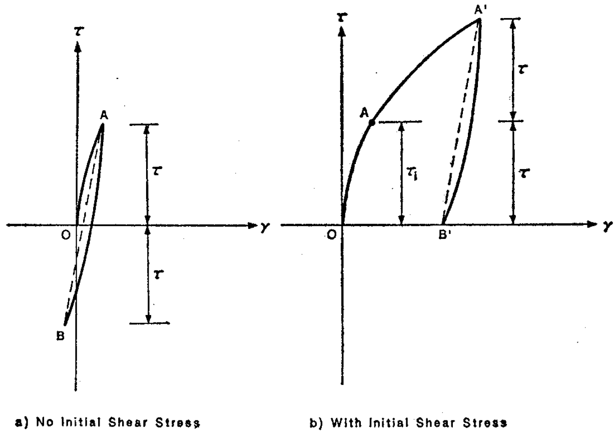

Fig. 8. Conventional interpretation of shear modulus in cyclic loading tests (schematic)

this unloading-reloading value of G that is found to be relatively unaffected by the initial state of stress, as shown by the test results in Fig. 7. However, this can be a misleading interpretation when the behavior in undrained shear is of interest, as illustrated in Figs. 8 and 9.

Fig. 8(a) shows schematically the first three quarters of a loading cycle applied to a sand specimen that has not been subjected to any prior shear stress. The slope AB is the conventional value of G. Fig. 8(b) shows the result if the same level of shear stress, τ, is applied to a specimen with an initial shear stress, τ_i. The slope A′B′ is now the interpreted value of G. It turns out that the slope A′B′ is not significantly different

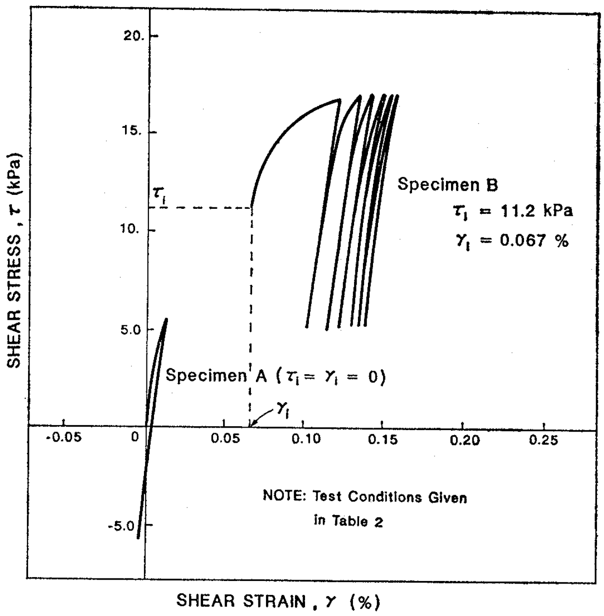

Fig. 9. Influence of initial stress conditions on the undrained cyclic shear stress-shear strain relationship

from the slope AB, however the strain-stress behavior in the first quarter of the loading cycle, AA′, is very different from that under initial loading, OA. The difference increases as τ_i increases, and this has a drastic influence on the behavior in undrained shear. The importance of the initial state of stress on the behavior in undrained shear has often been reported (e. g. Ishihara and Li, 1972 ; Lee and Seed, 1967 ; Vaid and Chern, 1983, 1985 ; Alarcon-Guzman, 1986), and is clearly demonstrated in Fig. 9. Specimens A and B of Ottawa 20-30 sand (Batch 1) had similar relative densities, mean effective stresses, and maximum shear moduli prior to the application of cyclic shear stress under un-

Table 2. Comparison of test data for specimens A and B in Fig. 9

	Specimen A	Specimen B
Relative Density (%)	44.6	43.2
Mean Confining Stress (kPa)	101.0	100.1
Maximum Shear Modulus (MPa)	106.0	101.0
Initial Shear Stress (kPa)	—	11.2
Initial Shear Strain (%)	—	0.067
Cyclic Shear Stress (kPa)	5.80	6.20
Additional Shear Strain After 5 Cycles (%)	0.0085	0.081

drained conditions (Table 2); however, specimen B had been subjected to an initial (static) shear stress of 11.2 kPa. The measured stress-strain curves during undrained cyclic loading (results are given for five cycles of loading for both tests in Fig. 9) are very different. The additional shear strain experienced by specimen B is about 10 times larger than that of specimen A (Table 2). Of particular importance is the large strain increment observed during the first quarter of the loading cycle for specimen B, which was accompanied by a sharp increase in pore water pressure. Using conventional interpretation, G_0 and the drained modulus degradation curves of the two specimens would be nearly identical (Fig. 7). These test results show that an initial anisotropic state of stress (induced in this test by applying a static shear stress, $\tau_{z\theta}$) has an important effect on the undrained cyclic behavior of the sand, with little reflection in drained shear modulus degradation curves.

3. *Monotonic prestressing*-The effects of isotropic prestressing on the conventional modulus degradation curve are negligible (Tatsuoka et al., 1979 b; Alarcon-Guzman, 1986). However, prestressing along a stress path with anisotropic states of stress decreases the shear modulus particularly in the early reloading range. For example, prestressing a dry hollow specimen of Ottawa 20-30 sand (Batch 1) along an anisotropic stress path to an OCR of 4.0 (Stress path

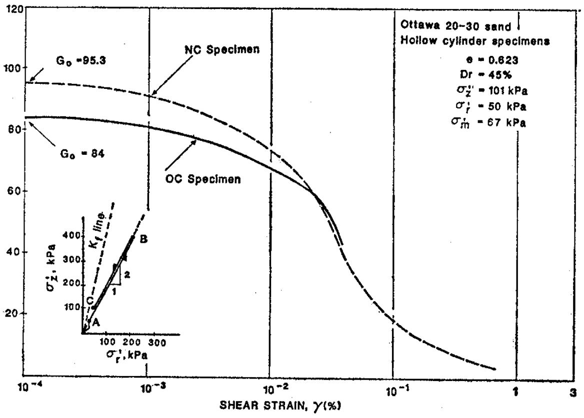

Fig. 10. Effect of axial compression prestressing on modulus reduction curve

ABC in Fig. 10) caused a degradation of the shear modulus of the specimen at low shear strains when compared to a normally consolidated specimen at the same state of stress (stress path AC in Fig. 10), but the degradation was not significant for shear strain amplitudes above $2\times10^{-2}\%$. The unfavorable effect of the applied prestressing can be attributed, in part, to the fact that the stress path during prestressing differs from the one followed during cyclic reloading. However, it will be shown later that the actual influence of prestressing on the shear modulus may be obscured by the stiffening effect associated with the large number of strain cycles applied during the resonant column tests.

The results in Fig. 10 show that, although both the normally and the overconsolidated specimens have been subjected to the same maximum stress ratio of about 2.0, their G_0 values differ by about 12% (95.3 and 84 MPa, respectively) due to the stress history effects. Thus, since prestressing along a stress path with constant stress ratio also causes a degradation of the shear modulus, stress ratio alone does not appear to account fully for stress history effects, as was suggested by Yu and Richart (1984)

It is important to recognize that the effects of monotonic prestressing on G_0 are small when compared to the effects of void ratio (Fig. 1) and mean confining stress (Fig. 4). This is clearly illustrated in Fig. 11 for saturated solid specimens of Ottawa 20-30 sand (Batch 2), with relative densities ranging between 21% and 95%, prestressed along a stress path with a stress ratio of 1.5. The effect on G_0 ranges from an increase of 1% (D_r=95%) to a decrease of 11% (D_r=21%) at an OCR of 8. The modulus values in this figure were corrected to account for void ratio variations during prestressing by using a void ratio function (Iwasaki et al., 1978; Yu and Richart, 1984).

Although the effect of monotonic prestressing on G or G_0 is small (Fig. 11), the effect on the pore water pressure response observed in undrained tests (Ishihara and Takatsu,

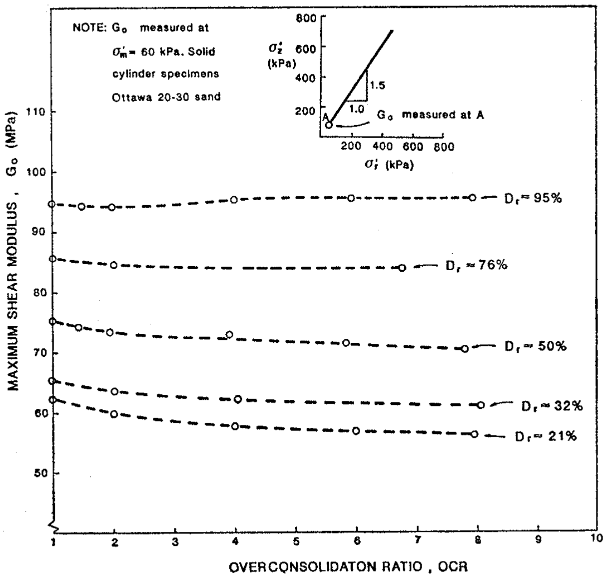

Fig. 11. Effect of axial compression prestressing on maximum shear modulus

1979 ; Seed, 1979) is very significant. This is due to the fact that the tendency for pore pressure build-up during undrained shear is related not so much to changes in stiffness but to the changes in dilatancy (volumetric strain) characteristics that result from prestressing (Lee and Farhoomand, 1967 ; Lee and Seed, 1967). Accordingly, neither G_0 nor G adequately account for either stress ratio (Fig. 6) or stress history (Fig. 11) effects on the stress-strain behavior of sands in undrained shear.

4. *Cyclic Strain (or Stress) Repetitions*–The data in Fig. 12 were obtained from dry

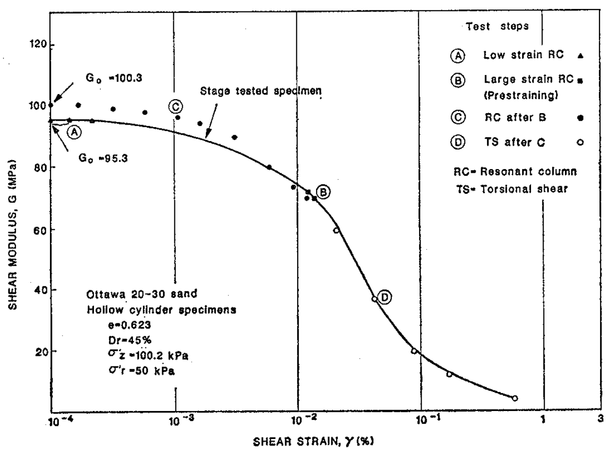

Fig. 12. Effect of cyclic prestraining on modulus reduction curve

hollow specimens of Ottawa 20–30 sand (Batch 1) with the same initial conditions as in some of the previous tests ($D_r \simeq 45\%$ and stress ratio $\sigma'_z/\sigma'_r = 2.0$). The maximum shear modulus of about 95.3 MPa, measured before prestraining the specimen (step A), compares very well with that for a previous specimen that was stage tested without prestraining (solid curve). In step B, the specimen was subjected to cyclic prestraining using the resonant column mode with a shear strain amplitude of about $1.3 \times 10^{-2}\%$. Following step B, the complete modulus degradation curve was determined by means of resonant column tests (step C) and torsional shear tests (step D), beginning again at $\gamma = 10^{-4}\%$.

The torsional prestraining (many cyclic strain repetitions in step B) caused the subsequent shear moduli to increase slightly (about 5% for G_0) above the value corresponding to the stage tested specimen without prestraining for shear strains below the previous maximum strain of $1.3 \times 10^{-2}\%$. A similar behavior was observed in previous investigations (Drnevich and Richart, 1970 ; Silver and Park, 1975 ; Tatsuoka et al., 1979 b). Thus, the effect of prestraining in the resonant column mode on the maximum shear modulus is small, and reflects in part the reduction in void ratio due to prestraining. A similar behavior was observed in specimens prestrained in the resonant column mode at shear strain levels ranging from 10^{-3} to $1.5 \times 10^{-2}\%$ (Shen et al., 1985 ; Alarcon-Guzman, 1986).

It is significant that the shear modulus measured during step B (about 71 MPa) agrees with the value for the specimen that had undergone the full range of stage testing with strains increasing from 10^{-4} to $1.3 \times 10^{-2}\%$, and with the value for the prestrained specimen at the end of step C. Moreover, the modulus degradation curve for the prestrained specimen coincides with that of the stage tested specimen at shear strains larger than the precycling strain (i. e., strain at B). Based on these results, one would tend to conclude that : (a) stage testing has little effect on the shear modulus values (Chung et al., 1984), and (b) shear moduli at shear strain amplitudes larger than the previous maximum strain are not affected by cyclic prestraining (Drnevich and Richart, 1970; Silver and Park, 1975; Tatsuoka et al., 1979 b). However, the apparent agreement in G values at shear strains larger than $1.3\times 10^{-2}\%$ is due to the fact that all three tests involved resonant column tests, which inherently include stage testing effects, as shown below.

Fig. 13 summarizes the results of several tests (with initial conditions similar to test 3 presented in Fig. 4) in which the shear strain amplitudes shown by the data points were applied to virgin specimens with no prior cyclic strain history. It is noted that at a given shear strain, say $2.2\times 10^{-2}\%$, the shear modulus of the "virgin" specimens having no prior cyclic stain history is much smaller (32–36 MPa) than the values (55–60 MPa) obtained in the stage tested specimen at the same shear strain amplitude. When compared with the results shown in Fig. 12 at the same level of shear strain, it is seen that this behavior reflects the considerable effects of prior strain repetitions on the shear modulus. It is significant that specimens with the same measured G_0 ($\simeq$95 MPa) exhibit very different behavior at larger strains, even under drained conditions, due to differences in the number of prior strain repetitions.

In the light of the results in Fig. 13 for

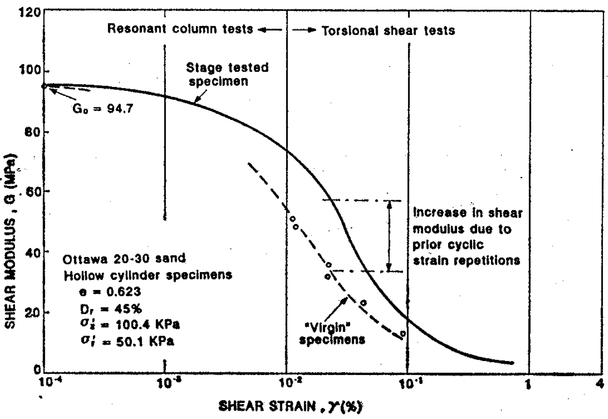

Fig. 13. Effect of cyclic strain history

Ottawa 20–30 sand (Batch 1), the magnitude of the effects of stage testing or cyclic prestraining on the shear modulus of some sands can be more significant than was concluded by Chung et al. (1984) and Silver and Park (1975). However, it should be noted that Silver and Park studied the effects of stage testing by means of cyclic triaxial tests involving a smaller number of cycles than resonant column tests. On the other hand, Chung et al. (1984) based their conclusion on resonant column tests. The shear modulus measured in resonant column tests corresponds to the unloading-reloading secant modulus after several hundred cycles: as shown in Fig. 12, the modulus determined in this manner is not significantly affected by prior stage testing. On the other hand, as shown in Fig. 13, stage testing involving a number of repeated strain cycles can lead to a significant overestimation of shear modulus for virgin sands with no prior cyclic strain history. Stage testing also obscures the effects of other factors such as stress ratio, monotonic prestressing, etc. on the shear modulus. According to the data in Fig. 13, resonant column tests at shear strains of about $10^{-4}\%$ apparently do not affect the shear modulus of virgin specimens, while stage testing up to shear strains of $10^{-2}\%$ increased G by about 38%. This is consistent with the concept of a threshold shear strain that must be exceeded before pore pressure buildup is initiated in undrained cyclic shear tests, as reported by Dobry et al. (1981), except that the threshold strain can be less than $10^{-2}\%$ for some sands. Consequently, until further experimental evidence is obtained, the authors recommend that cyclic shear tests should be conducted on virgin specimens of sand, or at least specimens not previously subjected to repeated strain cycles whose amplitude exceeds about $10^{-3}\%$.

5. *Number of repeated strain cycles*–To examine further the effects of the number of applied strain cycles, a specimen of Ottawa 20–30 sand (Batch 1) with no prior strain history was subjected to cyclic loading directly in the torsional shear mode. Typical

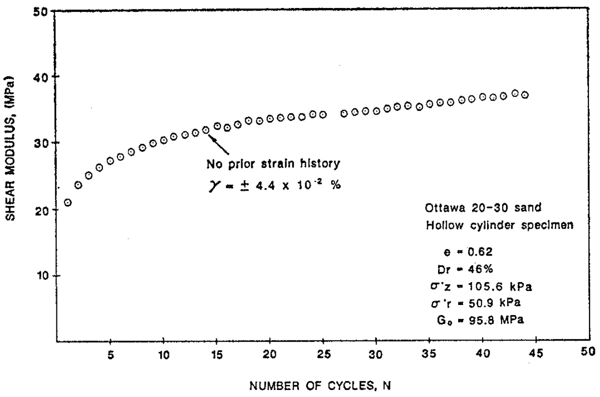

Fig. 14. Variation of shear modulus with number of cycles

results are given in Fig. 14 (G is the secant modulus for each cycle). Similar results were obtained from other specimens (D_r between 40 and 50%) with shear strains ranging between 10^{-2} and 10^{-1}%. The number of stress repetitions has an important stiffening effect on virgin sand specimens. This stiffening effect is more significant during the first 10 to 20 cycles. The results shown in Fig. 14 indicate that after about 40 cycles of a given shear strain amplitude G approaches an asymptotic value. The number of cycles to an asymptotic G value ranged between 30 and 50 for tests performed with γ between 10^{-2} and 10^{-1}% and D_r between 40 and 50% (Alarcon-Guzman, 1986). It is significant that as few as 10 cycles of prior straining at a given amplitude of shear strain will increase the shear modulus by 30 to 60% over that of a virgin sand specimen, with the higher percentages being associated with the larger shear strain amplitudes. In a recent study, Ray and Woods (1988) reported a small increase (less than 20%) in shear modulus due to cyclic strain repetitions. This is consistent with the present findings since their experiments involved stage testing in resonant column and torsional shear modes. As discussed above, stage testing stiffens the specimen significantly and thus obscures most of the stiffening effect that would be observed if the tests at each strain level were performed on virgin specimens.

Behavior in Undrained Shear

The difficulty of establishing a relationship between G_0 and cyclic drained shear behavior of sands was illustrated by the results discussed in the previous sections. It was also indicated that this difficulty is much greater in the case of cyclic undrained behavior, and this is clearly demonstrated by the results presented in Figs. 15 and 16. An isotropically consolidated solid specimen of Ottawa 20-30 sand (Batch 1) with an initial G_0 of 102 MPa, was subjected to undrained cyclic loading in the torsional shear mode with a cyclic shear stress amplitude of about 15 kPa. The shear strains that developed during the first nine cycles were small (up to point A in Fig. 15) while the pore water pressure increased progressively with each loading cycle, as indicated by the stress path moving towards the left in Fig. 15. During the first part of the 10th cycle the specimen "collapsed" (point A in Fig. 15) ; the collapse is

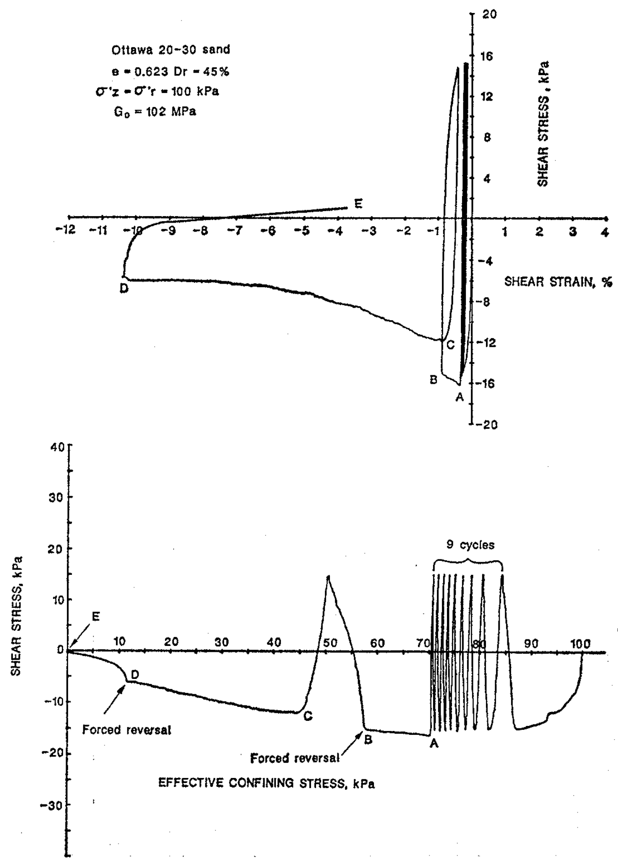

Fig. 15. Cyclic torsional shear test-virgin specimen

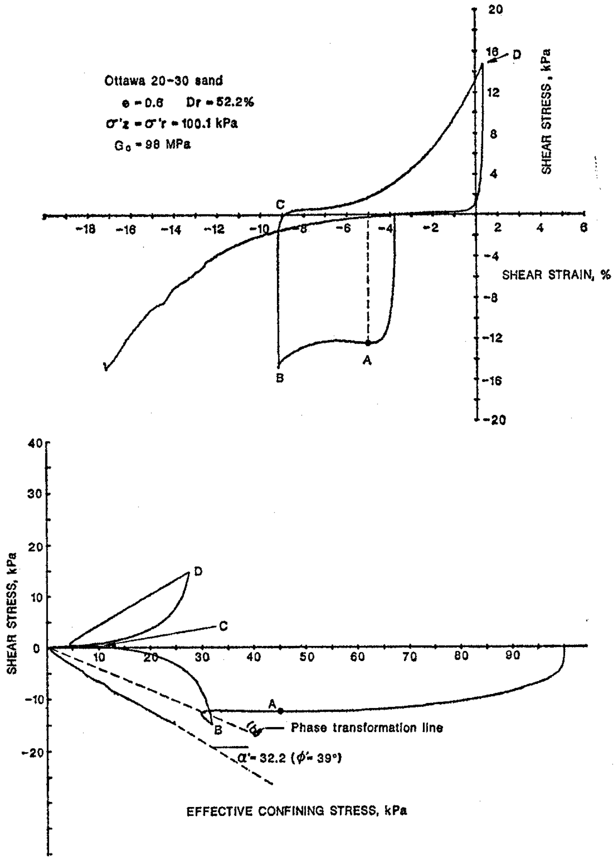

Fig. 16. Cyclic torsional shear test-reconsolidated specimen

manifested by a sharp increase in pore water pressure and subsequent strain softening behavior (from point A to B) and was initiated because the stress path reached the state boundary surface that defines collapse conditions in undrained monotonic tests (Alarcon-Guzman et al., 1988). After a shear strain of about 1% was reached, a reversal in direction of shearing was forced at point B and the specimen was subjected to an additional loading cycle. At point C the specimen experienced a second state of strain softening on loading, as the stress path again reached the state boundary surface. This is reflected in an additional large increase in pore pressure build-up accompanied by a large increase in shear strain (from point C to D in Fig. 15). After a strain of about 10.5% at point D, a second reversal in the direction of shearing was forced to occur, which resulted in the development of a state of essentially zero effective stress as the shear stress approached zero (point E in Fig. 15). Rotation was continued until the shear strain was reduced to about 3.8% with the specimen exhibiting almost negligible stiffness. At this stage the specimen was reconsolidated under the initial state of stress and subjected to a second stage of undrained cyclic loading (Fig. 16).

During reconsolidation the relative density of the specimen increased from 45% to about 52% whereas the measured value of G_0 reduced slightly from 102 to 98 MPa. These results indicate that the degrading effects of large prior shear strains (10.5% at point D, followed by a reduction to 3.8% at point E) on G_0 overrode the stiffening associated with the seven percent increase in relative density of the reconsolidated specimen. It is significant that during the second stage of cyclic loading (Fig. 16), a shear strain increment of about 5.5% developed in the first quarter of the loading cycle (up to point B in Fig. 16) when the shear stress amplitude reached 15 kPa, whereas the pore pressure increased to about 70% of the confining pressure (Fig. 16). This contrasts with a shear strain of only 0.3% and a pore pressure increase to about 30% of the confining pressure in the case of the looser virgin specimen after nine cycles of loading (point A in Fig. 15). Upon reversal in the direction of shearing at point B, the pore water pressure increased sharply and the stress path moved towards the origin of the stress space (Fig. 16). At this stage a drastic reduction in stiffness is observed as the shear stress reverses sign (point C in Fig. 16). Thereafter, the specimen builds up strength as a result of dilation with further straining, until reversal in the direction of shearing at point D. Further straining caused dilation to occur and the stress path moved up the large strain strength envelope, but only after a shear strain of about 10 percent had occurred.

An important feature illustrated in Figs. 15 and 16 is that the reconsolidated specimen developed strain softening during the first quarter of the loading cycle, as compared to more than 10 cycles that were required to bring the virgin specimen into a comparable condition. Thus, cyclic prestraining involving relatively large shear strains creates a particle packing that, although denser, may be more susceptible to pore pressure buildup under undrained cyclic loading, as was also pointed out previously (e. g. Finn et al., 1970 ; Ishihara and Okada, 1982 ; Tobita, 1983 ; Suzuki and Toki, 1984). However, it is noted that the reconsolidated specimen exhibits a lesser degree of strain softening on loading, which is likely due to the increase in density. Hence, after reconsolidation the structure of the sand is initially more compressible but less brittle.

The observed difference in cyclic undrained behavior, before and after reconsolidation, cannot be accounted for by the small difference in the G_0 values of the two specimens. Similar results were obtained on replicate specimens (Ottawa 20-30 sand with relative densities ranging between 40 and 55%) even for residual shear strains at the end of the first loading sequence smaller than that of Fig. 15. Hence, specimens with comparable values of G_0 may exhibit very different undrained behaviors. Consequently, inherent difficulties exist when trying to correlate G_0 with cyclic undrained behavior.

SUMMARY AND CONCLUSIONS

A comprehensive examination of the factors controlling the stress-strain behavior of sand subject to cyclic shear tests has been presented. The results both confirmed and added to the existing knowledge of this behavior. The tests were conducted on Ottawa 20-30 and 50-70 sands, with relative densities (D_r) generally ranging between 40 and 60 percent, although some of the tests were performed with D_r as low as 18%, and as high as 95%. Comparisons with other existing data suggest that the results may be representative of the behavior of different reconstituted sands over a wide range of relative densities ; however, the corresponding behavior of aged or cemented sands remains to be studied. The main findings of this part of the study are summarized as follows :

1. The shear modulus, G_0, of sand interpreted from resonant column tests after several hundred loading-unloading cycles at shear strain amplitudes less than or equal to 10^{-4}% is sensitive to changes in void ratio and especially to the effective normal stress level. G_0 is insensitive to sand fabric and is not directly correlated to relative density. This means that :

a. For sands subject to anisotropic states of normal stress, G_0 is stress path dependent.

b. G_0 is not a sensitive index of the degree to which fabric has been replicated in the preparation of sand specimens.

c. As the shear wave velocity measured in situ is directly correlatable to G_0, and G_0 is not correlated to relative density, inferences regarding the state of compaction of granular soils from geophysical measurements alone may be subject to considerable error.

2. The shear modulus, G, as conventionally interpreted from cyclic torsional or triaxial shear tests, is an unloading-reloading modulus even when determined from the first loading cycle. In drained shear tests, its value is affected only slightly (typically by less than 10 percent) by such factors as monotonic prestressing, a single cycle of prestraining, or the extant initial principal stress ratio. On the other hand, the shear modulus in monotonic loading, or in the first quarter of a load cycle, is strongly affected by all these factors. Accordingly, as the pore water pressure induced by the first quarter of the loading cycle in undrained shear is a large fraction of the pore pressure generated in the entire cycle, the insensitivity of G to prestraining and stress ratio in drained cyclic shear tests can lead to misleading interpretations of their effects on the behavior of sands in undrained shear.

3. For specimens with no prior strain history, the conventionally interpreted values of G, at shear strain amplitudes larger than about 10^{-3}%, are sensitive to the number of

applied repetitions of cyclic loading. The effect is most pronounced during the first 10 to 20 cycles ; typically, an asymptotic value is reached in 30 to 50 cycles. As few as 10 cycles of prior straining at a given amplitude of shear strain can increase the shear modulus by 30 to 60% over that interpreted from the first loading cycle, with the higher percentages being associated with the larger shear strain amplitudes.

4. Repeated shear strain amplitudes of less than 10^{-4}% have no effect on the value of G in drained tests and do not generate pore water pressures in undrained tests. This is consistent with the concept of a threshold shear strain. However, stage testing involving repeated values of strain equal to 10^{-2}% results in an increase in G of about 40% over that obtained from a virgin sand specimen. Accordingly, stage testing involving more than a few strain cycles, even at amplitudes as low as 10^{-2}%, can lead to important overestimations of the shear modulus of virgin specimens. It is significant that the effect of cyclic prestraining persists even when G is measured at values of shear strain larger than those extant during cyclic prestraining.

5. G_0 of reconstituted sands is insensitive to sand fabirc, initial principal stress ratio, monotonic prestressing and cyclic strain history because the effects of these factors become apparent only at shear strain amplitudes large enough to cause rearrangements of sand particles. Thus, the same sand can have the same G_0 but behavior in undrained shear can be drastically different (Figs. 15 and 16), due to different strain histories. The undrained behavior of different sands having the same G_0 can differ even more drastically than is the case for the same sand. Therefore, there are inherent limitations to obtaining general correlations between G_0 and behavior in undrained shear.

6. The subsequent behavior of sand subjected to a sufficient number of strain cycles to induce strain softening in undrained shear, and then allowed to consolidate fully, cannot readily be predicted. The prior strain cycles in undrained shear can alter the sand structure to make it more susceptible to build-up of pore water pressure in subsequent undrained loading while reconsolidation will densify the sand and cause the opposite effect. Thus, sands that have liquefied due to prior loadings can be either more, or less, susceptible to subsequent liquefaction, depending upon which of these effects dominates.

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NOTATION

D = damping ratio
D_r = relative density
e = void ratio
e_{max} = maximum void ratio
e_{min} = minimum void ratio
f = frequency
G = shear modulus
G_0 = maximum shear modulus
K = principal stress ratio
K_c = consolidation stress ratio
N = number of cycles
NC = normally consolidated
OC = overconsolidated
OCR = overconsolidation ratio
$p' = (\sigma'_1 + \sigma'_3)/2$, stress path parameter
$q = (\sigma_1 - \sigma_3)/2$, stress path parameter
γ = shear strain amplitude
ϕ' = angle of shearing resistance
σ_r = radial stress
σ_z = axial stress
σ_1 = major principal stress
σ_3 = minor principal stress
σ' = effective stress
σ'_m = mean effective confining stress
τ = shear stress
$\tau_{z\theta}$ = torsional shear stress component

REFERENCES

1) Alarcon-Guzman, A. (1986) : "Cyclic stress-strain and liquefaction characteristics of sands," Ph. D. Thesis, Purdue University, West Lafayette, IN.
2) Alarcon-Guzman, A., Chameau, J. L. and Leonards, G. A. (1986) : "A new apparatus for investigating the stress-strain characteristics of sands," Geotechnical Testing Journal, GTJODJ, Vol. 9, No. 4, pp. 204-212.
3) Alarcon-Guzman, A., Leonards, G. A. and Chameau, J. L. (1988) : "Undrained monotonic and cyclic strength of sands," Journal of the Geotechnical Engineering Division, ASCE, Vol. 114, No. 10, pp. 1089-1109.
4) Baldi, G., Bellotti, R., Crippa, V., Fretti, C., Ghionna, V., Jamiolkowski, M., Ostricati, D., Pascualini, E. and Pedroni, S. (1985) : "Laboratory validation of in situ tests," Geotechnical Engineering in Italy-An Overview, published on the occasion of the ISSMFE Golden Jubilee, 11 th International Conference on Soil Mechanics and Foundations Engineering, San Francisco, pp. 251-270.
5) Chen, Y. C., Ishibashi, I. and Jenkins, J. T. (1988) : "Dynamic shear modulus and fabric : Part I, Depositional and induced anisotropy," Geotechnique, Vol. 38, No. 1, pp. 25-32.
6) Chung, R. M., Yokel, F. Y. and Drnevich, V. P. (1984) : "Evaluation of dynamic properties of sands by resonant column testing," Geotechnical Testing Journal, GTJODJ, Vol. 7, No. 2, pp. 60-69.
7) Clayton, C. R. I., Hababa, M. B. and Simons, N. E. (1985) : "Dynamic penetration resistance and the compressibility of a fine grained sand--A laboratory study," Geotechnique, Vol. 35, No. 1, pp. 19-31.
8) Dobry, R., Stokoe II, K. H., Ladd, R. S. and Youd, T. L. (1981) : "Liquefaction susceptibility from S-wave velocity," Preprint 81-544, In Situ Testing to Evaluate Liquefaction Susceptibility, ASCE, National Convention, St. Louis, Missouri.
9) Drnevich, V. P. and Richart, Jr., F. E. (1970) : "Dynamic prestraining of dry sand," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 96, No. SM 2, pp. 453-469.
10) Finn, W. D. L., Bransby, P. L. and Pickering, D. J. (1970) : "Effect of strain history on liquefaction of sand," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 96, No. SM 6, pp. 1917-1934.
11) Hardin, B. O. and Drnevich, V. P. (1972) : "Shear modulus and damping in soils : Measurement and parameter effects," Journal of the Soil Mechanics and Foundation Division, ASCE, Vol. 98, No. SM 6, pp. 603-624.
12) Ishihara, K. and Li, S.-I. (1972) : "Liquefaction of saturated sand in triaxial torsion shear test," Soils and Foundations, Vol. 12, No. 2, pp. 19-39.
13) Ishihara, K. and Okada, S. (1982) : "Effects of large preshearing on cyclic behavior of sand," Soils and Foundations, Vol. 22, No. 3, pp. 109-125.
14) Ishihara, K. and Takatsu, H. (1979) : "Effects of overconsolidation and K_0 conditions on the liquefaction characteristics of sands," Soils and Foundations, Vol. 19, No. 4, pp. 60-68.
15) Iwasaki, T., Tatsuoka, F. and Takagi, Y. (1978): "Shear moduli of sands under cyclic torsional shear loading," Soils and Foundations, Vol. 18, No. 1, pp. 40-56.
16) Jamiolkowski, M., Ladd, C. C., Germaine, J. T. and Lancellotta, R. (1985) : "New developments in field and laboratory testing of soils," Proceedings of the 11 th International Conference on Soil Mechanics and Foundation Engineering, San Francisco, Vol. 1, pp. 57-153.
17) Kokusho, T. (1980) : "Cyclic triaxial test of dynamic soil properties for wide strain range," Soils and Foundations, Vol. 20, No. 3, pp. 46-60.
18) Lambrechts, J. R. and Leonards, G. A. (1978) : "Effects of stress history on deformation of sand," Journal of the Geotechnical Engineering Division, ASCE, Vol. 104, No. GT 11, pp. 1371-1387.
19) Lee, K. L. and Farhoomand, I. (1967) : "Compressibility and crushing of granular soil in anisotropic triaxial compression," Canadian Geotechnical Journal, Vol. 4, No. 1, pp. 68-99.
20) Lee, K. L. and Seed, H. B. (1967) : "Dynamic strength of anisotropically consolidated sand," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 93, No. SM 5, pp. 169-189.
21) Lee, K. L. and Seed, H. B. (1967) : "Drained strength characteristics of sands," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 93, NO. SM 6, pp. 117-141.
22) Leonards, G. A., Alarcon, A., Frost, J. D., Mohamedzein, Y. E., Santamarina, J. C., Thevanayagam, S., Thomaz, J. E. and Tyree, J. L. (1986) : "Dynamic penetration resistance and the prediction of the compressibility of a

fine-grained sand--A laboratory study," Discussion, Geotechnique, Vol. 36, No. 2, pp. 275-279.
23) Oda, M. (1972): "The mechanism of fabric changes during compressional deformation of sand," Soils and Foundations, Vol. 12, No. 2, pp. 1-18.
24) Ray, R. P. and Woods, R. D. (1988): "Modulus and damping due to uniform and variable cyclic loading," Journal of Geotechnical Engineering Division, ASCE, Vol. 114, No. 8, pp. 861-876.
25) Richart, Jr., F. E. (1977): "Field and laboratory measurements of dynamic soil properties," Proceedings of DMSR 77, Karlsruhe, Vol. 1, pp. 3-35.
26) Schmertmann, J. H. (1985): "Stress ratio effects on shear modulus of dry sands," Discussion, Journal of Geotechnical Engineering Division, ASCE, Vol. 111, No. 9, pp. 1153-1154.
27) Seed, H. B. (1979): "Soil liquefaction and cyclic mobility evaluation for level ground during earthquakes," Journal of the Geotechnical Engineering Division, ASCE, Vol. 105, No. GT 2, pp. 201-255.
28) Seed, H. B. and Idriss, I. M. (1970): "Soil moduli and damping factors for dynamic response analyses," Report No. EERC 70-10, University of Califormia, Berkeley.
29) Seed, H. B., Wong, R. T., Idriss, I. M. and Tokimatsu, K. (1984): "Moduli and damping factors for dynamic analyses of cohesionless soils," Report No. UCB/EERC-84/14, Earthquake Engineering Research Center, University of California, Berkeley.
30) Shen, C. K., Li, X. S. and Gu, Y. F. (1985): "Microcomputer based free torsional vibration test," Journal of the Geotechnical Engineering Division, ASCE, Vol. 111, No. 8, pp. 971-986.
31) Silver, M. L. and Park, T. K. (1975): "Testing procedure effects on dynamic soil behavior," Journal of the Geotechnical Engineering Division, ASCE, Vol. 101, No. GT 10, pp. 1061-1083.
32) Suzuki, T. and Toki, S. (1984): "Effects of preshearing on liquefaction characteristics of saturated sand subjected to cyclic loading," Soils and Foundations, Vol. 24, No. 2, pp. 16-28.
33) Tatsuoka, F., Iwasaki, T., Yoshida, S., Fukushima, S. and Sudo, H. (1979 a): "Shear modolus and damping by drained tests on clean sand specimens reconstituted by various methods," Soils and Foundations, Vol. 19, No. 1, pp. 39-54.
34) Tatsuoka, F., Iwasaki, T., Fukushima, S. and Sudo, H. (1979 b): "Stress conditions and stress histories affecting shear modulus and damping of sand under cyclic loading," Soils and Foundations, Vol. 19, No. 2, pp. 29-43.
35) Tobita, Y. (1983): "Effects of fabric change on deformation characteristics of sand in cyclic shearing," Advances in the Mechanics and the Flow of Granular Materials, Mohsen Shahinpoor, Editor, Trans. Tech. Publications, Vol. II, pp. 885-901.
36) Tokimatsu, K., Yamazaki, T. and Yoshimi, Y. (1986): "Soil liquefaction evaluations by elastic shear moduli," Soils and Foundations, Vol. 26, No. 1, pp. 25-35.
37) Vaid, Y. P. and Chern, J. C. (1983): "Effect of static shear on resistance to liquefaction," Soils and Foundations, Vol. 23, No. 1, pp. 47-60.
38) Vaid, Y. P. and Chern, J. C. (1985): "Cyclic and monotonic undrained response of saturated sands," Advances in the Art of Testing Soils under Cyclic Conditions, Session 52, ASCE Annual Convention and Exposition, Detroit, pp. 120-147.
39) Yu, P. and Richart, Jr., F. E. (1984): "Stress ratio effects on shear modulus of dry sands," Journal of Geotechnical Engineering Division, Vol. 110, No. 3, ASCE, pp. 331-345.
40) Yu, P. and Richart, Jr., F. E. (1985): "Stress ratio effects on shear modulus of dry sands," Closure, Journal of Geotechnical Engineering Division, Vol. 111, No. 9, ASCE, pp. 1157-1159.