SOILS AND FOUNDATIONS Vol. 30, No. 1, 155-166, Mar. 1990
Japanese Society of Soil Mechanics and Foundation Engineering

STRESS DILATANCY OF SAND AT SMALL STRESS RATIO STATES

DAWIT NEGUSSEY[i] and Y. P. VAID[ii]

ABSTRACT

The validity of the stress dilatancy equation for representing the small stress ratio state response of sand is examined. Test equipment development and procedures that were followed aimed at precise loading control and reliable monitoring of small displacements and volume changes. A variety of load controlled stress paths were applied to conventional triaxial test samples of Ottawa sand. The results obtained show that at small stress ratio states a relationship between stress state and strain increment direction as prescribed by the stress dilatancy equation does not apply. Stress increment directions were found to be related to strain increment directions for low stress ratio states. The relationship has some resemblance to the behavior of a cross anisotropic elastic material with a stiffer vertical to radial response. The range of low stress ratio states over which a stress dilatancy relationship does not apply tends to increase with density. In stress ratio regions where the stress dilatancy equation began to apply, the relationship between R and D tends toward K_{cv} with increasing confining stress for conventional triaxial but not for constant mean normal stress paths.

Key words: density, dilatancy, sand, shear, stress path, stress ratio (**IGC**: D 6)

INTRODUCTION

The stress dilatancy theory proposed by Rowe (1962) treats deformation characteristics of sand from fundamental considerations at the particulate level. The stress dilatancy equation

$$R=KD \tag{1}$$

relates the ratio, R, of major and minor principal stress to the incremental ratio of major and minor principal strains, D, through a constant K. The constant represents the ratio of the rate of energy input to the rate of energy output in coincident principal stress and strain directions and is presumed to be bounded between lower and upper limits, K_μ and K_{cv}. Both limits are considered to depend on the mineral constituting the particulate material.

For the stress conditions of the triaxial compression test, Eq. (1) takes the following form:

[i] Associate Professor, Department of Civil and Environmental Engineering, Syracuse University, Syracuse, New York 13244-1190, USA.
[ii] Professor, Department of Civil Engineering, University of British Columbia, Vancouver, B. C., V6T 1W5, Canada.
Manuscript was received for review on January 23, 1989.
Written discussions on this paper should be submitted before October 1, 1990, to the Japanese Society of Soil Mechanics and Foundation Engineering, Sugayama Bldg. 4 F, Kanda Awaji-cho 2-23, Chiyoda-ku, Tokyo 101, Japan. Upon request the closing date may be extended one month.

$$\frac{\sigma_a'}{\sigma_r'}=K\left(1-\frac{\delta\varepsilon_v}{\delta\varepsilon_a}\right)=K\frac{-2\delta\varepsilon_r}{\delta\varepsilon_a} \qquad (2)$$

in which σ_a' and σ_r' are the principal axial and radial effective stresses, $\delta\varepsilon_a$ and $\delta\varepsilon_r$ the principal axial and radial slip strain increments and $\delta\varepsilon_v=\delta\varepsilon_a+2\,\delta\varepsilon_r$, the volumetric strain increment. Eq. (2) relates instantaneous stress state to strain increment direction and thus resembles a flow rule as in plasticity theory but without stipulation on normality. Stress dilatancy theory has therefore attracted the interest of researchers as an alternative but fundamental basis for an incremental stress-strain model for sand. Development of realistic soil stress strain models are essential for solving geotechnical boundary value problems using numerical techniques.

The stress dilatancy theory is based on considerations of sliding between two particles. Sliding along a contact plane would be initiated when the corresponding Mohr-Coulomb shear strength is exceeded. Rowe (1962) postulated that the orientation of the plane of sliding would be so as to minimize the ratio of incremental energy input to output along the principal directions of stress and strain increment. This mechanism was then extended to describe the deformation of a random assembly of irregular particles in contact. The particles were considered to be rigid and deformations to be a result of non recoverable slip.

Reported agreement with experimental results has been the mainstay for the stress dilatancy theory. The bulk of supporting experimental evidence has come from conventional triaxial tests. The stress dilatancy equation has been shown to describe sand behavior for conditions of increasing stress ratio, starting from a hydrostatic state to peak and ultimate state (Barden and Khayatt, 1966). Confirmations were found most favorable for dense states, and upon reloading for loose conditions. Additional agreement with stress dilatancy was reported from plane strain and simple shear test results (Barden et al., 1969; Cole, 1967). These further verifications together with previous confirmations have led to the view that the stress dilatancy relationship is independent of stress path, stress ratio and confining stress levels, as long as the stress probe remains in a direction of increasing stress ratio. Previously reported experimental verifications of the stress dilatancy theory have been based on representation of total strains as slip strains. For conditions of increasing stress ratio, sliding deformations have been presumed to be predominant and hence equivalent to total deformations both during loading and reloading stages (Rowe, 1971).

The first objective of this paper is to undertake a closer examination of stress dilatancy at small stress ratio ($R<2$) and hence small strains (10^{-4} to 10^{-2}) in conventional triaxial paths. This issue has been raised by Nova and Wood (1979) and Nova (1982) and reservations as to the form of the stress dilatancy relationship at small stress ratios have been expressed on the basis of conceptual arguments. The second objective is to examine the stress path independence of the stress dilatancy equation at small stress ratios and the associated small strain response region. As in previous studies of stress dilatancy, total strains will be for the most part regarded as slip strains. However, as non recoverable deformations may not be negligible in small strain regions, an approximate correspondence between slip and total strains may not be justified. In accordance with the fundamental assumptions of the stress dilatancy theory, separation of total strains into recoverable and nonrecoverable slip components will be attempted and the possible unique association of nonrecoverable strain increment directions to stress states will be investigated. The emphasis on small strain response region stems from the fact that in many situations working stresses under drained conditions generally induce strains in sands that are less than 10^{-2}.

EXPERIMENTATION

Tests were carried out on Ottawa sand ASTM C-109, a medium uniform quartz sand

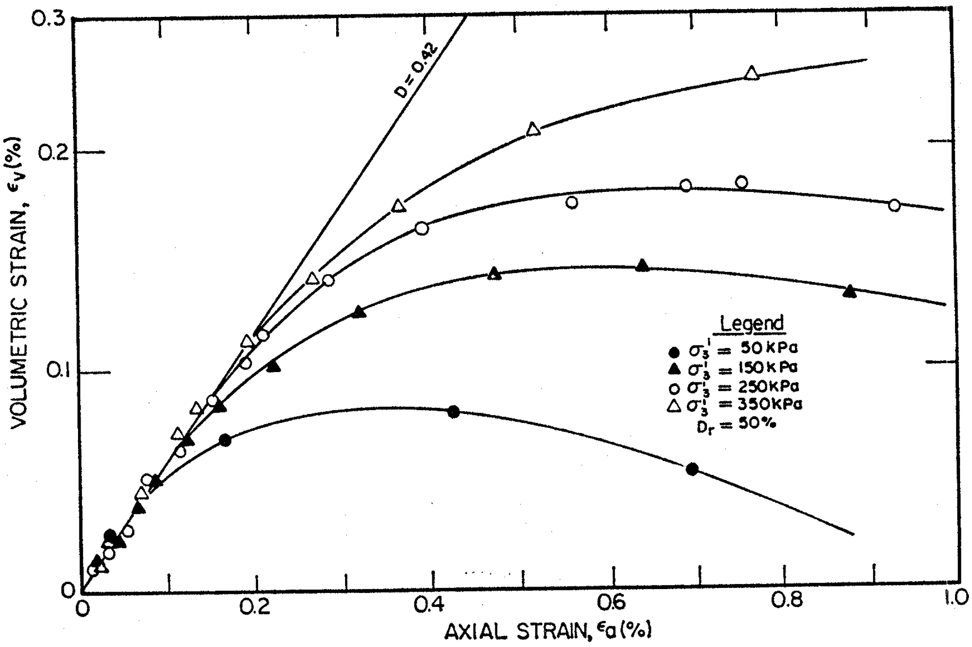

Fig. 1. (a) Strain paths for conventional triaxial stress paths

having $D_{50}=0.40$ mm, $C_u=1.5$ and $e_{max}=0.82$ and $e_{min}=0.50$. A constant volume friction angle ϕ_{cv} of 30 degrees was obtained for this sand from ring shear testing (Negussey et al., 1988). There is generally no consensus on a precise value of ϕ_μ, the angle of inter-particle friction of quartz (Rowe, 1971). An average value of $\phi_\mu=25$ degrees was selected for the sand tested. Triaxial test specimens were 63.5 mm diameter and 130 mm high and were reconstituted by water pluviation. Smooth anodized aluminum end plattens with centrally located 20 mm diameter porous discs were used in order to minimize end restraint and bedding errors. Lubricated end plattens were not used because of associated large bedding errors (Sarsby et al., 1980) and marginal usefulness for small strain considerations as would be inferred from the reported results by Barden and Khayatt, 1966 ; and Rowe and Barden, 1964. For stress paths involving changes in effective confining stress, necessary membrane penetration corrections were estimated as suggested by Vaid and Negussey (1984). The selected sample geometry and steps taken to minimize apparatus compressibility, bedding errors and improved membrane penetration corrections together with the use of high resolution data system enabled confident and consistent measurement of both axial and volumetric strain increments in the order of 1×10^{-5}.

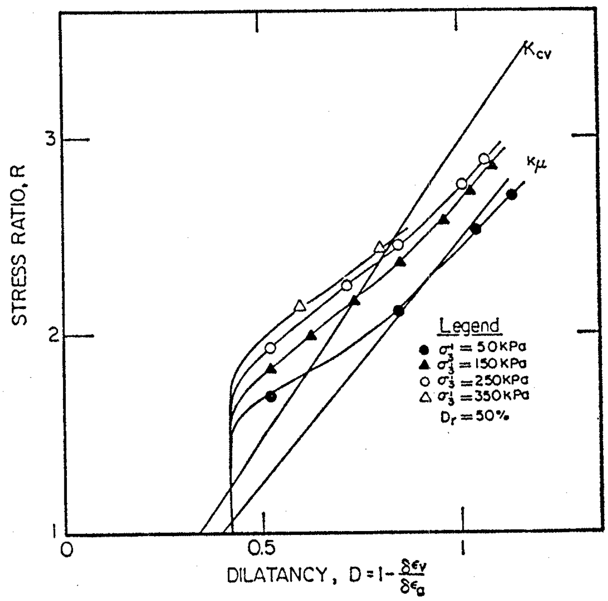

Fig. 1. (b) Stress dilatancy plot of conventional triaxial test results

TEST RESULTS

Stress Dilatancy in Conventional Triaxial Paths

(a) *The influence of confining stress level*

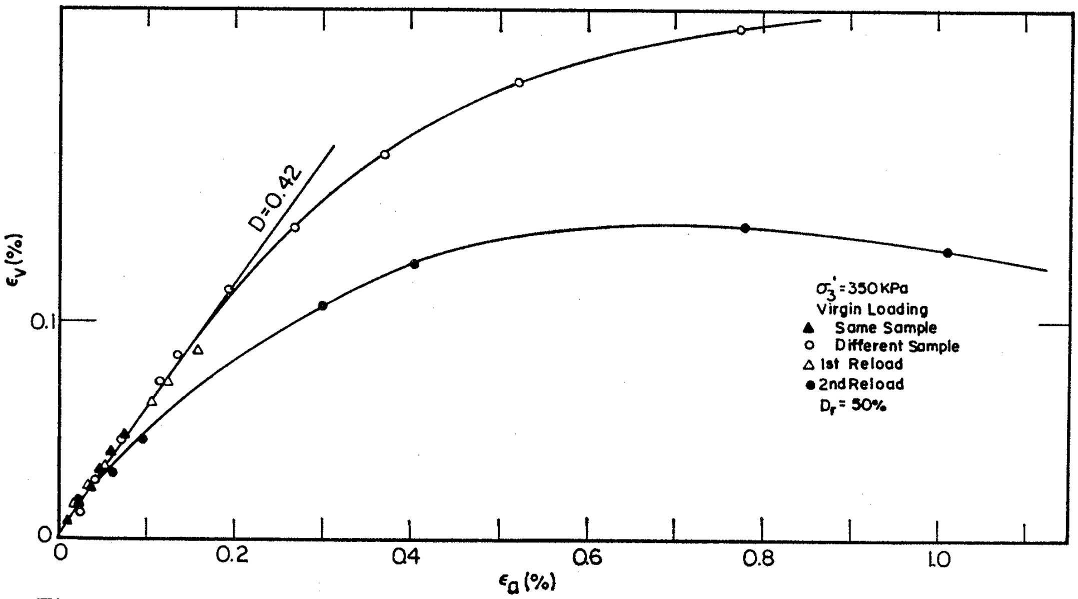

Fig. 2. (a) Strain paths for conventional triaxial loading and reloading

A plot of volumetric strain against axial strain from the results of a series of conventional triaxial tests (σ_r'=constant) at the same relative density of 50 percent but different levels of confining stress are presented in Fig. 1(a). Incremental strain ratios were obtained as tangent slopes in Fig. 1(a) from which dilatancy, $D=\left(1-\frac{\delta\varepsilon_v}{\delta\varepsilon_a}\right)$, was determined to produce the plot in Fig. 1(b). Test data in Fig. 1(a) show that at low strain levels (ε_a<about 0.1%), D is relatively constant and independent of confining pressure. The strain range over which D tends to remain constant increases with confining stress level from ε_a of about 0.1% for σ_r'=50 kPa to ε_a of about 0.2% for σ_r'=450 kPa. Thus, in the R–D space of Fig. 1(b), resulting dilatancy plots have an initial vertical segment over this range of R values for which strain increment ratios remain constant. This implies strain increment ratios are characteristic to the stress path rather than the stress state (R) in the initial stages of shear loading. Hence, the stress-dilatancy equation is not a suitable flow rule for modeling sand behavior at small strains. Fig. 1(b) also shows that with increasing confining pressure, the onset of increase in initial dilatancy gets delayed progressively, and the data for each confining stress follow separate

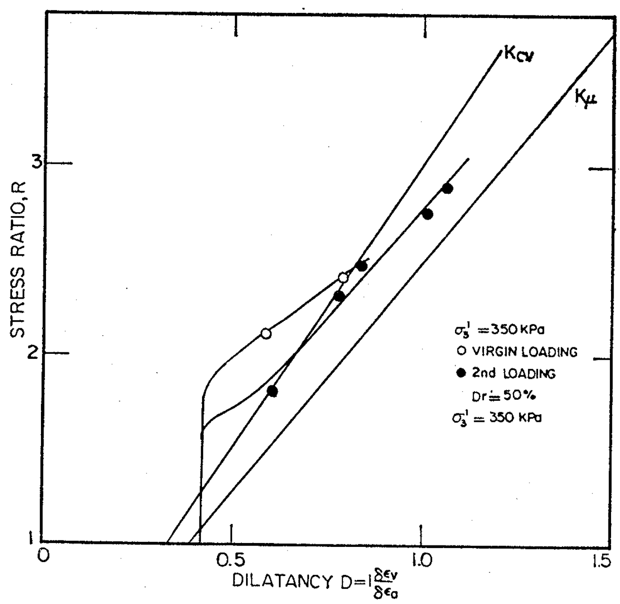

Fig. 2. (b) Stress dilatancy plot for conventional triaxial loading and reloading

lines that tend from K_μ towards K_{cv}. There is thus a slight, but discernible, dependence on confining stress in the relationship between R and D beyond the small stress ratio response region.

It is generally recognized that at a given void ratio dilation becomes suppressed with increasing confining stress (Vesic and Clough, 1968). Therefore, at least in a qualitative sense, increasing density and confin-

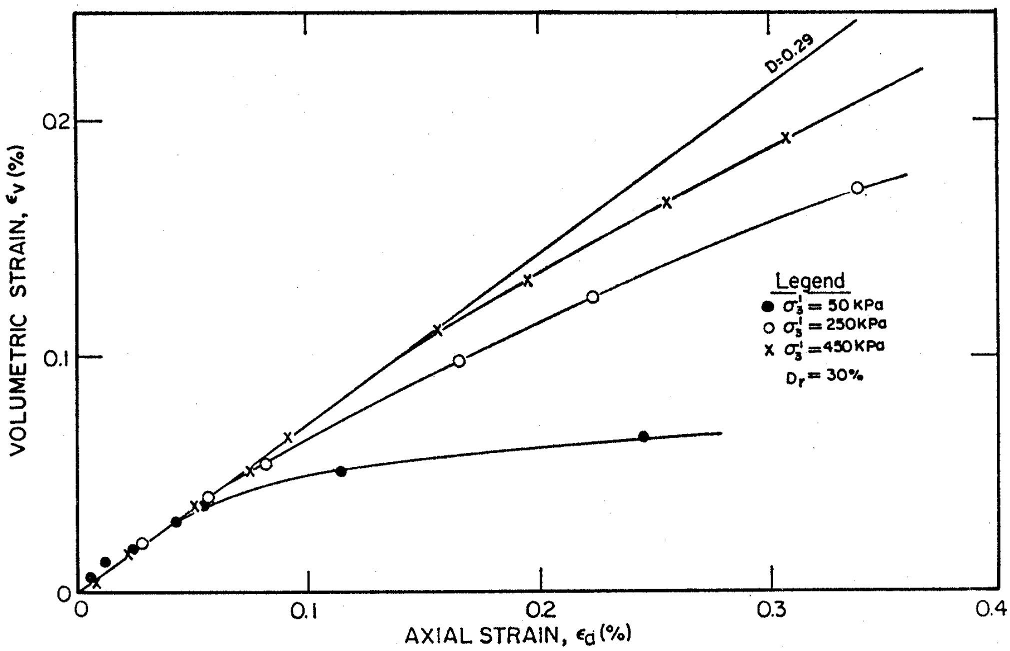

Fig. 3. (a) Strain paths for conventional triaxial tests on loose sand

ing stress appear to have compensating effects. The influence of density has been recognized in the stress dilatancy expression by extreme settings relative to K_{cv} and K_μ under triaxial conditions. However, the possible existence of order on the basis of confining stress and within the extreme limits of K_μ and K_{cv} has neither been suggested previously nor identified experimentally.

(b) *Loading and reloading*

An initial common linear strain increment response may also be observed not only during loading but also on reloading, as shown in Fig. 2(a) for test results at a confining pressure of 350 kPa. Thus dilatancy D in the small stress ratio region is indicated to be constant. In the stress dilatancy plot (Fig. 2 (b)) the initial vertical and subsequent transition segments are still in evidence and the initial dilatancy is the same for loading and subsequent reloading. Although strain increment magnitudes associated with loading exceed those during reloading, both share a common initial strain increment ratio. This implies that the association of stress increment and strain increment directions for loading is the same as for reloading.

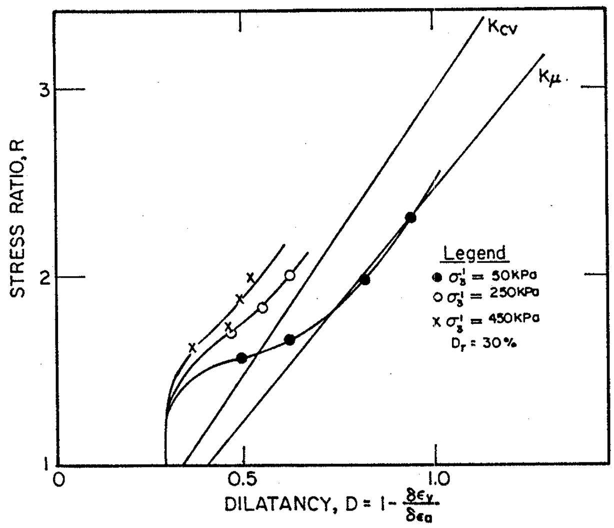

Fig. 3. (b) Stress dilatancy plot of conventional triaxial tests on loose sand

(c) *The influence of density*

A common initial strain path is also followed in loose sand, as illustrated in Fig. 3 (a) by test results at a relative density of 30 percent at three confining stresses. In a stress dilatancy plot, Fig. 3(b), this again corresponds to an initial vertical segment. However, this stress dilatancy relationship initiates to the left of that for medium dense sand (D_r=50%), and with increasing strain transitions towards parallel alignment to K_μ

and K_{cv} at lower and higher confining stresses, respectively. Thus, the influence of confining stress in a stress dilatancy plot is similar but more significant for loose as opposed to medium dense sand. The low stress ratio range over which a stress dilatancy relationship does not apply is smaller for the loose density. Therefore, sand behavior in stress dilatancy terms shows some dependence on density as well as confining stress level.

Dilatancy in Different Stress Paths

The preceding experimental results at small stress ratio and in conventional triaxial paths indicated that the stress dilatancy relationship did not continue as a straight line to a state of hydrostatic compression. Constant total strain directions that were independent of

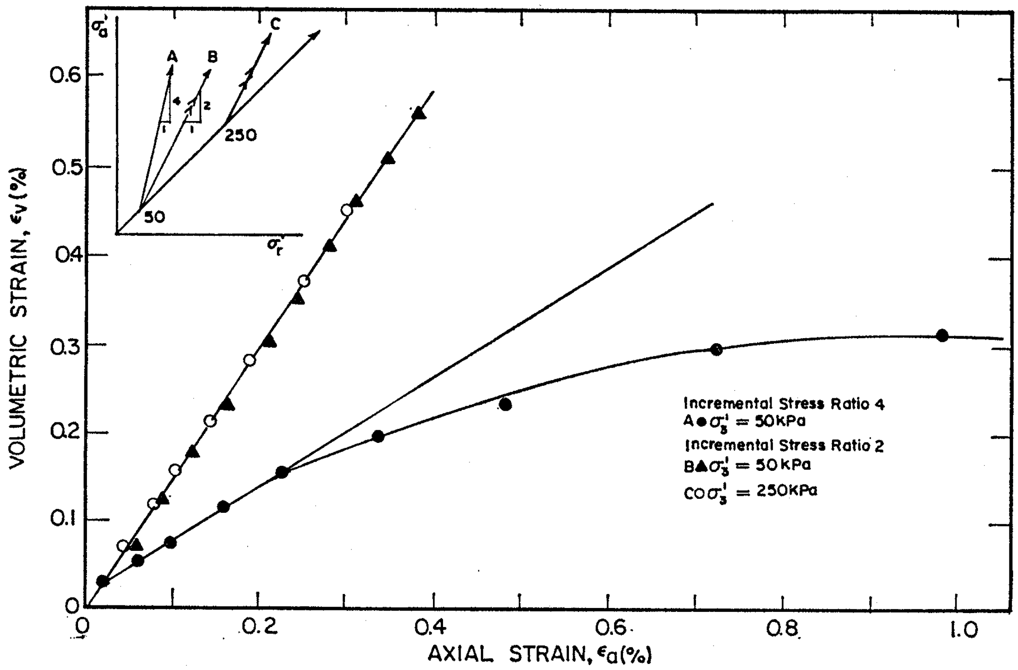

Fig. 4. (a) Strain paths for constant incremental stress ratio paths

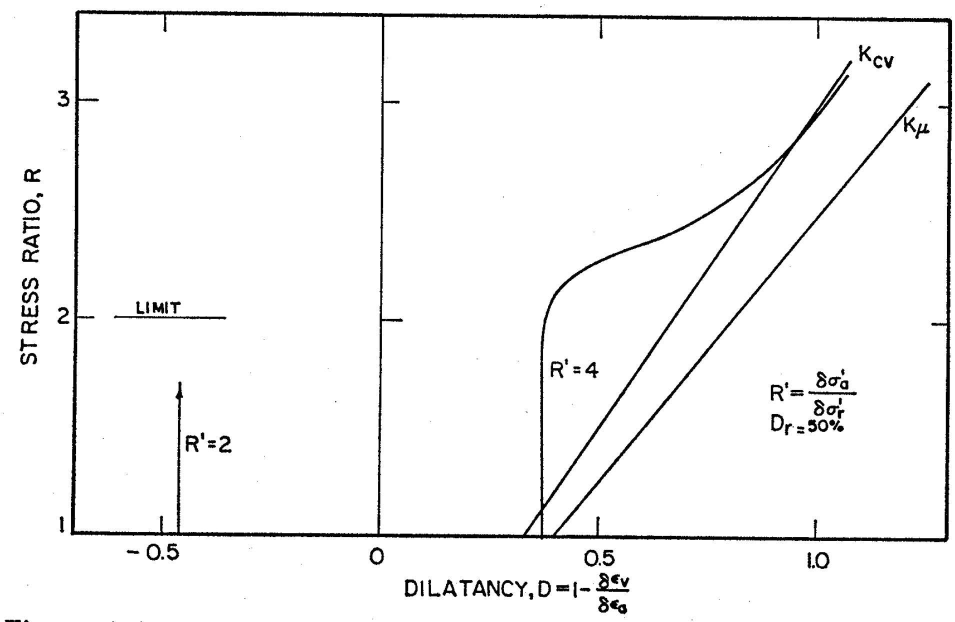

Fig. 4. (b) Stress dilatancy plot for constant incremental stress ratio paths

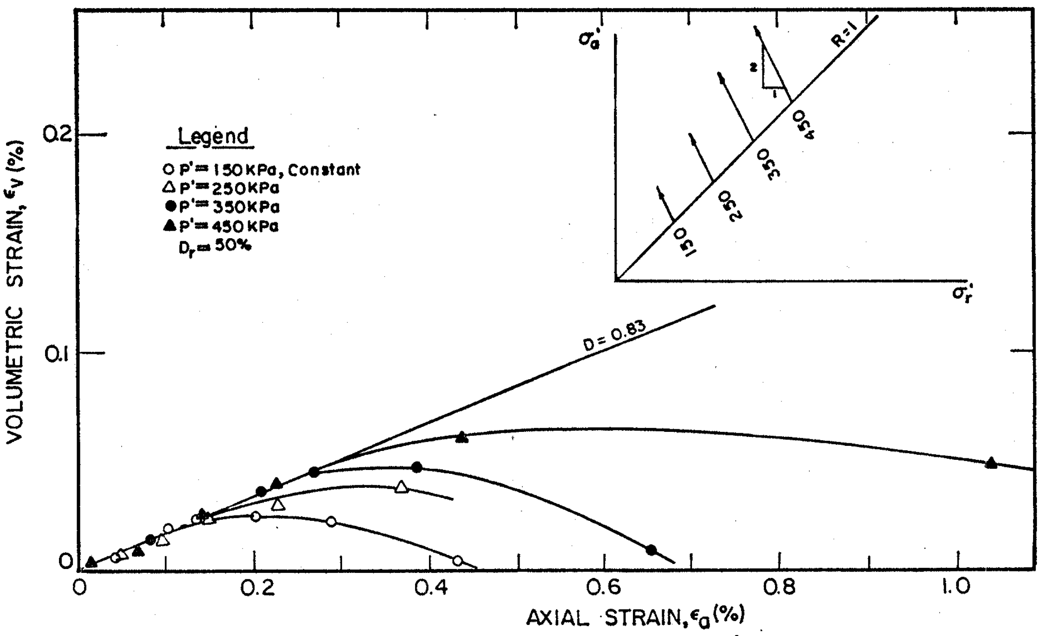

Fig. 5. (a) Strain paths for constant mean normal stress paths

confining stress but dependent on density were observed for low stress ratio states. Thus the hypothesis and subsequent experimental verifications advanced to justify the validity of the stress dilatancy equation for all states of increasing stress ratio starting from a state of $R=1$ could not be supported. Further consideration of diverse linear stress paths in which the requirement of monotonically increasing stress ratio, as stipulated by stress dilatancy theory are satisfied, are presented below to provide additional clarification.

Results from constant incremental stress ratio ($\Delta\sigma_a'/\Delta\sigma_r'$=constant) paths at an initial relative density of 50 percent, are shown in Fig. 4 in terms of volumetric against axial strain and stress dilatancy plots. The specimens were hydrostatically consolidated to $\sigma_r'=50$ and 250 kPa. It may be noted in Fig. 4(a) that constant incremental stress ratio paths of 2 maintain a constant dilatancy factor of -0.5 regardless of confining as well as shear stress level. A negative value of D implies both axial and radial strain increments are positive (compressive). Hence energy is input in both principal directions whether or not total or nonrecoverable strains are considered, and thus the concept

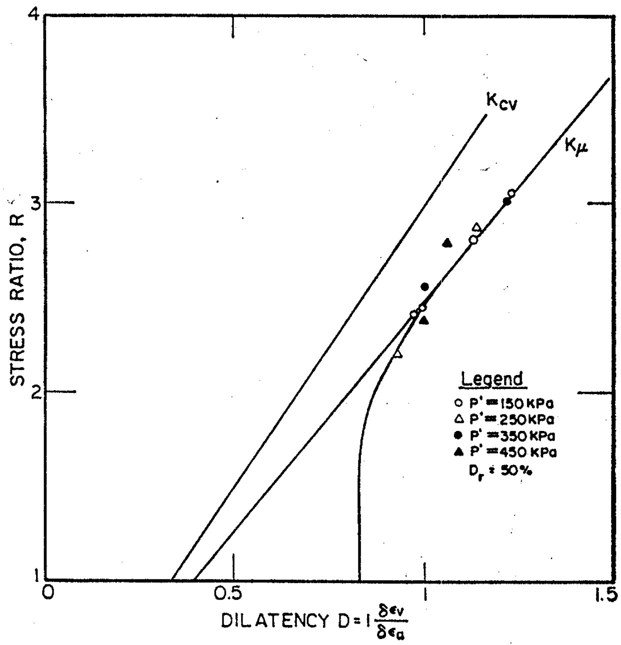

Fig. 5. (b) Stress dilatancy plot for constant mean normal stress paths

of stress dilatancy should not apply for these cases. For an incremental stress ratio path of 4 also, an initial constant dilatancy factor of 0.37 may be noted, that holds until R in excess of about 2.1.

Test result from constant mean normal stress paths, [$p'=1/3(\sigma_a'+2\sigma_r')$] shown in Fig. 5 indicate a constant initial dilatancy of 0.83. For these stress paths, the influence of stress level appears to be less evident in comparison to conventional triaxial paths.

The hypothesis that stress dilatancy is valid in all paths of increasing stress ratio, from hydrostatic to failure state, thus does not appear to be supported by experimental evidence presented herein. Previous verifications of stress dilatancy were based on test results from conventional triaxial, simple shear and plane strain tests. Of these, however, a hydrostatic stress state is accessible only to the conventional triaxial test. Both plane strain and simple shear tests initiate from K_0 consolidation states that correspond to stress ratio states generally in excess of

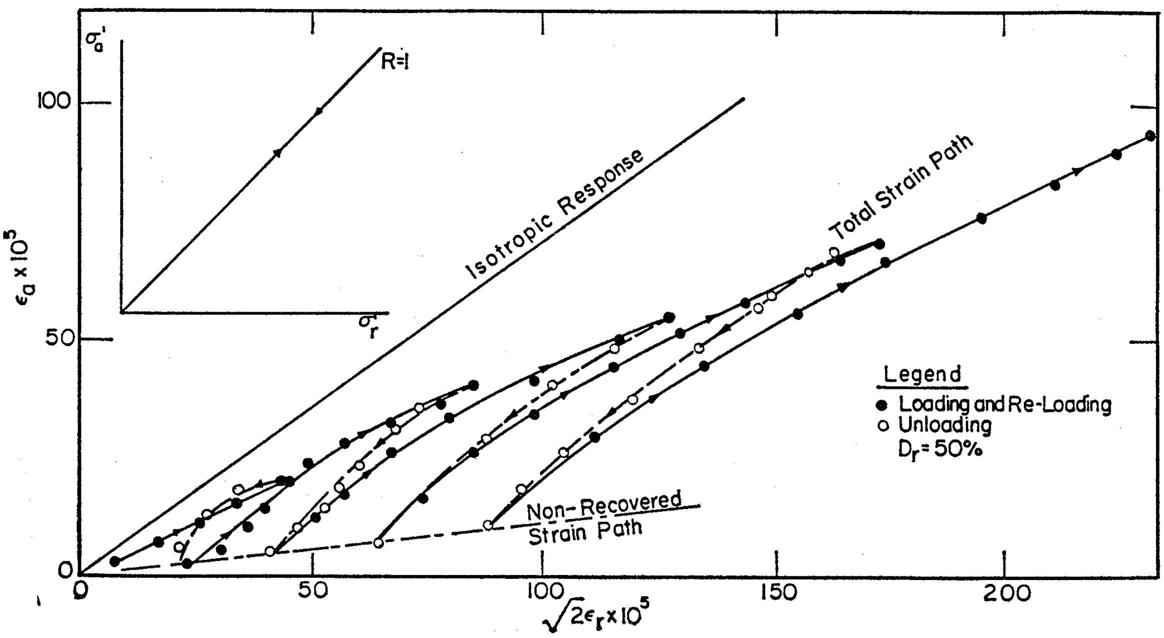

Fig. 6. (a) Strain paths for conditions of hydrostatic loadings and unloadings

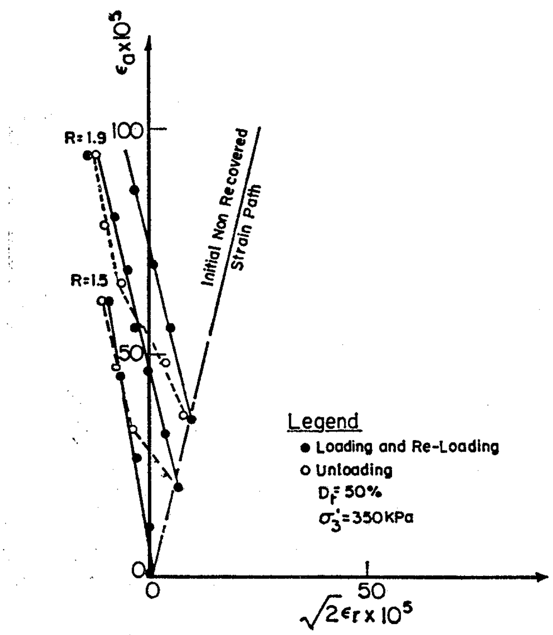

Fig. 6. (b) Strain component directions for conventional triaxial loadings and unloadings

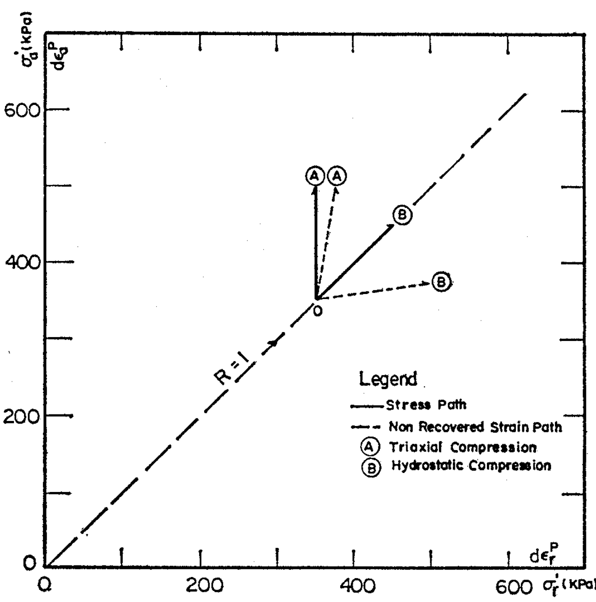

Fig. 6. (c) Comparison of stress and non-recovered strain directions at low stress ratio states

2. Hence validity of stress dilatancy below a stress ratio of 2 could only be assessed on the basis of conventional triaxial test data. As may be noted in Figs. 2(b) and 3(b), the initial dilatancy factor for conventional triaxial paths is located within close proximity of the upper K_{cv} and lower K_μ bounds specified by stress dilatancy. This coincidence may have encouraged unjustifiable extrapolation of experimental evidence in previous studies in support of stress dilatancy in the region of small stress ratios.

Stress path test results and comparisons presented herein have been made subsequent to application of appropriate membrane compliance corrections to volume change observations. The observed deviations from the stress dilatancy equation for stress paths that involve changing confining stress states, whether increasing or decreasing, would be more pronounced if volume changes were not corrected for effects of membrane compliance.

Consideration of Non Recoverable (Slip) Strain Directions

Results of loading to and unloading from increasing magnitudes of hydrostatic effective stress have been plotted in strain space in Fig. 6(a). An approximately linear strain path may be noted for accumulated residual strain. Accumulated residual strains following loading and unloading are generally considered to be a result of nonrecoverable (slip) deformations.

Fig. 6(b) presents results in strain space for loading and unloading of a specimen at $D_r=50\%$ consolidated hydrostatically under $\sigma_r'=350$ kPa to increasing levels of deviator stress along a conventional triaxial path (σ_r'=constant). For stress ratios maintained below approximately 1.9, a distinct nonrecoverable strain increment direction may be noted. This uniqueness of slip strain directions along fixed stress paths implies relative independence of strain directions on stress level but dependence on stress direction. Thus, whether or not total or nonrecoverable strains are considered, the stress-dilatancy requirement of unique association of strain increment direction to stress state along paths of increasing stress ratio does not hold in a small stress ratio region.

Association of Stress Increment and Strain Increment Directions

In a summary stress ratio-dilatancy plot, three zones A, B and C as shown in Fig. 7(a) can be identified. Zone A contains stress paths whose orientations are parallel to proportional loading paths that lie between an approximate K_0 and hydrostatic stress directions. These paths result in contraction in both principal strain directions and lead to a progressive stiffening with stress but not to failure. Energy is therefore input along both principal directions, and thus the concepts of stress-dilatancy would not be applicable. Linear strain paths are maintained and the correspondence between stress and strain increment directions is fixed at all stages. The upper limit line A would be defined by proportional loading paths. Relatively independent of confining stress level, parallel constant incremental stress ratio paths tend towards the corresponding proportional loading states on line A. States above line A are not accessible and the position of line A shifts upward with decreasing density.

Stress path orientations that lead to a strain hardening type and failure response would be initially contained within Zone B. For these paths, the minor principal strain increments are expansive. Hence, energy is input and output in the major and minor principal directions, respectively. Below threshold stress ratios approximately delineated by line B, the relationship between stress increment and strain increment ratios within Zone B, remain fixed. Beyond line B out into Zone C, the correspondence between stress increment and strain increment ratio is gradually altered to be between stress ratio and strain increment ratio. Even though minor refinements relative to stress level and density may be possible for some stress paths, as indicated by the reported results, Rowe's stress dilatancy equation $R=KD$ is a reasonable basis for describing behavior in Zone C.

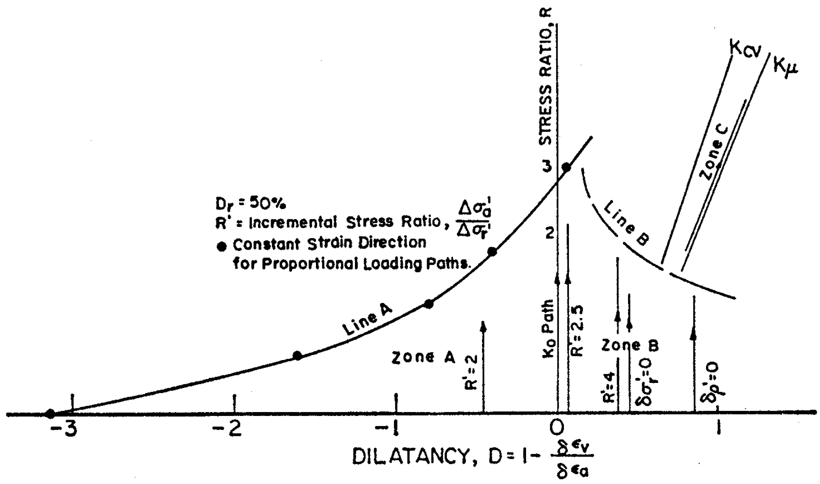

Fig. 7. (a) Overview of the test results in a stress dilatancy plot

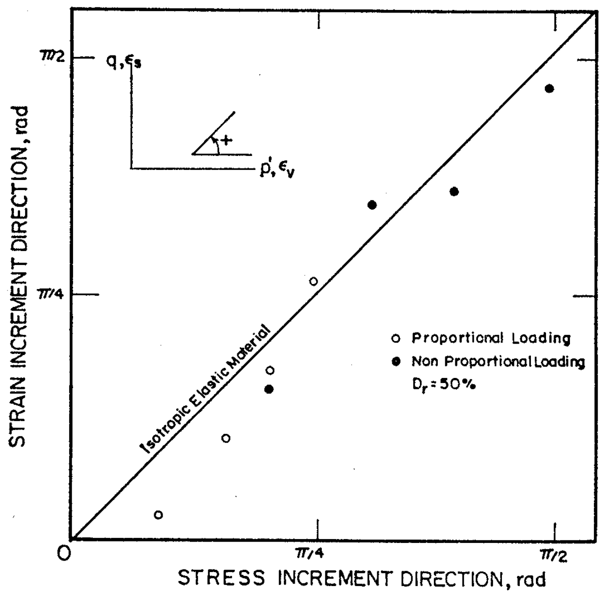

Fig. 7. (b) Stress increment and strain increment directions at small stress ratio regions

For states in Zones A and B in Fig. 7(a), stress increment and strain increment directions are found to be related as shown in Fig. 7(b). A line representing relationships between stress increment and strain increment directions for deformation of an ideal isotropic elastic material is also shown in Fig. 7 (b). The experimental results are predominantly below but closely follow the trend of stress increment and strain increment relationships for the ideal material. This suggests the small stress ratio behavior of the sand samples has resemblance to a cross anisotropic elastic material with a vertical response that is stiffer than the horizontal.

Further Remarks

Experimental results presented in this paper indicate that the stress dilatancy equation would not describe sand behavior at small stress ratio states. An assembly of particles subjected to an external load would deform simultaneously in rolling and sliding between grains as well as a consequence of elastic compression of the constituent grains. Slip deformations would require attainment of a threshold obliquity state. Sliding within an assembly of particles is considered to occur between clusters rather than individual grains (Horne, 1965). The size of sliding groups has been postulated to increase with density and gross sliding would appear to be restrained until a large number of contacts attain limiting equilibrium states. At that time, the predominant mode of deformation becomes sliding. This view emanates from consideration of an assembly of rigid particles in regular packing wherein no deformation would occur as R is increased from initial value of one until peak (Rowe, 1962). Simultaneous sliding would be initiated at all contacts upon attainment of peak stress ratio. Rowe's contention that stress dilatancy would be applicable at all stages of R was questioned on the basis of this idealized framework (Roscoe and Schofield, 1964). In the case of sand, with decreasing density, a larger number of unstable contacts would exist and sliding groups would tend to be smaller. There would be more freedom for rotation and rearrangement and sliding would be more localized for small stress ratio loadings. The overall stress ratio at which sliding deformations become prominent would increase with density. In this respect, previous observations (Rowe, 1971) relative to improved agreement with stress dilatancy upon reloading, for loose sands, may be interpreted to be a consequence of reducing the relative significance of rolling and local sliding. However, predominance of gross sliding deformation through unloading and re-loading leads to ambiguity in that a larger magnitude of threshold stress ratio and thus less agreement with stress dilatancy would be implied.

At large strains, experimental results (Poorooshasb et al., 1966 ; Lade and Duncan, 1976 ; Roscoe, 1970 ; Rowe, 1962 ; Barden and Khayatt, 1966) have shown that association of strain increment ratios to stress states would be reasonably justified and hence plastic characterization. Indeed this latter opinion holds true regardless of considering total or nonrecoverable strains. At large stress ratio states, recoverable strains are sufficiently small compared to slip strains that neither their inclusion nor omission has much significance. In this sense, the work of Rowe and Poorooshasb et al. are essentially the same even though they make appeal to different beginnings.

Both recoverable and nonrecoverable strains may assume relative significance in the approximate small strain range of 1×10^{-4} to 1×10^{-2}. Recoverable as well as slip and counter slip deformations take place during loading and unloading. As was proposed by Zytanski et al. (1978), successful separation of elastic and plastic strains may be impossible. The experimental results presented herein indicate that total as well as non recovered strain increment directions at small stress ratio regions maintain dependence on stress increment direction and not on stress state as implied by the stress dilatancy equation.

ACKNOWLEDGEMENTS

Financial assistance of the National Science and Engineering Research Council of Canada is gratefully acknowledged. The authors thank Ms. Heathere Booth for typing the manuscript and Mr. Rolf Ziemer for assistance with the drafting of the figures.

REFERENCES

1) Barden, L. and Khayatt, A. J. (1966) : "Incremental strain rate ratios and strength of sand in the triaxial test," Geotechnique, Vol. 16, No. 4, pp. 338-357.
2) Barden, L., Ismail, H. and Tong, P. (1969) : "Plane strain deformation of granular material at low and high pressures," Geotechnique, Vol. 19, No. 4, pp. 441-452.
3) Cole, E. R. L. (1967) : "The behavior of soils in the simple shear apparatus," Ph. D. Thesis, Cambridge University.
4) Horne, M. R. (1965) : "The behavior of an assembly of rotund, rigid, cohesionless particles, I and II," Proceedings, Royal Society, A 286, pp. 62-97.
5) Lade, P. V. and Duncan, J. M. (1976) : "Stress-path dependent behavior of cohesionless soil," ASCE, Journal of the Geotechnical Engineering Division, Vol. 102, pp. 51-68.
6) Negussey, D., Wijewickreme, W. K. D. and Vaid, Y. P. (1988) : "Constant volume friction angle of granular materials," Canadian Geotechnical Journal, Vol. 25, No. 1, pp. 50-55.
7) Nova, R. and Wood, D. M. (1979) : "A constitutive model for sand in triaxial compression," International Journal for Numerical and Analytical Methods in Geomechanics, Vol. 3, pp. 255-278.
8) Nova, R. (1982) : "A constitutive model for soil under monotonic and cyclic loading," Soil Mechanics-Transient and Cyclic Loads, John Wiley & Sons Ltd.
9) Poorooshasb, H. B., Holubec, I. and Sherbourne, A. N. (1966) : "Yielding and flow of sand in triaxial compression, Part I," Canadian Geo-

technical Journal, Vol. 3, No. 4, pp. 179-190.
10) Roscoe, K. H. and Schofield, A. N. (1964) : "Discussion," ASCE, Journal of the Soil Mechanics and Foundations Division, Vol. 90, No. 2, pp. 136-150.
11) Roscoe, K. H. (1970) : "The influence of strains in soil mechanics, Tenth Rankine Lecture," Geotechnique, Vol. 20, No. 2, pp. 129-170.
12) Rowe, P. W. (1962) : "The stress-dilatancy relation for static equilibrium of an assembly of particles in contact," Proceedings of the Royal Society, A 269, pp. 500-527.
13) Rowe, P. W. and Barden, L. (1964) : "Importance of free ends in triaxial testing," ASCE, Journal of Soil Mechanics and Foundations Division, Vol. 90, No. SM 1, pp. 1-27.
14) Rowe, P. W. (1971) : "Theoretical meaning and observed values of deformation parameters for soil," Proceedings of the Roscoe Memorial Symposium, Cambridge University, pp. 143-194.
15) Sarsby, R. W., Kalteziotis, N. and Haddad, E. H. (1980) : "Bedding error in triaxial tests on granular media," Geotechnique, Vol. 30, No. 3, pp. 302-309.
16) Vaid, Y. P. and Negussey, D. (1984) : "A critical assessment of membrane penetration in the triaxial test," ASTM, Geotechnical Testing Journal, Vol. 7, No. 2, pp. 70-76.
17) Vesic, A. S. and Clough, G. W. (1968) : "Behavior of granular materials under high stresses," ASCE, Journal of the Soil Mechanics and Foundations Division, Vol. 94, No. SM 3, pp. 661-688.
18) Zytynski, M., Randolph, M. F., Nova, R. and Wroth, C. P. (1978) : "On modelling the unloading-reloading behavior of soils," International Journal for Numerical and Analytical Methods in Geomechanics, Vol. 2, pp. 87-94.