

APPLICATION OF FORCHHEIMER'S FORMULA TO DEWATERED EXCAVATION AS A LARGE CIRCULAR WELL

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ABSTRACT

This paper describes not only the application of Forchheimer's formula, which is a discharge equation for flat, open-bottom impermeable casing wells, to the estimation for the discharge of dewatered excavation as a large circular well but also the practical utility of a proposed method for estimating the radius of influence which was derived from Forchheimer's formula. The investigation was conducted by comparing the observed results of an actual dewatered excavation with the calculated results. Also, the study was made of the application of the existing methods for estimating the radius of influence in the dewatered excavation as a large circular well with open-bottom impermeable casing.

The results of the investigation are as follows: (1) The calculated results of discharge by Forchheimer's formula are in good agreement with the observed results; the error between these two results is under 14 per cent. (2) The calculated results of the radius of influence by the proposed equation with $s_R=0.2$ m agree approximately with the observed results; the error between these two results is under 20 per cent. (3) The calculated results of the radius of influence by the existing methods are quite different from the observed results.

Key words: design, drawdown, excavation, groundwater, shaft, site investigation, well (IGC : E7 / K1)

INTRODUCTION

Recently, many big underground construction works with a large diameter shaft such as underground aeration shafts for a sewer-system and underground petroleum storage tanks have been planned and carried out in Japan. The structures are usually to be constructed in

lowlands near the seashore, which are often composed of sandy soil layers with a high groundwater level. Excavation in such grounds requires a large capacity pump-equipment and drainage, and furthermore some measures against expected piping or boiling in the open-bottom of excavation have to be taken. Therefore, it is important to evaluate accurately

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the discharge into an excavation shaft in order to save additional construction costs. At the same time, lowering of water level in the excavation on the adjacent area has to be assessed.

In the excavation, groundwater may flow into the shaft only through the bottom, because a casing of the shaft is made of impermeable steel sheet-pile wall or reinforced concrete wall.

The formula to calculate the discharge into an open-bottomed imperfect well was proposed by Forchheimer (1905). This formula is applicable to an ordinary well with a diameter of several meters, but it is unknown whether the formula can also be applied to a large well of 76 m in diameter. Therefore, the investigations were made experimentally on the application of Forchheimer's formula and the practicality of a proposed equation, derived from Forchheimer's formula, for estimating the radius of influence to a dewatered excavation as a large circular well.

The investigations were carried out by comparing the observed results in an actual dewatered excavation with calculated ones, and the conclusions are as follows: Forchheimer's formula is of great use to estimate the discharge; while the proposed equation is useful in estimating the radius of influence for dewatered excavation as a large circular well.

FORCHHEIMER'S FORMULA FOR THE DISCHARGE Q INTO A FLAT, OPEN-BOTTOM WELL WITH IMPERMEABLE CASING

In 1905, Forchheimer presented a theoretical solution for the calculation of the discharge into a flat, open-bottomed artesian well just penetrating the top surface of a semi-infinite pervious homogeneous and isotropic medium, which is overlaid by impervious layer deposits of finite thickness (Forchheimer, 1905). In the paper, Forchheimer described the following two cases as the application of the above solution. (1) Even if the pervious medium has finite thickness due to underlying impervious layer, the solution is applicable as far as the

thickness of the pervious layer is several times larger than the diameter of a well. (2) The solution is practically applicable to the calculation of the discharge in a flat open-bottom well with an impermeable casing in case of the overlying impervious layer being absent. The item (2) in the above-mentioned cases does hold good without serious errors for the following reason: assuming that the groundwater above the level of well-bottom acts merely as energy head, and that only the groundwater under the level of well-bottom flows into the well, the boundary condition is considered to be the same as that in the basic theory for an open-bottomed artesian well just penetrating the top surface of a pervious medium. Forchheimer's formula approximately expresses the relation between the discharge into a well and the drawdown in a well by making use of the coefficient of permeability in an aquifer and the radius of a well as parameters. Assuming that flow lines are indicated by the family of confocal hyperbolas with both ends of the well-bottom diameter being focal points, equipotential lines are indicated by the family of confocal ellipses with both ends of well-bottom diameter being focal points. The center of bottom in flat bottom well is chosen as the origin of coordinates as shown in Fig. 1.

Forchheimer's formula for the flow of confined groundwater into a well is shown as follows (Tamachi, 1938 a):

$$Q = 4 \cdot k \cdot r_w (H - h_w) = 4 \cdot k \cdot r_w \cdot s_w \quad (1)$$

where Q is the discharge into a flat, open

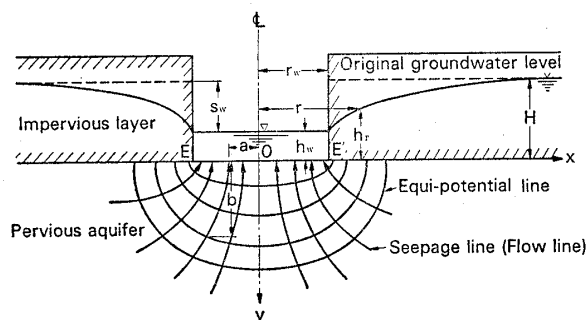


Fig. 1. Groundwater flow to an artesian well with flat, open-bottom

bottom well with an impermeable casing, k is the coefficient of permeability in an aquifer, r_w is the radius of the well, H is the head of water which shows depth from original groundwater level in the aquifer, h_w is the water level in the well, and $H-h_w$, i.e. s_w is the head-loss (=drawdown at well).

Forchheimer's formula has nothing to do with the radius of influence R which is used in other well-formulas on steady flow; the reason is as follows: Other well-formulas are based on the axisymmetric radial flow theory, and are made for an ordinary perfect well in which the screen-setting penetrates fully the entire thickness of an aquifer down to the impervious layer. In this case, water enters the well only through the perforation screen. Therefore, those formulas require the radius of influence R as the boundary condition corresponding to the headwaters of flows into the well. On the other hand, Forchheimer's formula is in use for a flat, open-bottom imperfect well installed in a semi-infinite permeable aquifer, and is based on the assumption that the seepage lines around the open-bottom of the imperfect well in an aquifer are indicated by the family of confocal hyperbolas with both ends of the well-bottom being two focal points. In this case, the groundwater under the level of well-bottom in an aquifer is considered to flow into the well. Therefore, the head of water H , represented by the depth from the original groundwater level to any elevation in the aquifer, is introduced in the theory as the headwaters of flows into the well. Namely, the head of water H in Forchheimer's formula corresponds to the radius of influence R in other formulas, and thereby Forchheimer's formula does not include the coefficient R in it. If Forchheimer's formula is applicable to the calculation of the discharge into an open-bottom large well with an impermeable casing, the calculation of discharge becomes very easy, because it does not have to consider the radius of influence R .

EXISTING FORMULAS FOR ESTIMATING THE RADIUS OF INFLUENCE R

The radius of influence R is defined as a radial distance from the center of the well to the point where the drawdown and seepage velocities are assumed to be zero, and the circle with R shows the area of the groundwater level drawdown by pumping up in an aquifer.

Many methods are proposed to estimate the R . The examples of equation for estimating the R are shown as follows:

Sichardt's equation:

$$R = 3000 s_w \sqrt{k} \quad (\text{m}) \quad (2)$$

Weber's equation:

$$R = 3 \sqrt{H \cdot k (t/n)} \quad (\text{m}) \quad (3)$$

Kozeny's equation

$$R = \sqrt{(12 \cdot t/n) \sqrt{Q \cdot k/\pi}} \quad (\text{m}) \quad (4)$$

where s_w is the maximum drawdown at a well (m), k is the coefficient of permeability in an aquifer (m/sec), H is the head of water which shows depth from original groundwater level to the top surface of impervious stratum underlying the aquifer (m), t is the pumping duration (sec), n is the porosity of the aquifer (%), and Q is the discharge into a well (m^3/sec).

Eqs.(2), (3) and (4) are considered to be made for an ordinary perfect well, but these equations are not related to the radius of the well r_w . It is clear that the well-model of Eqs.(3) and (4) is different from the well-model of an actual excavation, hence, Eqs.(3) and (4) are taken up as a simple approach.

PROPOSED METHOD FOR ESTIMATING THE RADIUS OF INFLUENCE R IN THE OPEN-BOTTOM WELL WITH IMPERMEABLE CASING

As mentioned above, the discharge in a flat, open-bottom well with an impermeable casing can be calculated by Forchheimer's formula.

However, it is not obvious whether the formula is applicable in estimating the discharge

in a dewatered excavation as a large circular well. Therefore, the applicability of the formula will be confirmed by the measurement of pumping rate in the actual excavation.

On the other hand, the radius of influence R is necessary for establishing the outer boundaries of fluid systems, and cannot be known exactly before pumping is started. Consequently, trial is made of establishing the formula for estimating the radius of influence R in a flat, open-bottom well with impermeable casing as follows:

Forchheimer presented the relation between the drawdown $s_r (= H - h_r)$ of groundwater level at radial distance r from the center of a well and the distance r in time of the pumping discharge rate and the total seepage rate into the well being in equilibrium, and the relational expression was obtained in the process of deriving Forchheimer's formula Eq.(1) (Tamachi, 1938 b).

$$\therefore H - h_r = \frac{Q}{2\pi k r_w} \sin^{-1} \frac{r_w}{r} \quad (5)$$

where h_r is the head of water at a point r , r is the radial distance from the center of the well as in Fig. 2.

In Eq.(5), using $s_R (>0)$ for $H - h_r$ at the distance of $r=R$, the radius of influence R is approximately expressed by the following formula:

$$R = \frac{r_w}{\sin (2\pi k r_w s_R / Q)} \quad (6)$$

Substituting Eq.(1) into Eq.(6), the following equation is obtained:

$$R = \frac{r_w}{\sin \{(\pi/2) \times (s_R/s_w)\}} \quad (7)$$

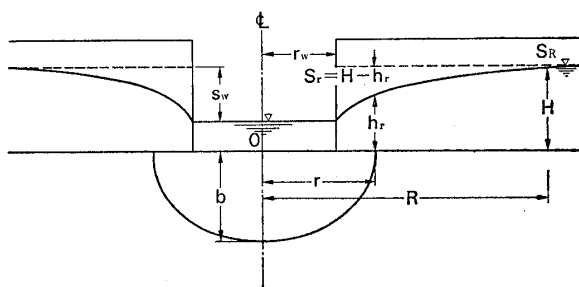


Fig. 2. Explanation drawing for Eq.(5)

where Eqs.(6) and (7) are applicable only to the area where s_R is more than zero ($s_R > 0$). The value of s_R that is seemingly the most reasonable will be subsequently determined on the basis of observed results.

OBSERVED RESULTS IN DEWATERED EXCAVATION AS LARGE CIRCULAR WELL

Observation was made of a dewatered excavation as a large circular well on a reclaimed ground along the water-front of Tokyo Bay, and a profile of the excavation, dewatered for the major portion of a year, is shown in Fig. 3. The area is characterized by a high water level as well as thick pervious sands with the average coefficient of permeability k_{ave} being 2.3×10^{-3} m/min. The circular excavation was 76 m in diameter and 22 m in depth. During excavation the water level was lowered 22 m at most by pumping at the rate of 4.24 to $7.72 \text{ m}^3/\text{min}$.

Figs. 4 and 5 show the observed results of the pumping discharge and the amount of drawdown in the excavation, and the drawdown of groundwater level obtained through observation wells set in the surrounding areas of the excavation.

Fig. 4 shows the relationship between the pumping discharge Q and the amount of drawdown s_w in the excavation; the regression equation and the corresponding correlation

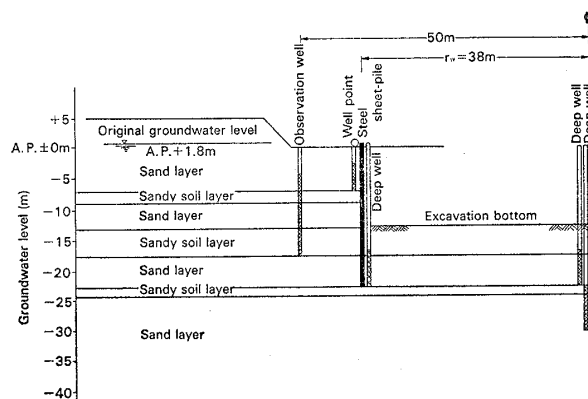
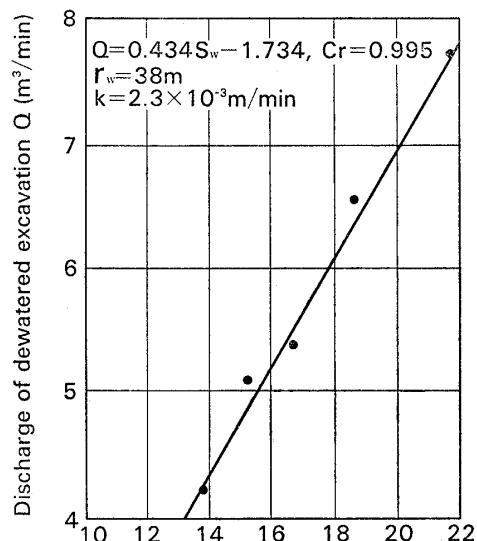


Fig. 3. Cross section of excavation as large circular well

coefficient C_r are given as follows :

$$Q = 0.434s_w - 1.734, \quad C_r = 0.995, \\ \text{at } 12 \text{ m} < s_w < 22 \text{ m} \quad (8)$$

Meanwhile, substituting $k = 2.3 \times 10^{-3} \text{ m/min}$ and $r_w = 38 \text{ m}$ to Eq.(1), the coefficient of Eq.(1) $4kr_w$ is 0.35. Hence, Q in Eq.(1) of



Amount of drawdown at dewatered excavation $S_w(\text{m})$

Fig. 4. Relationship between pumping discharge and amount of drawdown in excavation as large circular well (observed result)

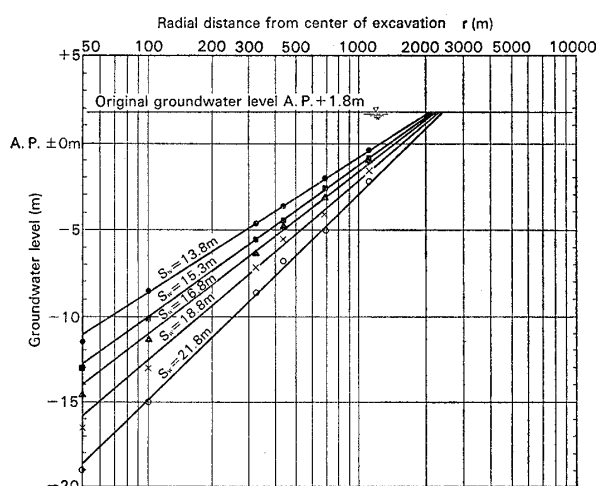


Fig. 5. Relationship between groundwater level and radial distance from center of excavation as large circular well (observed result)

the coefficient $4kr_w$ being 0.35 and Eq.(8) resemble each other in the range of $12 \text{ m} < s_w < 22 \text{ m}$.

Fig. 5 shows the relationship between the drawdown of groundwater level in the environs of the excavation and the radial distance r from the center of circular excavation at five excavating stages. As seen in Fig. 5, the plots in the graph show approximately a linear relation for each excavating stage. Further, the straight lines intersect the original groundwater level line at the points of 2050~2300 m away from the center of excavation. Namely, it is estimated from the observed results that the radius of influence R is about 2300 m in this case.

DISCUSSION OF PROPOSED EQUATION FOR ESTIMATING THE DISCHARGE Q AND THE RADIUS OF INFLUENCE R

Comparative Study of Calculated Result with Observed Result for The Discharge Q

Rearranging Forchheimer's formula for the discharge into a flat, open-bottom well with an impermeable casing by substituting $k = 2.3 \times 10^{-3} \text{ m/min}$ and $r_w = 38 \text{ m}$ into Eq.(1), the following Eq.(9) is obtained.

$$Q = 4 \times 2.3 \times 10^{-3} \times 38 \times s_w \\ = 3.50 \times 10^{-1} \times s_w \quad (\text{m}^3/\text{min}) \quad (9)$$

The calculated results of the discharge for each excavation stage by Eq.(9) are shown in Table 1. The results agree well with the observed results, and the error between both results is under 14%. Such a good results are obtained for the following reason; the thickness of previous layer is large enough as compared with the radius of large circular excavation.

Comparative Study of Calculated and Observed Results for The Radius of Influence R

The radii of influence for each excavation stage are calculated through both Eqs.(6) and (7). Table 2 shows these results together with the corresponding observed results: The results calculated by Eq.(7) with $s_R = 0.2 \text{ m}$ are closer to observed ones than in other cases

Table 1. Comparison of calculated and observed results of the discharge rate from excavation, Q

Amount of Drawdown in Excavation s_w (m)	Calculated Results of Discharge Rate from Excavation Q (m ³ /min)	Observed Results of Discharge Rate from Excavation Q (m ³ /min)	Ratio of Cal. Results to Obs. Results
13.8	4.83	4.24	1.14
15.3	5.36	5.08	1.06
16.8	5.88	5.37	1.09
18.8	6.58	6.52	1.01
21.8	7.63	7.72	0.99

Table 2. Comparison of calculated and observed results of the radius of influence, R

Amount of Drawdown in Excavation s_w (m)	Calculated Results of R (m)						Observed Results of R (m)
	$R = \frac{r_w}{\sin (2 \pi k r_w s_R / Q)}$			$R = \frac{r_w}{\sin \{(\pi / 2) \cdot (s_R / s_w)\}}$			
	s_R (m)			s_R (m)			
	0.10	0.15	0.20	0.10	0.15	0.20	
13.8	2,934	1,956	1,468	3,339	2,225	1,667	2,050
15.3	3,515	2,343	1,758	3,700	2,468	1,851	2,150
16.8	3,718	2,477	1,858	4,064	2,708	2,032	2,180
18.8	4,513	3,006	2,257	4,548	3,033	2,274	2,280
21.8	5,345	3,561	2,672	5,273	3,515	2,637	2,300

Table 3. Calculated results of the radius of influence by existent formulas

Amount of Drawdown in Excavation s_w (m)	Accumulation of Pumping Duration Σt		Discharge from Excavation Q		Calculated Results of Radius of Influence R by Existing Formulas R (m)		
	days	sec	m ³ /min	m ³ /sec	Eq.(11)	Eq.(12)	Eq.(13)
13.8	90	7.776×10^6	4.24	0.0707	256	626	471
15.3	120	10.368×10^6	5.08	0.0847	284	722	647
16.8	300	25.920×10^6	5.37	0.0895	312	1142	1039
18.8	330	28.512×10^6	6.52	0.1087	349	1198	1143
21.8	360	31.104×10^6	7.72	0.1287	405	1251	1246

of s_R , and the error between the calculated results and the observed results ranges from 18.7% to 0.3%. Judging from the above results, first of all, Eq.(1) can be in use for estimating the discharge of the dewatered excavation as a large circular well and Eq.(7)

is applicable to estimating the radius of influence R . And, there are many cases where the drawdown s_w becomes more than 4 m in the actual large excavation. Accordingly, the following Eq.(10) is proposed in stead of Eq.(7) as the estimating equation of the radius of

the influence R : in the case that s_R is 0.2 m and s_w is more than 4 m, $\sin \{(\pi/2) \cdot (s_R/s_w)\}$ can be approximated by $\{(\pi/2) \cdot (s_R/s_w)\}$.

$$R = \frac{r_w}{\{(\pi/2) \times (0.20/s_w)\}} \quad (10)$$

Eq.(10) is proposed as a new estimating equation of the radius of influence R for dewatered excavation as a large circular well.

Second, the radii of influence R calculated by the existing methods are as follows:

Sichardt's equation: Substituting $k=2.3 \times 10^{-3}$ m/min $= 3.83 \times 10^{-5}$ m/sec into Eq.(2),

$$R = 3000 s_w \sqrt{k} = 18.566 \times s_w \quad (11)$$

Weber's equation: Substituting $H=43.8$ m (the operated depth of test boring), $k=3.83 \times 10^{-5}$ m/sec and $n=0.3$ (an assumed value) into Eq.(3),

$$R = 3\sqrt{H \cdot k(t/n)} = 0.2243\sqrt{t} \quad (12)$$

Kozeny's equation: Substituting $n=0.3$ (an assumed value) and $k=3.83 \times 10^{-5}$ m/sec into Eq.(4),

$$R = \sqrt{(12 \cdot t/n) \sqrt{Q \cdot k/\pi}} = 0.373\sqrt{t \sqrt{Q}} \quad (13)$$

The calculated results of the radius of influence R by Eqs.(11), (12) and (13) are shown in Table 3. These calculated results dose not agree with the observed results; then the existing equations are not applicable to the estimation of the radius of influence R for dewatered excavation as a large circular well with an open-bottom impermeable casing.

CONCLUSION

The results in the investigation are as follows:

(1) The discharge calculated by Forchheimer's formula agree well with the observed results; the error between both results is under 14%. Therefore, Forchheimer's formula is of great use to estimate the discharge of dewatered excavation as a large circular well with an open-bottom impermeable casing.

(2) The calculated results of the radius of influence by the proposed equation with $s_R=0.2$ m agree approximately with the observed results; the relative error between both results is under 20%. Therefore the proposed equation can be applied to estimating the radius of influence for dewatered excavation as a large circular well with an open-bottom impermeable casing.

(3) The calculated results of the radius of influence by the existing equations are not applicable to the estimation of the radius of influence R in the case of the dewatered excavation as a large circular well with an open-bottom impermeable casing.

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