SEISMIC COEFFICIENT CIRCLE METHOD IN STABILITY ANALYSIS OF ROCK-FILL DAMS

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SUMMARY

This paper presents a new analysis method for designing rock-fill dams which are resistant to earthquakes; rock-fill dams mentioned in this paper are those consisting wholly of rock materials.

When a rock-fill dam is subjected to a large earthquake force, it collapses, and the shape of the collapsing rock-fill dam approaches a circular arc in cross-section. The seismic coefficient circle method gives the relationship between the earthquake force, the cross-section of the circular arc and the stable cross-section formed in the circular arc at this moment.

It is possible to obtain a seismic coefficient circle for any earthquake force (seismic coefficient) from theoretical considerations and thereby the cross-section of the dam required for any seismic coefficient can be designed.

1. CRITICAL SLIDING SURFACE FOR FILL-TYPE DAMS

To date, there has been no satisfactory method of stability analysis of rock-fill dams. At present, stability analysis has generally been carried out using the slip circle method which is also used for earth dams. However, as there are essential differences between materials used for rock-fill dams and earth dams, i.e., the former is rock materials having only angle of internal friction ϕ, the latter is soil materials having both angle of internal friction ϕ and cohesion c, it is unreasonable from the engineering point of view to use the same stability calculation for both.

In general, the stability of earth dams constructed with the materials having ϕ and c can be shown by equation (1)

$$F=\frac{\Sigma N \tan\phi + cL}{\Sigma T} \quad \cdots\cdots\cdots\cdots (1)$$

where

F: safety factor

ϕ: angle of internal friction of the slope-forming material

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c: cohesion of the slope-forming material
L: length of the circular arc of a slip surface
ΣN: sum of the normal components of the body forces of slices
ΣT: sum of the tangential components of the body forces of slices

If we assume $F = F_n$ and indicate the scale ratio of the length of the dam cross-section by $s(s \geqq 1)$, the following equation can be established for any cross-section using the same materials.

$$F_n = \frac{s^2 \Sigma N \cdot \tan\phi + s \cdot cL}{s^2 \Sigma T} = \frac{1}{\Sigma T}\left(\Sigma N \cdot \tan\phi + \frac{1}{s} \cdot cL\right) \quad \cdots\cdots\cdots\cdots(2)$$

From equation (2), it is clear that for earth dams constructed with the same soil materials, the cohesion value decreases in inverse proportion to the increase of the scale ratio of the cross-section.

Accordingly, the slope angle should be changed if c is a constant in order to keep F_n constant.

Now, in the case of $F_n = \text{const.}$, putting $s^2 \Sigma N = N_n$ and $s^2 \Sigma T = T_n$, the following calculation is carried out after they are composed on the line of the center of gravity. (Refer to Fig. 1-1)

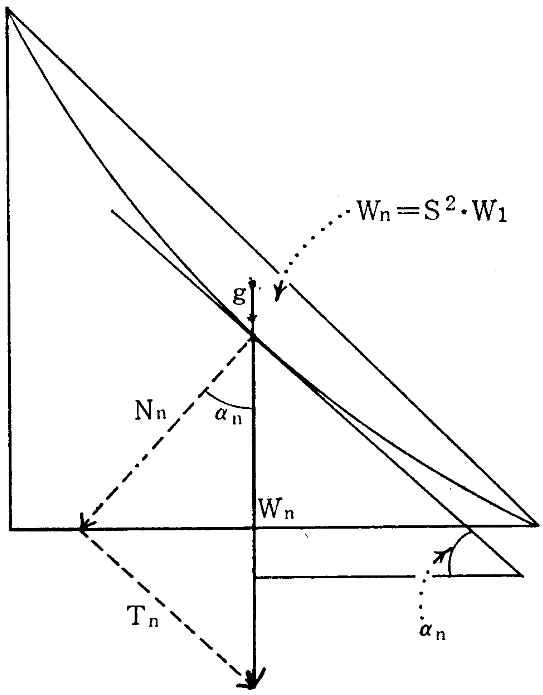

Fig. 1-1. Explanation of N_n, T_n, W_n and α_n (g: center of gravity)

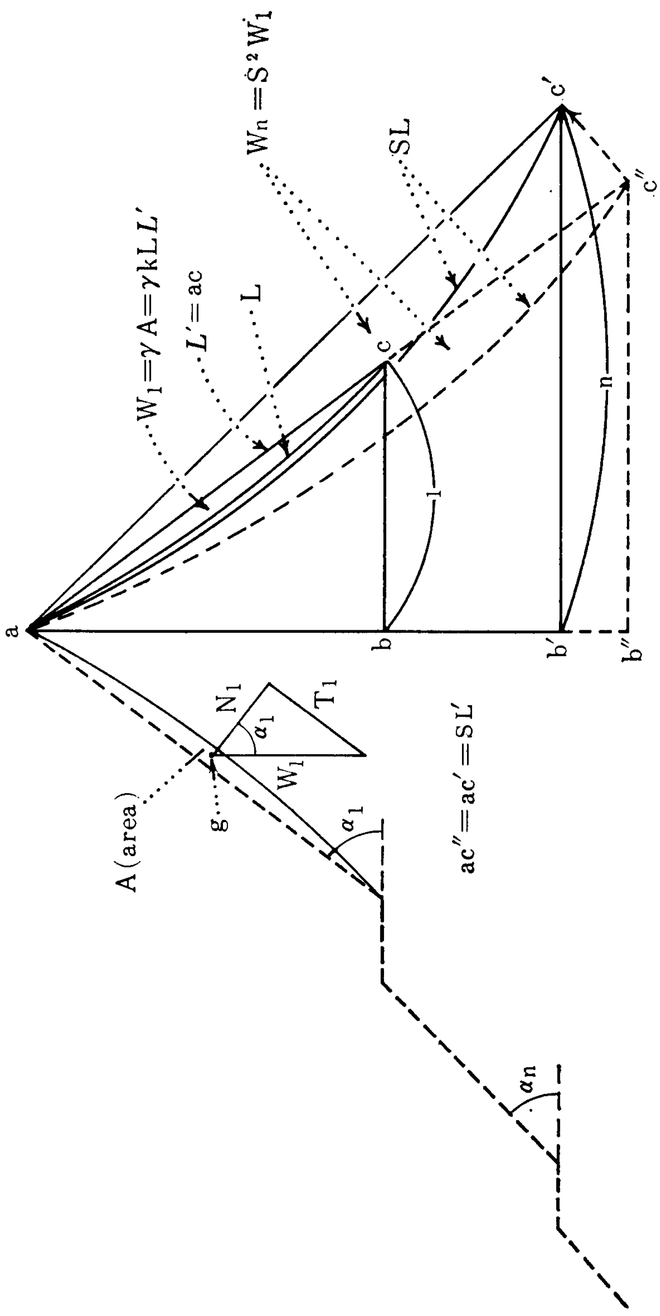

Fig. 1-2. Explanation of equations

When $F_n = 1$, the slope of any cross-section is obtained with equation (2) as follows:

$$\frac{T_n}{N_n} = \tan\phi + s\cdot\frac{cL}{N_n} \quad \cdots\cdots(3)$$

where

$$T_n = s^2 W_1 \sin\alpha_n$$
$$N_n = s^2 W_1 \cos\alpha_n$$

W_1 and L are the values when $s = 1$ is considered too small. α_n is the slope angle of the sliding surface.

$$\frac{T_n}{N_n} = \frac{s^2 W_1 \sin\alpha_n}{s^2 W_1 \cos\alpha_n} = \frac{\sin\alpha_n}{\cos\alpha_n} = \tan\alpha_n \quad \cdots\cdots(3')$$

From equations (3) and (3′)

$$\begin{aligned}\tan\alpha_n &= \tan\phi + s\cdot\frac{cL}{s^2 W_1 \cos\alpha_n} \\ &= \tan\phi + \frac{cL}{s W_1 \cos\alpha_n}\end{aligned} \quad \cdots\cdots(4)$$

If equation (4) is applied to an arbitrary cross-section, since $\cos\alpha_n = n/sL'$ (Fig. 1–2),

$$\begin{aligned}\tan\alpha_n &= \tan\phi + \frac{cL}{sW_1\cdot\dfrac{n}{sL'}} = \tan\phi + \frac{cL}{n\cdot W_1\cdot\dfrac{1}{L'}} \\ &= \tan\phi + \frac{cL}{n\cdot W_1\cdot\cos\alpha_1} = \tan\phi + \frac{1}{n}\cdot\frac{cL}{N_1}\end{aligned} \quad \cdots\cdots(5)$$

where

$$\alpha_n = \alpha_1 - \varepsilon$$

ε is a variable. The value of ε becomes large in accordance with an increase in n, therefore if Δn is considered, the following differential equation is obtained:

$$\frac{d\xi}{dn} = \tan\phi + \frac{1}{n}\cdot a \quad \cdots\cdots(6)$$

$$a = \frac{cL}{N_1} = \frac{cL}{W_1\cdot\cos\alpha_1} = \frac{cL}{\gamma A\cos\alpha_1} = \frac{cL}{\gamma kLL'\dfrac{1}{L'}} = \frac{c}{\gamma\cdot k}$$

where

γ: unit weight

k: A/LL' (area ratio when circular arc is given)

A: W_1/γ

[Note]

$$\frac{T_n}{N_n}=\frac{s^2 W_1 \sin\alpha_n}{s^2 W_1 \cos\alpha_n}=\frac{s\cdot\tan\alpha_n}{s}=\frac{n\tan\alpha_n}{n}=\frac{\xi}{n}$$

Therefore, $\xi=f(n)$ is obtained. Consequently, n and height $-\xi$ are plotted on abscissa and ordinate as shown in Fig. 2.

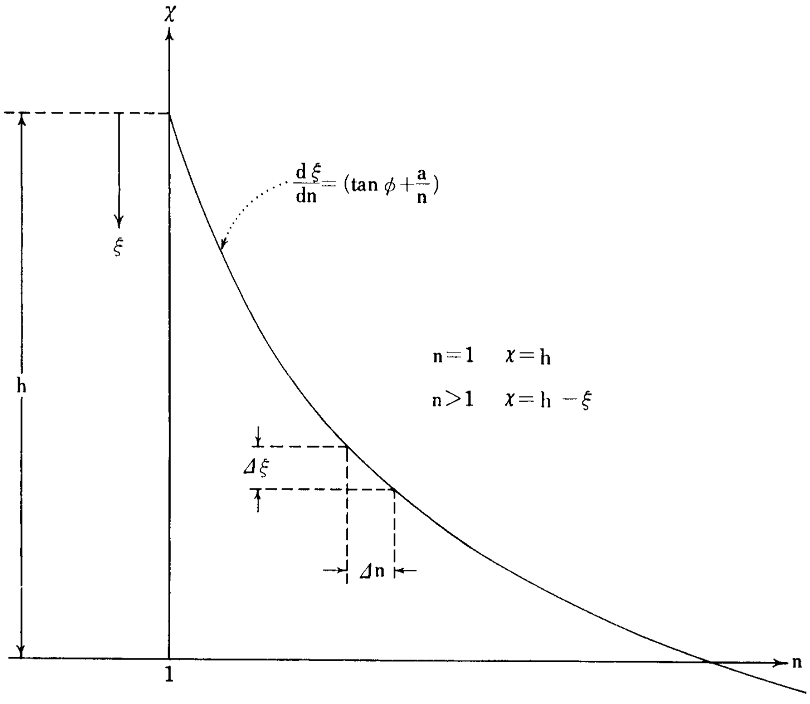

Fig. 2. The relation between height, base width and slope shown in equation (7)

Integrating equation (6),

$$\begin{aligned}\xi &= \int_0^\xi d\xi=\int_1^n\left(\tan\phi+\frac{a}{n}\right)dn\\ &= \left|n\cdot\tan\phi\right|_1^n+\left|a\cdot\log n\right|_1^n\\ &= (n-1)\cdot\tan\phi+a\cdot\log n\end{aligned}$$

If χ is taken upwards from the bottom and the height of the dam for a certain unit length $n=1$ is h. Thus,

$$\xi = h - \chi$$
$$\chi = h - (n-1)\tan\phi - a\log_e n \qquad (7)$$

This equation gives a stable critical slope for the dam and expresses a logarithmic curve ($a>0$) and a straight line ($a=0$) as the scale ratio n on the abscissa increases.

Equation (7) was examined with Toyoura fine sand as shown in Photo. 1, and also was proved by actual slides of earth dams during the Tokachioki earthquake of May 16, 1968, as shown in Figs. 4 through 6. It is noted that although equation (7) was derived by two-dimensional analysis, it holds true even in axially symmetrical case of Photo. 1.

The characteristic of equation (7) gives the configuration of the critical slope on the slope AC, as height χ for each base width $n(\chi=0$ when $\mathrm{BC}=n)$ is obtained when the height of dam, $h(\mathrm{AB}=h)$ is given (Fig. 3).

Accordingly, we can save troubles of obtaining the critical slip circle by trial calculations and by introducing $F>1$ into the equation. The cross-section which has any value of safety factor can also be determined.

$$\chi = h - \{(n-1)\cdot\tan\phi + a\cdot\log_e n\}/F \qquad (8)$$

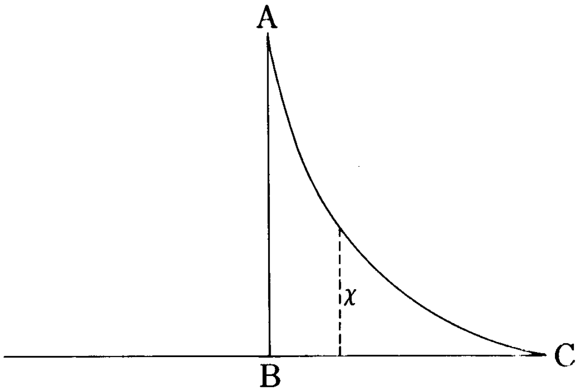

Fig. 3. Explanation of the characteristic of equation (7)

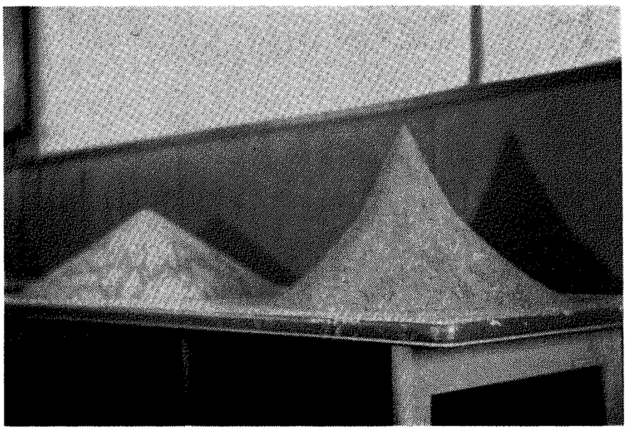

Photo. 1. Left side: Dry Toyoura fine sand ($a=0$)
Right side: Wet Toyoura fine sand ($a>0$)

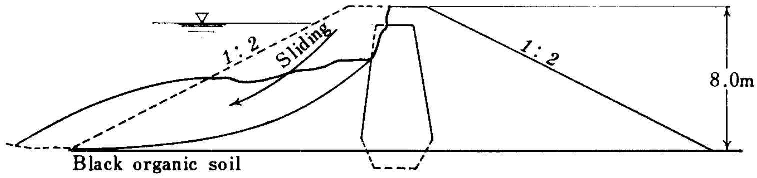

Fig. 4. Ichirigoya earth dam which failed during the Tokachioki earthquake of 1968.

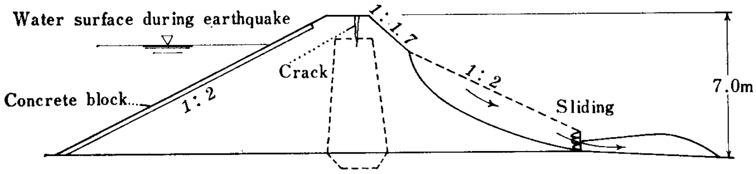

Fig. 5. Hiyamizu earth dam which failed during the Tokachioki earthquake of 1968.

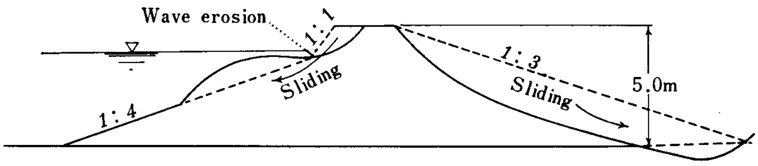

Fig. 6. Odanainuma earth dam which failed during the Tokachioki earthquake of 1968.

For rocks in a rock-fill dam for which the value of c is zero, the value of a becomes zero as well. Hence equation (7) becomes

$$\chi = h-(n-1)\cdot\tan\phi \quad\text{(9)}$$

Equation (9) is a linear equation of n, i.e., it has the same characteristic as the slope rule, $i \leqq \tan\phi$, for conventional cohesionless and free draining materials.

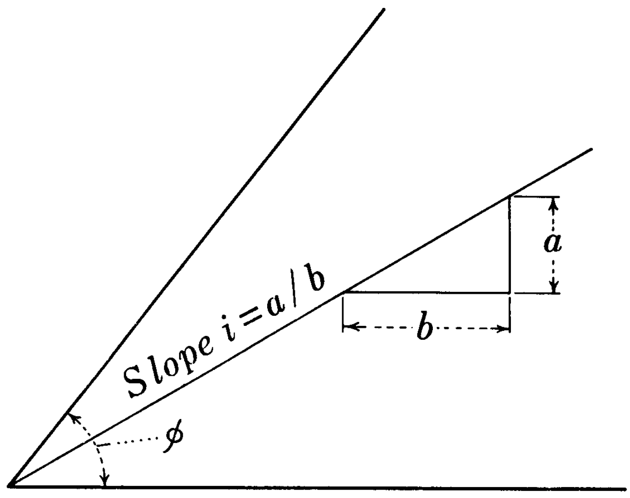

Fig. 7. Slope rule for non-cohesive soils

Therefore, the critical slope is not a curve but a straight line.

For this reason, it will be clear that there are significant differences between earth dams and rock-fill dams, and that it is not possible to apply a slip circle method such as shown by equation (1) for a stability analysis of rock-fill dams.

2. THEORY OF SEISMIC COEFFICIENT CIRCLE

When an earthquake force acts on a rock-fill dam having a straight line slope as mentioned above, the shape of the slope changes to a curve which is convex upwards. This phenomenon results from the increasing vibration from the bottom to the top of the dam. The angle from the horizontal under earthquake condition (the angle A is given by $A = \tan^{-1} K_0$, K_0 being seismic coefficient) considered in the seismic coefficient method increases as vibration increases, and this fact has been confirmed on existing dams.

Accordingly, the dynamic angle of repose decreases from the bottom to the top of the dam. The slope angle of the dam decreases and the convex curve approaches a circular arc with an increase in the seismic coefficient.

Finally, the shape of the slope of the dam shows a circular arc (this circle is referred to as the seismic coefficient circle) when subjected to the forces corresponding to the maximum seismic coefficient for the design of the rock-fill dam (Refer to Photos. 2 through 5 and Fig. 9). Half of the center angle for the circular arc (called the angle of the seismic coefficient circle, θ) equals the dynamic angle of repose α of the slope at the bottom.

Using the seismic coefficient circle, all earthquake-resistant cross-sections ranging from multi-step cross-sections to triangular cross-sections can be obtained graphically. (Refer to 3.2, Fig. 8 and Fig. 20.)

The basic consideration in obtaining these earthquake-resistant cross-sections is to keep the stability of the dam within a certain limit of seismic coefficient, limiting the slope of the earthquake-resistant cross-section to be

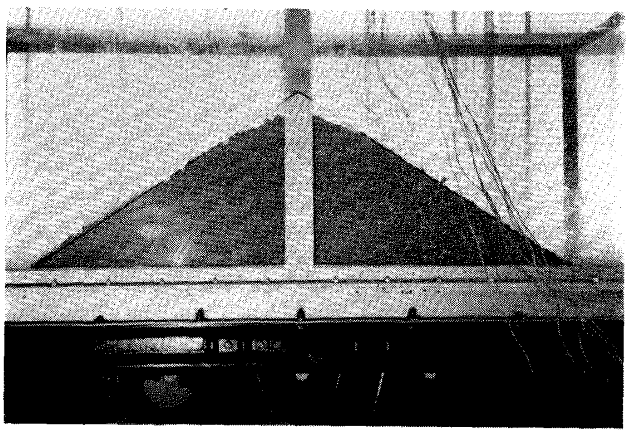

Photo. 2. Model before vibration

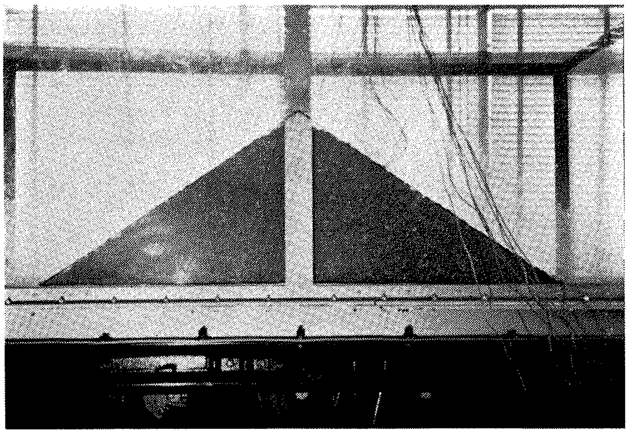

Photo. 3. Seismic coefficient = 0.204

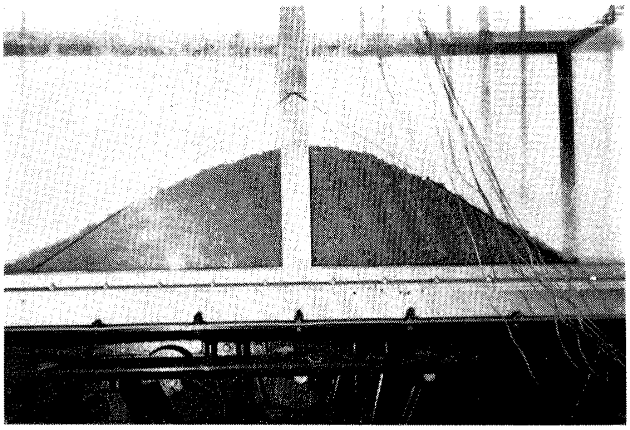

Photo. 4. Seismic coefficient = 0.340

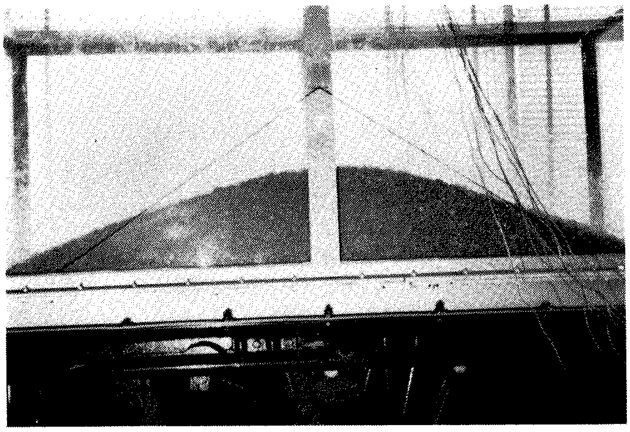

Photo. 5. Seismic coefficient = 0.460

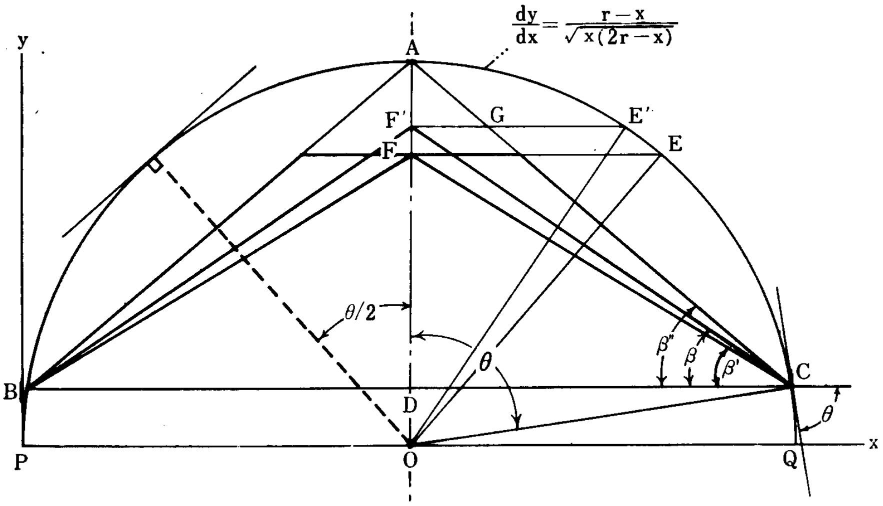

Fig. 8. Seismic coefficient circle

equal to or less than the slope of the tangent at point E' on the arc BAC of the seismic coefficient circle at the same height as the top F' of the earthquake-resistant cross-section ($\Delta BF'C$) formed within the seismic coefficient circle (Fig. 8).

Now using the seismic coefficient circle drawn at $\alpha = \theta$, the formula for obtaining the slope angle of a triangular earthquake-resistant cross-section, β, mentioned above, is as follows:

if $\angle F'CD = \beta$

$\angle F'OE' = \beta$

$$\overline{DF'} = \overline{OF'} - \overline{OD} = r\cos\beta - r\cos\theta$$

$$\tan\beta = \frac{\overline{DF'}}{\overline{DC}} = \frac{\cos\beta - \cos\theta}{\sin\theta}$$

$$\cos(\theta - \beta) = \cos^2\beta \quad \cdots\cdots\cdots\cdots\cdots (10)$$

Next, the slope of the triangle being approximated by the triangular cross-section as mentioned above,

if $\angle FCD = \beta'$

$$\overline{DF} = \overline{OF} - \overline{OD} = r\left(\cos\frac{1}{2}\theta - \cos\theta\right)$$

$$\overline{DC} = r\sin\theta$$

$$\beta' = \tan^{-1}\frac{\overline{DF}}{\overline{DC}} = \tan^{-1}\frac{\cos\frac{1}{2}\theta - \cos\theta}{\sin\theta} \quad \cdots\cdots\cdots (11)$$

and for the cross-section of a trapezoid which has the same height as the center point of the circular arc,

if $\angle ACD = \beta''$

$$\overline{AD} = \overline{AO} - \overline{OD}$$

$$\overline{AD} = r - r\cos\theta$$

$$\beta'' = \tan^{-1}\frac{\overline{AD}}{\overline{DC}} = \tan^{-1}\frac{r(1-\cos\theta)}{r\sin\theta} = \tan^{-1}\frac{1-\cos\theta}{\sin\theta} = \frac{\theta}{2} \qquad \text{(12)}$$

thus β, β' and β'' for practical use can be obtained as functions of θ by equations (10), (11) and (12) as follows:

$$\beta = 0.416\,\theta$$
$$\beta' = 0.375\,\theta \sim 0.392\,\theta \quad (\theta = 10^\circ \sim 90^\circ) \qquad \text{(13)}$$

Since β' may be used as an approximate solution for β,

$$\beta' \fallingdotseq 0.4\,\theta \qquad \text{(14)}$$

$$\beta'' = 0.5\,\theta \qquad \text{(15)}$$

The coefficient of equation (15) is constant for any θ. Therefore if the angle of seismic coefficient "θ" for any seismic coefficient is obtained, the earthquake-resistant cross-section can immediately be obtained by means of equations (10) through (15). (Refer to Figs. 14 and 15)

The process by which the seismic coefficient circle is formed will be explained subsequently. The circular arc does not actually exist. However,

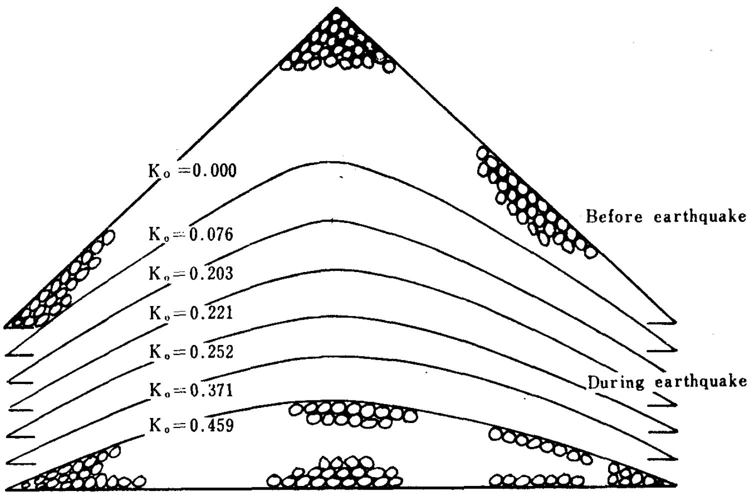

Fig. 9. Change in cross-section during vibration

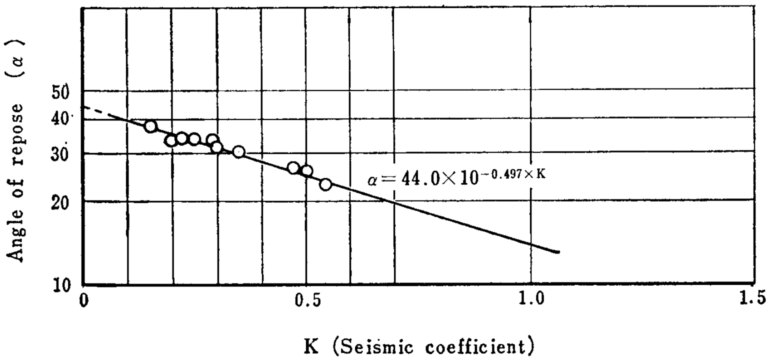

Fig. 10. The relation between the seismic coefficient and the static and dynamic angle of repose

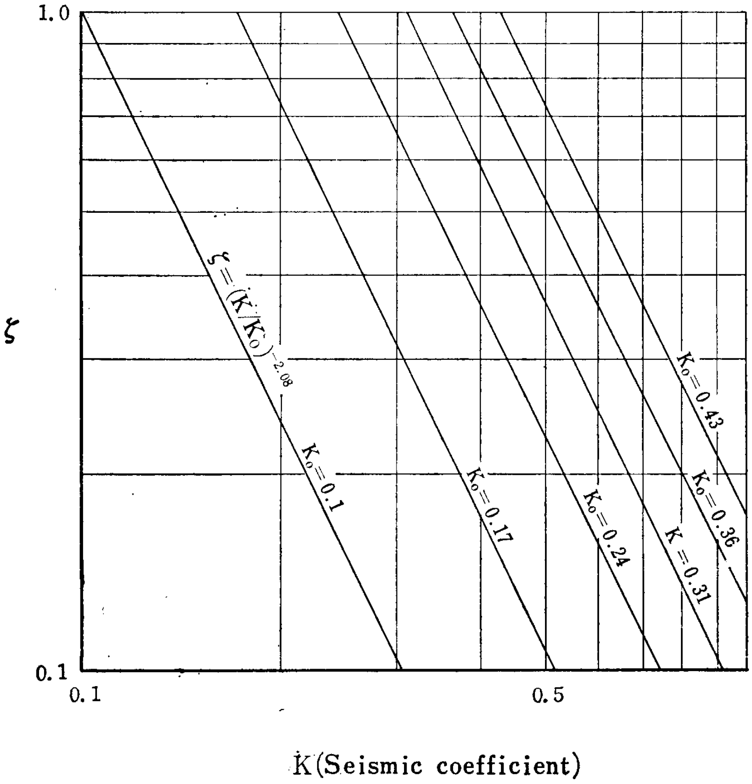

Fig. 11. The relation between seismic coefficient *K* and ζ

the triangular cross-section can always be obtained from the above theory. For instance, when the earthquake force is zero, the triangular cross-section ($\beta = \alpha_0$) is formed with static angle of repose α_0, and then the angle of seismic coefficient circle, "θ_0" can be obtained by equation (13) using this value of α_0. Therefore, it is possible to obtain the seismic coefficient circle for any seismic coefficient from the theoretical point of view.

Therefore, the triangle obtained from both the existing triangular cross-section and equations (13) and (14) is called the basic triangle.

Changes of the shape of the slope to the arc of this seismic coefficient circle for any seismic coefficient can be obtained experimentally and there are following relations among the angle of repose, the seismic coefficient, and the height of dam (Figs. 10 and 11).

$$\alpha = \alpha_0 \exp(b \cdot K) \quad \text{(16)}$$

where

α : dynamic angle of repose
α_0 : static angle of repose
b : an experimental constant

$$\zeta = (K/K_0)^{\omega} \quad \text{(17)}$$

where

ζ : 1−(height of dam at a certain place/height of dam)
K : seismic coefficient of dam
K_0 : seismic coefficient of the base of dam (K when $\zeta = 1$)
ω : an experimental constant

Equations (16) and (17) and Figs. 10 and 11 are obtained with the following experimental method.

First of all, the changing processes of the shape of the slope to the existing seismic coefficient circle for any seismic coefficient are measured with the experiment described below.

After drawing 5 cm mesh graph on the glass which is the front face of the vibration apparatus, the changing shapes of the cross-section for each seismic coefficient are recorded during a vibration test. The dynamic angles of repose at the bottom of the dam are measured with a slope gauge, and at the same time, the seismic coefficients are also measured by means of an accelerometer buried in the bottom of the dam.

From the readings of cross-section figures in this graph, the dynamic angles

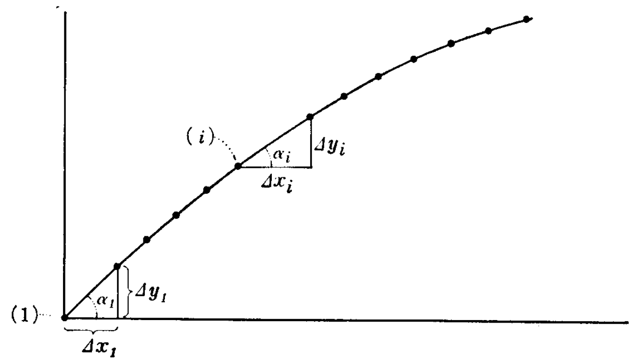

Fig. 12. Measurement of α_i from slope figure in any seismic coefficient K_0

of repose for each height are determined using the expression: $\Delta y_i/\Delta x_i = \tan\alpha_i$. The value of α_1 in $\Delta y_1/\Delta x_1 = \tan\alpha_1$ is the same as the value measured by the slope gauge. (Refer to Fig. 12)

In Fig. 10, the relation between α_1 and the seismic coefficients determined by the accelerometer is plotted in a semi-logarithmic scale, from which equation (16) is obtained.

If K is obtained by substituting α in equation (16) for the dynamic angle of repose α_i of each height, K corresponds to K_i for each height. The relation between the seismic coefficient K and the height is shown in Fig. 11; in this case, ζ on ordinate axis and K on abscissa are plotted in a logarithmic scale. Equation (17) is a formulation of Fig. 11. The term ω is an experimental constant which depends upon the angle of internal friction and the grain shape of materials to be used. From equations (16) and (17),

$$\alpha = \alpha_0 \exp(b\cdot K_0\cdot\zeta^{1/\omega})$$

and also from $d\zeta/d\psi = \tan\alpha$, the configuration of the dam from the toe to the top is given by

$$\psi = \int_1^{\zeta} \frac{d\zeta}{\tan(\alpha_0\cdot\exp(b\cdot K_0\cdot\zeta^{1/\omega}))} \quad \cdots\cdots\cdots\cdots\cdots\cdots(18)$$

where ψ: length of base from toe/height of dam.

From the configuration of the dam, the slope β of the cross-section of the basic triangle can be obtained in a similar way to the basic consideration in obtaining the earthquake-resistant cross-section formed in the seismic coefficient circle. Figs. 14 and 15 show calculated values of β of the configuration of the dam shown in Fig. 9, and the value of α in Figs. 14 and 15 is the same value as α in Fig. 10. Also, from the result mentioned above, the relation between θ and α is shown as follows:

$$\theta = f(\alpha) = \beta/0.416 \cdots\cdots\cdots\cdots\cdots\cdots\cdots(19)$$

where

$$\beta = \alpha_0\cdot\exp(-m\cdot\alpha\cdot K_0)$$

Accordingly, θ for static and earthquake conditions can be obtained as shown in Figs. 14 and 15.

As shown in Figs. 14 and 15, if a semi-logarithmic scale is used, θ can be expressed by a straight line. Therefore, the theoretical seismic coefficient circle can be drawn, and from the engineering point of view, if θ is obtained, the earthquake resistant cross-section (basic triangle) for any seismic coefficient may be determined. The practical method of analysis will be described subsequently.

3. STABILITY ANALYSIS OF SEISMIC COEFFICIENT CIRCLE METHOD

3.1 *The method to obtain the angle of seismic coefficient circle "θ"*

For practical stability analysis, a graph is to be drawn for obtaining the angle of seismic coefficient circle, θ, for any seismic coefficient as mentioned before.

1) First of all, the static angle of repose α_0 of rock material is measured. As the cohesion is zero, the angle of internal friction ϕ is equal to α_0. Rock materials to be actually used for the fill should be used to obtain this value.

2) A laboratory experiment is carried out using a similar material which has the same ϕ as that of the rock material, or the actual material if possible. In this case, there is no significant relationship between the size of stones and the scale of model, but only a relationship between α and α_0.

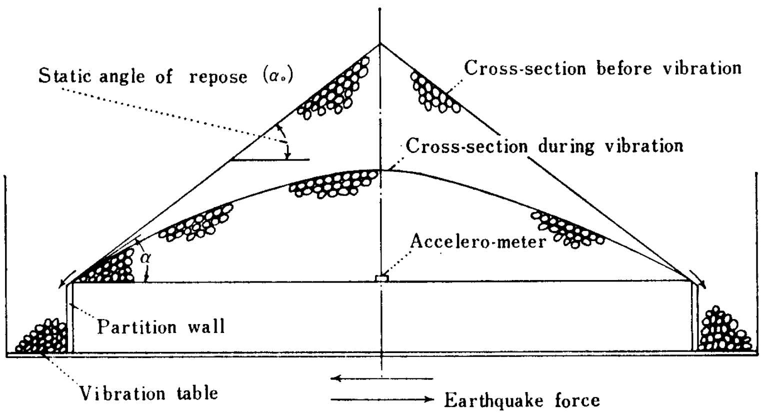

Fig. 13. Front view of model test apparatus

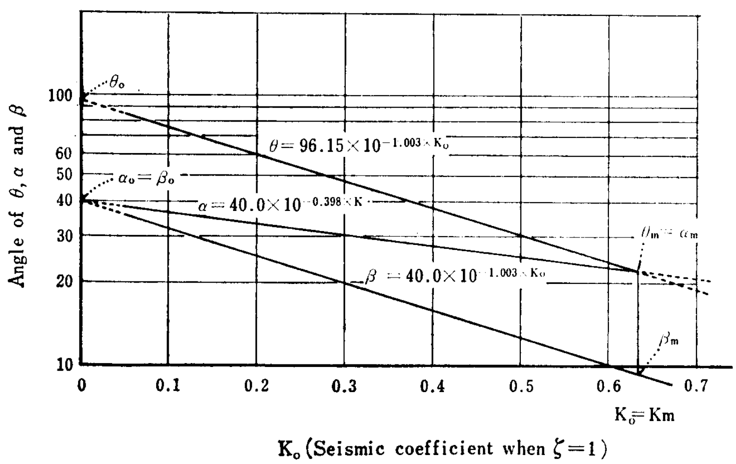

Fig. 14. The relation between seismic coefficient and θ, α and β (Zone 3)

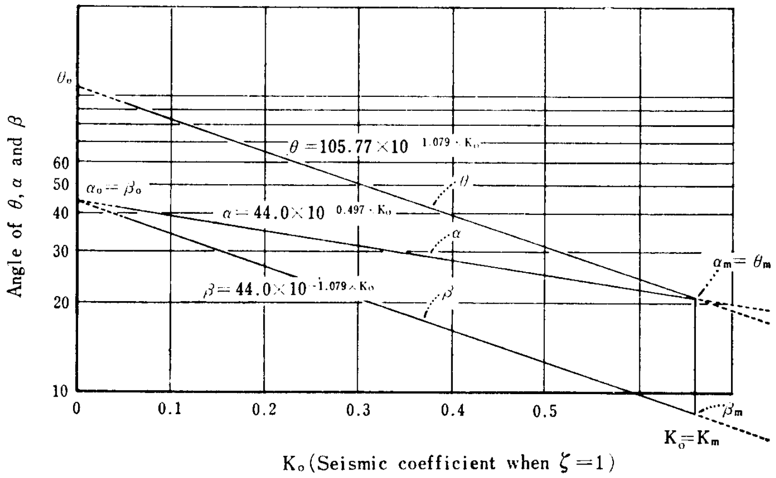

Fig. 15. The relation between seismic coefficient and θ, α and β (Zone 2)

3) The dynamic angle of repose α when the seismic coefficient circle is actually formed is obtained. The authors used the apparatus shown in Fig. 13 and measured the dynamic angle of repose α_m at the base of a model dam when a circular arc was formed after the application of horizontal vibration. At the same time vertical vibration was applied, but its influence was small as compared with that of the horizontal vibration. As a result α_0 and α_m can be obtained.

4) The relations among θ, α, β, and the seismic coefficient are indicated in Figs. 14 and 15. In this case, β in equation (13) is approximated by $\beta \fallingdotseq \beta'' \fallingdotseq 0.4\theta$. Now, if θ is θ_0 when the seismic coefficient is $K_0 = 0$, and θ and β are θ_m and β_m, respectively. When α_m was obtained for $K_0 = K_m$, $\theta_0 = \alpha_0/0.4$, $\alpha_0 = \beta_0$, $\theta_m = \alpha_m$ and $\beta_m \fallingdotseq 0.4\alpha_m$ are obtained. Refer to equations (13), (14), and (19).

Initially, the seismic coefficient K_0 is on the abscissa and the angle is on the ordinate.

After α_0 and θ_0 were plotted at $K_0 = 0$ and α_m and β_m were plotted at $K_0 = K_m$, β for any seismic coefficient is obtained from the straight line connecting points of $\alpha_0 = \beta_0$ and β_m, and α and θ are obtained by connecting points of α_0 and α_m, and θ_0 and θ_m, respectively, as shown in Figs. 14 and 15. From the graph in Fig. 14, it is easy to design an earthquake-resistant cross-section (basic triangle).

However, from the practical point of view it is necessary to design not only triangluar cross-sections but also other types of cross-sections consisting of more than two kinds of rock materials. Various sections have to be examined also for economic comparisons. Therefore, both this graph and the following method are used for the design of a rock-fill dam.

3.2. *Graphical method to draw a seismic coefficient circle*

1) For a given width of the base of the dam (Fig. 16):

i) Draw a straght line BC representing the width of the base and determine the mid-point D.

ii) Draw a vertical line at right angle to BC through the mid-point D.

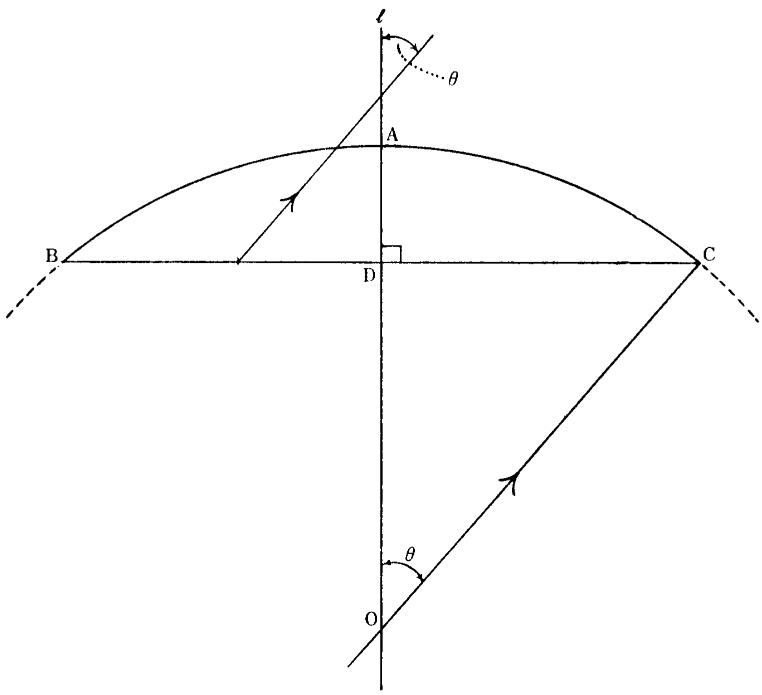

Fig. 16. Drawing the seismic coefficient circle for a given width of the base of the dam

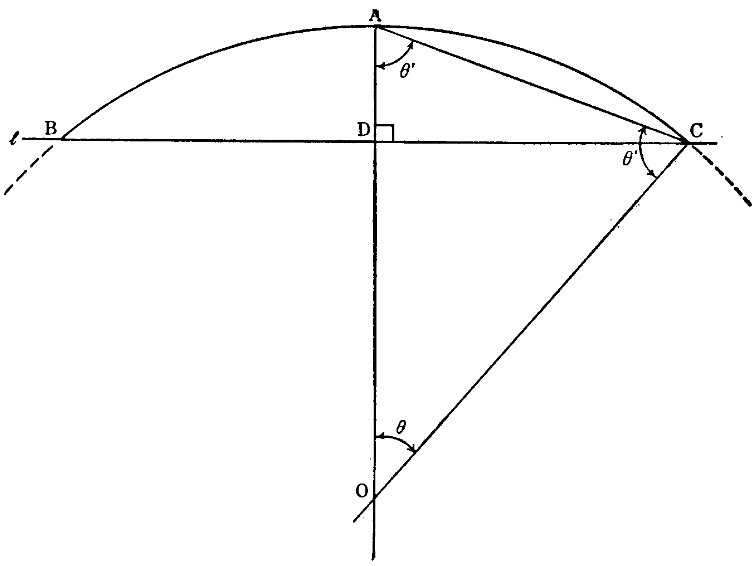

Fig. 17. Drawing the seismic coefficient circle for a given height of the dam

iii) Draw a straight line crossing the vertical line l at angle θ.
iv) Draw a line through point C in parralel with the straight line.
v) The point where the straight line crosses the vertical line l is assumed point O. (O is the origin of the seismic coefficient circle).
vi) Draw a circle with radius OC in which O is the center. This is the seismic coefficient circle.

2) For a given height of the dam (Fig. 17):
i) Draw a straight line AD representing the height of the dam.
ii) Draw a straight line at right angle to AD through point D.
iii) At point A draw a straight line with angle $\theta' = (180^\circ - \theta)/2$ to AD, and the point where it crosses the straight line l is C.
iv) At point C, draw a straight line with angle θ' to AC, and determine the point O where it crosses the extension of AD.
v) Draw a circle of radius OC, with its center at O. This is the seismic coefficient circle.

4. EXAMPLES OF APPLICATION OF THE THEORY
(Design of the rock-fill dam with asphalt facing)

The Ministry of Agriculture and Forestry has decided to construct a rock-fill dam with asphalt facing as a surface waterproofing layer at Peipenai in Hokkaido.

The dam for irrigation purpose will store about 200,000 tons of water and will be 57.4 m in height. The design calls for the use of gravel available from a river bed and the safety of the dam against earthquake forces with the seismic coefficient $K_0 = 0.15$. This dam was first designed by the slip circle method. However, as the result obtained from model tests was not satisfactory,[9] the new theory was applied to the design work of the dam.

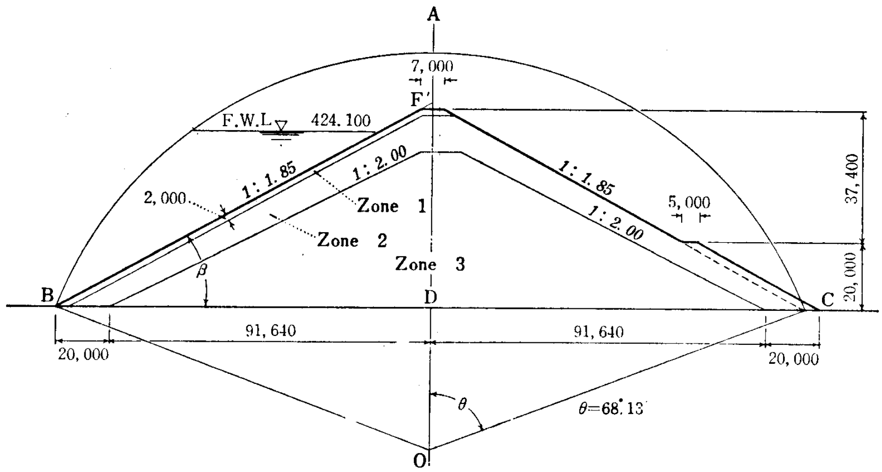

Fig. 18. The seismic coefficient circle and the earthquake-resistant cross-section of Peipenai Dam

This dam is not yet constructed, but a model test of the new design has given satisfactory results. In application of the theory to the design of this dam, the stability analysis and experiments were carried out at a seismic coefficient $K_0 = 0.18$, differing from $K_0 = 0.15$ originally decided, in order to provide safety. The static angle of repose α_0 of the materials of Zone 2 and Zone 3 shown in Fig. 18 were found to be 44° and 40°, respectively, and of course cohesions were zero.

The dynamic properties of materials are shown in Figs. 14 and 15. From Fig. 15, the slope β of Zone 2 at $K_0 = 0.18$ is 28°23′, and from Fig. 14, the slope β of Zone 3 is 26°34′. Finally, an earthquake-resistant cross-section as shown in Fig. 18 was obtained by both the calculation and graphical methods.

The result of a model test in the laboratory coincided with the value of stability analysis, i.e., the seismic coefficient when the top began to collapse was $K_0 = 0.18$, and this theory was confirmed to provide safety. As mentioned above, the design which calls for the use of gravel from the river bed was accomplished rationally and economically.

5. CONCLUSIONS

The design of earthquake resistant dams has been extensively studied, but has not reached the point where its use is practical in detailed design work. This paper presents a new method of analysis for rock-fill dams consisting of cohesionless material. It is pointed out that there is a contradiction when the slip circle method such as the Swedish Method, which is primarily for earth dams, is applied to rock-fill dams, and that in model tests in the laboratory, the slip circle method does not indicate the slope failure correctly. Materials having c and ϕ should, therefore, be differentiated from materials having only ϕ, i.e., different methods of analysis should be applied to soil materials and rock materials.

If a dam is a combined structure such as an earth and rock-fill dam, it is desirable to use both the analytical method expressed by equation (7) and the seismic coefficient circle method.

The seismic coefficient circle method, newly proposed in this paper, can easily be used to determine earthquake-resistant cross-section, if the static angle of repose and the dynamic angle of repose when the seismic coefficient circle is actually formed are determined, and if also the seismic coefficient at that time is known.

The authors hope that this theory is able to give a new guidance for the earthquake-resistant design of rock-fill dams for which there is no satisfactory stability analysis method, and also that it can be applied to other problems such as shown in the appendix given herein.

NOTATIONS

b : an experimental constant.
c : cohesion of the slope-forming material.
F : safety factor.
h : height of dam (height at $n=1$).
k : area ratio when circular arc is given.
K : seismic coefficient of dam.
K_0 : seismic coefficient of base (design seismic coefficient).
L : length of the circular are of a slip surface.
n : abscissa ($n \geqq 1$) indicating the width of dam in which 1 is origin.
N_n : vertical component of the weight of sliding mass.
s : scale ratio of cross-section of dam.
T_n : tangential component of the weight of sliding mass.
α : dynamic angle of repose.
α_0 : static angle of repose.
α_n : angle of the slope of the sliding surface.
γ : unit weight.
ε : a variable.
ζ : $1-$(height of dam at a certain place/height of dam.)
ϕ : angle of internal friction of the slope-forming material.
χ : vertical distance to slope (indicating height on the ordinate).
ψ : base length from the toe of slope/height of dam.
ω : an experimental constant.

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APPENDIX. ANALYSIS OF ANCIENT PYRAMIDS IN EGYPT AND MEXICO

Pyramids are good examples of existing structures which have resisted earthquakes for a long time. Fig. 19 shows an analysis of an earthquake-

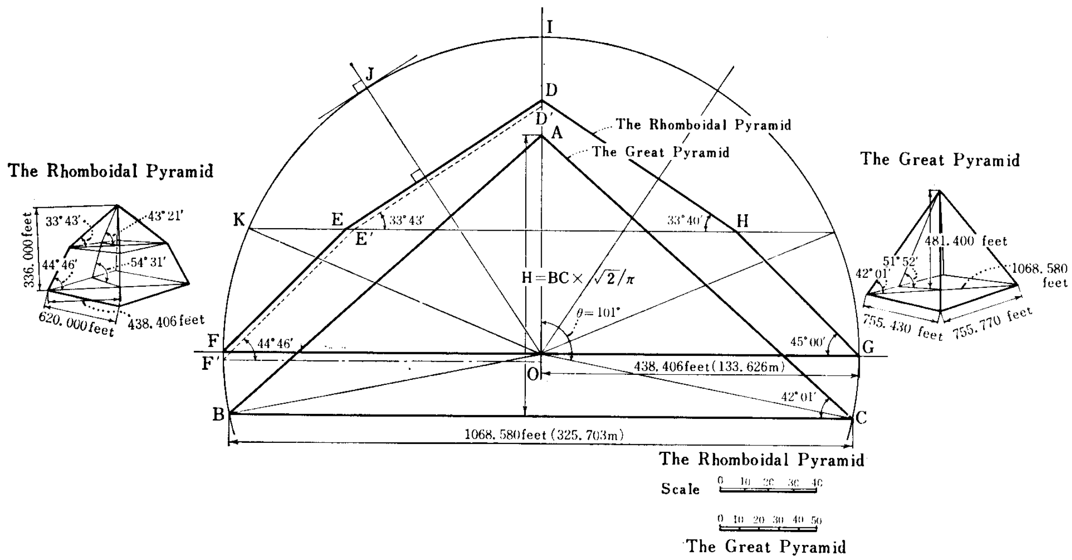

Fig. 19. An analysis of Pyramids in Egypt by the seismic coefficient circle method

resistant structure. The diagonal cross-section $\triangle ABC$ of the largest pyramid in the world constructed about 4500 years ago in Giza in Egypt is equal to the basic triangle obtained by the seismic coefficient circle with the angle of seismic coefficient circle of 101°. The slope β of the diagonal cross-section is $\beta = 0.416 \times 101° = 42°01'$ by equation (13), and it coincides with the value calculated from the measured slope angle of $51°52'$. The height of 481.4 feet also coincides with the measured value.

Furthermore, rhomboidal pyramids constructed in Dahchur before the above pyramid and having two step slopes can be analyzed with the combination of the angles of seismic coefficient circle, 101° and 90°. The circular arc B I for $\theta = 101°$ is equally divided into three parts by points J and K. The tangent at point J on the circular arc is transferred in parallel so that ED (the slope of the first step of the pyramid) is determined. point E can be obtained by transferring point K in parallel with straight line FO.

Next, the angle β'' of the basic trapezoid cross-section obtained from the seismic coefficient circle FOGI with the angle of seismic coefficient circle of 90° becomes 45° according to equation (15). If with this value of β'' a straight line is drawn through point F, FE (the slope of the second step of

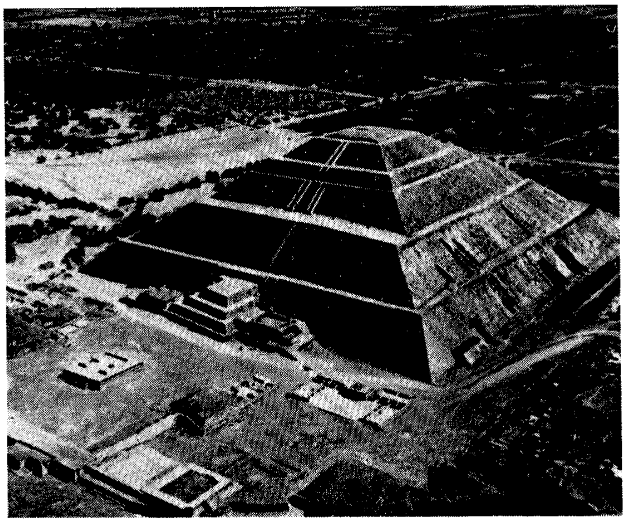

Photo. 6. Solar Pyramid of Teotihuacan in Mexico

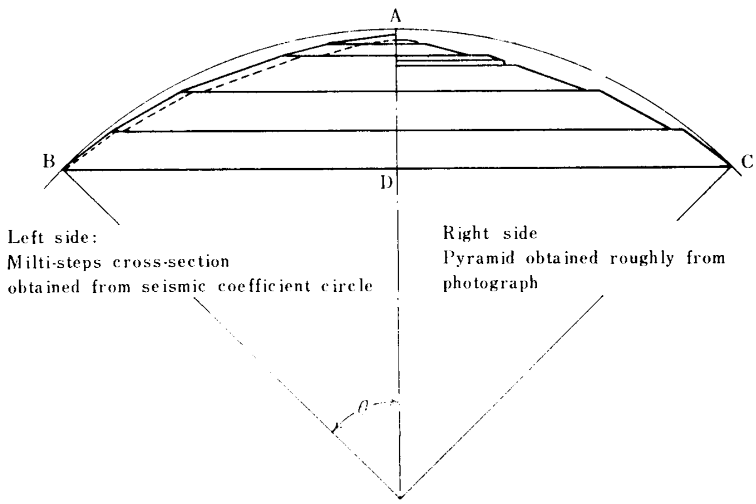

Fig. 20. The seismic coefficient circle obtained roughly from photograph

the pyramid) is determined. The pentagon FOGHDE shown by solid lines in Fig. 19 becomes the diagonal cross section of this pyramid. The dotted lines give the actually measured angles. The difference between the solid and the dotted lines is considered to be probably due to the surveying errors and settlements of the pyramid, considering the reference (1) page 98.

Accordingly, of these two pyramids, the largest in Giza is more stable to earthquakes than the one in Dahshur. Solar pyramid of Teotihuacan in Mexico shown in Photo. 6 can also be analyzed by the multi-step earthquake-resistant cross-section of the seismic coefficient method as shown in Fig. 20.

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