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STRESSES AND DISPLACEMENTS DUE TO RECTANGULAR LOAD ON A LAYER OF FINITE THICKNESS

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SUMMARY

In this study are shown the expressions for calculating stresses and displacements beneath the rectangular flexible foundation resting on an elastic isotropic layer of limited thickness, underlain by a rigid base. The solution is given for the vertical and horizontal load. The contact between the compressible layer and rigid base has been assumed to be perfectly rough.

Stresses and displacements have been calculated for the ratio $H/B=1.0$; 2.0; 3.0 and 5.0 (where H is the thickness of the compressible layer and B is the width of the foundation), for $L/B=1.0$; 2.0 and 5.0 (where L and B are the dimensions of the foundation) and for the Poisson's ratio $\mu=0.15$; 0.30 and 0.45.

INTRODUCTION

The stress distribution in an isotropic layer of finite thickness resting on a rigid base was treated by several investigators, Marguerre (1933), Biot (1935), Sovinc (1961), Poulos (1967), Milovic and Tournier (1970).

In this study, stresses and displacements in an elastic layer of finite thickness produced by a rectangular flexible foundation are shown. The solution is given for the vertical and horizontal load.

Using Fourier's series, stresses and displacements have been determined for the ratio $H/B=1.0$; 2.0; 3.0 and 5.0 (where H is the thickness of the compressible layer and B is the width of the rectangular foundation), and for the ratio $L/B=1.0$; 2.0 and 5.0, where L and B are the dimensions of the foundation.

The calculation has been programmed on the computer IBM–360–40.

ASSUMPTIONS AND DEFINITIONS

The following assumptions and definitions have been made in this study:

—the rectangular foundation is flexible and has a rough base;

—the soil behaves as an isotropic, homogeneous continuum which obeys Hooke's

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law. The mechanical properties of soil are defined by two elastic constants, Young's modulus of elasticity, E, and Poisson's ratio, μ;

—the compressible layer with a horizontal upper boundary is of finite thickness H, limited by a rigid base (Fig. 1);

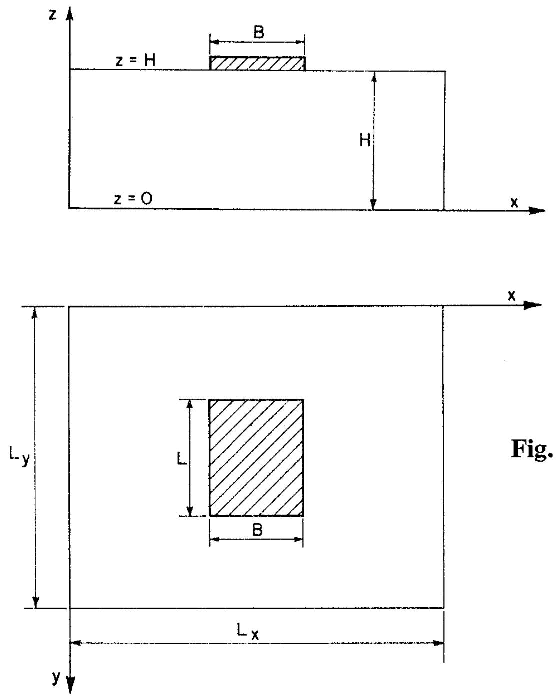

Fig. 1. Geometrical description

—the contact between the compressible layer and the rigid base is perfectly rough;
—the direction of the applied load is vertical or horizontal to the foundation;
—there are no body forces.

The calculation of stresses and displacements is made for a rectangular foundation of width $B = 1.0$ m and $p_v = 1.0$ kg/cm^2 or $P_h = 1.0$ kg/cm^2, with the following sign conventions (Fig. 2):

—the vertical axis Az is positive downwards;
—the axis Ax is horizontal and positive towards the right from the point A;
—the settlement w is positive if in the direction of increasing z;
—the horizontal displacement u is positive in the positive x direction;
—the normal stresses are positive if compressive;
—the inclination, δ, is the angle between the vertical axis and load. This angle is positive in the trigonometric sense;
—the compressive normal load, p_v, is positive if it is in the positive z direction;
—the horizontal load, p_h, is positive if it is in the positive x direction.

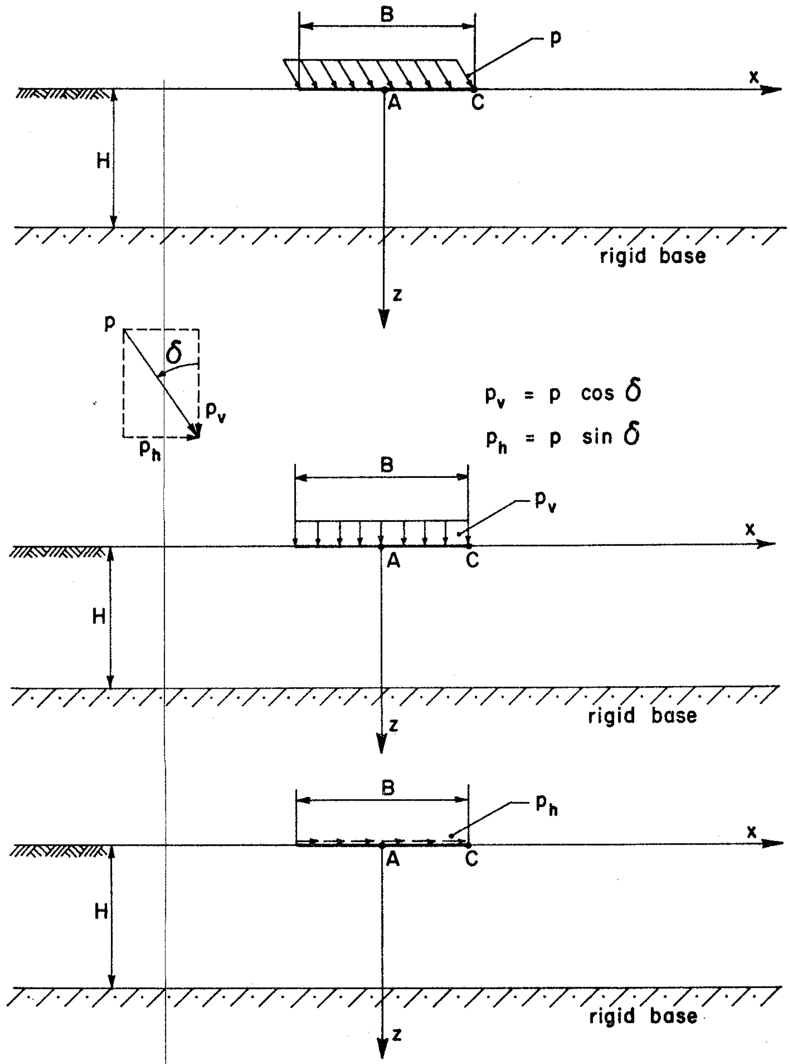

Fig. 2. Sign conventions

THEORICAL STUDY

a) Vertical load

The displacement functions for u, v and w are given by the following double trigonometric series:

$$u = \sum_m \sum_n U_{mn} \sin \alpha x \cos \beta y \tag{1}$$

$$v = \sum_m \sum_n V_{mn} \cos \alpha x \sin \beta y \tag{2}$$

$$w = \sum_m \sum_n W_{mn} \cos \alpha x \cos \beta y \tag{3}$$

with:

$$\alpha = \frac{m\pi}{L_x}; \quad \beta = \frac{n\pi}{L_y}$$

By substituting these expressions in the differential equations of equilibrium:

$$(1-2\mu)\nabla u+\frac{\partial\theta}{\partial x}=0 \qquad (4)$$

$$(1-2\mu)\nabla v+\frac{\partial\theta}{\partial y}=0 \qquad (5)$$

$$(1-2\mu)\nabla w+\frac{\partial\theta}{\partial z}=0 \qquad (6)$$

where μ is the Poisson's ratio and $\theta=(\partial u/\partial x)+(\partial v/\partial y)+(\partial w/\partial z)$ the volume expansion, one obtains:

$$2(1-\mu)\frac{\partial^2 u}{\partial x^2}+(1-2\mu)\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 w}{\partial x\partial z}=0 \qquad (7)$$

$$2(1-\mu)\frac{\partial^2 v}{\partial y^2}+(1-2\mu)\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2}\right)+\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 w}{\partial y\partial z}=0 \qquad (8)$$

$$2(1-\mu)\frac{\partial^2 w}{\partial z^2}+(1-2\mu)\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)+\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 v}{\partial y\partial z}=0 \qquad (9)$$

The Fourier's coefficients are expressed as follows:
if $m=n=0$

$$U_{oo}=0;\quad V_{oo}=0;\quad W_{oo}=-\frac{a_{oo}z}{A_{11}} \qquad (10)$$

if $m\neq 0$ or $n\neq 0$

$$U_{mn}=\frac{1}{2G}\{[C_1\alpha+C_5\beta+\alpha zC_4]\sinh\gamma z+[C_2\alpha+C_6\beta+\alpha zC_3]\cosh\gamma z\} \qquad (11)$$

$$V_{mn}=\frac{1}{2G}\{[C_1\beta+C_5\alpha+\beta zC_4]\sinh\gamma z+[C_2\beta+C_6\alpha+\beta zC_3]\cosh\gamma z] \qquad (12)$$

$$W_{mn}=\frac{1}{2G}\{[(3-4\mu)C_4-C_2\gamma-\gamma zC_3]\sinh\gamma z+$$
$$+(3-4\mu)C_3-C\gamma_1-\gamma zC_4]\cosh\gamma z\} \qquad (13)$$

For the rough rigid base, the boundary conditions are:

$$w=0;\quad u=0;\quad v=0\quad \text{when}\quad z=0$$

$$\sigma_z=-p=-\sum_m\sum_n a_{mn}\cos ax\cos\beta y\quad \text{when}\quad z=H$$

$$\tau_{zx}=0;\quad \tau_{zy}=0$$

Using these conditions, the following expressions for constants are obtained:

$$C_2=C_5=C_6=0$$

$$C_1 = \frac{(3-4\mu)C_3}{\gamma}$$

$$C_3 = a_{mn}[\gamma H \cosh \gamma H - (1-2\mu) \sinh \gamma H]/DEN$$

$$C_4 = -a_{mn}[\gamma H \sinh \gamma H + 2(1-\mu) \cosh \gamma H]/DEN$$

with:

$$DEN = \gamma[\gamma^2 H^2 + 4(1-\mu)^2 \cosh^2 \gamma H - (1-2\mu)^2 \sinh^2 \gamma H]$$

$$\gamma^2 = \alpha^2 + \beta^2$$

The coefficients a_{mn} are given by the following expressions:
if $m = 2M+1$ and $n = 2N+1$

$$a_{mn} = 0$$

if $m = 2M$ and $n = 2N$

$$a_{oo} = \frac{BL}{L_x L_y}; \quad a_{mo} = (-1)^M \cdot \frac{4L}{L_y m\pi} \sin \frac{m\pi B}{2L_x}$$

$$a_{on} = (-1)^N \cdot \frac{4B}{L_x n\pi} \sin \frac{n\pi L}{2L_y}$$

$$a_{mn} = (-1)^{M+N} \cdot \frac{16}{\pi^2 mn} \sin \frac{m\pi B}{2L_x} \sin \frac{n\pi L}{2L_y}$$

Using the stress-strain relationship:

$$\begin{vmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{zy} \\ \tau_{zx} \\ \tau_{xy} \end{vmatrix} = \begin{vmatrix} A_{11} & A_{12} & A_{12} & 0 & 0 & 0 \\ A_{12} & A_{11} & A_{12} & 0 & 0 & 0 \\ A_{12} & A_{12} & A_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{55} \end{vmatrix} \begin{vmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{zy} \\ \gamma_{zx} \\ \gamma_{xy} \end{vmatrix} \tag{14}$$

where:

$$A_{11} = \frac{E(1-\mu)}{(1+\mu)(1-2\mu)}; \quad A_{12} = \frac{E\mu}{(1+\mu)(1-2\mu)}; \quad A_{55} = G = \frac{E}{2(1+\mu)}$$

the following expressions for stresses are obtained:

$$\sigma_x = \sum_m \sum_n \left(\alpha A_{11} U + \beta A_{12} V + A_{12} \frac{dW}{dz}\right) \cos \alpha x \cos \beta y \tag{15}$$

$$\sigma_y = \sum_m \sum_n \left(\alpha A_{12} U + \beta A_{11} V + A_{12} \frac{dW}{dz}\right) \cos \alpha x \cos \beta y \tag{16}$$

$$\sigma_z = \sum_m \sum_n \left(\alpha A_{12} U + \beta A_{12} V + A_{11} \frac{dW}{dz}\right) \cos \alpha x \cos \beta y \tag{17}$$

$$\tau_{zy} = \sum_m \sum_n A_{55}\left(\frac{dV}{dz} - \beta W\right) \cos \alpha x \sin \beta y \tag{18}$$

$$\tau_{zx} = \sum_m \sum_n A_{55}\left(\frac{dU}{dz} - \alpha W\right) \sin \alpha x \cos \beta y \tag{19}$$

$$\tau_{xy} = \sum_m \sum_n - A_{55}(\alpha V + \beta U) \sin \alpha x \sin \beta y \tag{20}$$

b) Horizontal load

In this case, the displacement functions for u, v and w are given by the following trigonometric series:

$$u = \sum_m \sum_n U_{mn} \cos \alpha x \cos \beta y \tag{21}$$

$$v = \sum_m \sum_n V_{mn} \sin \alpha x \sin \beta y \tag{22}$$

$$w = \sum_m \sum_n W_{mn} \sin \alpha x \cos \beta y \tag{23}$$

The boundary conditions are:

$$w = 0; \quad u = 0; \quad v = 0 \quad \text{when} \quad z = 0$$

$$\tau_{xz} = p(x, y) = \sum_m \sum_n a_{mn} \cos \alpha x \cos \beta y \quad \text{when} \quad z = H$$

$$\sigma_z = 0; \quad \tau_{yz} = 0$$

The Fourier's coefficients are given by:

if $m = 0$ and $n = 0$

$$U_{oo} = \frac{a_{oo}}{A_{55}} z; \quad V_{oo} = 0; \quad W_{oo} = 0 .$$

if $m = 0$ and $n \neq 0$

$$U_{on} = \frac{1}{G\beta} \frac{a_{on}}{\cosh \beta H} \sinh \beta z; \quad V_{on} = 0; \quad W_{on} = 0 .$$

if $m \neq 0$ and $n = 0$

$$U_{mo} = \frac{1}{2G}\{[(3 - 4\mu)D_4 + \alpha D_2 z] \sinh \alpha z + \alpha D_4 z \cosh \alpha z\}$$

$$V_{mo} = 0$$

$$W_{mo} = \frac{1}{2G}\{[-(3 - 4\mu)D_2 + \alpha D_4 z] \sinh \alpha z + \alpha D_3 z \cosh \alpha z\}$$

$$D_2 = a_{mo}[\alpha H \cosh \alpha H + (1 - 2\mu) \sinh \alpha H]/DEN$$

$$D_4 = - a_{mo}[\alpha H \sinh \alpha H - 2(1 - \mu)\cosh \alpha H]/DEN$$

$$DEN = \alpha^2[\alpha^2 H^2 + 4(1 - \mu)^2 \cosh^2 \alpha H - (1 - 2\mu)^2 \sinh^2 \alpha H] .$$

if $m \neq 0$ and $n \neq 0$

$$U_{mn} = \frac{\alpha}{2G}\{[(3-4\mu)C_2 + \beta^2 C_6 + \gamma C_4 z]\sinh\gamma z + \gamma C_2 z\cosh\gamma z\}$$

$$V_{mn} = -\frac{\beta}{2G}\{[(3-4\mu)C_2 - \alpha^2 C_6 + \gamma C_4 z]\sinh\gamma z + \gamma C_2 z\cosh\gamma z\}$$

$$W_{mn} = \frac{\gamma}{2G}\{[-(3-4\mu)C_4 + \gamma C_2 z]\sinh\gamma z + \gamma C_4 z\cosh\gamma z\}$$

$$C_2 = \alpha a_{mn}[\gamma H\sinh\gamma H - 2(1-\mu)\cosh\gamma H]/DEN$$

$$C_4 = -\alpha a_{mn}[\gamma H\cosh\gamma H + (1-2\mu)\sinh\gamma H]/DEN$$

$$C_6 = \frac{2\,a_{mn}}{\alpha\gamma^3\cosh\gamma H}$$

$$DEN = \gamma^3[(1-2\mu)^2\sinh^2\gamma H - 4(1-\mu)^2\cosh^2\gamma H - \gamma^2 H^2]$$

Using the stress-strain relationship given by eq. (14), the following expressions for stresses are obtained:

$$\sigma_x = \sum_m\sum_n\left(-\alpha A_{11}U + \beta A_{12}V + A_{12}\frac{dW}{dz}\right)\sin\alpha x\cos\beta y \qquad (24)$$

$$\sigma_y = \sum_m\sum_n\left(-\alpha A_{12}U + \beta A_{11}V + A_{12}\frac{dW}{dz}\right)\sin\alpha x\cos\beta y \qquad (25)$$

$$\sigma_z = \sum_m\sum_n\left(-\alpha A_{12}U + \beta A_{12}V + A_{11}\frac{dW}{dz}\right)\sin\alpha x\cos\beta y \qquad (26)$$

$$\tau_{zy} = \sum_m\sum_n A_{55}\left(\frac{dV}{dz} - \beta W\right)\sin\alpha x\sin\beta y \qquad (27)$$

$$\tau_{zx} = \sum_m\sum_n A_{55}\left(\frac{dU}{dz} + \alpha W\right)\cos\alpha x\cos\beta y \qquad (28)$$

$$\tau_{xy} = \sum_m\sum_n A_{55}(\alpha V - \beta U)\cos\alpha x\sin\beta y \qquad (29)$$

The coefficients a_{mn} are identical to those obtained for the vertical load.

RESULTS

The dimensionless coefficients for stresses and displacements have been calculated for the ratio $(H/B) = 1.0$; 2.0; 3.0 and 5.0, for the ratio $(L/B) = 1.0$; 2.0 and 5.0, and for three values of the Poisson's ratio.

The stresses and displacements, produced by a vertical load, p_v, are now expressed in the following form:

Table 1. Influence values I_{zv}, I_{xv} and I_{xzh}

$(L/B)=1.00$ Point A

H/B	z/B	$\mu=0.15$			$\mu=0.30$			$\mu=0.45$		
		I_{zv}	I_{xv}	I_{xzh}	I_{zv}	I_{xv}	I_{xzh}	I_{zv}	I_{xv}	I_{xzh}
1.0	0.00	1.000	0.441	—	1.000	0.549	—	1.000	0.638	—
	0.10	0.974	0.310	0.738	0.974	0.425	0.739	0.974	0.520	0.740
	0.20	0.941	0.214	0.544	0.943	0.308	0.545	0.947	0.397	0.548
	0.40	0.837	0.040	0.276	0.842	0.113	0.275	0.855	0.192	0.274
	0.60	0.682	−0.005	0.167	0.690	0.062	0.164	0.712	0.149	0.164
	0.80	0.563	0.013	0.131	0.570	0.093	0.122	0.595	0.211	0.114
	1.00	0.473	0.083	0.117	0.468	0.201	0.096	0.478	0.391	0.067
2.0	0.00	1.000	0.527	—	1.000	0.649	—	1.000	0.766	—
	0.10	0.970	0.385	0.726	0.970	0.500	0.726	0.970	0.590	0.726
	0.20	0.931	0.261	0.521	0.931	0.355	0.522	0.931	0.446	0.522
	0.40	0.802	0.061	0.236	0.802	0.122	0.236	0.804	0.181	0.237
	0.80	0.462	−0.027	0.062	0.464	0.003	0.062	0.469	0.034	0.063
	1.20	0.282	−0.023	0.033	0.286	−0.005	0.033	0.294	0.027	0.033
	1.60	0.200	−0.005	0.030	0.204	0.020	0.028	0.215	0.057	0.025
	2.00	0.157	0.027	0.028	0.155	0.067	—	0.161	0.132	0.011
3.0	0.00	1.000	0.545	—	1.000	0.672	—	1.000	0.795	—
	0.10	0.970	0.400	0.724	0.970	0.502	0.724	0.970	0.635	0.724
	0.20	0.930	0.275	0.518	0.930	0.370	0.519	0.930	0.464	0.519
	0.40	0.799	0.070	0.230	0.799	0.131	0.230	0.799	0.191	0.231
	0.80	0.452	−0.024	0.052	0.453	0.003	0.052	0.454	0.030	0.053
	1.20	0.263	−0.025	0.020	0.264	−0.009	0.020	0.266	0.007	0.021
	1.60	0.170	−0.018	0.013	0.172	−0.006	0.013	0.175	0.007	0.014
	2.00	0.122	−0.011	0.012	0.124	0.000	0.012	0.129	0.013	0.012
	2.50	0.091	−0.001	0.013	0.093	0.011	0.011	0.099	0.029	0.010
	3.00	0.073	0.13	0.012	0.073	0.031	0.009	0.0076	0.063	0.004
5.0	0.00	1.000	0.555	—	1.000	0.684	—	1.000	0.811	—
	0.10	0.970	0.432	0.723	0.970	0.521	0.723	0.970	0.625	0.723
	0.20	0.930	0.282	0.517	0.930	0.380	0.517	0.930	0.476	0.517
	0.40	0.798	0.076	0.228	0.798	0.138	0.228	0.798	0.200	0.223
	0.80	0.450	−0.020	0.048	0.450	0.007	0.049	0.450	0.034	0.049
	1.20	0.258	−0.023	0.015	0.258	−0.009	0.015	0.258	0.006	0.015
	1.60	0.162	−0.017	0.007	0.162	−0.009	0.007	0.163	0.000	0.007
	2.00	0.110	−0.013	0.005	0.111	−0.007	0.005	0.112	0.000	0.005
	2.50	0.075	−0.008	0.004	0.075	−0.004	0.004	0.077	0.001	0.004
	3.00	0.055	−0.006	0.004	0.056	−0.002	0.004	0.057	0.003	0.004
	3.50	0.043	−0.004	0.004	0.044	0.000	0.004	0.046	0.006	0.004
	4.00	0.036	−0.002	0.005	0.037	0.003	0.004	0.039	0.009	0.004
	4.50	0.031	0.001	0.005	0.032	0.006	0.004	0.034	0.015	0.004
	5.00	0.027	0.005	0.005	0.027	0.012	0.003	0.029	0.023	0.002

Table 2. Influence values I_{zv}, I_{xv} and I_{xzh}

$(L/B) = 2.00$ Point A

H/B	z/B	$\mu = 0.15$			$\mu = 0.30$			$\mu = 0.45$		
		I_{zv}	I_{xv}	I_{xzh}	I_{zv}	I_{xv}	I_{xzh}	I_{zv}	I_{xv}	I_{xzh}
	0.00	1.000	0.511	—	1.000	0.571	—	1.000	0.607	—
	0.10	0.992	0.364	0.775	0.992	0.445	0.775	0.992	0.501	0.777
	0.20	0.976	0.245	0.589	0.977	0.314	0.589	0.981	0.377	0.593
1.0	0.40	0.919	0.047	0.337	0.924	0.122	0.334	0.936	0.207	0.337
	0.60	0.821	−0.006	0.233	0.827	0.080	0.226	0.847	0.193	0.224
	0.80	0.732	0.020	0.198	0.734	0.130	0.181	0.754	0.291	0.165
	1.00	0.651	0.115	0.181	0.638	0.273	0.145	0.639	0.523	0.099
	0.00	1.000	0.656	—	1.000	0.731	—	1.000	0.796	—
	0.10	0.985	0.502	0.754	0.985	0.545	0.755	0.985	0.620	0.764
	0.20	0.963	0.325	0.552	0.963	0.390	0.552	0.964	0.449	0.554
	0.40	0.877	0.084	0.271	0.878	0.138	0.272	0.880	0.189	0.274
2.0	0.80	0.615	−0.039	0.090	0.619	0.001	0.091	0.627	0.044	0.093
	1.20	0.436	−0.036	0.058	0.441	0.000	0.057	0.455	0.046	0.057
	1.60	0.334	−0.007	0.054	0.340	0.036	0.050	0.356	0.100	0.046
	2.00	0.271	0.048	0.052	0.269	0.115	0.038	0.277	0.227	0.020
	0.00	1.000	0.691	—	1.000	0.772	—	1.000	0.848	—
	0.10	0.982	0.508	0.751	0.982	0.580	0.752	0.982	0.675	0.752
	0120	0.961	0.350	0.546	0.962	0.418	0.546	0.962	0.482	0.547
	0.40	0.872	0.102	0.261	0.872	0.156	0.261	0.873	0.207	0.263
	0.80	0.598	−0.032	0.073	0.599	0.002	0.073	0.602	0.036	0.074
3.0	1.20	0.403	−0.040	0.033	0.405	−0.016	0.033	0.409	0.010	0.035
	1.60	0.286	−0.030	0.024	0.289	−0.001	0.024	0.295	0.012	0.025
	2.00	0.217	−0.019	0.023	0.220	0.000	0.022	0.229	0.025	0.023
	2.50	0.168	−0.002	0.024	0.171	0.020	0.022	0.180	0.055	0.020
	3.00	0.137	0.024	0.024	0.137	0.059	0.017	0.142	0.116	0.008
	0.00	1.000	0.710	—	1.000	0.795	—	1.000	0.879	—
	0.10	0.981	0.525	0.750	0.981	0.605	0.750	0.981	0.702	0.750
	0.20	0.961	0.366	0.544	0.961	0.436	0.544	0.961	0.505	0.544
	0.40	0.870	0.114	0.257	0.870	0.170	0.257	0.870	0.224	0.258
	0.80	0.594	−0.024	0.066	0.594	0.009	0.066	0.594	0.043	0.066
	1.20	0.393	−0.036	0.024	0.394	−0.014	0.024	0.395	0.007	0.025
5.0	1.60	0.270	−0.030	0.012	0.271	−0.015	0.013	0.272	0.000	0.013
	2.00	0.195	−0.023	0.009	0.195	−0.012	0.009	0.197	0.000	0.010
	2.50	0.138	−0.017	0.008	0.139	−0.008	0.008	0.141	0.002	0.008
	3.00	0.104	−0.001	0.008	0.105	−0.004	0.008	0.108	0.006	0.008
	3.50	0.083	−0.007	0.008	0.085	0.000	0.008	0.088	0.011	0.008
	4.00	0.070	−0.003	0.009	0.071	0.005	0.008	0.075	0.018	0.008
	4.50	0.060	0.002	0.009	0.062	0.012	0.008	0.065	0.029	0.007
	5.00	0.053	0.009	0.009	0.053	0.023	0.007	0.056	0.045	0.004

Table 3. Influence values I_{zv}, I_{xv} and I_{xzh}

$(L/B) = 5.00$ Point A

H/B	z/B	$\mu = 0.15$			$\mu = 0.30$			$\mu = 0.45$		
		I_{zv}	I_{xv}	I_{xzh}	I_{zv}	I_{xv}	I_{xzh}	I_{zv}	I_{xv}	I_{xzh}
	0.00	1.000	0.525	—	1.000	0.566	—	1.000	0.592	—
	0.10	0.996	0.365	0.796	0.996	0.425	0.796	0.996	0.462	0.797
	0.20	0.980	0.252	0.610	0.981	0.303	0.608	0.983	0.358	0.609
1.0	0.40	0.920	0.050	0.367	0.922	0.117	0.360	0.930	0.200	0.358
	0.60	0.830	−0.005	0.271	0.832	0.081	0.257	0.843	0.199	0.246
	0.80	0.753	0.022	0.239	0.751	0.138	0.212	0.760	0.307	0.183
	1.00	0.688	0.121	0.221	0.672	0.288	0.171	0.665	0.544	0.108
	0.00	1.000	0.715	—	1.000	0.745	—	1.000	0.760	—
	0.10	0.990	0.505	0.772	0.990	0.565	0.772	0.990	0.590	0.773
	0.20	0.971	0.362	0.565	0.971	0.390	0.566	0.972	0.410	0.568
	0.40	0.889	0.104	0.290	0.890	0.135	0.290	0.893	0.163	0.293
2.0	0.80	0.667	−0.040	0.119	0.670	−0.004	0.118	0.677	0.039	0.120
	1.20	0.524	−0.043	0.092	0.528	0.002	0.088	0.539	0.063	0.088
	1.60	0.441	−0.008	0.092	0.443	0.054	0.082	0.455	0.144	0.072
	2.00	0.385	0.068	0.088	0.377	0.162	0.064	0.379	0.310	0.032
	0.00	1.000	0.778	—	1.000	0.812	—	1.000	0.836	—
	0.10	0.990	0.541	0.767	0.990	0.610	0.767	0.990	0.649	0.768
	0.20	0.969	0.408	0.556	0.969	0.436	0.556	0.970	0.458	0.557
	0.40	0.884	0.137	0.272	0.884	0.165	0.273	0.885	0.188	0.274
	0.80	0.649	−0.027	0.089	0.650	0.000	0.089	0.653	0.026	0.091
3.0	1.20	0.489	−0.049	0.052	0.492	−0.022	0.052	0.498	0.008	0.054
	1.60	0.391	−0.043	0.045	0.395	−0.015	0.044	0.404	0.020	0.046
	2.00	0.329	−0.029	0.046	0.333	0.003	0.044	0.344	0.046	0.043
	2.50	0.278	−0.002	0.049	0.281	0.038	0.043	0.292	0.099	0.038
	3.00	0.241	0.042	0.047	0.238	0.102	0.034	0.242	0.198	0.016
	0.00	1.000	0.819	—	1.000	0.859	—	1.000	0.895	—
	0.10	0.990	0.603	0.765	0.990	0.642	0.765	0.990	0.703	0.765
	0.20	0.969	0.442	0.511	0.969	0.473	0.551	0.969	0.503	0.552
	0.40	0.881	0.164	0.264	0.881	0.194	0.264	0.882	0.221	0.265
	0.80	0.641	−0.010	0.075	0.641	0.015	0.075	0.642	0.039	0.076
	1.20	0.474	−0.040	0.033	0.475	−0.018	0.033	0.476	0.003	0.034
5.0	1.60	0.367	−0.041	0.021	0.368	−0.023	0.021	0.370	−0.003	0.023
	2.00	0.294	−0.037	0.018	0.296	−0.020	0.018	0.299	−0.001	0.019
	2.50	0.233	−0.029	0.017	0.235	−0.014	0.017	0.239	0.005	0.018
	3.00	0.191	−0.022	0.018	0.193	−0.007	0.018	0.199	0.013	0.019
	3.50	0.162	−0.014	0.019	0.165	0.002	0.019	0.172	0.024	0.019
	4.00	0.141	−0.006	0.021	0.144	0.012	0.019	0.152	0.040	0.018
	4.50	0.126	0.005	0.022	0.128	0.027	0.019	0.135	0.062	0.016
	5.00	0.113	0.020	0.021	0.113	0.048	0.015	0.117	0.096	0.008

Table 4. Influence values I_{zv}, I_{xv} and I_{xzh}

$(L/B) = 1.00$ Point C

H/B	z/B	$\mu = 0.15$			$\mu = 0.30$			$\mu = 0.45$		
		I_{zv}	I_{xv}	I_{xzh}	I_{zv}	I_{xv}	I_{xzh}	I_{zv}	I_{xv}	I_{xzh}
1.0	0.00	0.250	0.083	0.000	0.250	0.112	0.000	0.250	0.134	0.000
	0.20	0.250	0.061	0.121	0.250	0.921	0.119	0.250	0.122	0.114
	0.40	0.250	0.039	0.105	0.250	0.072	0.103	0.250	0.110	0.098
	0.60	0.250	0.028	0.079	0.250	0.064	0.079	0.250	0.112	0.075
	0.80	0.241	0.028	0.059	0.238	0.071	0.064	0.239	0.133	0.070
	1.00	0.227	0.040	0.056	0.220	0.094	0.073	0.215	0.176	0.096
2.0	0.00	0.250	0.131	0.000	0.250	0.163	0.000	0.250	0.190	0.000
	0.20	0.250	0.089	0.136	0.250	0.117	0.136	0.250	0.142	0.135
	0.40	0.243	0.052	0.128	0.244	0.076	0.127	0.245	0.098	0.126
	0.80	0.210	0.010	0.080	0.211	0.028	0.079	0.214	0.048	0.077
	1.20	0.170	0.001	0.044	0.172	0.016	0.043	0.178	0.037	0.042
	1.60	0.141	0.003	0.024	0.142	0.023	0.025	0.149	0.053	0.025
	2.00	0.118	0.021	0.019	0.117	0.050	0.024	0.120	0.098	0.030
3.0	0.00	0.250	0.146	0.000	0.250	0.181	0.000	0.250	0.213	0.000
	0.20	0.249	0.100	0.138	0.249	0.129	0.138	0.249	0.156	0.137
	0.40	0.241	0.060	0.131	0.241	0.083	0.131	0.242	0.106	0.130
	0.80	0.203	−0.013	0.084	0.203	0.028	0.084	0.204	0.044	0.084
	1.20	0.157	−0.003	0.049	0.158	0.008	0.049	0.160	0.020	0.048
	1.60	0.121	−0.007	0.028	0.122	0.003	0.028	0.125	0.014	0.027
	2.00	0.096	−0.005	0.017	0.098	0.004	0.016	0.102	0.016	0.016
	2.50	0.077	0.001	0.009	0.078	0.012	0.009	0.083	0.028	0.010
	3.00	0.064	0.011	0.007	0.064	0.027	0.009	0.066	0.054	0.011
5.0	0.00	0.250	0.156	0.000	0.250	0.192	0.000	0.250	0.227	0.000
	0.20	0.249	0.107	0.138	0.249	0.137	0.138	0.249	0.167	0.138
	0.40	0.241	0.066	0.132	0.241	0.090	0.132	0.240	0.113	0.131
	0.80	0.200	0.016	0.086	0.200	0.032	0.086	0.201	0.047	0.086
	1.20	0.152	−0.001	0.051	0.153	0.009	0.051	0.153	0.019	0.051
	1.60	0.114	−0.006	0.030	0.114	0.001	0.030	0.115	0.008	0.030
	2.00	0.086	−0.007	0.019	0.087	−0.002	0.019	0.087	0.004	0.018
	2.50	0.063	−0.006	0.011	0.064	−0.002	0.011	0.065	0.003	0.011
	3.00	0.049	−0.005	0.007	0.050	−0.001	0.006	0.051	0.004	0.006
	3.50	0.040	−0.003	0.004	0.040	0.001	0.004	0.042	0.006	0.004
	4.00	0.034	−0.001	0.003	0.034	0.003	0.003	0.036	0.009	0.003
	4.50	0.029	01001	0.002	0.030	0.006	0.002	0.032	0.014	0.002
	5.00	0.026	0.005	0.002	0.026	0.011	0.002	0.027	0.022	0.003

Table 5. Influence values I_{zv}, I_{xv} and I_{xzv}

$(L/B) = 2.00$ Point C

H/B	z/B	$\mu = 0.15$			$\mu = 0.30$			$\mu = 0.45$		
		I_{zv}	I_{xv}	I_{xzv}	I_{zv}	I_{xv}	I_{xzv}	I_{zv}	I_{xv}	I_{xzv}
	0.00	0.250	0.085	0.000	0.250	0.108	0.000	0.250	0.129	0.000
	0.20	0.250	0.062	0.119	0.250	0.089	0.118	0.250	0.118	0.113
1.0	0.40	0.250	0.041	0.104	0.250	0.072	0.103	0.250	0.112	0.097
	0.60	0.250	0.029	0.079	0.250	0.067	0.081	0.250	0.118	0.078
	0.80	0.248	0.030	0.061	0.244	0.076	0.069	0.240	0.141	0.075
	1.00	0.241	0.042	0.061	0.232	0.099	0.080	0.223	0.183	0.107
	0.00	0.250	0.147	0.000	0.250	0.164	0.000	0.250	0.176	0.000
	0.20	0.250	0.100	0.136	0.250	0.116	0.136	0.250	0.129	0.135
	0.40	0.248	0.059	0.130	0.249	0.076	0.129	0.250	0.092	0.127
2.0	0.80	0.230	0.012	0.086	0.231	0.030	0.085	0.234	0.051	0.082
	1.20	0.205	−0.002	0.051	0.207	0.020	0.051	0.212	0.048	0.049
	1.60	0.183	0.005	0.031	0.183	0.033	0.032	0.188	0.072	0.033
	2.00	0.163	0.029	0.026	0.160	0.068	0.033	0.160	0.131	0.042
	0.00	0.250	0.172	0.000	0.250	0.192	0.000	0.250	0.207	0.000
	0.20	0.250	0.118	0.139	0.250	0.135	0.139	0.250	0.149	0.139
	0.40	0.246	0.072	0.134	0.246	0.088	0.134	0.246	0.102	0.133
	0.80	0.222	0.017	0.093	0.222	0.031	0.092	0.224	0.045	0.091
3.0	1.20	0.190	−0.004	0.059	0.191	0.009	0.058	0.194	0.024	0.057
	1.60	0.162	−0.009	0.037	0.163	01004	0.036	0.167	0.020	0.035
	2.00	0.139	−0.007	0.023	0.141	0.007	0.023	0.146	0.026	0.022
	2.50	0.119	0.001	01014	0.120	0.019	0.014	0.125	0.046	0.015
	3.00	0.103	0.018	0.012	0.102	0.044	0.014	0.104	0.085	0.019
	0.00	0.250	0.189	0.000	0.250	0.211	0.000	0.250	0.232	0.000
	0.20	0.250	0.131	0.140	0.250	0.150	0.140	0.250	0.168	0.140
	0.40	0.245	0.083	0.135	0.245	0.100	0.135	0.245	0.116	0.135
	0.80	0.218	0.023	0.095	0.219	0.037	0.095	0.219	0.050	0.095
	1.20	0.183	−0.001	0.062	0.184	0.010	0.062	0.184	0.021	0.061
	1.60	0.151	−0.009	0.040	0.151	0.000	0.040	0.152	0.010	0.040
5.0	2.00	0.124	−0.011	0.027	0.125	−0.003	0.027	0.126	0.006	0.026
	2.50	0.099	−0.010	0.017	0.100	−0.003	0.016	0.102	0.005	0.016
	3.00	0.081	−0.008	0.011	0.082	−0.001	0.011	0.085	0.007	0.010
	3.50	0.068	−0.005	0.007	0.069	0.002	0.007	0.073	0.011	0.007
	4.00	0.059	−0.002	0.005	0.060	0.006	0.005	0.064	0.017	0.005
	4.50	0.053	0.002	0.004	0.053	0.011	0.004	0.057	0.026	0.004
	5.00	0.047	0.008	0.003	0.047	0.020	0.004	0.049	0.040	0.005

Table 6. Influence value I_{zv}, I_{xv} and I_{xzv}

$(L/B) = 5.00$ Point C

H/B	z/B	$\mu = 0.15$			$\mu = 0.30$			$\mu = 0.45$		
		I_{zv}	I_{xv}	I_{xzv}	I_{zv}	I_{xv}	I_{xzv}	I_{zv}	I_{xv}	I_{xzv}
1.0	0.00	0.250	0.082	0.000	0.250	0.103	0.000	0.250	0.124	0.000
	0.20	0.250	0.062	0.118	0.250	0.088	0.117	0.250	0.120	0.113
	0.40	0.250	0.041	0.103	0.250	0.072	0.103	0.250	0.114	0.098
	0.60	0.250	0.029	0.079	0.250	0.067	0.081	0.250	0.120	0.080
	0.80	0.247	0.030	0.061	0.244	0.076	0.069	0.242	0.143	0.077
	1.00	0.239	0.042	0.060	0.233	0.100	0.081	0.226	0.185	0.109
2.0	0.00	0.250	0.146	0.000	0.250	0.154	0.000	0.250	0.158	0.000
	0.20	0.250	0.101	0.135	0.250	0.112	0.134	0.250	0.121	0.133
	0.40	0.247	0.061	0.120	0.247	0.073	0.138	0.248	0.087	0.126
	0.80	0.230	0.012	0.086	0.230	0.029	0.085	0.232	0.050	0.082
	1.20	0.207	−0.001	0.052	0.208	0.020	0.052	0.211	0.050	0.050
	1.60	0.188	0.006	0.032	0.188	0.035	0.034	0.190	0.077	0.036
	2.00	0.172	0.030	0.028	0.168	0.072	0.035	0.166	0.136	0.046
3.0	0.00	0.250	0.177	0.000	0.250	0.182	0.000	0.250	0.184	0.000
	0.20	0.249	0.124	0.138	0.249	0.131	0.137	0.249	0.136	0.137
	0.40	0.245	0.077	0.133	0.246	0.086	0.133	0.246	0.093	0.132
	0.80	0.224	0.019	0.093	0.225	0.029	0.092	0.223	0.040	0.091
	1.20	0.197	−0.004	0.060	0.197	0.008	0.059	0.199	0.022	0.058
	1.60	0.173	−0.010	0.038	0.174	0.004	0.038	0.176	0.022	0.037
	2.00	0.155	−0.008	0.025	0.156	0.009	0.025	0.159	0.032	0.025
	2.50	0.139	0.002	0.016	0.138	0.024	0.017	0.141	0.056	0.018
	3.00	0.126	0.022	0.014	0.123	0.053	0.018	0.123	0.100	0.023
5.0	0.00	0.250	0.203	0.000	0.250	0.209	0.000	0.250	0.214	0.000
	0.20	0.249	0.146	0.139	0.249	0.153	0.139	0.249	0.158	0.139
	0.40	0.245	0.095	0.135	0.245	0.102	0.135	0.245	0.109	0.135
	0.80	0.221	0.031	0.096	0.221	0.038	0.096	0.222	0.045	0.096
	1.20	0.191	0.002	0.064	0.191	0.010	0.064	0.192	0.018	0.063
	1.60	0.164	−0.009	0.043	0.164	−0.001	0.043	0.166	0.009	0.042
	2.00	0.142	−0.012	0.030	0.143	−0.004	0.030	0.145	0.006	0.029
	2.50	0.122	−0.012	0.020	0.123	−0.004	0.019	0.125	0.008	0.019
	3.00	0.107	−0.010	0.013	0.108	−0.001	0.013	0.111	0.012	0.013
	3.50	0.096	−0.007	0.009	0.097	0.004	0.009	0.101	0.019	0.009
	4.00	0.088	−0.002	0.007	0.089	0.010	0.007	0.092	0.028	0.007
	4.50	0.081	0.004	0.005	0.082	0.019	0.006	0.085	0.042	0.007
	5.00	0.075	0.013	0.005	0.075	0.032	0.007	0.076	0.062	0.008

$$\sigma_{zv} = p_v \cdot I_{zv} \quad (30)$$

$$\sigma_{xv} = p_v \cdot I_{xv} \quad (31)$$

$$\tau_{xzv} = p_v \cdot I_{xzv} \quad (32)$$

$$w_{Av} = \frac{p_v B}{E} \cdot I_{wAv} \quad (33)$$

$$w_{Cv} = \frac{p_v B}{E} \cdot I_{wCv} \quad (34)$$

$$u_{Cv} = \frac{p_v B}{E} \cdot I_{uCv} \quad (35)$$

$$v_{Cv} = \frac{p_v B}{E} \cdot I_{vCv} \quad (36)$$

$$\sigma_{zh} = p_h \cdot I_{zh} \quad (37)$$

$$\sigma_{xh} = p_h \cdot I_{xh} \quad (38)$$

$$\tau_{xzh} = p_h \cdot I_{xzh} \quad (39)$$

$$w_{Ah} = \frac{p_h \cdot B}{E} \cdot I_{wAh} \quad (40)$$

$$w_{Ch} = \frac{p_h \cdot B}{E} \cdot I_{wCh} \quad (41)$$

$$u_{Ah} = \frac{p_h \cdot B}{E} \cdot I_{uAh} \quad (42)$$

$$u_{Ch} = \frac{p_h \cdot B}{E} \cdot I_{uCh} \quad (43)$$

$$v_{Ch} = \frac{p_h \cdot B}{E} \cdot I_{vCh} \quad (44)$$

where I values are dimensionless coefficients for the stresses or displacements due to vertical or horizontal load.

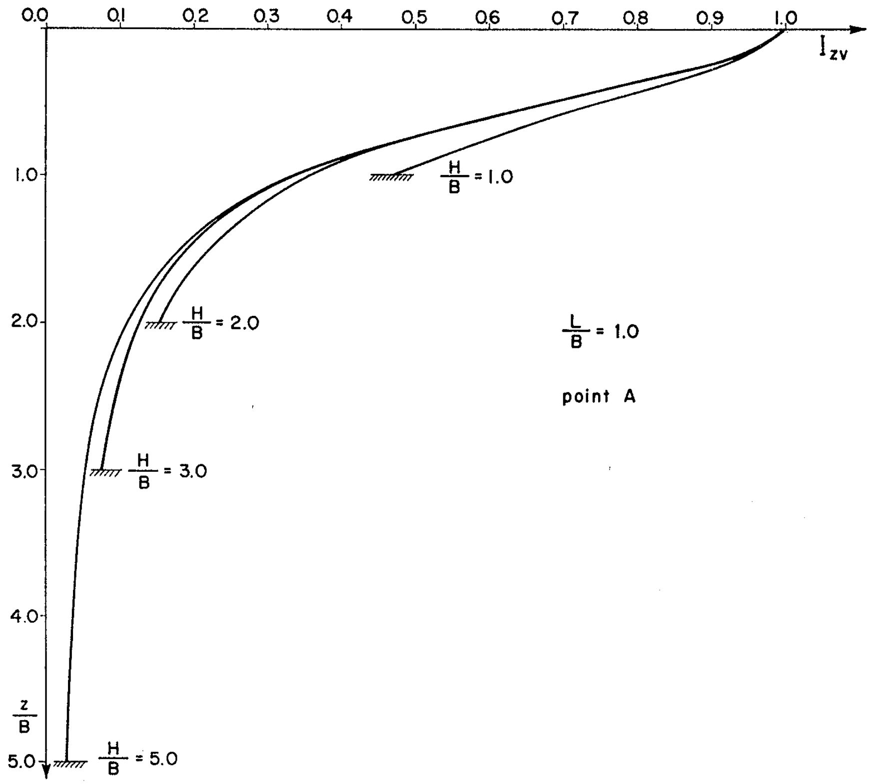

Fig. 3. Vertical stresses σ_z on the axis A_z; $(L/B) = 1.0$

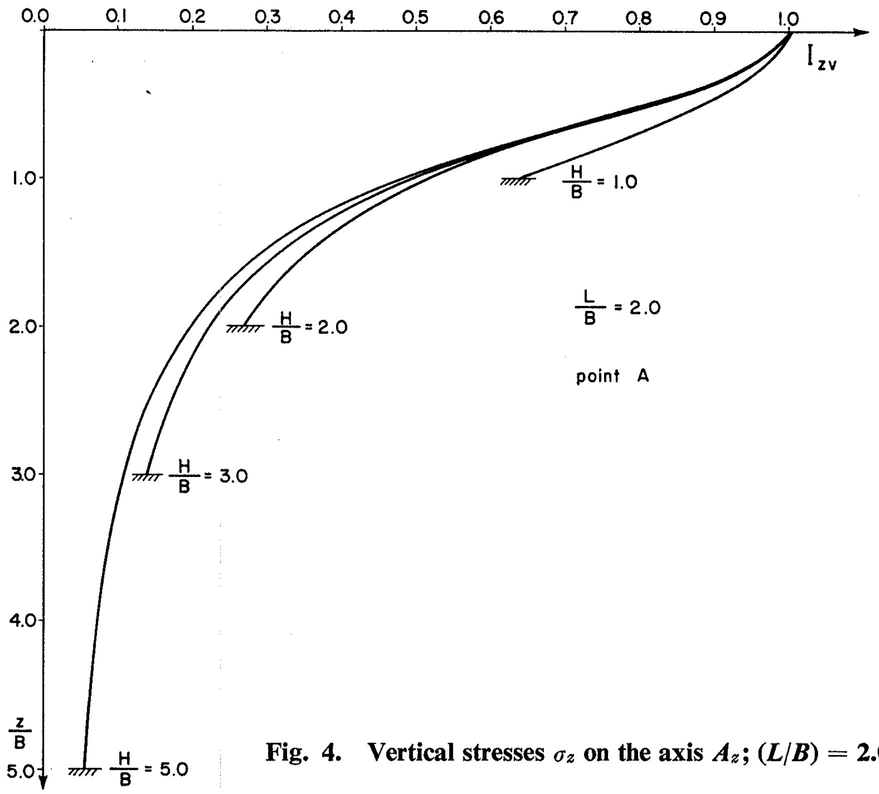

Fig. 4. Vertical stresses σ_z on the axis A_z; $(L/B) = 2.0$

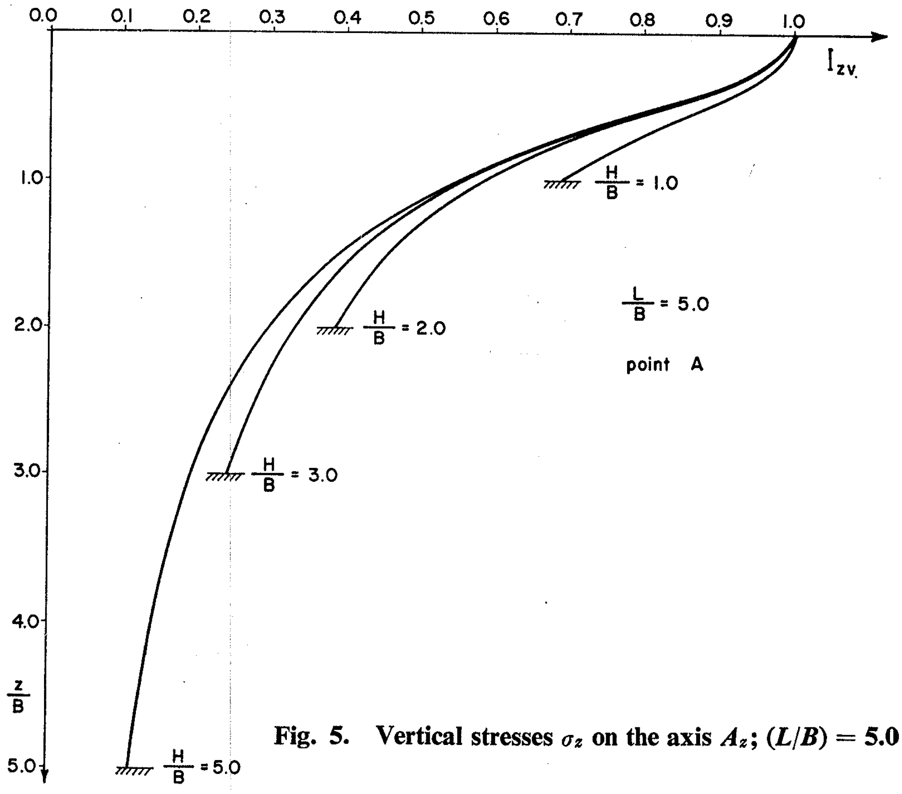

Fig. 5. Vertical stresses σ_z on the axis A_z; $(L/B) = 5.0$

The coefficients I for the stresses due to vertical load p_v are given in Tables 1–6. Some of the obtained results are presented graphically in Figs. 3–6. The coefficients for the displacements w, u and v are given in Table 7. The curves are shown in Figs. 7–9 for the Poisson's ratio $\mu = 0.30$.

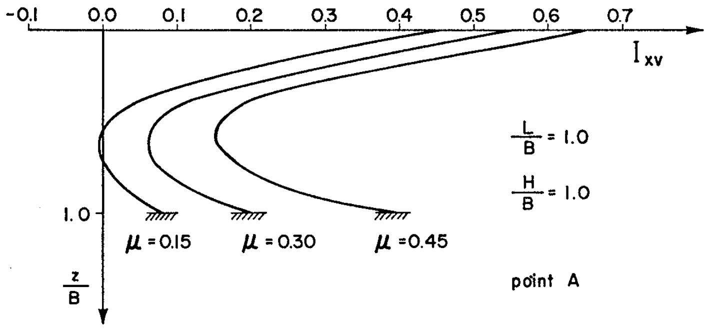

Fig. 6. Horizontal stresses σ_z on the axis A_z; $(L/B) = 1.0$ and $(H/B) = 1.0$

Table 7. Influence values I_{wAv}, I_{wCv}, I_{uCv} and I_{vCv}

L/B	H/B	$\mu = 0.15$				$\mu = 0.30$				$\mu = 0.45$			
		I_{wAv}	I_{wCv}	I_{uCv}	I_{vCv}	I_{wAv}	I_{wCv}	I_{uCv}	I_{vCv}	I_{wAv}	I_{wCv}	I_{uCv}	I_{vCv}
1.0	0.50	0.45	0.12	0.05	0.05	0.38	0.10	0.03	0.03	0.25	0.05	0.00	0.00
	1.00	0.71	0.23	0.09	0.09	0.63	0.20	0.05	0.05	0.50	0.13	0.00	0.00
	2.00	0.88	0.36	0.12	0.12	0.81	0.32	0.07	0.07	0.68	0.26	0.00	0.00
	3.00	0.95	0.42	0.13	0.13	0.87	0.38	0.08	0.08	0.74	0.31	0.00	0.00
	5.00	1.00	0.47	0.13	0.13	0.92	0.43	0.08	0.08	0.80	0.36	0.00	0.00
2.0	0.50	0.46	0.12	0.05	0.05	0.38	0.10	0.03	0.03	0.23	0.04	0.00	0.00
	1.00	0.81	0.24	0.10	0.10	0.71	0.20	0.05	0.05	0.53	0.12	0.00	0.00
	2.00	1.11	0.41	0.14	0.16	1.00	0.36	0.08	0.09	0.82	0.27	0.00	0.00
	3.00	1.23	0.51	0.15	0.19	1.12	0.45	0.09	0.11	0.94	0.36	0.00	0.00
	5.00	1.33	0.60	0.16	0.20	1.23	0.54	0.10	0.13	1.05	0.45	0.00	0.00
5.0	0.50	0.47	0.12	0.05	0.05	0.38	0.09	0.03	0.03	0.22	0.04	0.00	0.00
	1.00	0.82	0.24	0.10	0.11	0.70	0.19	0.05	0.06	0.49	0.11	0.00	0.00
	2.00	1.23	0.41	0.14	0.18	1.09	0.35	0.08	0.08	0.84	0.25	0.00	0.00
	3.00	1.45	0.53	0.15	0.23	1.31	0.46	0.09	0.12	1.06	0.35	0.01	0.01
	5.00	1.67	0.68	0.17	0.28	1.52	0.61	0.10	0.16	1.27	0.48	0.02	0.02

The coefficients I for the stresses due to horizontal load p_h are given in Tables 8–10. (The values of I_{xzh} on the axis Az are shown in Tables 1–3).
Some of the obtained results are also presented in Figs. 10–12. The coefficients for the displacements u, v and w are given in Table 11. The curves are shown in Figs. 13–15 for the Poisson's ratio $\mu = 0.30$.

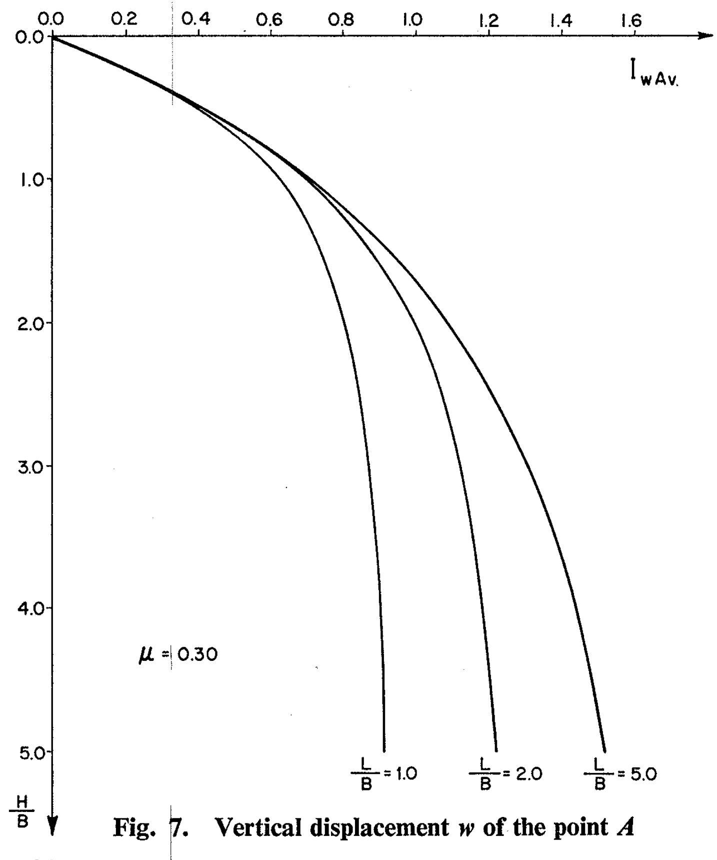

Fig. 7. Vertical displacement w of the point A

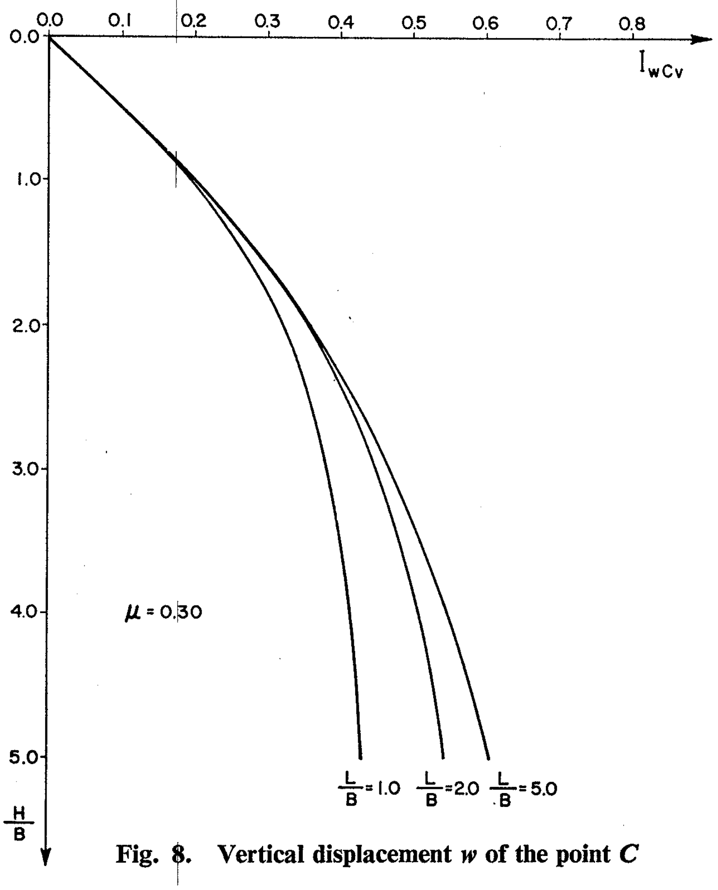

Fig. 8. Vertical displacement w of the point C

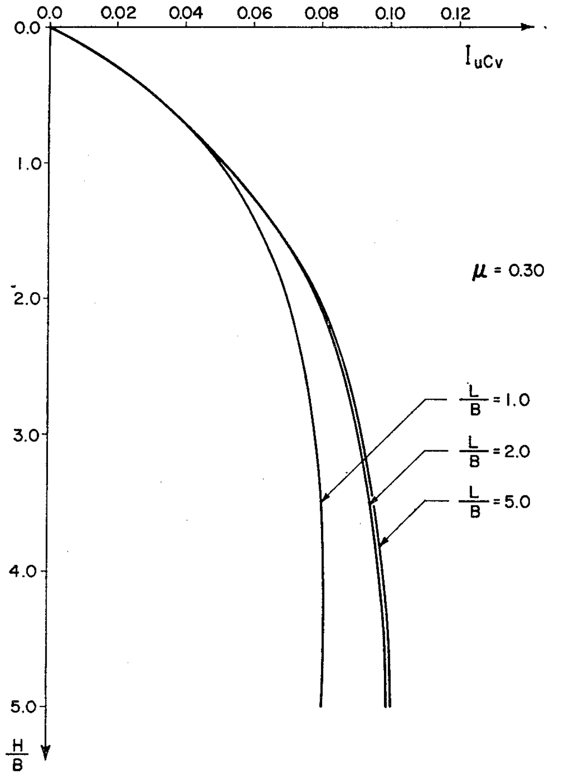

Fig. 9. Horizontal displacement *u* of the point *C*

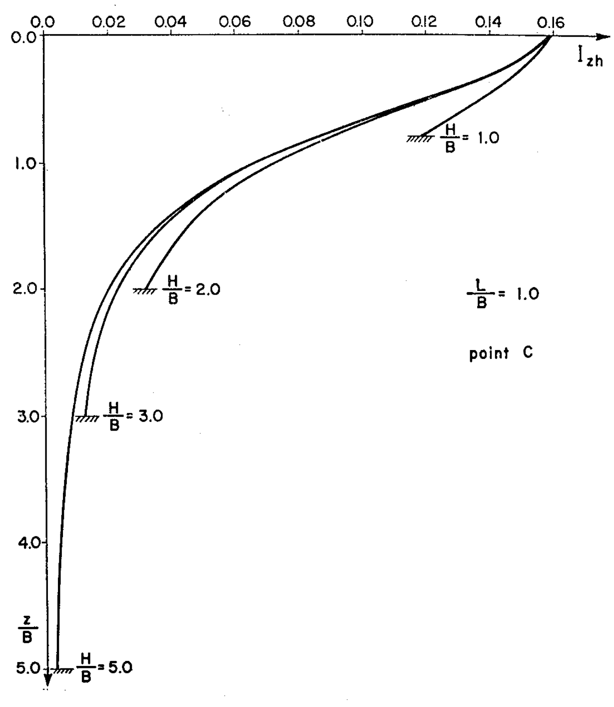

Fig. 10. Vertical stresses σ_z on the axis C_z; $(L/B) = 1.0$

Table 8. Influence values I_{zh}, I_{xh} and I_{xzh}

$(L/B) = 1.00$ Point C

H/B	z/B	$\mu = 0.15$			$\mu = 0.30$			$\mu = 0.45$		
		I_{zh}	I_{xh}	I_{xzh}	I_{zh}	I_{xh}	I_{xzh}	I_{zh}	I_{xh}	I_{xzh}
1.0	0.10	0.157	0.496	0.223	0.157	0.507	0.223	0.157	0.515	0.096
	0.20	0.152	0.303	0.200	0.151	0.313	0.200	0.151	0.321	0.104
	0.40	0.142	0.126	0.157	0.143	0.136	0.155	0.146	0.146	0.076
	0.60	0.130	0.053	0.126	0.131	0.064	0.123	0.135	0.077	0.051
	0.80	0.118	0.022	0.108	0.118	0.038	0.102	0.123	0.059	0.037
	1.00	0.108	0.019	0.100	0.105	0.045	0.090	0.105	0.086	0.044
2.0	0.10	0.155	0.527	0.217	0.155	0.537	0.217	0.155	0.547	0.217
	0120	0.150	0.328	0.188	0.150	0.337	0.189	0.150	0.346	0.189
	0.40	0.133	0.146	0.135	0.133	0.153	0.135	0.133	0.159	0.136
	0.80	0.089	0.033	0.069	0.089	0.037	0.069	0.090	0.041	0.069
	1.20	0.057	0.007	0.042	0.058	0.010	0.041	0.060	0.013	0.041
	1.60	0.040	0.001	0.033	0.041	0.005	0.031	0.044	0.010	0.028
	2.00	0.031	0.006	0.029	0.031	0.013	0.024	0.032	0.027	0.017
3.0	0.10	0.155	0.531	0.216	0.155	0.543	0.216	0.155	0.553	0.216
	0.20	0.150	0.332	0.186	0.150	0.342	0.186	0.150	0.351	0.186
	0.40	0.132	0.150	0.131	0.132	0.156	0.130	0.132	0.163	0.131
	0.80	0.087	0.036	0.061	0.087	0.039	0.061	0.087	0.042	0.062
	1.20	0.052	0.009	0.031	0.052	0.011	0.031	0.053	0.013	0.032
	1.60	0.032	0.002	0.019	0.033	0.003	0.019	0.033	0.004	0.020
	2.00	0.022	0.000	0.015	0.022	0.001	0.015	0.023	0.002	0.015
	2.50	0.015	0.000	0.014	0.015	0.001	0.013	0.017	0.003	0.011
	3.00	0.012	0.002	0.013	0.012	0.005	0.010	0.012	0.010	0.006
5.0	0.10	0.155	0.533	0.215	0.155	0.545	0.215	0.155	0.556	0.215
	0.20	0.150	0.334	0.185	0.150	0.344	0.185	0.150	0.353	0.185
	0.40	0.132	0.151	0.129	0.132	0.158	0.129	0.132	0.164	0.129
	0.80	0.086	0.037	0.058	0.086	0.040	0.058	0.086	0.044	0.058
	1.20	0.051	0.010	0.027	0.051	0.012	0.027	0.051	0.013	0.027
	16.0	0.031	0.003	0.014	0.031	0.004	0.014	0.031	0.005	0.014
	2.00	0.019	0.000	0.009	0.019	0.001	0.009	0.020	0.002	0.009
	2.50	0.012	0.000	0.006	0.012	0.000	0.006	0.012	0.001	0.006
	3.00	0.008	0.000	0.005	0.008	0.000	0.005	0.008	0.000	0.005
	3.50	0.005	0.000	0.005	0.006	0.000	0.005	0.006	0.000	0.005
	4.00	0.004	0.000	0.005	0.004	0.000	0.005	0.005	0.001	0.004
	4.50	0.003	0.000	0.005	0.003	0.000	0.004	0.004	0.001	0.004
	5.00	0.003	0.000	0.005	0.003	0.001	0.004	0.003	0.002	0.002

Table 9. Influence values I_{zh}, I_{xh} and I_{xzh}

$(L/B)=2.00$ Point C

HB	zB	$\mu=0.15$			$\mu=0.30$			$\mu=0.45$		
		I_{zh}	I_{xh}	I_{xzh}	I_{zh}	I_{xh}	I_{xzh}	I_{zh}	I_{xh}	I_{xzh}
1.0	0.10	0.158	0.509	0.230	0.158	0.515	0.229	0.158	0.517	0.229
	0.20	0.155	0.312	0.210	0.155	0.318	0.208	0.155	0.323	0.207
	0.40	0.147	0.132	0.172	0.148	0.140	0.168	0.150	0.149	0.165
	0.60	0.139	0.056	0.145	0.139	0.068	0.139	0.142	0.082	0.132
	0.80	0.131	0.024	0.128	0.130	0.042	0.119	0.133	0.066	0.018
	1.00	0.125	0.022	0.121	0.120	0.051	0.106	0.118	0.096	0.089
2.0	0.10	0.156	0.552	0.222	0.156	0.556	0.224	0.156	0.558	0.223
	0.20	0.151	0.348	0.195	0.151	0.352	0.195	0.151	0.354	0.196
	0.40	0.137	0.160	0.145	0.137	0.163	0.145	0.138	0.165	0.146
	0.80	0.100	0.039	0.084	0.100	0.042	0.083	0.102	0.045	0.084
	1.20	0.071	0.008	0.058	0.072	0.011	0.057	0.074	0.016	0.056
	1.60	0.055	0.001	0.049	0.056	0.007	0.045	0.059	0.015	0.041
	2.00	0.045	0.008	0.045	0.045	0.019	0.036	0.046	0.037	0.025
3.0	0.10	0.156	0.560	0.220	0.156	0.564	0.220	0.156	0.568	0.221
	0.20	0.151	0.355	0.191	0.151	0.359	0.191	0.151	0.362	0.192
	0.40	0.136	0.166	0.138	0.136	0.169	0.139	0.136	0.172	0.139
	0.80	0.096	0.043	0.072	0.096	0.046	0.072	0.097	0.048	0.072
	1.20	0.064	0.012	0.042	0.064	0.013	0.042	0.065	0.015	0.042
	1.60	0.044	0.002	0.030	0.044	0.004	0.029	0.045	0.006	0.030
	2.00	0.032	0.000	0.025	0.032	0.001	0.024	0.034	0.003	0.0234
	2.50	0.024	0.000	0.023	0.024	0.002	0.021	0.026	0.006	0.019
	3.00	0.019	0.003	0.022	0.019	0.008	0.017	0.020	0.016	0.010
5.0	0.10	0.156	0.563	0.219	0.156	0.568	0.219	0.156	0.572	0.219
	0.20	0.151	0.358	0.189	0.151	0.362	0.189	0.151	0.366	0.189
	0.40	0.136	0.168	0.135	0.136	0.172	0.135	0.136	0.175	0.135
	0.80	0.096	0.045	0.066	0.096	0.048	0.066	0.096	0.050	0.066
	1.20	0.063	0.014	0.034	0.063	0.015	0.034	0.063	0.016	0.035
	1.60	0.041	0.004	0.020	0.041	0.005	0.020	0.042	0.006	0.021
	2.00	0.028	0.001	0.014	0.028	0.002	0.013	0.028	0.002	0.014
	2.50	0.018	0.000	0.010	0.018	0.000	0.010	0.019	0.001	0.011
	3.00	0.013	−0.001	0.009	0.013	0.000	0.009	0.013	0.000	0.009
	3.50	0.009	−0.001	0.009	0.010	0.000	0.009	0.010	0.000	0.009
	4.00	0.007	0.000	0.009	0.008	0.000	0.009	0.008	0.001	0.008
	4.50	0.006	0.000	0.009	0.006	0.001	0.008	0.007	0.002	0.007
	5.00	0.005	0.001	0.009	0.005	0.002	0.007	0.006	0.005	0.004

Table 10. Influence values I_{zh}, I_{xh} and I_{wzh}

$(L/B)=5.00$ Point C

H/B	z/B	$\mu=0.15$			$\mu=0.30$			$\mu=0.45$		
		I_{zh}	I_{xh}	I_{xzh}	I_{zh}	I_{xh}	I_{xzh}	I_{zh}	I_{xh}	I_{xzh}
	0.10	0.158	0.510	0.231	0.158	0.514	0.230	0.158	0.516	0.229
	0.20	0.155	0.312	0.211	0.155	0.318	0.209	0.155	0.322	0.207
1.0	0.40	0.148	0.132	0.174	0.148	0.140	0.169	0.150	0.148	0.164
	0.60	0.140	0.056	0.147	0.140	0.067	0.140	0.143	0.082	0.131
	0.80	0.133	0.024	0.131	0.132	0.042	0.120	0.135	0.067	0.106
	1.00	0.127	0.022	0.123	0.122	0.052	0.107	0.120	0.098	0.087
	0.10	0.156	0.555	0.224	0.156	0.556	0.224	0.156	0.555	0.224
	0.20	0.152	0.351	0.197	0.152	0.352	0.197	0.152	0.351	0.198
	0.40	0.138	0.162	0.150	0.138	0.163	0.149	0.138	0.164	0.150
2.0	0.80	0.102	0.039	0.091	0.102	0.042	0.090	0.103	0.044	0.089
	1.20	0.074	0.008	0.068	0.075	0.011	0.064	0.077	0.016	0.061
	1.60	0.059	0.001	0.060	0.060	0.007	0.053	0.063	0.016	0.046
	2.00	0.051	0.009	0.055	0.050	0.021	0.043	0.051	0.041	0.027
	0.10	0.156	0.567	0.221	0.156	0.567	0.222	0.156	0.567	0.222
	0.20	0.151	0.316	0.193	0.151	0.361	0.193	0.151	0.361	0.193
	0.40	0.137	0.170	0.142	0.137	0.171	0.142	0.137	0.171	0.143
	0.80	0.098	0.046	0.078	0.098	0.046	0.078	0.099	0.047	0.079
3.0	1.20	0.067	0.013	0.050	0.068	0.014	0.050	0.068	0.015	0.050
	1.60	0.048	0.002	0.039	0.049	0.004	0.038	0.050	0.005	0.038
	2.00	0.037	−0.001	0.036	0.038	0.001	0.034	0.039	0.004	0.032
	2.50	0.029	0.000	0.035	0.030	0.002	0.030	0.031	0.007	0.026
	3.00	0.025	0.004	0.033	0.025	0.011	0.025	0.025	0.021	0.014
	0.10	0.156	0.572	0.220	0.156	0.573	0.220	0.156	0.573	0.221
	0.20	0.151	0.366	0.190	0.151	0.367	0.191	0.151	0.367	0.191
	0.40	0.136	0.175	0.137	0.136	0.175	0.137	0.136	0.175	0.138
	0.80	0.097	0.049	0.069	0.097	0.050	0.070	0.097	0.050	0.070
	1.20	0.065	0.016	0.039	0.065	0.016	0.039	0.066	0.016	0.040
	1.60	0.045	0.005	0.025	0.045	0.005	0.026	0.045	0.006	0.027
5.0	2.00	0.032	0.001	0.020	0.032	0.002	0.020	0.033	0.002	0.021
	2.50	0.023	0.000	0.017	0.023	0.000	0.017	0.023	0.000	0.018
	3.00	0.017	−0.001	0.017	0.017	0.000	0.016	0.018	0.000	0.017
	3.50	0.014	−0.001	0.017	0.014	0.000	0.016	0.014	01001	0.016
	4.00	0.011	−0.009	0.017	0.012	0.000	0.016	0.012	0.002	0.015
	4.50	0.010	0.000	0.017	0.010	01001	0.015	0.011	0.004	0.013
	5.00	0.009	0.002	0.017	0.009	0.004	0.013	0.009	0.007	0.008

Table 11. Influence values I_{uAh}, I_{uCh}, I_{wCh} and I_{vCh}

LB	HB	μ = 0.15				μ = 0.30				μ = 0.45			
		I_{uAh}	I_{uCh}	I_{wCh}	I_{vCh}	I_{uAh}	I_{uCh}	I_{wCh}	I_{vCh}	I_{uAh}	I_{uCh}	I_{wCh}	I_{vCh}
	0.50	0.53	0.20	0.05	0.04	0.56	0.21	0.03	0.06	0.58	0.23	0.01	0.09
	1.00	0.72	0.32	0.10	0.04	0.75	0.34	0.07	0.07	0.76	0.35	0.04	0.10
1.0	2.00	0.83	0.42	0.13	0.04	0.86	0.43	0.10	0.07	0.87	0.44	0.06	0.10
	3.00	0.86	0.45	0.13	0.04	0.90	0.47	0.10	0.07	0.91	0.48	0.07	0.10
	5.00	0.90	0.48	0.14	0.04	0.93	0.50	0.11	0.07	0.94	0.051	0.08	0.10
	0.50	0.60	0.20	0.05	0.04	0.63	0.22	0.03	0.07	0.63	0.23	0.01	0.10
	1.00	0.90	0.36	0.10	0.06	0.92	0.37	0.07	0.09	0.91	0.38	0.03	0.13
2.0	2.00	1.10	0.51	0.14	0.06	1.12	0.52	0.11	0.09	1.11	0.52	0.07	0.14
	3.00	1.18	0.58	0.15	0.06	1.20	0.59	0.12	0.09	1.19	0.58	0.08	0.14
	5.00	1.25	0.64	0.17	0.06	1.27	0.65	0.13	0.09	1.26	0.65	0.09	0.14
	0.50	0.62	0.20	0.05	0.04	0.63	0.22	0.03	0.07	0.63	0.23	0.01	0.10
	1.00	0.98	0.36	0.10	0.06	0.99	0.38	0.07	0.10	0.95	0.37	0.03	0.15
5.0	2.00	1.34	0.55	0.14	0.07	1.33	0.55	0.11	0.11	1.28	0.53	0.07	0.16
	3.00	1.50	0.66	0.16	0.07	1.50	0.66	0.12	0.11	1.43	0.64	0.08	0.17
	5.00	1.66	0.79	0.17	0.07	1.65	0.79	0.13	0.11	1.60	0.77	0.10	0.17

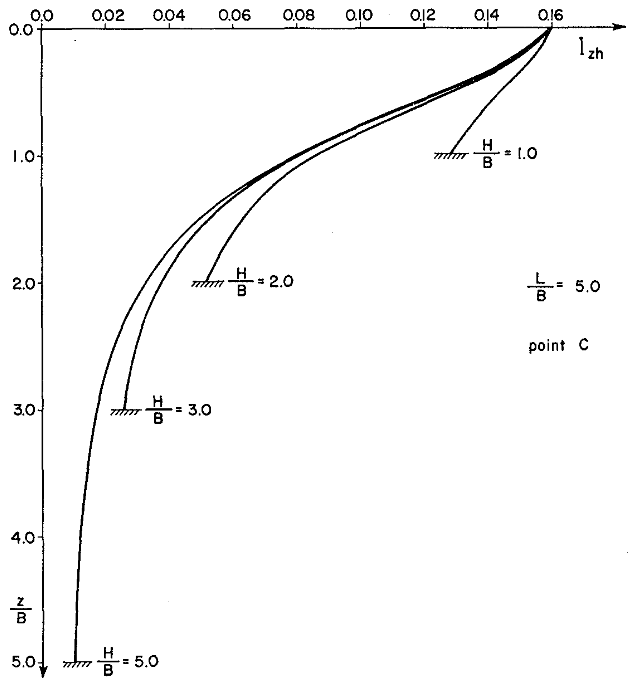

Fig. 11. Vertical stresses σ_2 on the axis C_z; $(L/B) = 2.0$

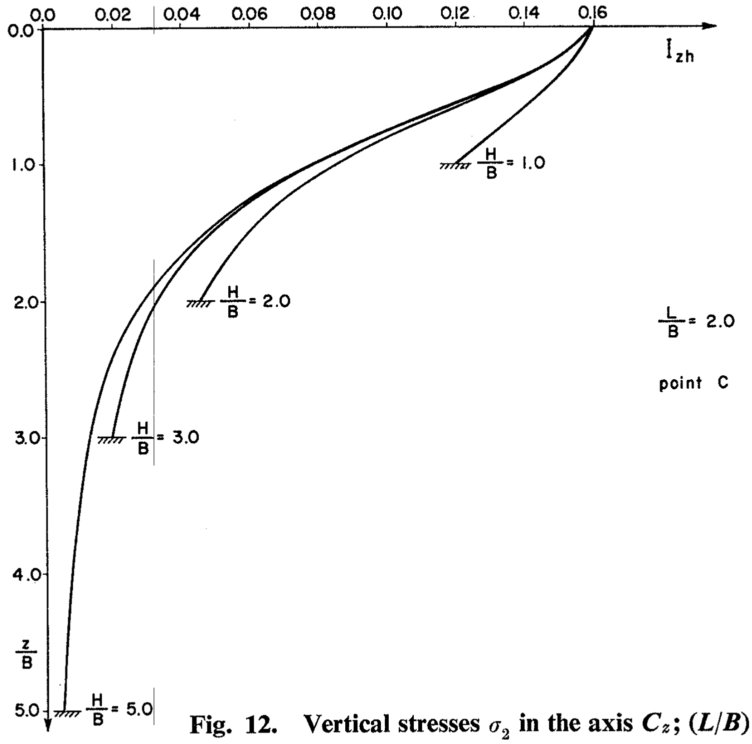

Fig. 12. Vertical stresses σ_2 in the axis C_z; $(L/B)=5.0$

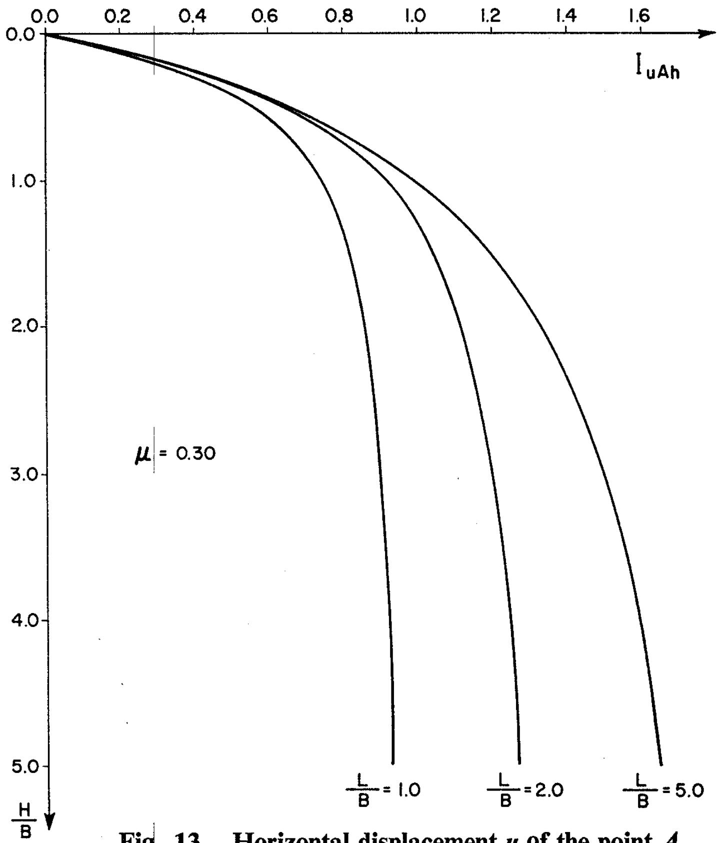

Fig. 13. Horizontal displacement *u* of the point *A*

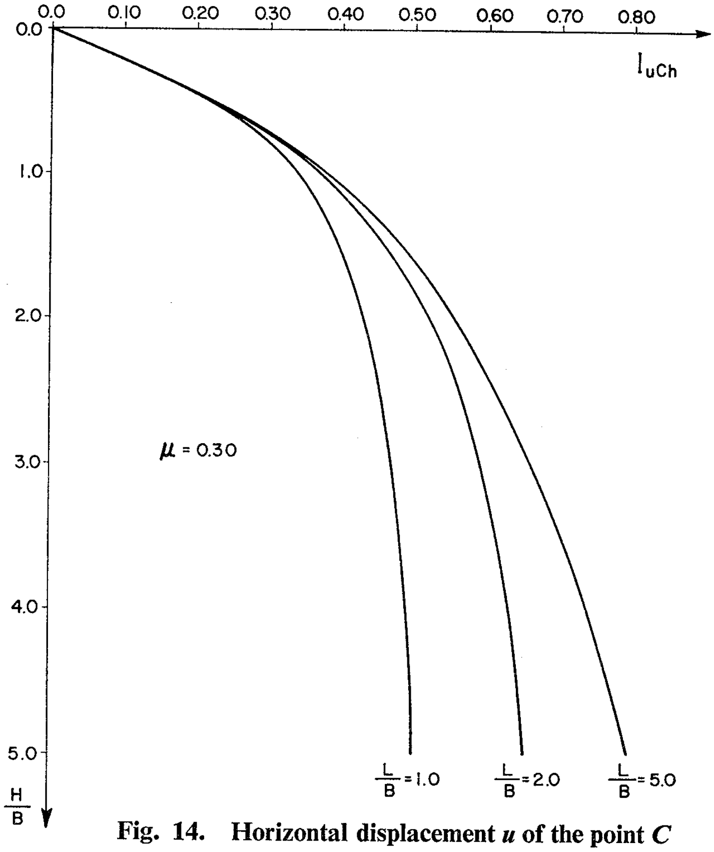

Fig. 14. Horizontal displacement *u* of the point *C*

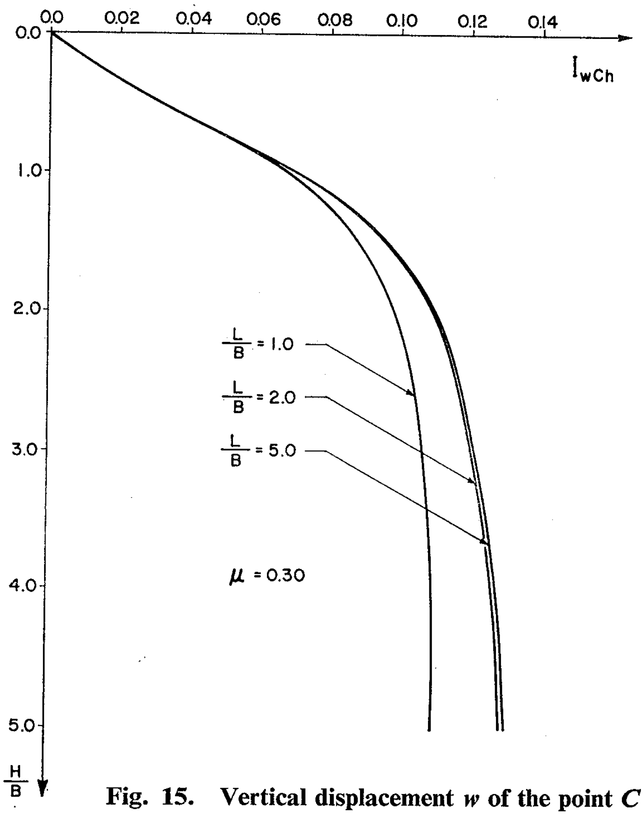

Fig. 15. Vertical displacement *w* of the point *C*

CONCLUSIONS

The dimensionless coefficients for the stresses and displacements produced by the vertical or horizontal rectangular load are calculated.

The tabulated values are easy to use and allow any engineer to calculate quickly the stresses and displacements in the layer of finite thickness.

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NOTATIONS

The following symbols are used in this paper:

B = Width of the rectangular foundation.
E = Young's modulus of elasticity.
H = Thickness of the compressible layer.
L = Length of the rectangular foundation.
p = Load.
p_h = Horizontal component of the load p
p_v = Vertical component of the load p
u = Horizontal displacement in the x direction.
u_{Ah} = Horizontal displacement of the point A in the x direction, due to horizontal load p_h
u_{Ch} = Horizontal displacement of the point C in the x direction, due to horizontal load p_h
u_{Cv} = Horizontal displacement of the point C in the x direction, due to vertical load p_v
v = Horizontal displacement in the y direction.
v_{Ch} = Horizontal displacement of the point C in the y direction, due to horizontal load p_h
v_{Cv} = Horizontal displacement of the point C in the y direction, due to vertical load p_v
w = Displacement in the z direction
w_{Ah} = Vertical displacement of the point A in the z direction, due to horizontal load p_h

w_{Av} = Vertical displacement of the point A in the z direction, due to vertical load p_v

w_{Ch} = Vertical displacement of the point C in the z direction, due to horizontal load p_h

w_{Cv} = Vertical displacement of the point C in the z direction, due to horizontal load p_h

w_{Cv} = Vertical displacement of the point C in the z direction, due to vertical load p_v

$\gamma_{zy}, \gamma_{zx}, \gamma_{xy}$ = Shearing strain components

$\varepsilon_x, \varepsilon_y, \varepsilon_z$ = Unit elongations in x, y and z directions.

δ = Angle between the vertical axis and the load p

μ = Poisson's ratio.

$\sigma_x, \sigma_y, \sigma_z$ = Normal stresses in x y and z directions.

σ_{xh} = Horizontal stress due to horizontal load p_h

σ_{xv} = Horizontal stress due to vertical load p_v

σ_{zh} = Vertical stress due to horizontal load p_h

σ_{zv} = Vertical stress due to vertical load p_v

$\tau_{zy}, \tau_{zx}, \tau_{xy}$ = Shearing stress components.

τ_{xzh} = Shear stress due to horizontal load p_h

τ_{xzv} = Shear stress due to vertical load p_v

APPENDIX

The use of the Tables is illustrated by the following example.

A rectangular foundation of width $B = 2.0$ m and length $L = 4.0$ m carries an inclined load $p = 2.0$ kg/cm^2. The angle of inclination is $\delta = 10°$. The thickness of a compressible layer is $H = 4.0$ m, for which the modulus of elasticity is $E = 100$ kg/cm^2 and Poisson's ratio $\mu = 0.30$.

Find the displacements w and u of points A and C (see Fig. 2).

Solution

The vertical component of the load is $p_v = p \cdot \cos\delta = 2.0 \cdot 0.986 = 1.972$ kg/cm^2. The horizontal component is $p_h = p \cdot \sin\delta = 2.0 \cdot 0.174 = 0.348$ kg/cm^2.

According to the equations (33), (34), and (35) the displacements w and u due to vertical component, p_v, are:

$$w_{Av} = \frac{P_v B}{E} \cdot I_{wAv}$$

$$w_{Cv} = \frac{P_v B}{E} \cdot I_{wCv}$$

$$u_{Cv} = \frac{P_v B}{E} \cdot I_{uCv}$$

For the ratio $(L/B) = (4.0/2.0) = 2.0$, $(H/B) = (4.0/2.0) = 2.0$ and $\mu = 0.30$, the Table 7 gives:

$I_{wAv} = 1.00$ (point A)

$I_{wCv} = 0.36$ (point C)

$I_{uCv} = 0.08$ (point C)

The displacements w and u are:

$$w_{Av} = \frac{1.972 \cdot 200}{100} \cdot 1.00 = 3.94 \text{ cm}$$

$$w_{Cv} = \frac{1.972 \cdot 200}{100} \cdot 0.36 = 1.42 \text{ cm}$$

$$u_{Cv} = \frac{1.972 \cdot 200}{100} \cdot 0.08 = 0.32 \text{ cm}$$

According to the equations (41) to (43), the displacements w and u due to horizontal component, p_h, are:

$$w_{Ch} = \frac{p_h \cdot B}{E} \cdot I_{wCh}$$

$$u_{Ah} = \frac{p_h \cdot B}{E} \cdot I_{uAh}$$

$$u_{Ch} = \frac{p_h \cdot B}{E} \cdot I_{uCh}$$

The Table 11 gives:

$I_{wCh} = 0.11$ (point C)

$I_{uAh} = 1.12$ (point A)

$I_{uCh} = 0.52$ (point C)

The displacements are:

$$w_{Ch} = \frac{0.348 \cdot 200}{100} \cdot 0.11 = 0.08 \text{ cm}$$

$$u_{Ah} = \frac{0.348 \cdot 200}{100} \cdot 1.12 = 0.78 \text{ cm}$$

$$u_{Ch} = \frac{0.348 \cdot 200}{100} \cdot 0.52 = 0.36 \text{ cm}$$

Consequently, the total displacements are:

$w_A = w_{Av} + w_{Ah} = 3.94 + 0 = 3.94$ cm (point A)

$w_C = w_{Cv} + w_{Ch} = 1.42 + 0.08 = 1.50$ cm (point C)

$u_A = u_{Av} + u_{Ah} = 0 + 0.78 = 0.78$ cm (point A)

$u_C = u_{Cv} + u_{Ch} = 0.32 + 0.36 = 0.68$ cm (point C)

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