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EXACT EQUIVALENT MODEL FOR A LATERALLY-LOADED LINEAR PILE-SOIL SYSTEM

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ABSTRACT

When using the substructure method for a pile-supported structure, it is common to adopt a simple element (equivalent model) to simulate the load-deflection behavior of a laterally loaded pile. Conventionally, two kinds of equivalent models, the uncoupled spring model and the equivalent cantilever model, are used to approximate the lateral pile-head response of a laterally loaded pile. These equivalent models can not work equally-well for different pile arrangements and loading conditions because the stiffness matrix (or flexibility matrix) of these equivalent models do not entirely match that of the original pile-soil model. The response obtained will never give correct displacements and forces simultaneously. To solve this problem, this study develops an exact equivalent model, in which an artificial lateral spring is added at the base of the cantilever to modify the fixed-base cantilever model so that it can completely represent the pile-head behavior of a laterally loaded pile-soil system. For verification, comparison studies between the proposed model and conventional equivalent models are conducted to show the effectiveness of the proposed model.

Key words: beam-spring Winkler model, equivalent cantilever model, equivalent model, laterally loaded pile, uncoupled spring model (**IGC**: E12/H1)

INTRODUCTION

In engineering practices, piles are often used as the foundations for buildings, bridges, and other structures. When modeling a structure with pile foundation, the substructure approach is usually adopted. A single pile with surrounding soils is first modeled as a substructure to deduce the stiffness matrix corresponding to the nodal degrees-of-freedom at the pile-head. This matrix can describe the complete force-displacement relationship of the pile-head response, and is usually called the pile-head stiffness. Then for the structural modeling, the pile-head stiffness can be added to the bottom of the structural model to represent the original pile. This technique can significantly simplify the modeling for a structure founded on a large number of piles, because each pile can be replaced by only a matrix.

The pile-head stiffness is a 6×6 matrix in general. When using the substructure technique for structural modeling, it is usual to replace each pile by a simple element to simplify the modeling. The vertical and torsional stiffness components in the pile-head stiffness matrix only have diagonal term (i.e., independent to others), and can be directly represented by equivalent spring, respectively. However, the lateral and rotational responses of a pile in two horizontal directions, respectively, are coupled with off-diagonal terms. For easy modeling in engineering applications, conventional analyses usually adopt two uncoupled springs to respectively represent the lateral and rotational stiffness of a pile. Since the influence of off-diagonal terms can not be disregarded theoretically, many researchers (e.g., Donovan, 1959; Gray, 1964; Francis, 1964; Nair et al., 1969; Poulos and Davis, 1980; Lam et al., 1998) have adopted various equivalent cantilever elements to model the coupled behavior of lateral and rotational responses of a pile. However, those equivalent cantilever models are not theoretically correct, and will present different degrees of error under different situations, such as different pile arrangements and loading conditions. To address this problem, this paper proposes a theoretically-equivalent cantilever element to completely describe the coupled behavior of lateral and rotational responses of a pile. Besides, case studies for comparing the proposed model and other conventional models are presented to investigate the applicability of all models.

WINKLER MODEL FOR SINGLE PILE-SOIL SYSTEM

The Winkler model (beam-spring model) is conventionally used in engineering practices to analyze the response of a laterally loaded pile. As shown in Fig. 1(a), the pile is modeled by beam elements and the surrounding soils are modeled by a series of independent lateral springs. Then the pile-head flexibility and stiffness matrixes

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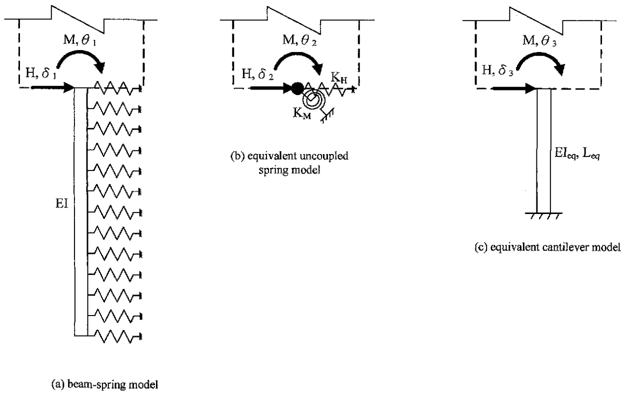

Fig. 1. Full beam-spring model and equivalent models for single pile

can be deduced and expressed as Eqs. (1) and (2), respectively,

$$\begin{bmatrix}\delta\\ \theta\end{bmatrix}=\begin{pmatrix}f_{xx} & f_{x\theta}\\ f_{x\theta} & f_{\theta\theta}\end{pmatrix}\begin{bmatrix}H\\ M\end{bmatrix} \tag{1}$$

$$\begin{bmatrix}H\\ M\end{bmatrix}=\begin{pmatrix}K_{xx} & K_{x\theta}\\ K_{x\theta} & K_{\theta\theta}\end{pmatrix}\begin{bmatrix}\delta\\ \theta\end{bmatrix} \tag{2}$$

in which H and M are the lateral load and moment applied at the pile head, respectively; δ and θ are the induced lateral displacement and rotation at the pile head, respectively; f_{xx}, $f_{\theta\theta}$ and $f_{x\theta}$ are the coefficients of the pile-head flexibility matrix; K_{xx}, $K_{\theta\theta}$ and $K_{x\theta}$ are the coefficients of the pile-head stiffness matrix.

For an infinitely long pile embedded in uniform soils, the pile-head flexibility and stiffness matrixes can be expressed as Eqs. (3) and (4), respectively (Chang, 1937):

$$\begin{bmatrix}\delta\\ \theta\end{bmatrix}=\begin{pmatrix}\dfrac{1}{2EI\beta^3} & \dfrac{1}{2EI\beta^2}\\ \dfrac{1}{2EI\beta^2} & \dfrac{1}{EI\beta}\end{pmatrix}\begin{bmatrix}H\\ M\end{bmatrix} \tag{3}$$

$$\begin{bmatrix}H\\ M\end{bmatrix}=\begin{pmatrix}4EI\beta^3 & -2EI\beta^2\\ -2EI\beta^2 & 2EI\beta\end{pmatrix}\begin{bmatrix}\delta\\ \theta\end{bmatrix} \tag{4}$$

where EI is the flexural rigidity of pile, and $\beta=\sqrt[4]{E_s/4EI}$ is the characteristic coefficient of the lateral pile-soil system, in which E_s denotes the horizontal subgrade reaction modulus and is equal to the product of the horizontal subgrade reaction coefficient k_s and the pile diameter D.

For a long pile embedded in Gibson soils with stiffness increasing linearly with depth, the pile-head flexibility and stiffness matrixes can be expressed as Eqs. (5) and (6), respectively (Poulos and Davis, 1980):

$$\begin{bmatrix}\delta\\ \theta\end{bmatrix}=\begin{bmatrix}\dfrac{2.4}{EI\eta^3} & \dfrac{1.6}{EI\eta^2}\\ \dfrac{1.6}{EI\eta^2} & \dfrac{1.74}{EI\eta}\end{bmatrix}\begin{bmatrix}H\\ M\end{bmatrix} \tag{5}$$

$$\begin{bmatrix}H\\ M\end{bmatrix}=\begin{pmatrix}1.077EI\eta^3 & -0.99EI\eta^2\\ -0.99EI\eta^2 & 1.485EI\eta\end{pmatrix}\begin{bmatrix}\delta\\ \theta\end{bmatrix} \tag{6}$$

in which $\eta=\sqrt[5]{n_h/EI}$ is the characteristic coefficient of the lateral pile-soil system in Gibson soils, where n_h denotes the variation of horizontal subgrade reaction modulus with depth.

CONVENTIONAL EQUIVALENT MODELS

For a lateral pile-soil system modeled by Winkler model, the pile-head stiffness deduced as above is a full 2 ×2 matrix with off-diagonal terms to correlate the coupled responses between the lateral displacement and rotation at the pile head. The full 2×2 matrix is not convenient to be incorporated into a conventional computer code. Therefore, it is very common to adopt a simpler element (i.e., the so-called equivalent model) to represent the original pile-head stiffness. Many equivalent models have been proposed in literature. The models commonly used in engineering practices are briefly described as follows:

Uncoupled Spring Model

The uncoupled spring model, as shown in Fig. 1(b), uses a set of uncoupled lateral and rotational springs to simulate the pile-head stiffness of a lateral pile-soil system, and neglects the coupling effects between the lateral displacement and rotation at the pile head. In engineering practices, the coefficients of lateral spring K_H and rotational spring K_M are taken directly from the diagonal terms of the single pile-head stiffness matrix of a lateral pile-soil system, i.e.,

$$\begin{bmatrix}K_H\\ K_M\end{bmatrix}=\begin{bmatrix}K_{xx}\\ K_{\theta\theta}\end{bmatrix} \tag{7}$$

Accordingly, the coefficients of K_H and K_M for a long pile embedded in uniform soils can be obtained from Eq. (4) and expressed as:

$$\begin{bmatrix}K_H\\ K_M\end{bmatrix}=\begin{bmatrix}4EI\beta^3\\ 2EI\beta\end{bmatrix} \tag{8}$$

Similarly, the coefficients of K_H and K_M for a pile embedded in Gibson soils can be obtained from Eq. (6) and expressed as:

$$\begin{bmatrix}K_H\\ K_M\end{bmatrix}=\begin{bmatrix}1.077EI\eta^3\\ 1.485EI\eta\end{bmatrix} \tag{9}$$

It is noted that the coupling coefficients shown in Eqs. (4) and (6) have negative values. Therefore, the uncoupled spring model will under-estimate the pile responses when subjected to a lateral load and moment at the pile head.

Equivalent Cantilever Models

The uncoupled spring model disregards the coupling effects between the translation and rotation at the pile head. In order to model the coupling effects, an equivalent cantilever is introduced to represent the lateral single

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pile-soil system, as shown in Fig. 1(c). This method was developed by Gray and utilized by Donovan (1959) to analyze pile groups in three dimensions. Francis (1964) used a similar method to analyze a group-pile foundation in two dimensions. Chai (2002) further applied this concept to develop a method for assessing the local ductility demand of a yielding pile-shaft when subjected to lateral loading.

The flexibility matrix corresponding to the top node of a cantilever when subjected to a lateral force and moment can be written as

$$\begin{bmatrix} \delta \\ \theta \end{bmatrix} = \begin{bmatrix} \dfrac{L^3}{3EI} & \dfrac{L^2}{2EI} \\ \dfrac{L^2}{2EI} & \dfrac{L}{EI} \end{bmatrix} \begin{bmatrix} H \\ M \end{bmatrix} \tag{10}$$

where L is the length of the cantilever.

Accordingly, the stiffness matrix corresponding to the top node of the cantilever is:

$$\begin{bmatrix} H \\ M \end{bmatrix} = \begin{bmatrix} \dfrac{12EI}{L^3} & -\dfrac{6EI}{L^2} \\ -\dfrac{6EI}{L^2} & \dfrac{4EI}{L} \end{bmatrix} \begin{bmatrix} \delta \\ \theta \end{bmatrix} \tag{11}$$

Although Eq. (10) or (11) has non-zero cross-coupling terms, the stiffness (or flexibility) matrix of a cantilever is not exactly the same as the pile-head stiffness (or flexibility) of the actual pile-soil system. In order to achieve some degree of equivalence, two parameters in the cantilever matrix, L and EI, can be adjusted to obtain the so-called equivalent cantilever model.

Nair's Model

Nair et al. (1969) suggested adopting a cantilever with an equivalent length L_{eq} in Eq. (10) to simulate the pile-head flexibility of a lateral single pile-soil system. Since only one parameter is used for equivalence, the simulation will be limited to fit some terms of the pile-head flexibility only. This formulation is actually an approximate equivalency. Different criteria will yield different equivalent length. Table 1 shows the equivalent lengths for the cases of a pile embedded in uniform soil and Gibson soil conditions, respectively. For example, to simulate a pile embedded in uniform soils, different loading conditions and different matching terms will yield $L_{eq}=1.0/\beta$ or $L_{eq}=1.142/\beta$. No unique equivalence can be achieved in this formulation. The applicability of either equivalence will depend on the loading pattern transmitted from the super-structure to the pile itself. It is not convenient for engineering analysis which usually consists of various loading combinations. For simplicity and practical application, Nair et al. suggested adopting $L_{eq}=1.0/\beta$ for complete embedment in uniform soils and $L_{eq}=1.85/\eta$ for complete embedment in Gibson soils.

Table 1. Equivalent length of cantilever model (By Nair et al., 1969)

Lateral support independent of depth (E_s = constant)			
$M=0,\ H\neq 0$		$M\neq 0,\ H=0$	
Equating slope of cantilever and pile at free head	Equating deflection of cantilever and pile at free head	Equating slope of cantilever and pile at free head	Equating deflection of cantilever and pile at free head
Complete embedment ($l=0$)			
$L_{eq}=\dfrac{1.0}{\beta}$	$L_{eq}=\dfrac{1.142}{\beta}$	$L_{eq}=\dfrac{1.0}{\beta}$	$L_{eq}=\dfrac{1.0}{\beta}$
Partial embedment ($l\neq 0$)			
$L_{eq}=\left(\dfrac{1+B}{B}\right)l$	$L_{eq}=\left(\dfrac{2B^3+6B^2+6B+3}{2B^3}\right)^{1/3}l$	$L_{eq}=\left(\dfrac{1+B}{B}\right)l$	$L_{eq}=\left(\dfrac{1+B}{B}\right)l$
Lateral support linearly increasing with depth			
$M=0,\ H\neq 0$		$M\neq 0,\ H=0$	
Equating slope of cantilever and pile at free head	Equating deflection of cantilever and pile at free head	Equating slope of cantilever and pile at free head	Equating deflection of cantilever and pile at free head
Complete embedment ($l=0$)			
$L_{eq}=\dfrac{1.712}{\eta}$	$L_{eq}=\dfrac{1.94}{\eta}$	$L_{eq}=\dfrac{1.752}{\eta}$	$L_{eq}=\dfrac{1.712}{\eta}$
Partial embedment ($l\neq 0$)			
$L_{eq}=[2(1.623G^2+1.75G+1/2)]^{1/3}l$	$L_{eq}=[3(2.435G^2+3.35G+1/3)]^{1/3}l$	$L_{eq}=(1.75G+1)l$	$L_{eq}=[2(1.623G^2+1.75G+1/2)]^{1/2}l$

Nair et al.'s note:

1. $B=\beta l$, $G=\eta l$ and l is free standing length.
2. These formulae are for long piles, that is, where $\beta L>2.5$ or $\eta L>2.0$, in which L is the pile length. Most practical cases fall within this category. For short piles, the algebra is more complicated, but the principle remains the same.

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Table 2. Parameters for equivalent cantilever model from Lam's model

Lateral support independent of depth (E_s = constant)			
Matching diagonal terms		Matching translational and cross-coupling terms	
$L_{eq}=\frac{1.224}{\beta}$	$EI_{eq}=0.612\,EI$	$L_{eq}=\frac{1.0}{\beta}$	$EI_{eq}=\frac{1}{3}EI$
Lateral support linearly increasing with depth			
Matching diagonal terms		Matching translational and cross-coupling terms	
$L_{eq}=\frac{2.034}{\beta}$	$EI_{eq}=0.755\,EI$	$L_{eq}=\frac{1.838}{\beta}$	$EI_{eq}=0.558\,EI$

Lam's Model

Alternatively, Lam et al. (1998) suggested adopting a cantilever with an equivalent flexural rigidity EI_{eq} and an equivalent length L_{eq} in Eq. (11) to simulate the pile-head stiffness of a lateral single pile-soil system. Obviously, although two parameters have been adopted in their formulation, they are still insufficient to simultaneously fit all three (two diagonal and one cross-coupling) coefficients in the corresponding stiffness matrix. Therefore, they proposed two approaches for approximation. The first approach is to match the two diagonal terms of the pile-head stiffness matrix (Eq. (2)), which will result as:

$$L_{eq}=1.732(K_{\theta\theta})^{0.5}(K_{xx})^{-0.5} \quad (12\text{–}1)$$

$$EI_{eq}=0.433(K_{\theta\theta})^{1.5}(K_{xx})^{-0.5} \quad (12\text{–}2)$$

The second approach is to match the translational and the cross-coupling terms of the pile-head stiffness matrix (Eq. (2)), which will result as:

$$L_{eq}=2(K_{\theta x})(K_{xx})^{-1} \quad (13\text{–}1)$$

$$EI_{eq}=0.667(K_{\theta x})^{3}(K_{xx})^{-2} \quad (13\text{–}2)$$

Table 2 shows the equivalent cantilever parameters for a long pile embedded in uniform soils and Gibson soils determined by the above formulae. Different criteria will yield different equivalent flexural rigidity EI_{eq} and equivalent length L_{eq}. For applications, Lam et al. also recommended adopting the first approach for the case of a pile-extension foundation (i.e., single-pile foundation) and the second approach for the case of a group-pile foundation, in which the rotational stiffness of the pile group is dominated by the axial stiffness of individual piles. However, their first recommendation can not be justified by the results of comparison studies shown in subsequent sections.

EXACT EQUIVALENT CANTILEVER MODEL

The above discussions indicate that neither the uncoupled spring model nor the conventional cantilever models can exactly represent the stiffness of a lateral pile-soil system. They should be categorized as approximate equivalent models because the stiffness matrix (or flexibility matrix) of those equivalent models do not entirely match that of the original Winkler model. The unmatched terms in the matrix would result in significant error in all terms when inversing the matrix. Thus, whether the equivalent process is based on the flexibility approach (Nair's model) or the stiffness approach (Lam's model), the responses obtained will not be good for both the displacements and the forces simultaneously. Therefore, they can not work equally-well for different pile arrangements and loading conditions. Based on the above discussions, the objective of this paper is to develop an exact equivalent model that can exactly represent the pile-head behavior of a laterally loaded pile-soil system.

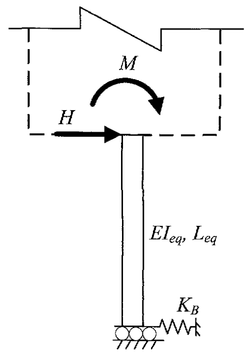

Fig. 2. Exact equivalent cantilever model

In order to entirely match the pile-head flexibility (or stiffness) of a lateral pile-soil system as shown in Eq. (1) or (2), it is obvious that three parameters are required. When the flexibility matrix of a cantilever (Eq. (10)) is compared with that of the Winkler model in uniform soils (Eq. (3)), it can be seen that the difference is limited to only one term (i.e., the translational displacement) if the equivalent length of the cantilever is set equal to $1/\beta$. It is therefore thought if a horizontal spring is artificially added on the bottom of the cantilever (as shown in Fig. 2) to compensate the observed difference between Eq. (10) and Eq. (3), both systems will have the same flexibility matrix all the time, i.e., an exact equivalent model can be established.

On the basis of the above idea, when an additional lateral spring with stiffness K_B is attached to the base of the cantilever, the flexibility matrix (Eq. (10)) of the equivalent cantilever becomes

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$$\begin{bmatrix}\delta\\ \theta\end{bmatrix}=\begin{bmatrix}\dfrac{L_{eq}^3}{3EI_{eq}}+\dfrac{1}{K_B} & \dfrac{L_{eq}^2}{2EI_{eq}}\\ \dfrac{L_{eq}^2}{2EI_{eq}} & \dfrac{L_{eq}}{EI_{eq}}\end{bmatrix}\begin{bmatrix}H\\ M\end{bmatrix} \tag{13}$$

Notably, the above equation differs from Eq. (10) only in the term corresponding to the coefficient of translational flexibility. The term of $1/K_B$ can be used to compensate the smaller translational flexibility coefficient of a fixed-base cantilever than that of a Winkler model.

For an infinite long pile embedded in uniform soils, equating Eq. (13) to Eq. (3) will result in

$$\frac{L_{eq}^3}{3EI_{eq}}+\frac{1}{K_B}=\frac{1}{2EI\beta^3} \tag{14-1}$$

$$\frac{L_{eq}^2}{2EI_{eq}}=\frac{1}{2EI\beta^2} \tag{14-2}$$

$$\frac{L_{eq}}{EI_{eq}}=\frac{1}{EI\beta} \tag{14-3}$$

Then, the equivalent parameters L_{eq}, EI_{eq} and K_B are obtained as

$$L_{eq}=\frac{1}{\beta} \tag{15-1}$$

$$EI_{eq}=EI \tag{15-2}$$

$$K_B=6EI\beta^3 \tag{15-3}$$

Similarly, for a pile embedded in Gibson soils, let Eq. (13) be equal to Eq. (5) and then the equivalent parameters L_{eq}, EI_{eq} and K_B are obtained as

$$L_{eq}=\frac{1.839}{\eta} \tag{16-1}$$

$$EI_{eq}=1.057EI \tag{16-2}$$

$$K_B=2.28EI\eta^3 \tag{16-3}$$

More generally, for a pile embedded in arbitrary soil profile, the equivalent parameters can be obtained based on the actual pile-head stiffness matrix (i.e., Eq. (2)) by using the following formulae:

$$L_{eq}=2\frac{K_{x\theta}}{K_{xx}} \tag{17-1}$$

$$EI_{eq}=2\frac{K_{x\theta}}{K_{xx}^2}\cdot(Det) \tag{17-2}$$

$$K_B=\frac{3K_{xx}}{3K_{xx}K_{\theta\theta}-4K_{x\theta}^2}\cdot(Det) \tag{17-3}$$

in which (Det) is the determinant of the pile-head stiffness matrix, i.e., $K_{xx}K_{\theta\theta}-K_{\theta x}^2$.

Table 3 summarizes the above parameters for the proposed equivalent model. This model is an exact equivalent model for modeling a lateral pile-soil system. Since all the stiffness coefficients of the equivalent model are exactly the same as those of the pile-soil system to be simulated, it can give correct responses for different pile arrangements and loading conditions. Furthermore, it is very important to notice that it is very simple and very easy to be applied in most available computer codes.

Table 3. Parameters for proposed exact equivalent cantilever model

Lateral support independent of depth (E_s=constant)		
$L_{eq}=\frac{1}{\beta}$	$EI_{eq}=EI$	$K_B=6EI\beta^3$
Lateral support linearly increasing with depth		
$L_{eq}=\frac{1.839}{\eta}$	$EI_{eq}=1.057EI$	$K_B=2.28EI\eta^3$
General form		
$L_{eq}=2\frac{K_{x\theta}}{K_{xx}}$	$EI_{eq}=2\frac{K_{x\theta}}{K_{xx}^2}\cdot(Det)$	$K_B=\frac{3K_{xx}}{3K_{xx}K_{\theta\theta}-4K_{x\theta}^2}\cdot(Det)$

COMPARISON OF EQUIVALENT MODELS WITH WINKLER MODEL

In order to investigate the applicability of above-mentioned conventional equivalent models with the proposed exact equivalent cantilever model, two simple cases as shown in Figs. 3 and 6 are chosen for comparison studies. Figure 3 is the case of a single-pile foundation, and Fig. 6 is the case of a group of two piles connected by a rigid cap.

Single-pile Foundation

For the case of a single pile subjected to a lateral load H and moment M at the pile head as shown in Fig. 3(b), the following four equivalent models are used to calculate the pile-head responses and compared with those of a Winkler model (complete beam-spring model) as shown in Fig. 1(a):

(1) Uncoupled spring model;
(2) Nair's model with $L_{eq}=1.0/\beta$;
(3) Lam's model with $EI_{eq}=0.612EI$ and $L_{eq}=1.224/\beta$; and
(4) Proposed exact equivalent cantilever model.

Assume that the pile is embedded in uniform soils and $\beta=0.3$ for simplicity. The pile-head responses calculated for all equivalent models are compared as shown in Figs. 4 and 5. In these two figures, the pile-head load is expressed by the load ratio M/H, and the pile-head responses are expressed by the lateral displacement ratio and rotation ratio which are the ratios with respect to the corresponding response of a complete Winkler model.

From Figs. 4 and 5, it can be seen that the proposed exact equivalent cantilever model gives lateral displacement ratio and rotation ratio equal to 1.0 for all load ratios, i.e., gives both responses exactly same as the original Winkler model. However, the conventional equivalent models will give different degrees of error depending on the load ratio M/H. The uncoupled spring model will significantly underestimate both the pile-head lateral displacement and rotation. Nair's model gives rotation ratio equal to 1.0 for all load ratios, however underestimates

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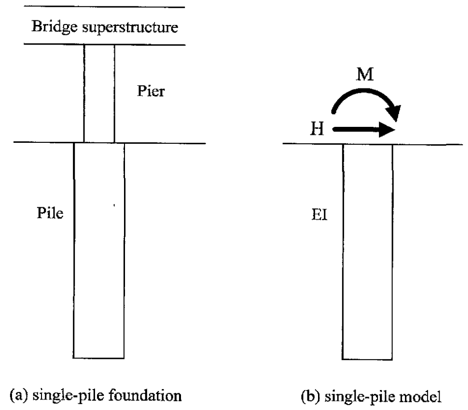

Fig. 3. Case of single-pile foundation

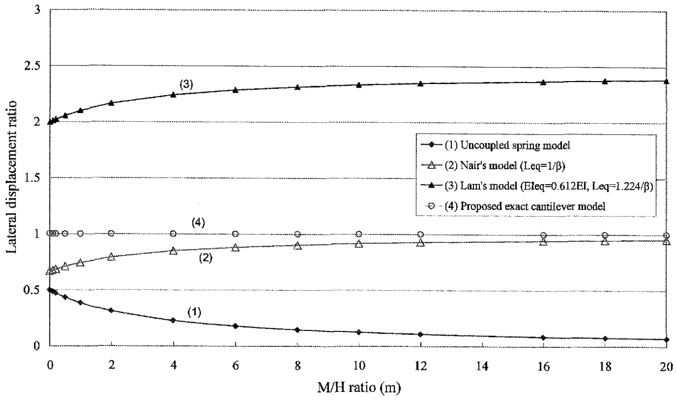

Fig. 4. Lateral displacement ratio of equivalent models

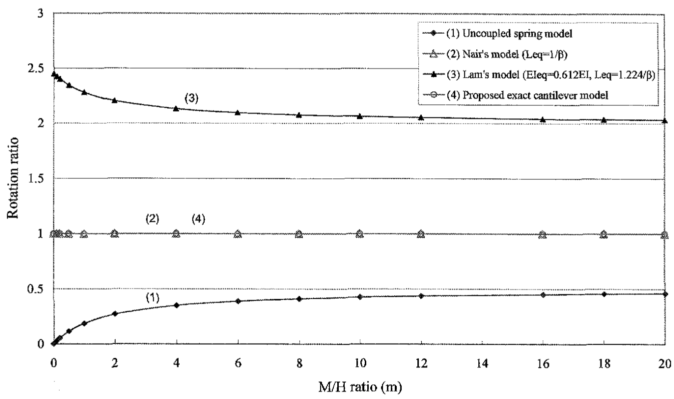

Fig. 5. Rotation ratio of equivalent models

the pile-head lateral displacement. Lam's model significantly overestimates both the pile-head lateral displacement and rotation with a factor larger than 2.0.

Two-pile Foundation

A simple two-pile group is adopted herein as an example of group-pile foundation. Consider a two-pile group with spacing s and rigidly connected to a rigid pile cap, as illustrated in Fig. 6(b). Assume the loads transmitted from superstructure to pile cap are expressed as the lateral load H_t and moment M_t at the base center of the pile cap. For this case, the Winkler (beam-spring) model is constructed as shown in Fig. 6(c), in which a vertical spring with coefficient K_v, is added to the tip of each pile to represent the axial stiffness of pile because it will participate in resisting the moment applied at the pile cap. According to Fig. 6(c), the stiffness of the Winkler model can be expressed as

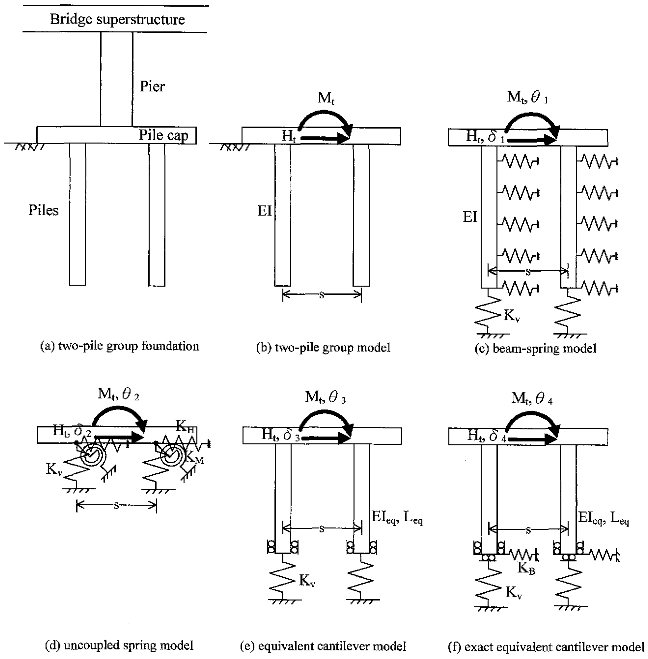

Fig. 6. Case of two-pile group foundation

$$\begin{pmatrix} H_t \\ M_t \end{pmatrix} = \begin{bmatrix} 8EI\beta^3 & -4EI\beta^2 \\ -4EI\beta^2 & 4EI\beta + \frac{1}{2}K_v s^2 \end{bmatrix} \begin{pmatrix} \delta_1 \\ \theta_1 \end{pmatrix} \tag{17}$$

where δ_1 and θ_1 represent the pile-cap lateral displacement and rotation of the pile group, respectively, and $(1/2)K_v s^2$ represents the contribution from the axial stiffness of individual piles to the rotational stiffness of the pile group. For this case, assume that the piles are embedded in uniform soils, $\beta = 0.3$ and $(1/2)K_v s^2 = 20K_{\theta\theta} = 40EI\beta$. The responses obtained from this Winkler model will be adopted as the reference solution. The following four equivalent models, as shown in Figs. 6(d) to (f), are chosen for comparison studies:

(1) Uncoupled spring model;
(2) Nair's model where $L_{eq} = 1/\beta$;
(3) Lam's model where $EI_{eq} = 1/3EI$ and $L_{eq} = 1/\beta$; and
(4) Proposed exact equivalent cantilever model.

The pile-cap responses calculated by using the equivalent models are compared with those of a Winkler model (beam-spring model) as shown in Figs. 7 and 8. Note that the lateral displacement and rotation of each pile are equal to those of pile cap since piles are rigidly connected by the pile cap. From Figs. 7 and 8, it can be seen that the proposed exact equivalent cantilever model gives exact pile-cap lateral displacement and rotation for all load ra-

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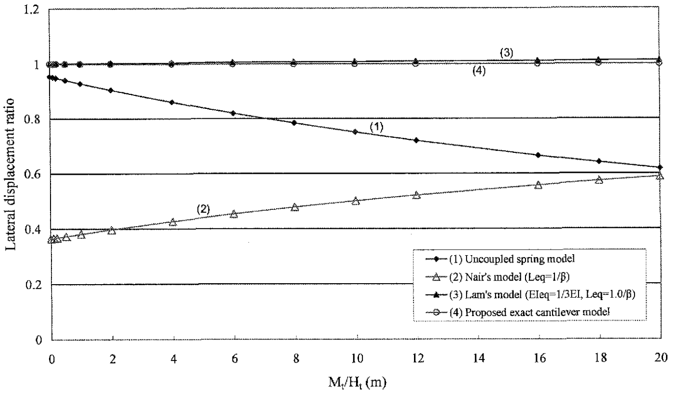

Fig. 7. Lateral displacement ratio of equivalent models for two-pile group ($(1/2)K_v s^2=20K_{\theta\theta}$)

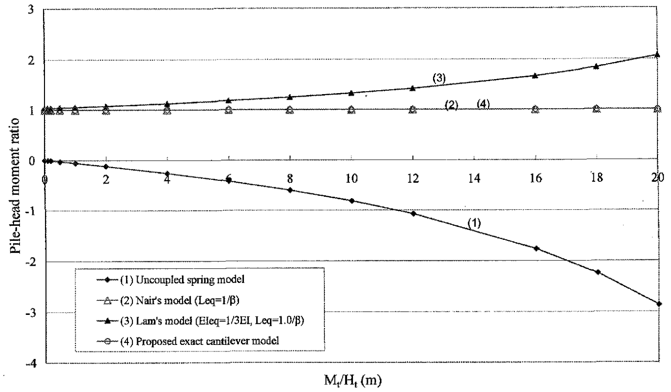

Fig. 9. Pile-head moment ratio of equivalent models for two-pile group ($(1/2)K_v s^2=20K_{\theta\theta}$)

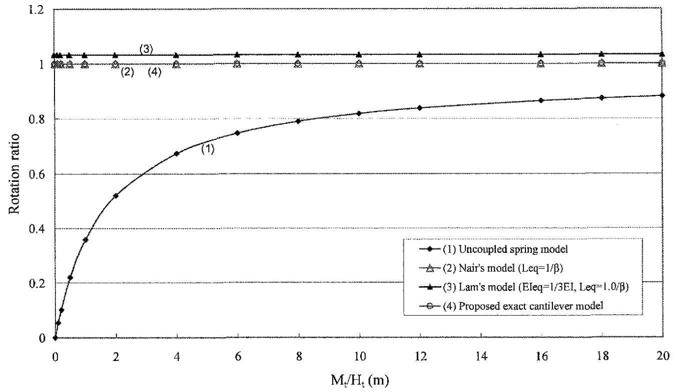

Fig. 8. Rotation ratio of equivalent models for two-pile group ($(1/2) K_v s^2=20K_{\theta\theta}$)

tios. However, the conventional equivalent models will give different degrees of error depending on the load ratio M_t/H_t. The uncoupled spring model gives poor prediction for both the pile-cap lateral displacement and rotation. Nair's model gives rotation ratio equal to 1.0 for all load ratios, however significantly underestimates the pile-cap lateral displacement. Lam's model gives the pile-cap lateral displacement very close to the original Winkler model, but overestimates by a little, the pile-cap rotation for all load ratios.

Furthermore, according to the pile-cap lateral displacement and rotation obtained from the equivalent models, the pile-head reaction forces can thus be calculated and compared with the results of the original Winkler model. The pile-head shear force will be same as the original Winkler model for all load ratios. The results of pile-head axial force ratio will be identical to that of the pile-cap rotation ratio because it is proportional to the pile-cap rotation. As for the pile-head moment, the relationship of pile-head moment ratio with respect to the M_t/H_t ratio is shown in Fig. 9. The proposed exact equivalent cantilever model gives exact pile-head moment for all load ratios. The uncoupled spring model yields the pile-head moment in the opposite direction. Nair's model gives an exact pile-head moment, but Lam's model will overestimate the pile-head moment as the load ratio M_t/H_t increases.

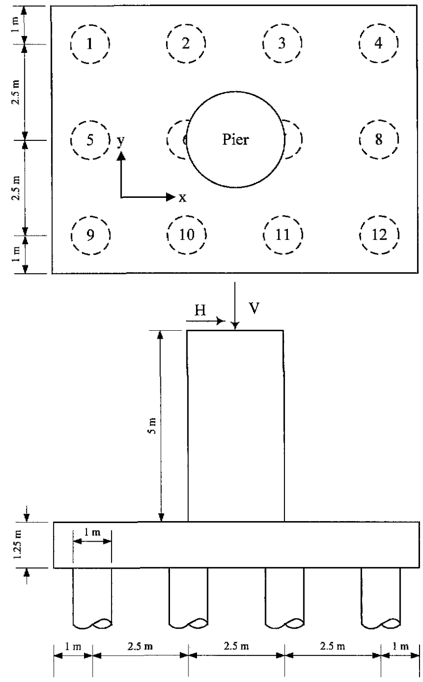

Fig. 10. Pile foundation for case study

CASE STUDY

A bridge pier supported by pile foundation as shown in Fig. 10 is selected for the case study. The bridge pier with a height of 5 m and a diameter of 2.5 m is subjected to a lateral load of 8000 kN (x-dir) and a vertical load of 15000 kN at the pier top. The pier is supported by a rigid cap with a height of 1.25 m underlain by 4 (x-dir) × 3 (y-dir) piles. Each pile has a diameter of 1 m and a length of 30 m. The spacing of piles is 2.5 m. All piles are concrete piles with Young's modulus $E=2.7\times10^7$ (kN/m^2). It is assumed that the soils are uniform with horizontal subgrade reaction modulus $E_s=23000$ (kN/m^2) and each pile has a vertical stiffness $K_v=551000$ (kN/m).

To analyze the structural response by using the Win-

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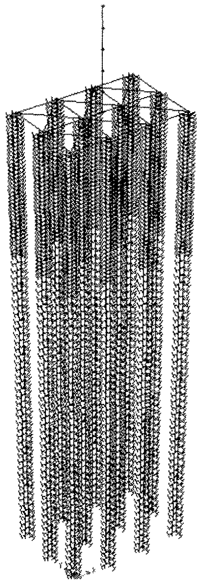

Fig. 11. Structural model of pier foundation by using Winkler springs

kler modeling, the global structural model can be established as shown in Fig. 11. The pier and piles are modeled by beam elements. The soils are modeled by spring elements. Each pile is assumed to be rigidly connected to the rigid pile cap. Under the specified pier-top loading, the structural responses including the pier-top displacements, pile-cap displacements, and pile-head reactions are shown in Table 5. Note that the horizontal displacement and rotation of all piles are equal to those of the pile-cap since the pile cap is assumed to be rigid and piles are rigidly connected to the pile cap. It is also noted that the pile-head moment and shear force of all piles are the same.

This problem can also be analyzed by using substructure approach. The piles are replaced by corresponding equivalent models in the global structural model to solve the structural responses. For the purpose of comparison, four equivalent models as mentioned above are adopted herein. For this case, the characteristic coefficient of the single pile-soil system is $\beta=\sqrt[4]{E_s/4EI}=0.257$ (1/m). Table 4 lists the equivalent parameters of these four models. By using the equivalent models, the amounts of elements are significantly reduced and the global structural model becomes much simpler. For example, Fig. 12 displays the global structural model in which piles are replaced by the proposed exact equivalent models.

The responses calculated by all four equivalent models are compared with those obtained from the original Winkler model as listed in Table 5. It can be seen that the proposed exact equivalent model will give identical responses to the original Winkler model. However, the responses obtained from other equivalent models are not satisfactory. For the uncoupled spring model, the results are very poor. For the Nair's model, the pile-head reaction forces are correct, but the horizontal displacements of the super-structure are significantly underestimated. At the pier top, the error is 21%. At the pile cap, the error can be as high as 50%. For the Lam's model, the horizontal displacements at the pier-top and the pile-cap are just a little overestimated, however, the pile-head reaction forces are not satisfactory, especially for the pile-head moments where the error can be as high as 29%.

Table 4. Parameters of equivalent models

No.	Equivalent model	Equivalent parameters	
(1)	Uncoupled spring model	$K_H=4EI\beta^3$	89618 (kN/m)
		$K_M=2EI\beta$	680295 (kN-m/rad)
(2)	Nair's equivalent cantilever model	$L_{eq}=1/\beta$	3.8964 (m)
		$EI_{eq}=EI$	1.325×10^6 (kN-m^2)
(3)	Lam's equivalent cantilever model	$L_{eq}=1/\beta$	3.8964 (m)
		$EI_{eq}=EI/3$	4.418×10^5 (kN-m^2)
(4)	Proposed exact cantilever model	$L_{eq}=1/\beta$	3.8964 (m)
		$EI_{eq}=EI$	1.325×10^6 (kN-m^2)
		$K_B=6EI\beta^3$	134427 (kN/m)

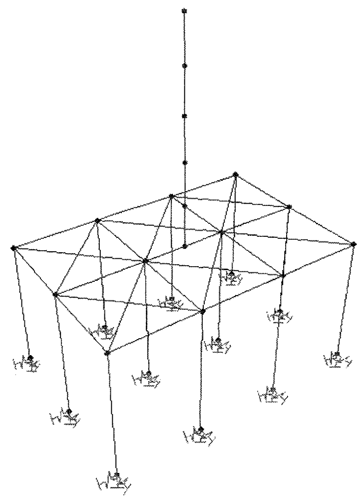

Fig. 12. Structural model of pier foundation by using exact equivalent cantilevers

From the above case studies, it can be found that the conventional equivalent models are not satisfactory and the errors induced will be changed when the loading conditions are changed. They are not suitable for engineering applications. However, the proposed equivalent cantilever model can always yield the exact results.

DISCUSSIONS

When using the substructure approach to deduce the pile-head stiffness for a pile-soil system, it is important to recognize that it is applicable only for a linear or equivalent-linear analysis. It is well known that soils are intrinsically nonlinear. If it is aimed at tracing the histories of nonlinear soil responses, none of the presented equivalent models can be applied and the complete beam-spring model with nonlinear soil springs (i.e., nonlinear

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Table 5. Comparison of responses for all models

Item		Winkler model	Uncoupled spring model	Nair's model	Lam's model	Proposed model
Pier-top displacement	settlement (m)	0.0029	0.0029	0.0029	0.0029	0.0029
	horizontal (m)	0.0235	0.0197 (−16%)	0.0186 (−21%)	0.024 (2%)	0.0235
	rotation (rad)	0.00311	0.00294 (−5%)	0.00311	0.00317 (2%)	0.00311
Pile-cap displacement	settlement (m)	0.0024	0.0024	0.0024	0.0024	0.0024
	horizontal (m)	0.0097	0.0074 (−24%)	0.0048 (−50%)	0.0098 (1%)	0.0097
	rotation (rad)	0.00118	0.00084 (−29%)	0.00118	0.00124 (5%)	0.00118
Pile 5 pile-head	axial (kN)	1133	429 (−29%)	1133	1258 (5%)	1133
	shear (kN)	667	667	667	667	667
	moment (kN)	−898	569 (−163%)	−898	−1158 (29%)	−898
Pile 6 pile-head	axial (kN)	−488	−722 (−29%)	−488	−446 (5%)	−488
	shear (kN)	667	667	667	667	667
	moment (kN)	−898	569 (−163%)	−898	−1158 (29%)	−898
Pile 7 pile-head	axial (kN)	−2109	−1874 (−29%)	−2109	−2150 (5%)	−2109
	shear (kN)	667	667	667	667	667
	moment (kN)	−898	569 (−163%)	−898	−1158 (29%)	−898
Pile 8 pile-head	axial (kN)	−3729	−3025 (−29%)	−3729	−3854 (5%)	−3729
	shear (kN)	667	667	667	667	667
	moment (kN)	−898	569 (−163%)	−898	−1158 (29%)	−898

Note: In computing the relative error of axial reactions for all equivalent models, the axial reaction of each pile is subtracted with average vertical reaction force caused by applied vertical loads (−1298 kN per pile)

p-y curves) has to be used. Alternatively, the technique of equivalent linearization has very often been used in engineering practices, such as the secant modulus method suggested in the Design Specifications of Japan Road Association (2002). The use of the equivalent cantilever model for modeling the equivalent-linearized soil and pile system will be very effective for engineering applications.

CONCLUSIONS

In order to simulate the load-deflection behavior of a laterally loaded pile, the conventional equivalent models such as the uncoupled spring model and cantilever models can be categorized as approximate models. Because the stiffness matrix (or flexibility matrix) of those equivalent models do not entirely match that of the original Winkler model, they can not work equally-well for different pile arrangements and loading conditions, and sometimes will induce very large errors in the responses obtained. This study develops an exact equivalent cantilever model, in which an artificial lateral spring is added at the base of a cantilever to modify the fixed-base cantilever model. The stiffness matrix of the proposed model can completely match that of the original Winkler model and gives exact responses for all cases. This model is simple and very useful for engineering applications.

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