[Paper]

Visual Appearance of Color and Image Quality of Computer-Generated 3-D Transparent Objects

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Translucent or transparent quality of computer-generated 3-D images with a complex of reflected scene among boundary surfaces may be visualized using a constructive solid geometry model (CSGM) and a ray tracing method (RTM) on a color CRT. Generally, the CSGM is composed of a combination of some primitives and quadratic surfaces. The RTM has been widely used as the principal means for displaying computer-generated 3-D images with reality.

The effects of additive and pseudo-subtractive color mixings are compared with a view to examining the visual appearance of color and image quality of transparent objects on a color CRT. The relation between the degree of transparency and the value of depth level : a kind of parameter for taking stock of the total number of times of specular reflection at the surface of opaque objects and/or that of reflection and refraction at the border of transparent media is discussed from a viewpoint of color rendering and color shift.

1. Introduction

Computer graphics, including computer art and computer animation, is concerned with the synthesis of images of real or imaginary objects. A ray tracing method has been known as one of the most powerful means for displaying 3-D objects in the field of computer graphics. M. Morikawa [1] made a survey of ray tracing techniques for opaque and/or transparent objects composed of fundamental primitives, and showed an elaborate and useful program list coded in BASIC and many quasi-color images displayed by a kind of dither method on a color CRT device. One of the main attraction of this technique includes an algorithm that determines which line or surface in a solid model is not visible from a particular viewpoint as a result of computation of intersection point, and that removes them from the computer-displayed images. Although there are the merits and demerits in the conventional (or reverse) ray tracing technique on the basis of the position of a viewer, i.e., the point of sight, it is possible to simply display opaque and/or transparent images in quasi-color or full color.

When even using a color CRT for 8 colors, quasi-fine color images may be generated by means of R·G·B bit expansions and the photographic multiple exposure techniques in place of using a random dither method. M. Iizuka et al. [2] had a discussion for and against the simulation results of 3-D opaque objects. But it is impossible to observe the final generated images on a color CRT directly in this method.

Recently, a new approach and many modified application techniques are proposed according to the aim of ray traced images with good quality. For example, a radiosity method [3], a rendering integral method [4], a bidirectional ray tracing method [5] etc. have been used in order to positively demonstrate the effects of ambient or global illumination arising out of complex interreflections in an open or closed environment system.

In this study, the three dimensional views of multiple transparent objects, the shift of color and the image quality are discussed by taking account of the depth level based on effects of reflection and refraction at the boundary surfaces from a practical viewpoint of color rendering techniques, i.e., additive and pseudo-subtractive color mixings.

2. Basic Expression for Ray Tracing Techniques

2.1 Formulation of Opaque and/or Transparent Objects

A ray of light passes through transparent

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media with refractive index or is reflected between opaque media with reflective components. Most common materials are compound reflectors and exhibit all three reflective components (spread or uniform, diffuse and specular). In some, one or two components predominate. If the surface is polished, a ray of light is reflected specularly.

A ray of reflected light from any point on an object is received by a viewer, and the intensity of reflected light depends on the angle between the direction of sight and the reflected light vector. The quantitative formulation for intensity computation has been proposed in many different ways in order to display computer-generated images of opaque objects by H. Gouraud [6], B.T. Phong [7], and J.F. Blinn [8] et al. T. Whitted [9] added the specular and refractive terms to the fundamental expression for the purpose of intensity computation of transparent objects.

Fig. 1 shows a simulation model for demonstrating the direction of a ray of light based on specular reflection and refraction at each surface of opaque objects with specular reflectance and/or transparent objects. The color intensity of light I at an arbitrary point on objects, i.e., an amount of light arriving at a viewer, is formulated quantitatively by the following expression irrespective of the wavelength λ.

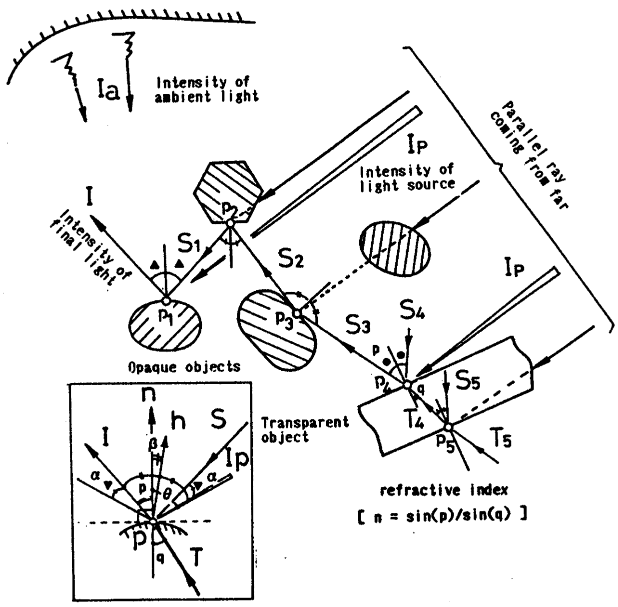


Fig. 1 *Simple simulation model for ray tracing*

See the lower left illustrative example in Fig. 1 except for the first and second terms of Eq. (1) in connection with the physical explanation and its symbol. Note that the ray traced component based on the diffuse reflection of opaque objects, i.e., the second term in Eq. (1) is not shown in Fig. 1 because this component (ray traced light) diffusingly reflected form other objects, i.e., the effect of interreflection between objects is omitted for simplicity of a ray tracing algorithm.

$$I = a \cdot I_a + A\{\rho_d \cdot \cos\theta \cdot I_c + w(\theta) \cdot \cos^g\alpha \cdot I_p\}/(C+D) + K_s \cdot S + K_t \cdot T \tag{1}$$

where,

ρ_d : diffuse reflectance of object
$w(\theta) = \rho_s$: specular reflectance of object
θ : angle between incident light and surface normal
α : angle between viewing line and specular direction of incident light for Phong model (β : angle between halfway direction and surface normal for Blinn model)
a : attenuation coefficient of ambient light
g : glossy index
I_a : color intensity of ambient light
I_c : color intensity of opaque and/or transparent objects
I_p : color intensity of light source
A : shadow coefficient (0 or 1)
C : constant
D : distance from perspective viewpoint to surface of object
S : color intensity of incident light from direction of specular reflection
T : color intensity of incident light from transparent direction

$$K_s = \frac{1}{2}\left\{\left[\frac{\sin(p-q)}{\sin(p+q)}\right]^2 + \left[\frac{\tan(p-q)}{\tan(p+q)}\right]^2\right\}$$
$$= \frac{1}{2}\left\{\left[\frac{Ci - n\cdot Cr}{Ci + n\cdot Cr}\right]^2 + \left[\frac{n\cdot Ci - Cr}{n\cdot Ci + Cr}\right]^2\right\}$$

$K_t = 1 - K_s$
$C_i = \cos(p)$; p : incident angle
$C_r = \cos(q)$; p : refractive angle
n : refractive index [$n = \sin(p)/\sin(q)$]

The third term in Eq. (1) corresponds to the highlight component to a viewer from a light source, or to specular reflective component of opaque objects. The denominator C + D, is often omitted on condition that the distance between a light source and a shading object is large, and a ray of light is parallel. The forth and fifth terms have an important role for simulating the effects of reflection and refraction for transparent objects in connection with an idea of the depth level, i.e., a tree construction.

The coefficient $\cos^g\alpha$ in the third term means the degree of highlight from a light source. The glossy index, i.e., the power g determines how glossy the object is for the case of the specular reflectance of opaque objects : $w(\theta) \neq 0$. For large g a very glossy highlight effect is obtained, while for smaller g the object appears less glossy. The component of specular reflectance : $w(\theta)$ for opaque objects is equivalent to that of reflectance : K_s for transparent objects, though the physical notation and definition are different from each other in the ray tracing algorithm. The two squared terms in K_s called Fresnel formula represent reflection of the "s" and "p" polarizations, respectively. Sun light, incandescent lamp, various discharge lamps, etc., which emit visible light, are regarded as unpolarized light. The specular reflectance: K_s of transparent media must be calculated from two kinds of polarization components. The refractive index : n of transparent objects is defined for calculating the specular reflectance in advance. The specular reflectance is not constant by ordinary and is affected by the incident angle of a ray of light in a RTM.

2.2 Recurrence Expression for Transparent Objects

For simplicity, the effect of multiple reflection is neglected between objects. A ray of light passing through the transparent objects is not absorbed due to the pass length of light. But T. Yasuda et al. [10] proposed the ray tracing algorithm based on the Lambert-Bouguer law, i.e., the simplified spectrum absorption model.

Table 1 shows the typical modeling techniques for simulating 3-D transparent objects. We use a modified idea of Whitted algorithm through this study.

Fig. 2 shows a simple tree construction grown in the process of tracing a particular ray of light backwards in connection with Fig. 1. It is very important to pay attention to the direction of each arrow. Each node of the tree shown as the symbol 《 ○ 》 corresponds to the point on boundary surfaces of Fig. 1. The sum of the forth and fifth terms in Eq. (1) at each node is computed after the tree is completely grown or

Table 1 *Typical modeling techniques for simulating transparent objects*

Techniques	Characteristics	Notes
(a) Simplified technique for transparent effects	τ : uniform irrespective of n	M.E.Newell & R.G.Newell [1]
		D.Kay [2]
(b) Standard technique for reflection & refraction effects	Snell law Fresnel formula Index of refraction	T.Whitted [3]
(c) Advanced technique for considering effects of wave length	$\rho(\lambda)$: $\tau(\lambda)$	R.A.Hall & D.P. Greenberg [4]
	Direction of refraction related to λ	S.W.Thomas [5]
(d) Modified technique for spectrum absorption model	Lambert & Bouguer law : $\tau(x) = \exp[-\alpha x]$	T.Yasuda et al. [6]

Notes :
(1) Proc. ACM. Nat.Conf.. (1972) 443
(2) Computer Graphics. Vol.13. No.3 (1979) 158
(3) Comm. ACM. Vol.23. No.6 (1980) 343
(4) IEEE CG&A. Vol.3. NO.8 (1983) 10
(5) Visual Computer. Vol.2. No.1 (1986) 3
(6) Trans. Inf. Pro. Society of Japan. Vol.26. No.4 (1985) 591 [in Japanese]

when arrived at till the specified maximum depth level in the ray tracing algorithm. The color intensity I incident to the viewpoint is

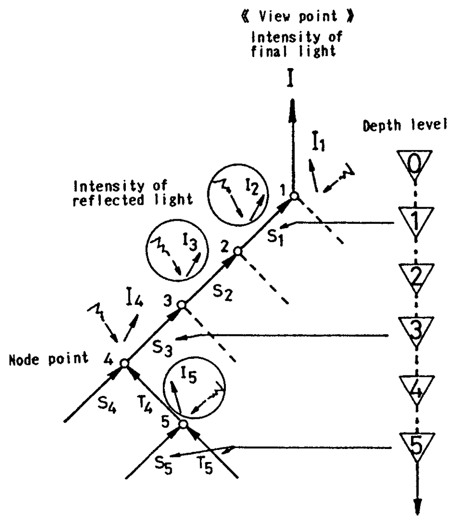


Fig. 2 *Tree construction for calculating intensity of light*

composed of two main components : the direct reflected intensity of light I_i and the secondary reflected and refracted intensity of light from the other objects indirectly. In the case of Fig. 2, the color intensity of light arriving at a viewer may be formulated by means of bottom-up procedure as follows.

$$
\begin{aligned}
T_4 &= (I_5) + \rho_5 \cdot S_5 + \tau_5 \cdot T_5 \\
S_3 &= I_4 + \rho_4 \cdot S_4 + \tau_4 \cdot T_4 \\
S_2 &= (I_3) + \rho_3 \cdot S_3 \\
S_1 &= (I_2) + \rho_2 \cdot S_2 \\
I &= I_1 + \rho_1 \cdot S_1
\end{aligned}
\qquad (2)
$$

where,

I : color intensity of light arriving at a viewer

I_i : reflected color intensity at node point i by ambient light and direct light

S_i : incident color intensity at node point i by secondary reflected light component

T_i : incident color intensity at node point i by refracted or transmitted light component

ρ_i : average specular reflectance on surface containing node point i

τ_i : average transmittance on surface containing node point i

Note that Eq. (2) changes together with the geometrical relationships between the direction of a viewing line and the intersection point of objects, even though the position of a viewer and a light source, and the surface property of a model are fixed as the constant parameters in the ray tracing algorithm.

In Eq. (2), the terms : I_5, I_3, and I_2 are omitted in the simulation model of Fig. 1, because color intensity of direct light at the points : P_5, P_3, and P_2 does not exist owing to the shadow or obstruction objects. The maximum depth level of the tree construction is equivalent to the number of product of reflectance and/or transmittance : $\Pi\ \rho_i\tau_j$. The ray tracing search is continuously carried out according to the value of depth level. To the contrary, if the amount of energy attenuation in relation to the product of reflectance and/or transmittance is less than the prespecified constant value, i.e., the threshold value in the ray tracing algorithm, the ray tracing search is stopped on the way irrespective of the value of depth level. As a result, the recurrence expression is influenced by surface property of model, viewer position, light source position, geometrical relation between viewing line and intersection point of objects, optical parameters etc.

Generally, the color intensity arriving to a viewer may be formulated as the recurrence expression for opaque and/or transparent models.

$$
\begin{aligned}
I &= I_i + \rho_i \cdot S_i + \tau_i \cdot T_i \\
S_i &= I_j + \rho_j \cdot S_j + \tau_j \cdot T_j \\
T_i &= I_k + \rho_k \cdot S_k + \tau_k \cdot T_k \\
&(i; j; k = 1,2,3, \ldots\ldots\ldots\ldots, m) \\
&(0 \leqq N \leqq m \ \text{ or } \ 0 < m \leqq N)
\end{aligned}
\qquad (3)
$$

where, m : node point (intersection point)
N : maximum depth level

The suffixs i ; j ; k are not equal to the number of fundamental objects composed of a few basic primitives, but corresponds to the node point in Fig. 2, i.e., the number of intersection point based on the ray tracing line at the boundary surfaces. It is very important to choose properly the value of depth level in connection with computation time of ray traced images as one of the main parameters in the ray tracing algorithm, because the shift of transparent color and the translucent or transparent quality of computer-generated images depend on the value of depth level.

3. Color Rendering Techniques

3.1 Additive and Subtractive Color Mixings [11]

The law of the additive color mixing depends on the three additive primaries such as Red, Green and Blue. The primary colors may be used to construct the color circle that permits a qualitative estimate of the hue and saturation. Generally, the additive color mixing can occur in two different ways. The first is known as partitive color mixing in the array of tiny dots in a color CRT. The second is used as superposition in stage lighting corresponding to simultaneous projection on a white screen from three kinds of RGB color slides. In the additive color mixing, the final SPD (spectral power distribution) is just the sum of the SPDs of the individual lights. The law of the subtractive color mixing is based on the three subtractive primaries such as Magenta, Yellow and Cyan which are the complements of the three additive primaries. The subtractive color mixing occurs when a ray of light passes through a series of colored glass (transparent medium) or when different colored paints or dyes are mixed together. Each glass, pigment or dye absorbs some portion of the incident light. The SPD of the remaining light which is not absorbed determines the color of the mixture under the illuminating conditions. Colored filters or transparent media are characterized by a spectral transmittance curve that specifies the percentage of incident light transmitted at each wavelength. When similar filters are stacked together or when the concentration of a dye is increased, the SPD of the emerging light may change. As a result, it is possible to distinguish the color (hue) of the reflected light from that of the original color. An effect called dichroism results from the dependence of absorption on increasing filter thickness governed by Bouguer's law and on increasing dye concentration governed by Beer's law. If the symbol Z represents the number of filters in the beam and the symbol T represents the transmittance (expressed as a decimal or fraction) at a white light or a wavelength, the synthetic net transmittance known as Bouguer's law is described as T^Z.

The computer generated-image may be generally displayed by means of the additive color mixture of RGB primary colors on a color CRT. In the case of the full color rendering, each intensity of RGB primary colors is normalized within the gamut from 0 to 255. As a result, it is possible to demonstrate an arbitrary color from $2^8 \times 2^8 \times 2^8$ colors per dot simultaneously. On the other hand, quasi-color images may be demonstrated by a dot combination of only 8 kinds of colors in the dither color rendering, on condition that the pixel resolution is less than the dot resolution of a color CRT.

3.2 Fine Color Rendering using SuperFrame Module Board [12]

The treatment of quasi-color and full color for computer graphics is possible by a commercially available workstation, for example, a 16 (or 32) bit microcomputer system with frame buffer boards. An input/output type interface board called SuperFrame developed of late by Sapience Co. may be set up in a part of the extension slot for a 16 bit PC-9801 microcomputer. The interface board is a frame memory of I/O type, and it is composed of four types of registers, i.e., control registers; x and y registers; R, G, and B registers; and mode registers.

Table 2 *Register Constructions of SuperFrame Module Board*

address \ bit	D 15 14 13 12 11 10 9 8	7 6 5 4 3 2 1 0 D
00DE	Don't care.	Mode register (W)
00DC	Prohibition for reading.	Blue register (R/W)
00DA	Prohibition for reading.	Green register (R/W)
00D8	Prohibition for reading.	Red register (R/W)
00D6	Don't care.	Y axis register (W)
00D4	Don't care.	X axis register (W)
00D2	Don't care.	Control register 2 (R/W)
00D0	Don't care.	Control register 1 (W)

Notes : $0 \leqq$ X axis register $\leqq 639$
$0 \leqq$ Y axis register $\leqq 399$

Table 2 shows the relation between the memory address and bit position in the SuperFrame module board. The main memory space is not decreased at all by installing the superframe board except for occupation of the I/O space from 00D0 address to 00DF address. The two control registers are used for initializing the superframe board. The x and y registers are used for writing data only on a CRT screen, and the bit size of register construction differs from between the two (See the notes in Table 2). The three kinds of R·G·B registers are used for reading and writing the current position of each pixel within the color intensity level : $0 \leqq IR$; IG ; $IB \leqq 255$. The dot resolution has 640 x 400 in this microcomputer system, and the memory size of this module board is 640 x 400 x 24 bits. Accordingly, it is possible to display $2^8 \times 2^8 \times 2^8$ colors. Full color images may be displayed simply and directly without using the dither method which has been used in the case of a color CRT for 8 colors. The mode register is used for carrying out superimpose functions of two kinds of displays : CRT screen images for PC-9801 and for SuperFrame.

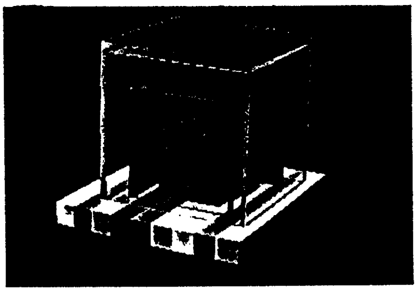
Usually, look-up tables are used in conjunction with a frame buffer memory in CG systems, and can be updated quickly for altering displayed colors under program control from the CPU. But this microcomputer system has no the look-up table. As a result, it is impossible to quickly and freely modify the effects of displayed colors on a color CRT.

4. Transparent Quality of Computer-Generated 3-D Images

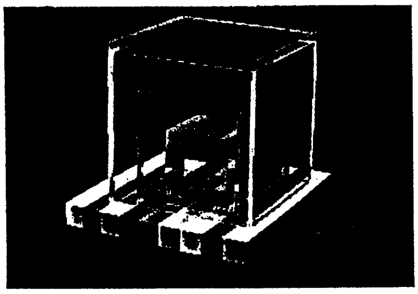
Fig. 3 (a) and (b) shows the simulation results of an inner small cube with no cavity and two types of outer transparent cubes with a cavity. The translucent or transparent quality and the shift of color may be clearly visualized by altering only the value of depth level. The outer cube with a cavity has green and blue color intensity, i.e., 32 and 64. On the other hand, the intermediate cube with a cavity has, in nature, red and green color intensity, i.e., 64 and 32. In Fig. 3 (a) for depth level : N = 5, the visual color aspect of an intermediate cube with the cavity and an inner cube looks like dull color as a result of a combination of RGB primary colors. It is impossible to discern perfectly the back of a striped checker board. In Fig. 3 (b) for N = 10, the whole of the striped checker board is clearly discerned from the position of a viewpoint, and the fine transparency and a complex of reflected view in the central transparent cube are demonstrated in comparison with Fig. 3 (a). The shift of apparent color is clearly distinguishable from an original color because of a combination of transparent media. Note that the object color of an inner small cube, i.e., an original color component such as R = 16, G = 32 and B = 16 is specified as one of the initial parameters in advance.

The color aspects of ray traced images are very complicated under even the same CSGM owing to the specified value of depth level and/or the threshold value for energy attenuation as a result of the ray tracing algorithm.

Fig. 4 (a) and (b) shows the simulation results in the case of the multiple transparent objects under the condition of 640 x 400 pixels. The original color of transparent objects from the outer side is specified as a combination of RGB primary colors : (0, 32, 64) ; (64, 32, 0) ; (16, 32, 16) ; (32, 32, 32), respectively. In Fig. 4 (b), the outer transparent object with a cavity is omitted, and the middle transparent object is demonstrated in dull red color. The difference of computation time between the two amounts to more than 2 times. Note that the value of depth level is specified as N = 15 and N = 10, respectively. A notion of depth level is introduced for specify the number of times by which a ray of light reflects and/or refracts at the surfaces of transparent media on the basis of the viewpoint. The superposition of transparent media with different object colors of the same refractive index : n = 1.5 promises the degradation of transparency according to the value of depth level. The visual color appearance of a white transparent ball looks like chromatic color at the center of a striped checker board, and the reflected view between the two differs from each other. A shift of color corresponding to the



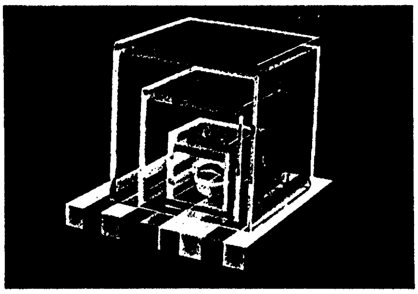
(a)



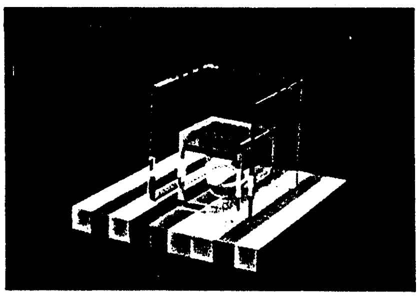
(b)

Fig. 3 *Ray traced images with cubical transparent cavities*

(a) Depth level : N = 5
(b) Depth level : N = 10



(a)



(b)

Fig. 4 *Ray traced images with multiple transparent objects*

(a) Three cubical cavities and small sphere
(b) Two cubical cavities and small sphere

pseudo-subtractive mixture may be realized as a result of the superposition of multiple transparent models. In Fig. 3 (b) and Fig. 4 (b), when the threshold value of depth level is specified as N = 10, good transparency will be visualized by judging from the simple CSGM.

Fig. 5 shows a complex of color rendering view of ray traced transparent objects. The main transparent object : a wineglass with an opaque sphere and a transparent cube is enclosed with another outer transparent object : parallelepiped cube made of glass. The full color rendering image using color intensity levels such as $2^8 \times 2^8 \times 2^8$ colors is directly displayed on a color CRT on the assumption that transparent objects are generally compounded of the constructive solid geometry as fundamental primitives in the ray tracing algorithm. This result brings us a realization of the shift of color and a complex of reflected views in comparison with the results of Fig. 3 and Fig. 4 which have multiple transparent objects.

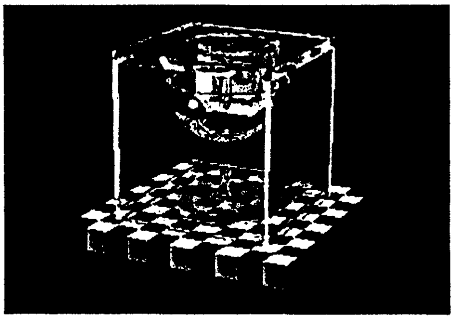
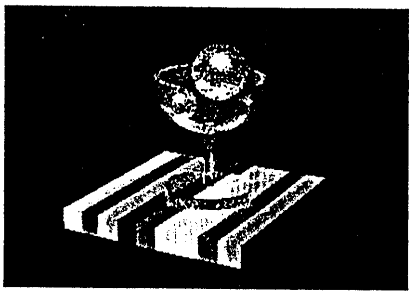
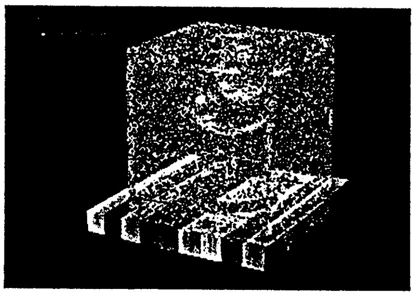


Fig. 5 *Full color rendering view of transparent objects*

Fig. 6 (a) and (b) shows the ray traced simulation results of transparent objects : the quasi-color images under the condition of 320 x 200 pixels. One pixel is composed of 2 x 2 dots within a field of view, and the computer-generated image with 5 color levels per each color is displayed as the dot combination of R, G and B primary colors by means of a random dither method. The difference of computation time between the two amounts to about 3 times because of the existance of an outer transparent cover. A number of granular and random speckles are very clear and dominant. The value of depth level : N = 10 and the refractive index of glass : n = 1.5 are introduced for the ray tracing algorithm, and these have relation to the degree of color intensity of light in a certain medium as shown in Fig. 1.



(a)



(b)

Fig. 6 *Quasi-color rendering view by means of random dither method*

(a) Wineglass with two different spheres
(b) Transparent object with cubical cavity and CSGM of Fig. 6 (a)

In the case of middle resolution such as 320 x 200 pixels except for the result of Fig. 4, jaggies which stand for the uneven appearance of vertical lines and diagonal or oblique lines in a matrix of pixels appear a little because of prespecified middle resolution in the ray tracing algorithm.

Though the computer-generated color images of 3-D transparent models may be displayed by means of the additive color mixture on a color CRT, the transparent quality and its visual color appearance of images depend on distinctly not only the value of depth level but also on the color rendering techniques. A pseudo-subtractive color mixture, i.e., the shift of color may be visually demonstrated. The shift of apparent chromatic color occurs in connection with the rate of decrease of synthetic transmittance.

Fig. 7 shows four kinds of typical curves for the relative computation time vs. the value

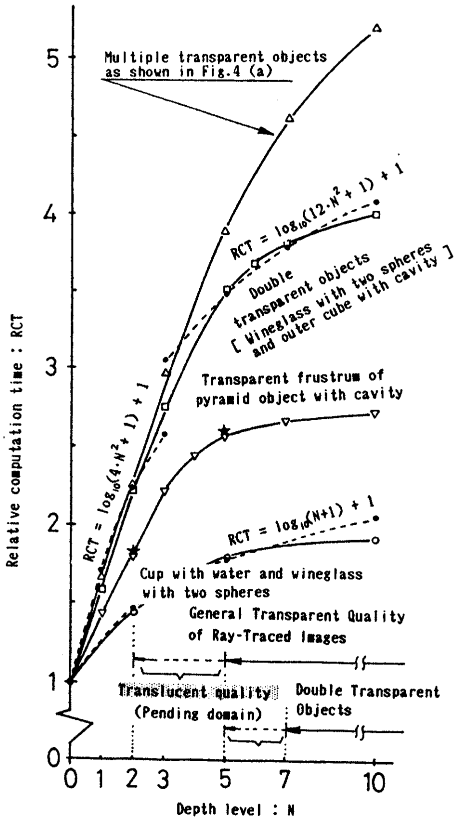


Fig. 7 *Relative computation time and value of depth level for ray traced images with 320 x 200 pixels*

of depth level under the pixel resolution : 320 x 200. The lower curve corresponds to the ray traced result which demonstrates two different and isolated transparent objects. The middle curve corresponds to the result which simulates the transparent frustrum of pyramid object with a cavity. A few data points such as the symbol 《 ★ 》 on this curve have almost the same value irrespective of the condition of high pixel resolution : 640 x 400. But the actual computation time amounts to about 4 times as contrasted with the middle pixel resolution : 320 x 200. The two kinds of upper curves show the results of multiple transparent objects similar to Fig. 4 (a) and Fig. 5, respectively.

Note that the computation time is not always in proportional to the value of depth level. But the relation between the depth level and relative computation time may be approximated using the logarithmic function such as $\log_{10}(C_1 \cdot N^{C_2} + 1) + 1$; where, C1 and C2 mean an integer number, though there are the clear differences on CSGM between the two. The translucent or transparent quality of computer-generated color images appears except for the special perspective domain, i.e., a part of the striped checker board, when the value of depth level is more than 2 in the case of simple CSGM.

It is possible, to some degree, to estimate the proper value of depth level in advance. For example, in Fig. 3 the clear transparency of 3-D ray traced images will be demonstrated if the value of depth level is specified as more than 10. The reason is that multiple transparent objects for simulation models are constructed from a combination of three cubical primitives, and the total number of times after refraction and transmission is considered at each border of glass as regards a certain direct incident light.

As the value of depth level increases in the ray tracing algorithm, the synthetic net transmittance constructed from a few kinds of transparent media is modified due to the total number of times of reflection and refraction in appearance. As a result, the shift of color based on additive color mixture on a color CRT and the improvement of translucent or transparent quality may be conveniently demonstrated using a notion of depth level.

5. Conclusions

The translucent or transparent quality and the visual color appearance of computer-generated 3-D transparent objects, i.e., the effects of the pseudo-subtractive color mixture may be simulated by taking account of the main parameters for ray tracing such as depth level, truncation of energy level, refractive index, absorption factor, original color intensity level etc.

When the value of depth level is specified as more than 4 ~ 5 in consideration of computation time, the reflected view and transparent quality of ray traced images become clear for simple transparent models.

To sum up, the transparency and the shift of color of the ray traced 3-D images may be quantitatively estimated to some degree by using only a notion of depth level in the ray tracing algorithm.

Acknowledgement

A ray tracing algorithm and its software program have been provided and published by Morikawa, and continuously modified by the

authors and research members. Many ray traced images were produced using a 16 bit microcomputer (PC-9801) with a 640 x 400 x 24 bit frame buffer (Sapience Co.) and a color CRT.

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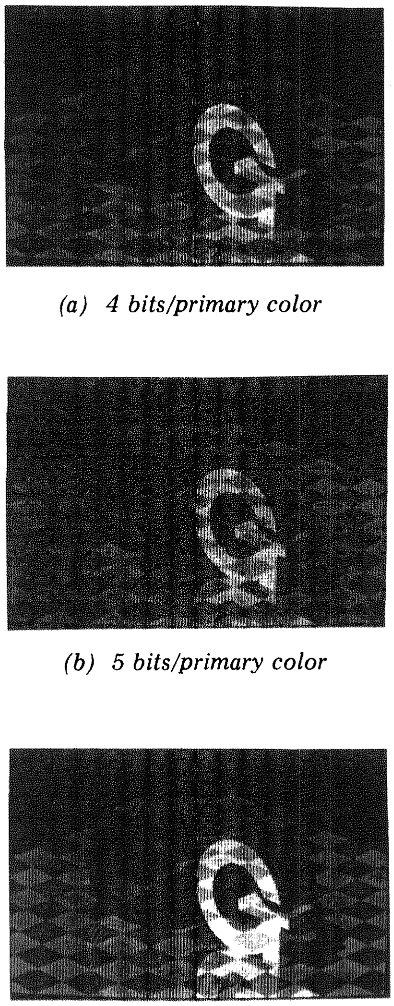
Appendix

In explanation of image quality, for example, color shift of ray traced opaque objects, the Mach band effect is demonstrated in **Fig. A** by altering the truncation number of significant upper bit of ray traced image data. A subtle difference in hue is intuitively discriminated between Fig. A (a) and (b). Each band in the original color photography is displayed under almost the constant color intensity. To the eye, however, the difference of brightness pattern in the color images seems to be emphasized particularly around the boundaries. It should be noted that the number of bit per primary color is defined here as the specified number for truncation from significant upper bit when using a SuperFrame module board for full color display such as 2^8 x 2^8 x 2^8 colors.



(a) 4 bits/primary color

(b) 5 bits/primary color

(c) 8 bits/primary color (in full color)

Fig. A *Bit number per primary color for color rendering and color shift of ray traced opaque objects in the case of 320 x 200 pixels*

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