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A new trial of estimating the equivalent characteristic of sound propagation for the actual road traffic noise field based on the mixing model of two idealized propagation environments

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It is well-known that the level fluctuations appearing in the road traffic noise environment are brought on by various type causes and can be roughly classified into two great divisions on the road traffic flow pattern and the surrounding sound field. Accordingly, if the equivalent models for the above two divisions—traffic flow and its surrounding field are well established in a flexible form of reflecting the complex situation of actual road traffic noise field, it will be possible to predict precisely and/or to control effectively the road traffic noise. In this paper, from the above point of view, a stochastic model of road traffic noise with a generalized Poisson type flow is first introduced with many informations of actual situation. Next, an equivalent type sound propagation model is newly proposed in a form of mixing of two idealized sound propagation environments—a free sound field and a diffused sound field. Furthermore, a methodological trial to estimate the above equivalent sound propagation characteristics of the compound environment by measuring the actual road traffic noise data is derived from the system-theoretical point of view. In this case of estimation, the well-known stochastic approximation method is used in order to overcome the difficulty of non-Gaussian and non-linear characteristics existing in the actual environmental noise phenomena. Finally, the legitimacy of this estimation method is confirmed by applying it to the traffic noise data actually observed in a big city.

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1. INTRODUCTION

Recently, an environmental noise pollution problem caused by road traffic flow has been aggravated with the consolidation and expansion of roadway systems. Furthermore, the level fluctuation appearing in the actual environment of road traffic noise is brought on by various type causes of fluctuation. It is obvious that the ultimate causes of fluctuation are originated to a variety of noise sources, i.e., uncertain behavior of individual vehicles, and additionally to the surrounding effect of the noise propagation characteristics. It is definitely necessary for engineers to obtain a better knowledge about future nuisance situation caused by road traffic noise before improving the traffic control system. For purpose of predicting precisely and controlling effectively the road traffic noise, the effectively equivalent models for traffic flow and sound propagation field must be first established. Especially, in almost of the previous studies on road traffic noise, it is very often assumed that the sound wave diverges hemispherically in a free sound field. Conversely, the road traffic noise under the actual complex sound propagation environment is not yet sufficiently studied. Accordingly, even if it is in an equivalent or approximate form of study, the actual noise propagation characteristics in road traffic noise system must be studied more positively. In general, the actual noise propagation paths between noise sources and an observer are too complex to be expressed in the form of an artificially simplified model or a pure physical model constructed from the base on the internal mechanism of sound propagation environment. Furthermore, because there are various types of actual sound propagation environment, a lot of models for them must be established individually in order to investigate each of them in detail only from the physical viewpoint. Therefore, a unified investigation or a systematic classification for the actual sound propagation environment can not be expected only from the above approach. On the other hand, the practical method of introducing an equivalent model which has the same characteristics as the actual sound propagation between the noise sources and an observation point is a powerful method in order to investigate the actual sound propagation environment especially from the systematical view point of study. As one of equivalent models, the actual surrounding sound field can be expressed to consist of mixed environment of two typically idealized fields with a free sound field and a diffused sound field.

In this paper, an effectively equivalent model which is applicable commonly to arbitrary sound propagation environment on the above mixed environment of road traffic noise system is proposed by considering two idealized geometrical situations of surrounding sound propagation—a series of buildings in a city and a rural area without any kind of building. Furthermore, a methodological trial for estimating the actual sound propagation characteristics by measuring the road traffic noise data is proposed.

More concretely, first, after generalizing the well-known road traffic noise model with a Poisson distribution type traffic flow in a free sound field, a new model for noise evaluation due to the road traffic under an arbitrary sound propagation environment is introduced. Especially, considering that the actual sound propagation does not take an idealized field such as a free sound field in a rural area and/or some diffused sound field in a series of tall building and mostly takes an intermediate propagation character between these two idealized sound fields, two models of new equivalent type on sound propagation are introduced. One is an additive compound propagation model which is assumed that the above two idealized sound field are mixed with an additive property, and another is a multiplicative compound propagation model which is modeled an intermediate propagation field by mixing the above two idealized sound fields with a multiplicative property. In the above two compound type sound propagation models, the mixed rate of two idealized sound fields is recognized as a proper parameter characterizing the actual complex sound field.

Moreover, by combining the above compound type sound propagation model with the generalized Poisson type traffic noise model (which is newly introduced in this paper), a systematical method to estimate this characteristic parameter of sound propagation by use of the statistical information obtained from the measurement data of the actual road traffic noise is proposed. In this case, both of non-Gaussian and non-linear characteristics of road traffic noise system must be considered, because the level fluctuations of road traffic noise exhibit various kinds of probability distribution form apart from the usual Gaussian distribution. Also the road traffic noise is in a complex, non-linear relation with a traffic flow and its propagation surrounding. Therefore, the stochastic approximation method is used in order to overcome the difficulty of such non-Gaussian and non-linear characteristics in case of parameter estimation. After all, a new type of on-line estimation method for the sound propagation parameter is derived with help of the above stochastic approximation method based on the sequential observation of actual road traffic noise.

Finally, though the main point of this paper is focussed on new establishment of methodology since it is in an earlier stage of study, the validity of this estimation method is basically confirmed by applying it on trial to the actual road traffic noise data experimentally observed in a big city.

2. ESTABLISHMENT OF PROBLEM

As shown in Fig. 1, the road under consideration is first divided into a suitable number of blocks P. Furthermore, two variables t and K are introduced as two kinds of time parameters, where K denotes

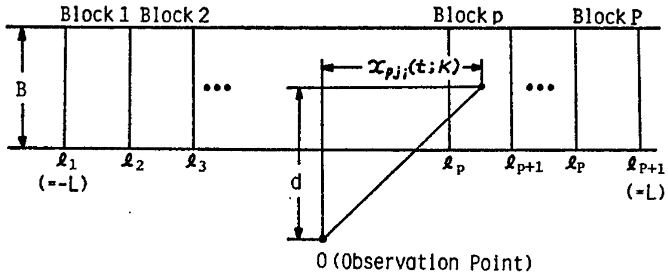

Fig. 1 Traffic noise model with straight road divided into P blocks.

the Kth time interval of observation and t indicates an instantaneous time in its interval. Then, by using the additive property of sound intensity, the instantaneous sound intensity at the fixed observation point O is

$$\left.\begin{aligned} I(t;\boldsymbol{a},K)&=\sum_{p=1}^{P} I_p(t;\boldsymbol{a},K)+v(t;K)\ ,\\ I_p(t;\boldsymbol{a},K)&=\sum_{j=1}^{J}\sum_{i=1}^{n_{pj}(t;K)} W_{pji}(t;K)\\ &\quad\cdot f(x_{pji}(t;K);\boldsymbol{a})\ , \end{aligned}\right\} \qquad (1)$$

where $n_{pj}(t;K)$ is the number of the jth type of vehicles observed in the Pth block at time t of the Kth time interval and $f(x_{pji}(t;K);\boldsymbol{a})$ denotes an propagation factor at the observation point O connected with an acoustic power $W_{pji}(t;K)$ generated by the ith vehicle with the jth type located at a position $x_{pji}(t;K)$ in the pth block along the road. Thus, $I_p(t;\boldsymbol{a},K)$ denotes the instantaneous sound intensity generated by all types of vehicles in the pth block, and $v(t;K)$ denotes the background noise. Furthermore, vector $\boldsymbol{a}$ denotes a parameter characterizing the actual sound propagation and will be concretely evaluated by some explicit expressions in the next section.

3. MODELING OF A ROAD TRAFFIC NOISE SYSTEM

In order to establish a mathematical model for the actual road traffic noise system, the following assumptions for basic parameters and statistics characterizing this traffic pattern are now introduced.

(i) The width of road with straight line considered here is B.

(ii) The vehicles which give a noise effect to the observation point O are located at arbitrary point in the road segment $[-L, L]$. Furthermore, this road segment is divided into P blocks and the pth block occupies a line segment $[l_p, l_{p+1}]$, and hereupon the length $l_{p+1}-l_p$ is enough long so that the road traffic noise in the pth block is statistically independent of the noise in other blocks.

(iii) The traffic flowing on the road is formed by J different types of vehicles.

(iv) The average number of vehicles flowing on the pth block in the Kth time interval of observation is $N_p(K)$ per unit time interval, where the average number of the jth type of vehicles is $N_{pj}(K)$ $(j=1, 2, \cdots, J)$.

(v) $n_p(t;K)$ is the random variable expressing the number of vehicles in the line road segment $[l_p, l_{p+1}]$ at time t in the Kth time interval and its probability distribution shows the Poisson type[1] as employed very often in the traffic model:

$$P(n_p(t;K))=\frac{1}{n_p(t;K)!}e^{-N_{op}(K)}\{N_{op}(K)\}^{n_p(t;K)}\ , \qquad (2)$$

where $N_{op}(K)=(l_{p+1}-l_p)N_p(K)/\bar{V}_p(K)$ and also $\bar{V}_p(K)$ denotes the average speed of vehicles flowing on the pth block.

(vi) The probability that $n_{pj}(t;K)$ vehicles with the jth type occupy in $n_p(t;K)$ vehicles is governed by the well-known multi-nomial distribution:

$$\begin{aligned} &P(n_{p1}(t;K),\ n_{p2}(t;K),\ \cdots,\ n_{pJ}(t;K)\,|\,n_p(t;K))\\ &\quad=\frac{n_p(t;K)!}{\prod_{j=1}^{J} n_{pj}(t;K)!}\prod_{j=1}^{J}\{\theta_{pj}(K)\}^{n_{pj}(t;K)}\ , \end{aligned} \qquad (3)$$

where $\theta_{pj}(K)$ denotes the mixture ratio of the jth type of vehicles defined by

$$\theta_{pj}(K)\triangleq N_{pj}(K)/N_p(K)\ . \qquad (4)$$

($N_p(K)$ is sufficiently large)

(vii) The location $x_{pji}(t;K)$ of the ith vehicle with the jth type in the pth block is governed by the probability density function $P(x_{pji}(t;K))$.

On the basis of the above assumptions, the moment generating function of the total noise intensity $I(t;\boldsymbol{a},K)$ can be easily expressed as follows:

$$g(s;\boldsymbol{a},K)=\left\{\prod_{p=1}^{P} g_p(s;\boldsymbol{a},K)\right\}g_v(s;K)\ , \qquad (5)$$

where $g_p(s;\boldsymbol{a},K)$ and $g_v(s;K)$ denote the moment generating functions of $I_p(t;\boldsymbol{a},K)$ and $v(t;K)$ respectively. Equation (5) is based on the natural assumption that the road traffic noise $I_p(t; \boldsymbol{a}, K)$ in the pth block ($p=1, 2, \cdots, P$) and the background noise $v(t; K)$ are statistically independent. Thus the nth order cumulant of $I(t; \boldsymbol{a}, K)$ observed in the Kth time interval is given by

$$\lambda_n(\boldsymbol{a}, K)=\sum_{p=1}^{P}\lambda_{pn}(\boldsymbol{a}, K)+\psi_n(K)\,, \tag{6}$$

where $\lambda_{pn}(\boldsymbol{a}, K)$ and $\psi_n(K)$ respectively denote the nth order cumulants of $I_p(t; \boldsymbol{a}, K)$ and $v(t; K)$ observed in the Kth time interval.

Next, let us derive concretely the above $\lambda_{pn}(\boldsymbol{a}, K)$ in an explicit form of expression reflecting the effect of internal mechanism of the traffic flow and the complex sound propagation. By using the well-known fundamental property of conditional probability distribution, the moment generating function $g_p(s; \boldsymbol{a}, K)$ can be shown as follows:

$$\begin{aligned}
g_p(s; \boldsymbol{a}, K) &\triangleq \langle \exp\{s\cdot I_p(t; \boldsymbol{a}, K)\}\rangle_{I_p(t;\, \boldsymbol{a},K)}\\
&=\left\langle\left\langle\left\langle \exp\left\{s\sum_{j=1}^{J}\sum_{i=1}^{n_{pj}(t;K)} W_{pji}(t;K)f(x_{pji}(t;K);\boldsymbol{a})\right\}\right\rangle_{W_{pji}(t;K),x_{pji}(t;K)\mid n_{pj}(t;K),n_p(t;K)}\right\rangle_{n_{pj}(t;K)\mid n_p(t;K)}\right\rangle_{n_p(t;K)}\\
&=\left\langle\left\langle \prod_{j=1}^{J}\left\langle \exp\{s W_{pji}(t;K)f(x_{pji}(t;K);\boldsymbol{a})\}\right\rangle^{n_{pj}(t;K)}_{W_{pji}(t;K),\ x_{pji}(t;K)\mid n_{pj}(t;K),\ n_p(t;K)}\right\rangle_{n_{pj}(t;K)\mid n_p(t;K)}\right\rangle_{n_p(t;K)}\,,
\end{aligned} \tag{7}$$

where $\langle\cdot\rangle_X$ denotes the averaging operation with respect to X, and $\langle\cdot\rangle_{X|Y}$ denotes the conditional averaging operation with respect to X when Y is set as a constant value. Moreover, by considering Eq. (3), the following equation can be obtained.

$$\begin{aligned}
&\left\langle \prod_{j=1}^{J}\left\langle \exp\{sW_{pji}(t;K)f(x_{pji}(t;K);\boldsymbol{a})\}\right\rangle^{n_{pj}(t;K)}_{W_{pji}(t;K),\ x_{pji}(t;K)\mid n_{pj}(t;K),\ n_p(t;K)}\right\rangle_{n_{pj}(t;K)\mid n_p(t;K)}\\
&\quad=\sum_{\sum_{j=1}^{J} n_{pj}(t;K)=n_p(t;K)} \frac{n_p(t;K)!}{\prod_{j=1}^{J} n_{pj}(t;K)!}\prod_{j=1}^{J}\left\{\theta_{pj}(K)\right\}^{n_{pj}(t;K)}\\
&\qquad\cdot\prod_{j=1}^{J}\left\langle \exp\{s W_{pji}(t;K)f(x_{pji}(t;K);\boldsymbol{a})\}\right\rangle^{n_{pj}(t;K)}_{W_{pji}(t;K),\ x_{pji}(t;K)}\\
&\quad=\left\{\sum_{j=1}^{J}\theta_{pj}(K)\left\langle \exp\{s W_{pji}(t;K)f(x_{pji}(t;K);\boldsymbol{a})\}\right\rangle_{W_{pji}(t;K),\ x_{pji}(t;K)}\right\}^{n_p(t;K)}\,.
\end{aligned} \tag{8}$$

After all, by taking Eq. (2) into consideration, $g_p(s; \boldsymbol{a}, K)$ can be derived as follows:

$$\begin{aligned}
g_p(s; \boldsymbol{a}, K) &= \sum_{n_p(t;K)=0}^{\infty}\frac{1}{n_p(t;K)!}e^{-N_{op}(K)}\{N_{op}(K)\}^{n_p(t;K)}\\
&\qquad\cdot\left\{\sum_{j=1}^{J}\theta_{pj}(K)\left\langle \exp\{s W_{pji}(t;K)f(x_{pji}(t;K);\boldsymbol{a})\}\right\rangle_{W_{pji}(t;K),\ x_{pji}(t;K)}\right\}^{n_p(t;K)}\\
&=\exp\{-N_{op}(K)\}\\
&\qquad\cdot\exp\left[N_{op}(K)\left\{\sum_{j=1}^{J}\theta_{pj}(K)\left\langle \exp\{s W_{pji}(t;K)f(x_{pji}(t;K);\boldsymbol{a})\}\right\rangle_{W_{pji}(t;K),\ x_{pji}(t;K)}\right\}\right]\\
&=\exp\{-N_{op}(K)\}\\
&\qquad\cdot\exp\left\{N_{op}(K)\sum_{j=1}^{J}\theta_{pj}(K)\sum_{n=0}^{\infty}\frac{s^n}{n!}\cdot\langle W_{pji}{}^n(t;K)\rangle_{W_{pji}(t;\,K)}\cdot\langle f^n(x_{pji}(t;K);\boldsymbol{a})\rangle_{x_{pji}(t;K)}\right\}\\
&=\exp\left\{N_{op}(K)\sum_{j=1}^{J}\theta_{pj}(K)\sum_{n=1}^{\infty}\frac{s^n}{n!}\langle W_{pji}{}^n(t;K)\rangle_{W_{pji}(t;\,K)}\cdot\langle f^n(x_{pji}(t;K);\boldsymbol{a})\rangle_{x_{pji}(t;K)}\right\}\,.
\end{aligned} \tag{9}$$

Thus, the nth order cumulant $\lambda_{pn}(\boldsymbol{a}, K)$ is given by

$$\lambda_{pn}(\boldsymbol{a}, K)=N_{op}(K)\sum_{j=1}^{J}\theta_{pj}(K) \cdot \langle W_{pji}{}^{n}(t;K)\rangle_{W_{pji}(t;K)}\cdot F_{pjn}(\boldsymbol{a}) , \quad (10)$$

$$F_{pjn}(\boldsymbol{a}) \triangleq \langle f^{n}(x_{pji}(t;K);\boldsymbol{a})\rangle_{x_{pji}(t;K)} = \int_{l_p}^{l_{p+1}} f^{n}(x_{pji}(t;K);\boldsymbol{a}) \cdot P(x_{pji}(t;K))\, dx_{pji}(t;K) . \quad (11)$$

The sound propagation characteristics is concretely reflected in this $F_{pjn}(\boldsymbol{a})$. Thus, in order to express the function $F_{pjn}(\boldsymbol{a})$ in an explicit form as a function of parameter vector $\boldsymbol{a}$, consider the sound propagation function $f(x_{pji}(t;K);\boldsymbol{a})$ in more detail.

Now consider the following two extremely idealized situations as typical models of sound propagation field. One is a free sound field (modelling rural situation in which there exists no building in both sides of road):

$$f_a(x;\boldsymbol{a})=1/2\pi(x^2+d^2) , \quad (12)$$

where d is the shortest distance between the observation point and the straight road (cf. Fig. 2(a)).

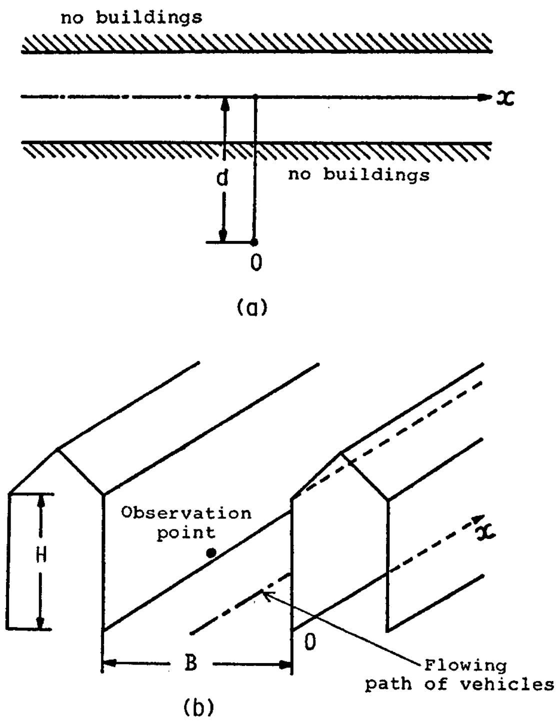

Fig. 2 Sound environmental model in two extremely idealized situations: (a) a free sound field, (b) some diffused sound field.

Another is some diffused sound field[2)] (modelling a typical city closely surrounded by the series of tall buildings in both sides of road):

$$f_b(x;\boldsymbol{a})=(1/2BH)\exp\{-|x|/H\} , \quad (13)$$

where B is the width of the straight road and H denotes the height of tall buildings close to both sides of the road (cf. Fig. 2(b)). Of course, the actual sound field shows an intermediate situation between the above two extremely idealized ones. Therefore, it is practically reasonable to express equivalently the actual propagation characteristics by combining successfully the above two idealized models. By combining Eqs. (12) and (13) in two fundamental forms, the compound type models of sound propagation field in the pth block can be expressed as follows as the first stage of this study:

(I) For the case of an additively mixed propagation model, it is assumed that the actual sound propagation field in the pth block occupies respectively the above two idealized models with a weighting rate α_p and $1-\alpha_p$ $(0\leq\alpha_p\leq 1)$. Accordingly, one can obtain

$$\begin{aligned} f_{\mathrm{I}}(x_{pji}(t;K);\boldsymbol{a}) &= \alpha_p f_a(x_{pji}(t;K);\boldsymbol{a}) + (1-\alpha_p) f_b(x_{pji}(t;K);\boldsymbol{a}) \\ &= \frac{\alpha_p}{2\pi\{x_{pji}{}^2(t;K)+d^2\}} + \frac{1-\alpha_p}{2BH_p}\exp\{-|x_{pji}(t;K)|/H_p\} . \end{aligned} \quad (14)$$

(II) For the case of a multiplicatively mixed propagation model, it is formed as an intermediate propagation field by mixing the above two idealized models in a multiplication form with power rate α_p and $1-\alpha_p$ $(0\leq\alpha_p\leq 1)$. Accordingly, one obtain

$$\begin{aligned} f_{\mathrm{II}}(x_{pji}(t;K);\boldsymbol{a}) &= \{f_a(x_{pji}(t;K);\boldsymbol{a})\}^{\alpha_p} \cdot \{f_b(x_{pji}(t;K);\boldsymbol{a})\}^{1-\alpha_p} \\ &= \frac{1}{\{2\pi(x_{pji}{}^2(t;K)+d^2)\}^{\alpha_p}} \frac{1}{\{2BH_p\}^{1-\alpha_p}} \cdot \exp\{-(1-\alpha_p)|x_{pji}(t;K)|/H_p\} . \end{aligned} \quad (15)$$

Of course, in a case with $\alpha_p=1$, Eqs. (14) and (15) express a free sound field, and in a case when $\alpha_p=0$, those equations express some diffused sound field. Therefore, in a case when $0<\alpha_p<1$, the above models can express the actual situation with intermediate sound propagation between a free sound field and a diffused sound field.

Thus, the conclusion of the above consideration can be summed up as follows:

$$\lambda_n(\boldsymbol{a},K)=\sum_{p=1}^{P}\lambda_{pn}(\boldsymbol{a},K)+\psi_n(K)\ , \tag{16}$$

$$\lambda_{pn}(\boldsymbol{a},K)=\sum_{j=1}^{J}Q_{pj}(K)\langle W_{pji}{}^n(t;K)\rangle_{W_{pji}(t;K)}\cdot F_{pjn}(\boldsymbol{a})\ , \tag{17}$$

$$Q_{pj}(K)\triangleq N_{op}(K)\theta_{pj}(K)\ . \tag{18}$$

Furthermore, by assuming that the location of vehicles in the pth block is governed by the uniform distribution in its block of road $[l_p, l_{p+1}]$ $(P(x_{pji}(t;K))=1/(l_{p+1}-l_p))$, the function $F_{pjn}(\boldsymbol{a})$ in Eq. (17) can be expressed concretely from Eqs. (11), (14) and (15), as follows:

(I) For the case of an additively mixed propagation model, one obtain (cf. Appendix 1)

$$\begin{aligned}F_{pjn}(\boldsymbol{a})&=\frac{1}{(2\pi d^2)^n 2L'}\Bigg[\alpha_p{}^n\Bigg\{\sum_{r=1}^{n-1}\frac{(2n-3)!!\,(2n-2r-2)!!}{(2n-2r-1)!!\,(2n-2)!!}\\&\quad\cdot\left\{\frac{l_{p+1}'}{(1+l_{p+1}'^2)^{n-r}}-\frac{l_p'}{(1+l_p'^2)^{n-r}}\right\}\\&\quad+\frac{(2n-3)!!}{(2n-2)!!}(\tan^{-1}l_{p+1}'-\tan^{-1}l_p')\Bigg\}\\&\quad+\frac{\pi^n}{\pi B'^n}\cdot\frac{(1-\alpha_p)^n}{H_p'^{n-1}}\{(1-e^{-n|l_{p+1}'|/H_p'})\\&\quad\cdot\mathrm{sgn}\,(l_{p+1}')-(1-e^{-n|l_p'|/H_p'})\,\mathrm{sgn}\,(l_p')\}\\&\quad+\sum_{r=1}^{n-1}\binom{n}{r}\frac{\pi^r}{B'^r}\frac{\alpha_p{}^{n-r}(1-\alpha_p)^r}{H_p'^r}\\&\quad\cdot\int_{l_p'}^{l_{p+1}'}\frac{e^{-r|y|/H_p'}}{(1+y^2)^{n-r}}dy\Bigg]\ ,\end{aligned} \tag{19}$$

where

$$\left.\begin{aligned}&B'\triangleq B/d\ ,\qquad H_p'\triangleq H_p/d\ ,\\&L'\triangleq(l_{p+1}-l_p)/2d\ ,\qquad l_p'\triangleq l_p/d\ ,\\&l_{p+1}'\triangleq l_{p+1}/d\ ,\\&\mathrm{sgn}\,(x)\triangleq\begin{cases}1 & (x>0)\\0 & (x=0)\\-1 & (x<0)\end{cases}\ .\end{aligned}\right\} \tag{20}$$

(II) For the case of a multiplicatively mixed propagation model, by substituting Eq. (15) into Eq. (11), one obtain

$$\begin{aligned}F_{pjn}(\boldsymbol{a})&=\frac{1}{(2\pi d^2)^n 2L'}\left(\frac{\pi}{B'H_p'}\right)^{n(1-\alpha_p)}\\&\quad\cdot\int_{l_p'}^{l_{p+1}'}\frac{e^{-[n(1-\alpha_p)/H_p']|y|}}{(1+y^2)^{n\alpha_p}}dy\ .\end{aligned} \tag{21}$$

In the above equations, $\boldsymbol{a}$ is the unknown parameter to be estimated. Hereupon, α_p and $H_p(p=1,2,\cdots,P)$ are chosen as parameters characterizing the sound propagation functions (14) and (15). That is, we can anew set $\boldsymbol{a}\triangleq[\alpha_1, H_1/d, \alpha_2, H_2/d, \cdots, \alpha_P, H_P/d]^T$. In Eqs. (16)~(18), the average number, $Q_{pj}(K)$, of the jth type vehicles following on the pth block in the Kth time interval and the nth order moment, $\langle W_{pji}{}^n(t;K)\rangle_W$, of acoustic power can be obtained experimentally from the measured data. Furthermore several order cumulants of the noise intensity at the observation point can be obtained by two ways. One is to calculate theoretically by using Eqs. (16)~(21), and another is to evaluate experimentally from the noise level data measured by sound level meter. An estimation algorithm of the parameter selected by minimizing the difference between the theoretical value and the experimental value on the several order cumulants of noise intensity will be derived in the next section.

4. ESTIMATION OF SOUND PROPAGATION PARAMETER BY USE OF STOCHASTIC APPROXIMATION METHOD

The estimation algorithm used in this section is based on the stochastic approximation method proposed by Robbins and Monro.[8] The stochastic approximation method is expected to have a consistent estimation in any cases with arbitrary non-Gaussian and non-linear characteristics appearing usually in the actual phenomena of road traffic noise. First, as a criterion for estimating the parameter $\boldsymbol{a}$, let us consider the difference between the probability density function established theoretically with use of the nth order cumulant in Eqs. (16)~(21) and the probability density function obtained experimentally from the actually observed data of noise intensity in a short time interval. Next, let us take the average of some weighting type integration of this difference in a long time interval as follows:

$$J(\boldsymbol{a})\triangleq\frac{1}{2}\langle J_1{}^2(\boldsymbol{a};K)|\boldsymbol{a}\rangle_K\ , \tag{22}$$

$$J_1(\boldsymbol{a};K)=\int_{-\infty}^{\infty}\varphi(I;\boldsymbol{\lambda}(\boldsymbol{a},K))\{P_1(I;\boldsymbol{\mu}(K))-P_2(I;\boldsymbol{\lambda}(\boldsymbol{a},K))\}^m dI\,, \tag{23}$$

where $\boldsymbol{\mu}(K)$ ($\triangleq[\mu_1(K), \mu_2(K), \cdots, \mu_N(K)]^T$) is a set of cumulants based on the actual noise intensity data observed experimentally in the Kth time interval, and $\boldsymbol{\lambda}(\boldsymbol{a},K)$ ($\triangleq[\lambda_1(\boldsymbol{a},K), \lambda_2(\boldsymbol{a},K), \cdots, \lambda_N(\boldsymbol{a},K)]^T$) denotes a set of cumulants given theoretically by Eqs. (16)$\sim$(21) reflecting the parameter $\boldsymbol{a}$. Furthermore $\varphi(I;\boldsymbol{\lambda}(\boldsymbol{a},K))$ and m denote an arbitrary type weighting function and a weighting constant parameter respectively, which operate as the weight in the performance index (23). The intensity of the actual road traffic noise exhibits various types of probability distribution form apart from the usual Gaussian distribution, while the probability distribution of noise level can be often approximate to the Gaussian distribution. Therefore, we can express the probability density function $P_1(I;\boldsymbol{\mu}(K))$ and $P_2(I;\boldsymbol{\lambda}(\boldsymbol{a},K))$ of noise intensity in a non-Guassian distribution form of Gram-charlier A type series expansion[4] which is universally applicable to arbitrary type distribution form:

$$P_1(I;\boldsymbol{\mu}(K))=\frac{1}{\sqrt{2\pi\mu_2(K)}}\exp\left[-\frac{1}{2}\left(\frac{I-\mu_1(K)}{\sqrt{\mu_2(K)}}\right)^2\right]\cdot\left\{1+\sum_{s=3}^{\infty}\xi_s(\boldsymbol{\mu}(K))H_s\left(\frac{I-\mu_1(K)}{\sqrt{\mu_2(K)}}\right)\right\}\,, \tag{24}$$

$$P_2(I;\boldsymbol{\lambda}(\boldsymbol{a},K))=\frac{1}{\sqrt{2\pi\lambda_2(\boldsymbol{a},K)}}\exp\left[-\frac{1}{2}\left(\frac{I-\lambda_1(\boldsymbol{a},K)}{\sqrt{\lambda_2(\boldsymbol{a},K)}}\right)^2\right]\cdot\left\{1+\sum_{s=3}^{\infty}\xi_s(\boldsymbol{\lambda}(\boldsymbol{a},K))H_s\left(\frac{I-\lambda_1(\boldsymbol{a},K)}{\sqrt{\lambda_2(\boldsymbol{a},K)}}\right)\right\}\,, \tag{25}$$

where $H_s(\cdot)$ denotes the Hermite polynomial. Taking the importance on the end of probability distribution curve like L_x ($x=10$, 90 etc.) in the actual noise evaluation and the facilities of analytical procedure into consideration, the following weighting function in Eq. (23) can be adopted as an example:

$$\varphi(I;\boldsymbol{\lambda}(\boldsymbol{a},K))\triangleq\sum_{r=0}^{R'}A_rH_r\left(\frac{I-\lambda_1(\boldsymbol{a},K)}{\sqrt{\lambda_2(\boldsymbol{a},K)}}\right)\,, \tag{26}$$

Hereupon, it must be noticed that the above weighting function is affected predominantly by the higher order statistics like 10 or 90 percentile points of noise level distribution in a case with high order of r, since the Hermite polynomial $H_r(x)$ takes some predominant value for large values of x. Furthermore, owing to the fact that when mean and variance of the probability density function $P_1(I;\boldsymbol{\mu}(K))$ take near values to those of $P_2(I;\boldsymbol{\lambda}(\boldsymbol{a},K))$, the difference $\{P_1(I;\boldsymbol{\mu}(K))-P_2(I;\boldsymbol{\lambda}(\boldsymbol{a},K))\}$ in Eq. (23) shows very often an odd function, it is reasonable to adopt only odd number order of the Hermite polynomial as the weighting function in Eq. (26) (i.e., $R'\triangleq 2R+1$, where R is a positive integer).

Next, by using Eqs. (24), (25) and (26), Eq. (23) in a special case with $m=1$ can be calculated concretely as follows (cf. Appendix 2):

$$J_1(\boldsymbol{a},K)=B_o(\boldsymbol{a},K)+\sum_{n=3}^{2R+1}B_n(\boldsymbol{a},K)\xi_n(\boldsymbol{\mu}(K))(\sqrt{\mu_2(K)})^n n!+\sum_{r=1}^{R}A_{2r+1}\xi_{2r+1}(\boldsymbol{\lambda}(\boldsymbol{a},K))(2r+1)!\,, \tag{27}$$

$$B_n(\boldsymbol{a},K)=\sum_{r=[n/2]}^{R}A_{2r+1}\binom{2r+1}{n}\frac{(\mu_1(K)-\lambda_1(\boldsymbol{a},K))^{2r+1-n}}{(\sqrt{\lambda_2(\boldsymbol{a},K)})^{2r+1}}\cdot\sum_{t=0}^{[(2r+1-n)/2]}\binom{2r+1-n}{2t}(2t-1)!!\cdot(\mu_1(K)-\lambda_1(\boldsymbol{a},K))^{-2t}\cdot(\mu_2(K)-\lambda_2(\boldsymbol{a},K))^t$$
$$(n=0, 3, 4, \cdots, 2R+1)\,. \tag{28}$$

As estimate $\hat{\boldsymbol{a}}$ is selected by minimizing the criterion function J in Eq. (22). Then, the following regression function $M(\boldsymbol{a})$ can be adopted to derive a recurrence algorithm based on the stochastic approximation method:

$$M(\boldsymbol{a})\triangleq\frac{\partial J(\boldsymbol{a})}{\partial\boldsymbol{a}}=\left\langle J_1(\boldsymbol{a},K)\frac{\partial J_1(\boldsymbol{a},K)}{\partial\boldsymbol{a}}\,\middle|\,\boldsymbol{a}\right\rangle=\boldsymbol{0}\,. \tag{29}$$

In above equation, the partial differentiation of $J_1(\boldsymbol{a},K)$ in terms of the unknown parameter $\boldsymbol{a}$ can be calculated as follows:

$$\frac{\partial J_1(\boldsymbol{a},K)}{\partial\boldsymbol{a}}=\Lambda(\boldsymbol{a},K)\boldsymbol{f}(\boldsymbol{a},K)\,, \tag{30}$$

where

$$\Lambda(\boldsymbol{a},K)\triangleq\frac{\partial\boldsymbol{\lambda}^T(\boldsymbol{a},K)}{\partial\boldsymbol{a}}\,, \tag{31}$$

$$
\begin{aligned}
f(\boldsymbol{a},K) &\triangleq \frac{\partial J_1(\boldsymbol{a},K)}{\partial \boldsymbol{\lambda}(\boldsymbol{a},K)} \\
&= \frac{\partial B_o(\boldsymbol{a},K)}{\partial \boldsymbol{\lambda}(\boldsymbol{a},K)} + \sum_{n=3}^{2R+1} \frac{\partial B_n(\boldsymbol{a},K)}{\partial \boldsymbol{\lambda}(\boldsymbol{a},K)} \xi_n(\boldsymbol{\mu}(K)) \\
&\quad \cdot (\sqrt{\mu_2(K)})^n n! \\
&\quad + \sum_{r=1}^{R} A_{2r+1} \frac{\partial \xi_{2r+1}(\boldsymbol{\lambda}(\boldsymbol{a},K))}{\partial \boldsymbol{\lambda}(\boldsymbol{a},K)} (2r+1)! \,. \qquad (32)
\end{aligned}
$$

Moreover, by using Eq. (28), the elements of $\partial B_n(\boldsymbol{a}, K)/\partial \boldsymbol{\lambda}(\boldsymbol{a}, K)$ ($n=0, 3, 4, \cdots, 2R+1$) are respectively given by

$$
\begin{aligned}
\frac{\partial B_n(\boldsymbol{a},K)}{\partial \lambda_1(\boldsymbol{a},K)} &= -\frac{1}{(\mu_1(K)-\lambda_1(\boldsymbol{a},K))^n} \\
&\quad \cdot \sum_{r=[n/2]}^{R} A_{2r+1} \binom{2r+1}{n} \frac{(\mu_1(K)-\lambda_1(\boldsymbol{a},K))^{2r}}{(\sqrt{\lambda_2(\boldsymbol{a},K)})^{2r+1}} \\
&\quad \cdot \sum_{t=0}^{[(2r+1-n)/2]} \binom{2r+1-n}{2t} (2t-1)!! \\
&\quad \cdot (2r+1-n-2t)(\mu_1(K)-\lambda_1(\boldsymbol{a},K))^{-2t} \\
&\quad \cdot (\mu_2(K)-\lambda_2(\boldsymbol{a},K))^t \,, \qquad (33)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial B_n(\boldsymbol{a},K)}{\partial \lambda_2(\boldsymbol{a},K)} &= -\frac{1}{(\mu_1(K)-\lambda_1(\boldsymbol{a},K))^n} \\
&\quad \cdot \sum_{r=[n/2]}^{R} A_{2r+1} \binom{2r+1}{n} \frac{(\mu_1(K)-\lambda_1(\boldsymbol{a},K))^{2r+1}}{(\sqrt{\lambda_2(\boldsymbol{a},K)})^{2r+3}} \\
&\quad \cdot \sum_{t=0}^{[(2r+1-n)/2]} \binom{2r+1-n}{2t} (2t-1)!! \\
&\quad \cdot (\mu_1(K)-\lambda_1(\boldsymbol{a},K))^{-2t} (\mu_2(K)-\lambda_2(\boldsymbol{a},K))^{t-1} \\
&\quad \cdot \left\{ t\lambda_2(\boldsymbol{a},K) + \frac{2r+1}{2} (\mu_2(K)-\lambda_2(\boldsymbol{a},K)) \right\} \,, \qquad (34)
\end{aligned}
$$

$$
\frac{\partial B_n(\boldsymbol{a},K)}{\partial \lambda_j(\boldsymbol{a},K)} = 0 \qquad (j=3, 4, \cdots, N) \,. \qquad (35)
$$

Thus, the estimation algorithm can be obtained from the above regression function as follows:

$$
\left.
\begin{aligned}
&\hat{\boldsymbol{a}}(K+1) = \hat{\boldsymbol{a}}(K) - \Gamma(K) \cdot \Lambda(\hat{\boldsymbol{a}}(K), K) \\
&\qquad \cdot J_1(\hat{\boldsymbol{a}}(K), K) \cdot f(\hat{\boldsymbol{a}}(K), K) \\
&\text{with} \\
&\Gamma(K) \triangleq \mathrm{diag}\,(\gamma_1(K), \gamma_2(K), \cdots, \gamma_{P+1}(K)) \,.
\end{aligned}
\right\} \qquad (36)
$$

The estimate $\hat{\boldsymbol{a}}$ is determined from a solution of this estimation algorithm by Eq. (36) in a sequential form of random variable, which hopefully converges to a solution of the regression function. Nevertheless, it can not show a uniqueness of the solution obtained from the regression function, because of difficulty due to the complexity of the function. However, it is reasonable to select a proper solution from various types of possible solutions calculated by using various initial values of $\hat{\boldsymbol{a}}$ under our consideration of its physical meaning. In the estimation algorithm given by Eq. (36), $\gamma_i(K)$ is a sequence of real numbers which must satisfy certain conditions with respect to the regression function in order to ensure a convergence of the algorithm. The contribution by Robbins and Monro showed previously that this convergence can be recognized under the following three constraint conditions:

$$
\gamma_i(K) > 0 \,, \quad \sum_{K=1}^{\infty} \gamma_i(K) \to \infty \,, \quad \sum_{K=1}^{\infty} \gamma_i^2(K) < \infty \,.
$$
$$
(i=1, 2, \cdots, P+1) \qquad (37)
$$

As well known, one of simple sequence of gain $\{\gamma_i(K)\}$ satisfying these three conditions is G_i/K (G_i: arbitrary constant).

5. EXPERIMENTAL CONSIDERATION

The estimation method proposed in section 4 is firstly applied on trial to the traffic noise data actually observed at the highway with almost free sound propagation characteristics. The situation for the measurement of actual road traffic noise is shown in Fig. 3. Road traffic noise data are measured at every five second by use of sound level meter of digital type with microphone located at the observation point O. The measured noise level data (dB value) are transformed into the noise intensity by use of the inverse logarithm transformation. Based on these noise intensity data, cumulants of noise intensity in the Kth time interval (this time interval is arbitrarily set to 250 s in our experiment) can be obtained. At the same time, the scope of traffic flow is continuously filmed by use of automatic camera synchronized with the sound level meter, and the average number of vehicles flowing on road in each short time interval is counted. Since the number of blocks P is originally arbitrary in the general theory in the previous sections, let us now consider only one block (i.e., $P=1$) in this section. Furthermore, for the purpose of obtaining the estimation of sound propagation with a high degree of accuracy, the data in a case when only light commercial vehicles

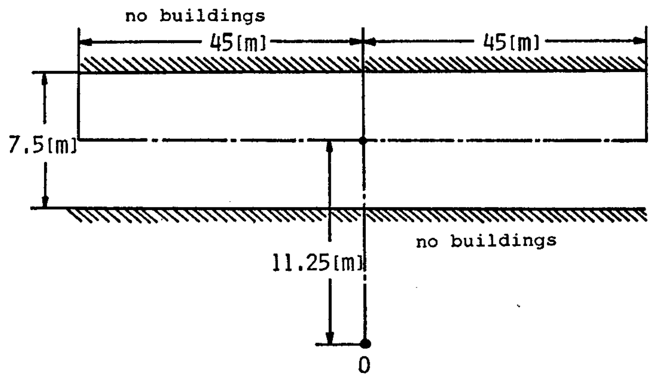

Fig. 3 Situation for the measurement of actual road traffic noise under free sound field.

flow are selected from total observation data on traffic flow.

As a convergence condition of the recurrence estimation algorithm, the following error function is used

$$e(K) \triangleq \|\hat{\boldsymbol{a}}(K)-\hat{\boldsymbol{a}}(K-1)\|/\|\hat{\boldsymbol{a}}(K-1)\| \tag{38}$$

where $\|\cdot\|$ denotes the norm with respect to a vector. A flow chart on the proposed estimation algorithm and the disposal of measurement data is shown in Fig. 4.

The estimated results of unknown parameter $\boldsymbol{a}$ are shown in Table 1 with several of initial values. In three cases with I-1, I-2 and I-3, the estimated results in the additive compound propagation model are shown, and three cases with II-1, II-2 and II-3 show the estimated results in the multiplicative compound propagation model. The initial value in the estimation of parameter α is tentatively set to 0.5 as the middle point in the possible existence region of a value α (i.e., $0\leq\alpha\leq 1$), and the initial value of parameter H/d is arbitrarily set to one-tenth of $\hat{\alpha}(0)$ as a trial. In a case with I-2, the final estimation values of the case I-1 are used especially as an initial values of estimation in this case. Furthermore, the cases with I-3 and II-2 show the estimation results when H/d is set to a constant value. The estimation result in the case when the road width B is set to 15 (m) by taking the distance from road edge to

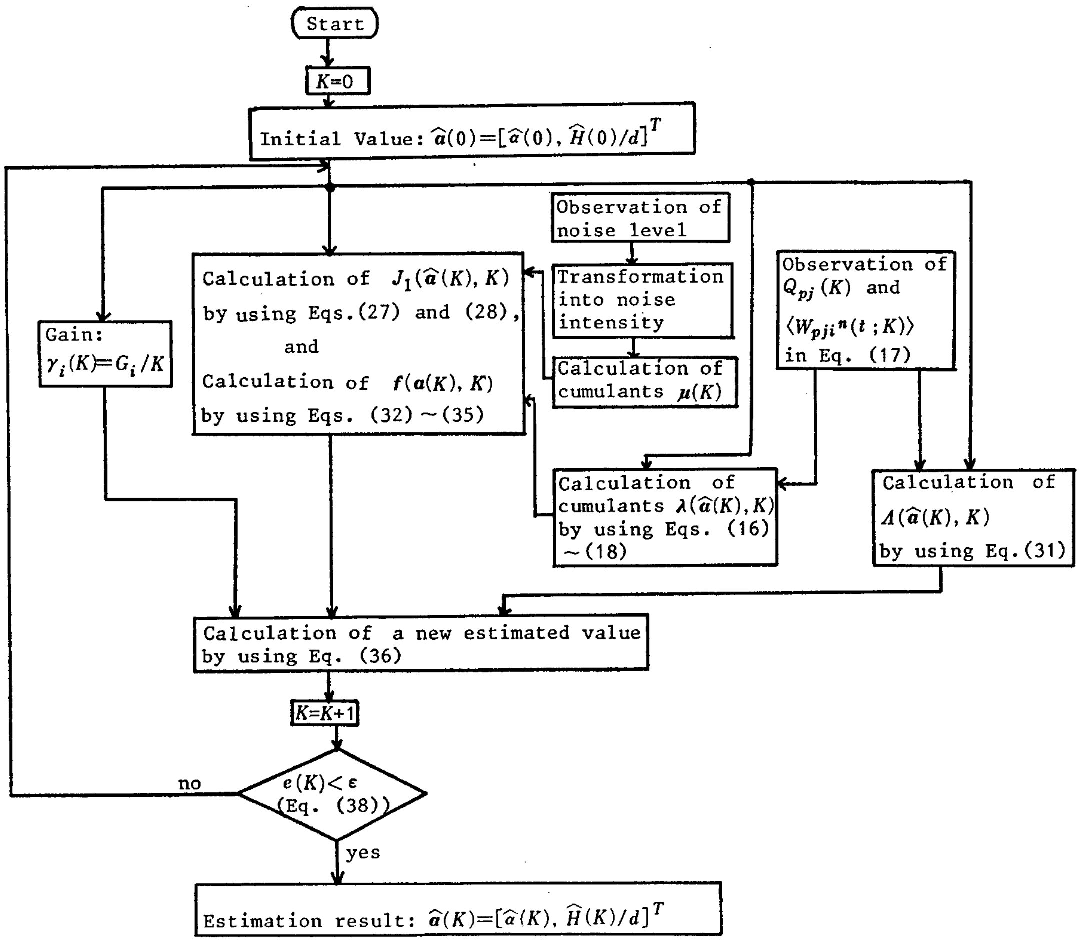

Fig. 4 A flow chart on estimation algorithm and the disposal of measurement data.

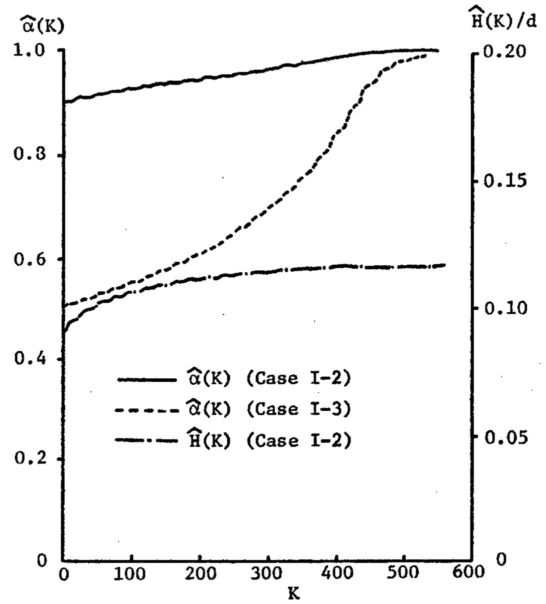

Fig. 5 Estimation process of parameter $\hat{\boldsymbol{a}}(K)$ based on the additive compound propagation model.

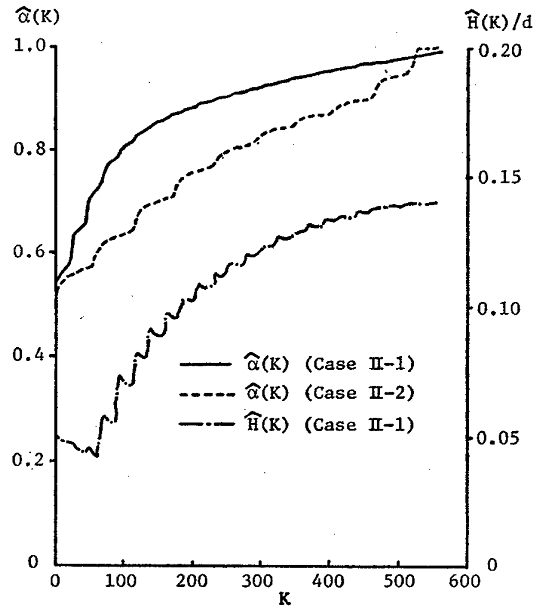

Fig. 6 Estimation process of parameter $\hat{\boldsymbol{a}}(K)$ based on the multiplicative compound propagation model.

Table 1 Estimated results of sound propagation characteristics by use of stochastic approximation method.

Case	Sound propagation model	Initial values		K	Estimated results		
		$\hat{\alpha}(0)$	$\hat{H}(0)/d$		$\hat{\alpha}(K)$	$\hat{H}(K)/d$	
I-1	Additive model	0.5	0.05	527	0.89	0.138	
I-2	Additive model	0.9	0.09	560	1.01	0.117	
I-3	Additive model	0.5	0.05	529	1.00	—	$\hat{H}(K)=\hat{H}(0)$ (constant)
II-1	Multiplicative model	0.5	0.05	562	0.99	0.140	
II-2	Multiplicative model	0.5	0.05	223	1.00	—	$\hat{H}(K)=\hat{H}(0)$ (constant)
II-3	Multiplicative model	0.5	0.05	1091	0.99	0.194	$B=15$ (m)

the observation point O into consideration is shown in case II-3. The estimation processes of parameter $\boldsymbol{a}$ in the additive compound propagation model and the multiplicative compound propagation model are shown in Figs. 5 and 6, respectively. These experimental results show that several estimation values of parameter α converge finally to an identical value near 1.0 as a proper value, because the actual sound field of noise measurement can obviously be regarded with almost free sound propagation characteristics.

Next, the estimation method is applied to another road traffic noise data measured in the actual case with semi-diffused sound field. The situation for the measurement of actual road traffic noise is shown in Fig. 7. Both sides of road are highly heaped up the soil and their tall walls are constructed by concrete. This environment seems to have the intermediate situation between a free sound field and a diffused sound field. Parameters α_p and H are estimated simultaneously by the same estimation algorithm of Eq. (36) (i.e., $\boldsymbol{a}=[\alpha_p, H]^T$). Figure 8 shows some estimation processes of parameter α_p by use of multiplicative compound propagation model in cases when initial values are set to be several values. Estimation values tend to converge to a value around 0.4$\sim$0.5 in spite of employing arbitrarily several initial values. This estimation result doesn't seem to be unreasonable judging from the situation of noise measurement shown in Fig. 7. Especially, Fig. 9 shows an estimation process of

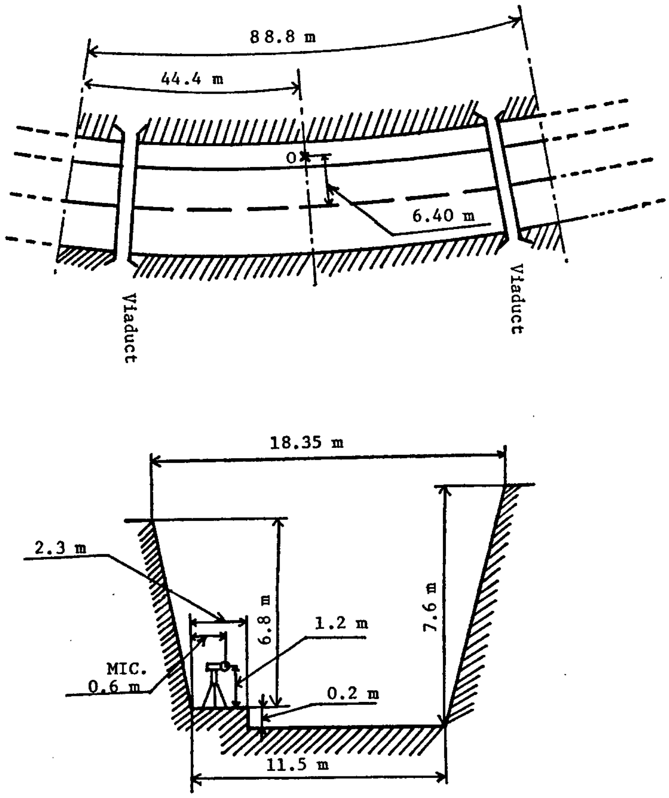

Fig. 7 Situation for the measurement of actual road traffic noise under semi-diffused sound field.

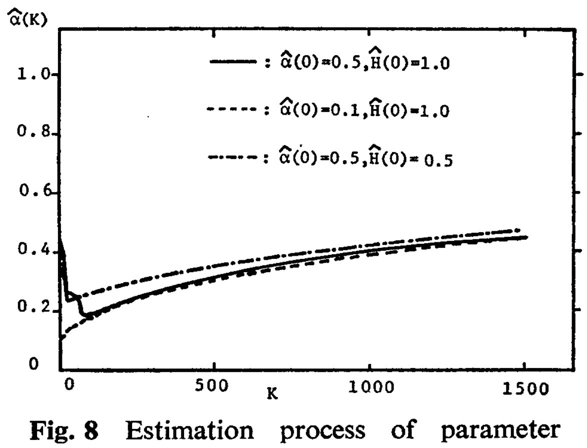

Fig. 8 Estimation process of parameter $\hat{\alpha}(K)$ based on the arbitrary choice of initial values (the multiplicative compound propagation model).

parameter H based on the multiplicative compound propagation model. Its estimation result $H=7.5$ m shows a good agreement with the actually measured value $H=7.6$ m calculated from the geographical situation shown in Fig. 7.

The above results show the legitimacy and the usefulness of the proposed estimation method.

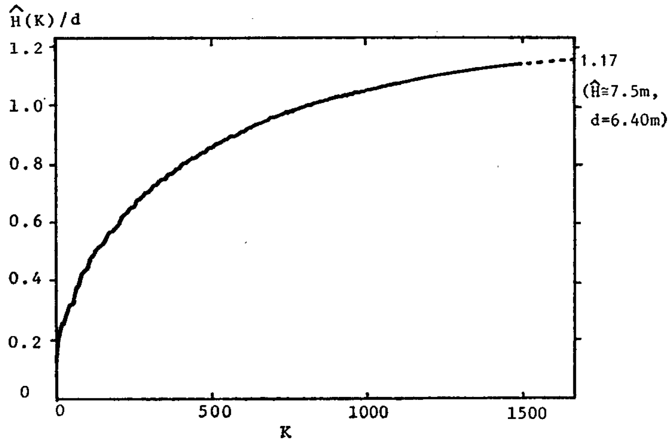

Fig. 9 Estimation process of parameter $\hat{H}(K)$ based on the multiplicative compound propagation model ($\hat{\alpha}(0)=0.1$, $\hat{H}(0)=1.0$).

6. CONCLUSION

First, a new model of road traffic noise which is applicable to the traffic flow with an arbitrary fluctuation pattern has been introduced by generalizing a well-known traffic noise model. And also, two kinds of models for the actual sound propagation field consisting of the intermediate environment between the high-rise buildings in a big city and the fields and gardens in a rural country have been introduced in the expression of functional form. Next, the cumulant of noise intensity at an observation point has been derived in the unified expression form with a sound propagation parameter characterizing the actual complex situation of the above intermediate environment. Furthermore, a new evaluation criterion for this parameter estimation has been established especially from a methodological viewpoint by newly introducing the difference idea between theoretical distribution form and experimental distribution form on the noise intensity, and then an algorithm to estimate the actual sound propagation parameter has been also derived concretely by use of the well-known stochastic approximation method under our newly introduced criterion. The legitimacy of the proposed estimation method has been experimentally confirmed on trial by applying it to the traffic noise data actually observed in a big city.

Though the main point of the present study is focussed on proposing a new trial of parameter estimation of the sound propagation especially from a methodological viewpoint since it is in an early stage of study, there remain many kinds of future researches of applying it to many other actual engineering cases, deriving a simplified estimation method for practical use based on this general theoretical approach, finding an appropriate values of gain parameter for speed convergency in stochastic approximation method and setting the optimum method of dividing the present road into a suitable number of blocks.

ACKNOWLEDGMENTS

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APPENDIX 1

By substituting Eq. (14) into Eq. (11), one can obtain

$$
\begin{aligned}
F_{pjn}(\boldsymbol{a}) &= \frac{1}{l_{p+1}-l_p}\int_{l_p}^{l_{p+1}} f^n(x_{pji}(t;K);\boldsymbol{a})\,dx_{pji}(t;K) \\
&= \frac{1}{l_{p+1}-l_p}\int_{l_p}^{l_{p+1}}\left\{\frac{\alpha_p}{2\pi(x_{pji}^2(t;K)+d^2)}\right. \\
&\quad \left.+\frac{1-\alpha_p}{2BH_p}\exp\left[-\frac{|x_{pji}(t;K)|}{H_p}\right]\right\}^n dx_{pji}(t;K) \\
&= \frac{1}{(2\pi d^2)^n\cdot 2L'}\cdot\int_{l_p'}^{l_{p+1}'}\left\{\frac{\alpha_p}{1+y^2}+\frac{\pi(1-\alpha_p)}{B'H_p'}\right. \\
&\quad \left.\cdot\, e^{-y/H_p'}\right\}^n dy \\
&= \frac{1}{(2\pi d^2)^n\cdot 2L'}\left[\alpha_p^{\,n}\int_{l_p'}^{l_{p+1}'}\frac{dy}{(1+y^2)^n}\right. \\
&\quad +\left\{\frac{\pi(1-\alpha_p)}{B'H_p'}\right\}^n\int_{l_p'}^{l_{p+1}'} e^{-ny/H_p'}\,dy \\
&\quad +\sum_{r=1}^{n-1}\binom{n}{r}\frac{\pi^r}{B'^r}\cdot\frac{\alpha_p^{\,n-r}(1-\alpha_p)^r}{H_p'^{\,r}} \\
&\quad \left.\cdot\int_{l_p'}^{l_{p+1}'}\frac{e^{-r|y|/H_p'}}{(1+y^2)^{n-r}}\,dy\right], \qquad \text{(A-1)}
\end{aligned}
$$

where B', H_p', L', l_p' and l_{p+1}' are defined by Eq. (20).

In Eq. (A-1), by adopting an integral formula[8]:

$$
\begin{aligned}
\int\frac{dx}{(x^2+c)^n} &= \sum_{r=1}^{n-1}\left[\frac{(2n-3)!!\,(n-r-1)!}{(2n-2r-1)!!\,(n-1)!}\right. \\
&\quad \left.\cdot\frac{x}{(2c)^r(x^2+c)^{n-r}}\right]+\frac{(2n-3)!!}{(n-1)!} \\
&\quad \cdot\frac{1}{(2c)^{n-i}}\cdot\frac{1}{\sqrt{c}}\tan^{-1}\frac{x}{\sqrt{c}} \quad (c>0),
\end{aligned}
\qquad \text{(A-2)}
$$

Eq. (19) can be derived.

APPENDIX 2

First, let us show the following theorem.

Theorem: The following formula on Hermite polynomial can be concluded:

$$
\begin{aligned}
H_n(ax+b) &= \sum_{s=0}^{n}\binom{n}{s}a^sH_s(x)\sum_{r=0}^{[(n-s)/2]}\binom{n-s}{2r} \\
&\quad \cdot (2r-1)!!\,b^{n-s-2r}(a^2-1)^r, \qquad \text{(A-3)}
\end{aligned}
$$

where Gauss' notation $[l]$ denotes the greatest integer less than l.

Proof: By use of the generating function on Hermite polynomial, first $H_n(x)$ can be expressed as follows[4]:

$$H_n(x)=\left[\frac{\partial^n}{\partial t^n}e^{tx-t^2/2}\right]_{t=0}. \qquad \text{(A-4)}$$

Thus, $H_n(ax+b)$ can be expressed as follows:

$$
\begin{aligned}
H_n(ax+b) &= \left[\frac{\partial^n}{\partial t^n}e^{t(ax+b)-t^2/2}\right]_{t=0} \\
&= \left[\frac{\partial^n}{\partial t^n}e^{tax-a^2t^2/2}\cdot e^{bt-(1-a^2)t^2/2}\right]_{t=0} \\
&= \sum_{s=0}^{n}\binom{n}{s}\left[\frac{\partial^s}{\partial t^s}e^{tax-a^2t^2/2}\right]_{t=0} \\
&\quad \cdot\left[\frac{\partial^{n-s}}{\partial t^{n-s}}e^{tb-(1-a^2)t^2/2}\right]_{t=0} \\
&= \sum_{s=0}^{n}\binom{n}{s}a^sH_s(x)\left[\frac{\partial^{n-s}}{\partial t^{n-s}}e^{tb-(1-a^2)t^2/2}\right]_{t=0},
\end{aligned}
\qquad \text{(A-5)}
$$

where the following Leibniz formula[7] is used in the derivation of Eq. (A-5):

$$
\begin{aligned}
\frac{\partial^n}{\partial x^n}(f(x)g(x)) &= \sum_{r=0}^{n}\binom{n}{r}\left(\frac{\partial^{n-r}}{\partial x^{n-r}}f(x)\right) \\
&\quad \cdot\left(\frac{\partial^r}{\partial x^r}g(x)\right). \qquad \text{(A-6)}
\end{aligned}
$$

Now, consider the last term of Eq. (A-5) in three cases with $a<1$, $a>1$ and $a=1$.

i) When $a<1$, one can obtain

$$\left[\frac{\partial^{n-s}}{\partial t^{n-s}}e^{tb-(1-a^2)t^2/2}\right]_{t=0}$$
$$=\left[\frac{\partial^{n-s}}{\partial t^{n-s}}\exp\left\{\sqrt{1-a^2}\,t\frac{b}{\sqrt{1-a^2}}-\frac{1-a^2}{2}t^2\right\}\right]_{t=0}$$
$$=(\sqrt{1-a^2})^{n-s}\cdot H_{n-s}\left(\frac{b}{\sqrt{1-a^2}}\right)$$
$$=(\sqrt{1-a^2})^{n-s}\sum_{r=0}^{[(n-s)/2]}(-1)^r\binom{n-s}{2r}\cdot(2r-1)!!\left(\frac{b}{\sqrt{1-a^2}}\right)^{n-s-2r}$$
$$=\sum_{r=0}^{[(n-s)/2]}\binom{n-s}{2r}(2r-1)!!\,b^{n-s-2r}(a^2-1)^r, \qquad \text{(A-7)}$$

where, in the derivation of the above equation, the following definition of Hermite polynomial is used

$$H_n(x)=\sum_{r=0}^{[n/2]}(-1)^r(2r-1)!!\binom{n}{2r}x^{n-2r}. \qquad \text{(A-8)}$$

ii) When $a>1$, one can obtain

$$\left[\frac{\partial^{n-s}}{\partial t^{n-s}}e^{tb-(1-a^2)t^2/2}\right]_{t=0}$$
$$=\left[\frac{\partial^{n-s}}{\partial t^{n-s}}\exp\left\{j\sqrt{a^2-1}\,t\frac{b}{j\sqrt{a^2-1}}+\frac{a^2-1}{2}t^2\right\}\right]_{t=0}$$
$$=(j\sqrt{a^2-1})^{n-s}H_{n-s}\left(\frac{b}{j\sqrt{a^2-1}}\right)$$
$$=(j\sqrt{a^2-1})^{n-s}\sum_{r=0}^{[(n-s)/2]}(-1)^r\binom{n-s}{2r}\cdot(2r-1)!!\left(\frac{b}{j\sqrt{a^2-1}}\right)^{n-s-2r}$$
$$=\sum_{r=0}^{[(n-s)/2]}\binom{n-s}{2r}(2r-1)!!\,b^{n-s-2r}(a^2-1)^r. \qquad \text{(A-9)}$$

iii) When $a=1$, Eq. (A-5) can be expressed as follows:

$$H_n(x+b)=\left[\frac{\partial^n}{\partial t^n}e^{t(x+b)-t^2/2}\right]_{t=0}$$
$$=\left[\frac{\partial^n}{\partial t^n}e^{tx-t^2/2}e^{bt}\right]_{t=0}$$
$$=\sum_{s=0}^{n}\binom{n}{s}\left[\frac{\partial^s}{\partial t^s}e^{tx-t^2/2}\right]_{t=0}\cdot\left[\frac{\partial^{n-s}}{\partial t^{n-s}}e^{bt}\right]_{t=0}$$
$$=\sum_{s=0}^{n}\binom{n}{s}H_s(x)b^{n-s}. \qquad \text{(A-10)}$$

Thus, the theorem is established from Eqs. (A-5), (A-7), (A-9) and (A-10).

Next, in Eq. (A-3), by setting

$$\left.\begin{aligned}&a=\sqrt{\mu_2(K)}/\sqrt{\lambda_2(\boldsymbol{a},K)},\\&x=(I-\mu_1(K))/\sqrt{\mu_2(K)},\\ \text{and}\\&b=(\mu_1(K)-\lambda_1(\boldsymbol{a},K))/\sqrt{\lambda_2(\boldsymbol{a},K)},\end{aligned}\right\} \qquad \text{(A-11)}$$

the following relation can be derived.

$$H_n\left(\frac{I-\lambda_1(\boldsymbol{a},K)}{\sqrt{\lambda_2(\boldsymbol{a},K)}}\right)$$
$$=H_n\left(\frac{\sqrt{\mu_2(K)}}{\sqrt{\lambda_2(\boldsymbol{a},K)}}\frac{I-\mu_1(K)}{\sqrt{\mu_2(K)}}+\frac{\mu_1(K)-\lambda_1(\boldsymbol{a},K)}{\sqrt{\lambda_2(\boldsymbol{a},K)}}\right)$$
$$=\sum_{s=0}^{n}\binom{n}{s}\left(\frac{\sqrt{\mu_2(K)}}{\sqrt{\lambda_2(\boldsymbol{a},K)}}\right)^s H_s\left(\frac{I-\mu_1(K)}{\sqrt{\mu_2(K)}}\right)\cdot\sum_{r=0}^{[(n-s)/2]}\binom{n-s}{2r}(2r-1)!!\cdot\left(\frac{\mu_1(K)-\lambda_1(\boldsymbol{a},K)}{\sqrt{\lambda_2(\boldsymbol{a},K)}}\right)^{n-s-2r}\cdot\left(\frac{\mu_2(K)-\lambda_2(\boldsymbol{a},K)}{\lambda_2(\boldsymbol{a},K)}\right)^r$$
$$=\frac{1}{(\sqrt{\lambda_2(\boldsymbol{a},K)})^n}\sum_{s=0}^{n}H_s\left(\frac{I-\mu_1(K)}{\sqrt{\mu_2(K)}}\right)\binom{n}{s}\cdot(\sqrt{\mu_2(K)})^s\sum_{r=0}^{[(n-s)/2]}\binom{n-s}{2r}(2r-1)!!\cdot(\mu_1(K)-\lambda_1(\boldsymbol{a},K))^{n-s-2r}(\mu_2(K)-\lambda_2(\boldsymbol{a},K))^r. \qquad \text{(A-12)}$$

Adopting Eq. (A-12) to the weighting function $\varphi(I;\boldsymbol{\lambda}(\boldsymbol{a},K))$, and using the orthogonal condition of Hermite polynomial, Eq. (23) in a special case with $m=1$ can be expressed as Eq. (27).

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