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Measure of complexity of wave motions and cross-correlation coefficient in reverberant fields

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In this paper, the complexity of wave motions has been formulated in order to more detailedly than previously consider the complexities of reverberant sound fields, and the difference of complexity between two separate points has been described in an attempt to find a measure of diffuseness in these fields. The reverberant fields are assumed to be sound fields where random plane waves propagate two- or three-dimensionally. Entropy H $(=h_2-h_1)$ and energy-entropy product K are used to define the complexity of wave motions, where h_1, h_2 are entropies by sound pressures p_1, p_2, respectively. Then H is compared with the cross-correlation coefficient R calculated from same sound pressures. Furthermore, K is obtained by considering energy dissipation with propagation, and its properties are discussed as well as those of H. The results show that the entropy of sinusoidal wave is in a complementary relationship with the cross-correlation coefficient; $H=1-R$, and that it may be convenient to measure since the cross-correlation coefficient is inseparable into two entropies. Either the entropy or the energy-entropy product is also found to be required at audio frequencies as a measure of the complexity of wave motions (i.e. approximately $K=A_0H$, $A_0=$const. for this case). Because of the lack of a quantity giving degrees of directional distribution, our introduced measure is as yet insufficient to accurately measure the diffuseness of sound fields, but it may be fairly near to the corrected measure.

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1. INTRODUCTION

Room acoustic criteria for evaluating various properties of sound fields have been investigated and measured by many researchers.[1-7] Although many arguments have been made for the diffuseness of sound fields since Sabine[8] and Eyring,[9] a measure fully formulating the concept of diffuseness has not been found yet. The reason for this is that the diffuseness of sound fields is stochastically defined at all positions in the fields. As an indirect method evaluating the diffuseness, reverberation time is frequently measured for multiple points in a room. There may be, of course, a measurable relationship between them. However, since the theory of reverberation is basically founded upon the total acoustic energy in the room, it is generally difficult to estimate the diffuseness from the reverberation times. In the preceeding paper,[10] the authors, as an approach to the concepts of reverberation and diffuseness, have introduced a concept similar to entropy and have considered macroscopic complexities of a reverberant sound field, from the contribution of the total energy to the vibrating components in the field. The complexity at every point in a complicated sound field, therefore, should be discussed by noting the positional dependence of diffuseness. A measure expressing the diffuseness may be required to contain information on density and direction of acoustic energy flow at a point. Presumably, the concept of cross-correlation is rather closely associated with that of diffuseness, but as the cross-correlation coefficient between two measuring points in the field is a relative quantity it is not the best criteria for estimating the diffuseness.

Hence in the present paper, the cross-correlation coefficients in reverberant fields are described first, and entropy and energy-entropy product concerning sound pressure at a point are defined so as to consider complexities of sound fields from a microscopic point of view. Then, the entropy is theoretically related to the cross-correlation coefficient, and superiority and inferiority among our introduced measures and cross-correlation coefficients are discussed for reverberant sound fields. The reverberant fields diffused two- and three-dimensionally are, here, assumed to be composed of myriadly random plane waves, as well made.

2. ENTROPY AND CROSS-CORRELATION COEFFICIENT

The cross-correlation coefficient R, for the sound pressures at two separate points in a reverberant sound field, was defined and measured by Cook *et al.*[11] Their definition is as follows:

$$R=\frac{1}{T}\int_0^T p_1(t)p_2(t)dt\bigg/\left\{\frac{1}{T}\int_0^T p_1^2(t)dt\cdot\frac{1}{T}\int_0^T p_2^2(t)dt\right\}^{1/2}, \qquad (1)$$

where $p_1(t)$, $p_2(t)$ are sound pressures at time t respectively, and T is time of observation. After their suggestion, many experiments and studies[12-14] have been made in the attempt to clarify the diffuseness of sound fields and the propagation direction of sound waves, using the above formula. Assuming that a plane progressive wave sinusoidally varying comes from the direction θ, we can observe the sound pressures p_1, p_2 at two points X_1, X_2 which r is the distance between, as shown in Fig. 1. For an accuracy, X_2 is designated as a reference point. The signs $\pm$ are the notations for representing the traveling waves,

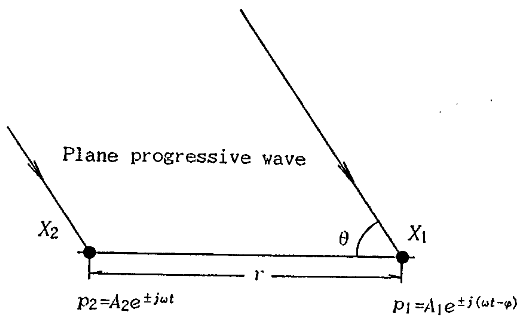

Fig. 1 Geometry of plane progressive wave and measuring points.

and correspond to double signs in the following equations. First, we start from the derivation of a complex cross-correlation coefficient between two points (distance r) for the plane progressive wave. In the case of single harmonic motion, Eq. (1) may be expanded to

$$R=\frac{1}{T}\int_0^T p_1^* p_2 dt\bigg/\left\{\frac{1}{T}\int_0^T |p_1^*|^2 dt\cdot\frac{1}{T}\int_0^T |p_2|^2 dt\right\}^{1/2}, \qquad (2)$$

without dividing into real and imaginary parts as is usually carried out, where * denotes a complex conjugate. Above equation is the result of direct insertion of two complex functions into Eq. (1). However, it can be replaced by complex coherence function[15] coh(ω) between Fourier transforms $p_1(\omega)$, $p_2(\omega)$ of p_1, p_2:

$$R=\mathrm{coh}(\omega)=\frac{S_{12}(\omega)}{\{S_{11}(\omega)S_{22}(\omega)\}^{1/2}}, \qquad (3)$$

where

$S_{12}(\omega)=\lim_{T\to+\infty}\frac{2\pi}{T}p_2^*(\omega)p_1(\omega)$: cross-spectral density function of p_1, p_2,

$S_{11}(\omega)=\lim_{T\to+\infty}\frac{2\pi}{T}|p_1(\omega)|^2$: autospectral density function of p_1,

$S_{22}(\omega)=\lim_{T\to+\infty}\frac{2\pi}{T}|p_2(\omega)|^2$: autospectral density function of p_2.

This is the reason that Eq. (2), representing the degree of coherence between p_1, p_2, is a cross-correlation coefficient at a single frequency. Now the entropy of the sound wave, by referring to the integral with respect to the time domain in Eq. (2), is written in the form:

$$h=-\frac{1}{T}\int_0^T\frac{p^*}{\left\{\frac{1}{T}\int_0^T|p^*|^2dt\right\}^{1/2}}\cdot\log_e\frac{p}{\left\{\frac{1}{T}\int_0^T|p|^2dt\right\}^{1/2}}dt, \qquad (4)$$

from the energy contribution of the waveform of sound pressure at time t to its total energy. The integration concerning with the logarithm in Eq. (4) is defined as the value calculated by consistently using only the principal value. The entropy h is expected to be one of the basic quantities describing a property at every point in a sound field. To elucidate the property of h dependent on the distance between two points (such as in the correlation method), we take the difference of entropies,

$$H=-\frac{1}{T}\int_0^T \frac{p_2^*}{\left\{\frac{1}{T}\int_0^T |p_2^*|^2 dt\right\}^{1/2}} \cdot \log_e \frac{p_2}{\left\{\frac{1}{T}\int_0^T |p_2|^2 dt\right\}^{1/2}} dt + \frac{1}{T}\int_0^T \frac{p_1^*}{\left\{\frac{1}{T}\int_0^T |p_1^*|^2 dt\right\}^{1/2}} \cdot \log_e \frac{p_1}{\left\{\frac{1}{T}\int_0^T |p_1|^2 dt\right\}^{1/2}} dt . \tag{5}$$

Substituting p_1, p_2 with $T=2\pi/\omega$ into Eq. (5), we obtain entropy H_1 for a plane progressive wave incident from a single direction:

$$H_1=\frac{1}{T}\int_0^T \{-\omega t \sin \omega t+(\omega t-\varphi)\sin(\omega t-\varphi)\}dt \pm \frac{j}{T}\int_0^T \{-\omega t \cos \omega t+(\omega t-\varphi)\cos(\omega t-\varphi)\}dt \tag{6}$$

$$=1-e^{\pm j\varphi}, \tag{7}$$

where

$\varphi=\pm kr\cos\theta$: phase difference between p_1, p_2,
k: wave number of plane wave.

The formula expressed in Eq. (7) can be regarded as the (complex) difference entropy of the plane wave at a single angular frequency ω. It may be useful for general signal analysis, since φ represents the phase difference between two sinusoidal signals at the same frequency not only in Fig. 1 but also in arbitrary signal analysis based on sinusoidal waves. In addition, from Eq. (2) or Eq. (3), the cross-correlation coefficient R_1 for the plane progressive wave is

$$R_1=e^{\pm j\varphi}, \tag{8}$$

whose real part apparently corresponds to the correlation coefficient by Cook *et al.*[11] Consequently, H_1 can be written using R_1 as

$$H_1=1-R_1 . \tag{9}$$

It is of interest, that the imaginary part of H_1 is equal to that of R_1.

Then, the entropy H_2 in two-dimensionally diffused sound fields is, from Eq. (6),

$$H_2=\frac{1}{2\pi}\int_0^{2\pi}(1-e^{\pm jkr\cos\theta})d\theta=1-J_0(kr), \tag{10}$$

due to superposition of random plane waves against θ-direction. $J_0(x)$ is the zero-order Bessel function of first kind. The imaginary part of H_2 is equal to zero as is that of R_2 ($=J_0(kr)$: cross-correlation coefficient in the sound fields). Evidently the relationship

$$H_2=1-R_2, \tag{11}$$

also holds.

Similarly, the entropy H_3 in three-dimensionally diffused sound fields, averaging H_1 with all directions (ψ, θ), becomes

$$H_3=\frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi}(1-e^{\pm jkr\cos\theta})\sin\theta\, d\psi d\theta = 1-\frac{\sin kr}{kr}, \tag{12}$$

in which the imaginary part vanishes. Also the imaginary part of R_3 ($=\sin kr/kr$: cross-correlation coefficient in the sound fields) is zero, while the relationship similar to Eq. (11) maintains. These entropies H_2, H_3 can be obtained from the frequently measured correlation coefficient.

3. ENERGY-ENTROPY PRODUCT

On the measure which expresses complexity of vibrations quantitatively, the authors previously stated that our energy-entropy product is a more general measure than entropy.[16] Here, we formulate this concept as a product of both the total energy of a waveform of sound pressure and its entropy, within definite time. The energy-entropy product in this case is expected to be a quantity adding degrees of energy change into the entropy, near the ceiling, floor, wall or at higher frequencies, and so on .... Thus the quantity in Fig. 1, multiplying the energies of waveform by the entropies determined from time records of two sound pressures, respectively, is written as

$$K=-\frac{1}{T}\int_0^T p_2^* \log_e \frac{p_2}{\left\{\frac{1}{T}\int_0^T |p_2|^2 dt\right\}^{1/2}} dt + \frac{1}{T}\int_0^T p_1^* \log_e \frac{p_1}{\left\{\frac{1}{T}\int_0^T |p_1|^2 dt\right\}^{1/2}} dt . \tag{13}$$

Hence, energy-entropy product K_1 for a plane progressive wave propagating in the direction θ, from the above and p_1, p_2, is

$$K_1 = A_2 - A_1 e^{\pm j\varphi}. \tag{14}$$

The solution of sound pressure for a plane progressive wave in ideal gas,[17] including an attenuation term, is

$$p = A_0 e^{-\alpha x} e^{\pm j(\omega t - kx)}. \tag{15}$$

If point X_2 is selected to be an origin, setting $x=0$, so that A_1, A_2 become

$$A_1 = A_0 e^{-\alpha r \cos\theta}, \tag{16}$$

$$A_2 = A_0, \tag{17}$$

respectively. The substitution of these equations into Eq. (14) yields a more concrete K_1,

$$K_1 = A_0(1 - e^{z\cos\theta}), \tag{18}$$

where

$$z = -\alpha r \pm jkr.$$

Therefore, in sound fields diffused two-dimensionally, energy-entropy product K_2 is obtained from K_1 as

$$K_2 = \frac{1}{2\pi}\int_0^{2\pi} A_0(1 - e^{z\cos\theta})d\theta = A_0\{1 - I_0(z)\}, \tag{19}$$

where $I_0(z)$ is the zero-order modified Bessel function of first kind.

Furthermore, energy-entropy product K_3 in sound fields diffused three-dimensionally, in the same manner as Eq. (12), is given by

$$K_3 = \frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi} A_0(1 - e^{z\cos\theta})\sin\theta\, d\psi d\theta = A_0\left(1 - \frac{\sinh z}{z}\right). \tag{20}$$

The considerations in 2.1 implies that we should discuss only the real parts of K_1, K_2, K_3 as well as those of the entropies, because their quantities are experimentally measurable.

4. NUMERICAL EXAMPLES AND DISCUSSION

Typical examples obtained as a function of kr have been plotted in Fig. 2 to demonstrate both the properties of the real parts of entropy (strong lines) and the cross-correlation coefficient (weak lines), with $\theta = 45°$ for H_1. If the distance between observations and/or the frequency of composite plane waves increase, except for H_1, R_1 the entropies approach 1, and the correlations zero while alternately increasing and decreasing around the respective limiting values. The correlation coefficient and the difference of our defined entropy have complementary properties, as is obvious from their theoretical consideration. The use of entropy has two advantages. First, the correlation coefficient may be estimated from the difference of entropy, however, the measured correlation coefficients may not be separated into the two entropies. This produces the advantage that for multiple points only the entropy at each point must be mea-

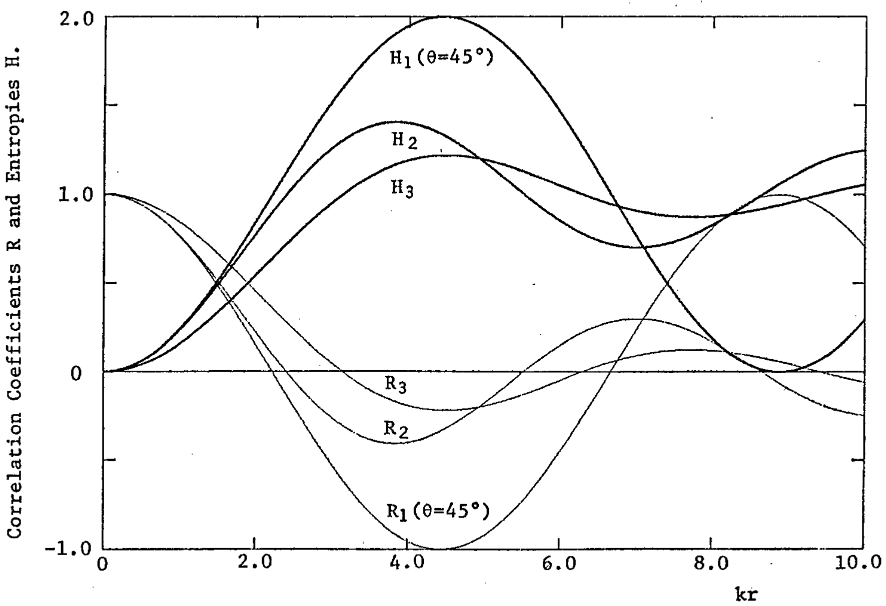

Fig. 2 Correlation coefficient and entropy in three kinds of sound fields (real parts).

sured rather than the correlation coefficient between each possible pair of points. Second, the equipment for measuring the entropies in a reverberant sound field may be simple* since sound pressures for two points are not required to be measured at the same time, as in the measurement of the correlation coefficient. The entropy is, therefore, a more essential quantity of the fields as compared to the cross-correlation coefficient.

In any case owing to energy dissipation with propagation, it is evident that the energy-entropy products K_2, K_3 approach 1 at smaller kr than the entropies H_2, H_3. Moreover, it is noted that the imaginary parts of K_2, K_3 are not equal to zero, although those of H_2, H_3 are zero. The entropies, normalized by their total energies according to the energy-entropy products, can not express the amplitudes and reactive components due to a propagation loss. Accordingly, the energy-entropy products are generally found to have more superior characteristics than the entropies. At audio frequencies, since the attenuation of the progressive wave in such sound fields is negligibly small, the relationship, $K_n = A_0 H_n$, proves to be approximately valid for each subscript ($n=1, 2, 3$). As a result, we only need either the entropy or the energy-entropy product, as a measure of complexity of wave motions in this case. However, the energy-entropy product introduced would become important at higher frequencies (ultrasonic) and for sound waves of finite amplitudes, near the central region of the room.

Finally, if the sound field has many frequency components by bandlimited noise excitation, H is calculated by integration of Eq. (6) with ω in the frequency band, and K may be given as an expansive quantity based on this entropy.

5. CONCLUSION

The measures of complexity of wave motions have been proposed, they and the cross-correlation coefficients have been related in reverberant sound fields. From the complementary characteristics of both difference of entropy and cross-correlation coefficient, it is concluded that the value of entropy expresses degrees of incoherence in the sound fields, and is convenient for discussing the diffuseness. The latter feature will enable us to build the measuring apparatus in handy sound level meter utilizing one microphone, and also to measure the cross-correlation coefficient by the level meter simultaneously. When the energy of a waveform is, however, extremely different between measuring points, the energy-entropy product should be employed instead of the entropy. Thus, the entropy and energy-entropy product are not mutual quantities, but may be nearly equivalent to kinds of informations at a point in a sound field. There is one possibility for finding a measure of diffuseness, that is, we might be able to include a factor of directional distributions into both the entropy and energy-entropy product. Equation (4) would be needed to be modified to perform it.

The entropy formulated in Eq. (6) will be applicable to signal analysis of many kinds, as will the energy-entropy product.

* This equipment could be constructed from a combination of a differentiator circuit, a clock pulse generator, an analog multiplier, and an integrator circuit, adjusted to each frequency.

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