J. Acoust. Soc. Jpn. (*E*) **14**, 1 (1993)

Determination of SEA total loss and coupling loss factors for built-up structures: Theory and numerical experiments

Shiping Zhang,*,† Takeshi Kihara,* and Shigeyasu Amada**

**Equipment and Marine Environment Division, Ship Research Institute, Ministry of Transport, 6–38–1, Shinkawa, Mitaka, Tokyo, 181 Japan*

***Department of Mechanical System Engineering, Gunma University, 1–5–1, Tenjin-cho, Kiryuu, 376 Japan*

(*Received 16 January 1992*)

It is the aim of this paper to develop a strategical method for the determination of SEA total loss factors (TLFs) and some of the coupling loss factors (CLFs) for built-up structures based on the recent development of *in situ* measurement techniques and the Statistical Energy Analysis theory. An obvious advantage of the method is that the determined TLFs would include the dissipation of coupling, and 'automatically' share the dissipation loss of the coupling to each coupled subsystem. The present strategy provides an applicable means for determination of total loss and some coupling loss factors of built-up structures for high frequency structure borne vibration and sound prediction. Satisfactory results of numerical experiments of the present strategy have been achieved.

Keywords: Statistical Energy Analysis, Total loss and coupling loss factors, Structure borne vibration and noise, Coupling dissipation

PACS number: 43. 20. Tb, 43. 40. At, 43. 40. Qi, 43. 40. Rj

1. INTRODUCTION

The Statistical Energy Analysis (SEA) developed by Lyon[1] and others over the past 30 years offers a powerful and strategical means for estimating the distribution of average vibrational energy levels and/or structure borne noise in a complex structure excited by broad band random forces or sound pressures. The SEA system is idealized into an assemblage of individual subsystems which are identified by classes or groups of resonant modes of a same nature.[2] The energy dissipated by a subsystem due to internal damping and radiation to the adjacent media is characterized by the loss factor (LF), which plays a role of proportionality constant between the stored vibrational energy and the dissipated energy of the subsystem. The energy transmission between spatially direct related subsystems is represented as to be proportional to the difference of their energy levels, the proportionality is the so-called coupling loss factor (CLF). SEA theory leads to the description of energy balance for each subsystem considered. The steady state energy balance of all the subsystems represented by a linear system of equations is indicated in terms of the loss and coupling loss factors. If the various coupling loss and loss factors are known then resulting vibrational energy levels in the structure elements and/or sound pressure levels in the acoustic spaces could be predicted for any arbitrary distribution of power injected into the system.

† On leave from the Jet Propulsion Dept., Beijing University of Aeronautics and Astronautics, Beijing, P. R. China.

Coupling loss factors have been dealt with by a lot of SEA researches, such as by Lyon and Scharton[3] or by Crandall and Lotz.[4,5] Conventionally, two basic strategies can be used for the evaluation of the coupling loss factors: wave-transmission method and natural frequency-shift method. The expressions of coupling loss factors between some typical subsystems are presented in the Lyon's book.[2] Recently, Wöhle, *et al.*[6] presented a generally valid method for calculating the coupling loss factors at a rectangular slab junction for incident bending, longitudinal and transverse waves. Langley[7,8] derived a general result of CLF between two connected subsystems which involves a space and frequency averaged Green function. Simmons[9] suggested that the SEA coupling loss factors can be calculated using the procedure described that is based on FEM. In this paper, it is supposed that most of (or some of) the coupling loss factors between the subsystems could be predicted with sufficient accuracy based on the theoretical calculations. A strategical method is developed for determining the built-up structure total loss factors (TLF) and some of the coupling loss factors which are difficult to be predicted with enough accuracy.

Loss factors of a SEA system represent the energy dissipations of every subsystems. However, the mechanism of dissipation identified as material damping and radiation to the adjacent media are very complex.[2,10] No reliable theoretical formulations are available for estimation of loss factors, the most commonly used method of determining the loss factor is to measure it. In the conventional SEA approach, LF is usually determined for each individual subsystem in the uncoupled condition from the structure, however, the dissipation can differ significantly from the uncoupled to the coupled condition.[2]

When considering a built-up structure, the dissipation introduced by the joints has also to be taken into account[2,10]: 1. For riveted joints due to the surface slip and plastic deformation of the overlapping surface, or due to viscous flow in the region between the metal surfaces. 2. For welded structures due to stress concentration and the damping of residual weld materials. Also, the change of the acoustical space into which the structure elements radiate energy and the alternating of edge conditions between coupled and uncoupled conditions may significantly influence the dissipations. As mentioned by Tratch,[10] not much progress has been made in the determination of loss factors for built-up structures since Ungar.[11] Ungar (1973)[11] pointed out the importance of the effects of structural joints on the damping of built-up structures, and concluded that the greatest need for future work exists with respect to the damping (and in turn loss factors) of high-frequency vibration of built-up structures.

It is the aim of this paper to develop an efficient strategical method for the determination of SEA total loss factors and some of the coupling loss factors for built-up structures based on the recent development of *in situ* measurement techniques[12,13] and SEA theory. The power injection method[12] can be used for the measurement of input power and the steady state vibration energy level of the subsystem. In case of determining total loss factors, for a system possesses N subsystems, only N times separate measurements is needed for determining the ratio between input power and vibrational energy of the subsystem into which power is injected. Based on the *in situ* determination of the ratios of input powers to the energy levels as well as the SEA linear system power balance equations, N nonlinear equations can be formulated with N unknown total loss factors of the system. Solving the N nonlinear equations, the N total loss factors could be determined. The accuracy of each determined total loss factors is not influenced much by the small errors of the measurements of the input power and the vibrational energies. The strategy is tested with some numerical examples of Woodhouse and Hodges[14,15] and compared with their results which was assumed to be based on N^2 times measurements. Satisfactory results of numerical experiments of the present strategy have been achieved. Experiments for the determination of total loss factors of some simple built-up structures with the present strategy are being performed in the Institute.

2. OUTLINE OF DETERMINATION OF SEA BASIC VARIABLES

Lyon [Ref. 2), see p. 217] suggested that experimental determination of the loss and coupling loss factors may be done by measuring resulting energy levels in the various parts of the structure under known power injection into each of the subsystems to obtain the necessary N^2 equations. Then the power balance equations may be inverted to determine *in situ* the coupling loss and loss factors. Of course, this method could be used for determination of total loss factors if the coupling loss factors are known, and only N times measurements of every subsystem energies under the known power injection into one subsystem are necessary. However, the resulting energies of some subsystems may be practically not sensitive to the input power to a subsystem, especially those subsystems far apart from the power injected subsystem, and the accuracy of the parameters are influenced by every small errors of energy measurements. As observed by Lyon,[2] this method has had limited success, small errors in measured energy and input power values may produce negative values of parameters which is physically impossible.

Craik[16] presented a means for calculating the total loss factor of wall and floors supposed that the material properties of the surrounding substructures are known.

Sun and Richards[17] derived a formula for estimating 'total loss factor' of a structure under the assumption of only one of the substructures is directly excited by external forces. However, a questionable concept was made, that was, taking the ratio of input power to the energy, $P_i/\omega E_i$, as the 'total loss factor.'

The measurement of mechanical input power from a vibration generator to a structure may be implemented in different ways.[12,13,18,19] Ottesen and Vigran[12] described a method for measuring power input to a structure by direct analogue multiplication and integration of force and velocity signals. The influence of phase errors is controlled through measurement of both the real and imaginary power components. Swift,[19] Bies and Hamid[13] also described a successful power injection measuring device, which measured the force and acceleration at the point of injection and the phase between these quantities. The power input was obtained from force and acceleration signals which were passed through parallel one-third octave filters. However, Fahy[20] suggested that power injected at a single point will result in modes which are not statistically independent, a fundamental requirement of SEA. This problem was solved by Bies and Hamid,[13] they showed that the simple expedient of injecting power at several points of the subsystem chosen at random may sufficiently well approximate the requirement of modal statistical independence.

Bies and Hamid[13] used the inversion of the linear power balance equations to determine the plate loss and the coupling loss factors *in situ*. To accomplish the determination power was injected and measured sequentially at five points chosen at random on each plate to ensure effective statistical independence of modes. In each case the response of both plates was measured at ten randomly chosen points and mean values of response and injected power were determined. Good agreement was obtained between predicted and measured results. However, the power balance equations inversion method is more appropriate for a test structure than a real complex structure with many subsystems, that is, not only because the results are sensitive to small errors in the measurements but also because the expense of a great increase in number of measurements.

Recently, Clarkson and Ranky[21,22] presented a further development of the experimental technique. They showed that for most structural applications bandwidth of one-third octave is too wide and a more suitable bandwidth is a constant 100 Hz (or 500 Hz for honeycomb structures.[20]) The results showed that when the decay is linear (on a logarithmic scale) the energy method and the decay method give very similar results. However, when the decay is not linear, the energy method gives a result which yields a better estimate of the band average vibration level.

The above outlined pieces of experimental work have established the feasibility of measuring reliably the input power and response vibrational energies *in situ*. This paper based on there works propose a strategy for the determination of built-up structure total loss and coupling loss factors *in situ*.

3. DETERMINATION OF TOTAL LOSS FACTORS

The SEA power balance equations for a system with N subsystems can be written as follows:

$$P_i=\omega\left(\sum_{\substack{j=1\\j\neq i}}^{N}\eta_{ij}+\eta_i\right)E_i-\omega\sum_{\substack{j=1\\j\neq i}}^{N}\eta_{ji}E_j\,,\quad (i=1,2,\dots,N) \tag{1}$$

where P_i and E_i are the time averaged power input to the ith subsystem and the energy stored in it, respectively. ω is the center frequency of the frequency band considered. η_i is the internal loss factor of the ith subsystem. The coupling loss factors η_{ij} are related by the consistency equations[2]

$$n_i\eta_{ij}=n_j\eta_{ji}\,, \qquad (2)$$

where n_i and n_j are the modal densities of the ith and jth subsystems, respectively. Equation (1) represents that the SEA approach is a set of linear equations which establish the relations between input power P_i and the energies of each subsystem. Equation (1) can be indicated in matrix form as:

$$\begin{bmatrix} \eta_{11} & -\eta_{21} & -\eta_{31} & \cdots & -\eta_{N1} \\ -\eta_{12} & \eta_{22} & -\eta_{32} & \cdots & -\eta_{N2} \\ -\eta_{13} & -\eta_{23} & \eta_{33} & \cdots & -\eta_{N3} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -\eta_{1N} & -\eta_{2N} & -\eta_{3N} & \cdots & \eta_{NN} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \\ \cdots \\ E_N \end{Bmatrix} = \begin{Bmatrix} P_1/\omega \\ P_2/\omega \\ P_3/\omega \\ \cdots \\ P_N/\omega \end{Bmatrix} \qquad (3)$$

or

$$[S]\{E\}=\{P\}\,, \qquad (4)$$

where the total loss factor η_{ii} is the sum of all the coupling loss factors and the internal loss factor:

$$\eta_{ii}=(\eta_i+\sum_{\substack{j=1\\j\neq i}}^{N}\eta_{ij})\,.$$

If the various coupling loss factors could be estimated with theoretical prediction, the question is how to determine the total loss factors η_{ii} of the built-up structures, and the determined TLFs would include the effect of the dissipation of the couplings. The current strategy is that injecting power to a subsystem in certain frequency band and measuring the input power as well as the resulting energy of the subsystem to which power is injected. Then the ratio of input power to the energy, $P_i/\omega E_i$, (let $\alpha_{ii}=P_i/\omega E_i$) could be determined. Under the condition of $P_j=0$, except $j=i$ ($j=1, 2, \ldots, N$), with the determined α_{ii}, Eq. (3) can be rewritten as:

$$\begin{bmatrix} \eta_{11} & -\eta_{21} & \cdots\cdots\cdots & -\eta_{N1} \\ -\eta_{12} & \eta_{22} & \cdots\cdots\cdots & -\eta_{N2} \\ \cdots & \cdots & \cdots\cdots\cdots & \cdots \\ -\eta_{1i} & -\eta_{2i} & \cdots\eta_{ii}-\alpha_{ii}\cdots & -\eta_{Ni} \\ \cdots & \cdots & \cdots\cdots\cdots & \cdots \\ -\eta_{1N} & -\eta_{2N} & \cdots\cdots\cdots & \eta_{NN} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ \cdots \\ E_i \\ \cdots \\ E_N \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ \cdots \\ 0 \end{Bmatrix} \qquad (5)$$

Therefore, the determinant of the matrix $[S(\alpha_{ii})]$ (Let $[S(\alpha_{ii})]$ denote matrix of $[S]$ with η_{ii} minus α_{ii}) must be zero to ensure non-zero $\{E\}$, *i.e.*:

$$\det([S(\alpha_{ii})])=0\,. \quad (i=1, 2, \ldots, N) \qquad (6)$$

With the above mentioned method, α_{ii} could be determined experimentally for each subsystem which has an unknown total loss factor. N simultaneous nonlinear equations indicated by Eq. (6) are then readily to be established with N unknown TLFs. Solving Eq. (6) with some iterative method or minimization method (see, for example, Ref. 23)), N unknown TLFs could be determined. The determined total loss factor with the current strategy can be deduced to be including the effect of the dissipation of the couplings to the subsystem, since the measured energy is related not only to the dissipation of the subsystem itself but also the coupling dissipation. It may be practically a very effective means of identifying the dissipation of couplings and sharing the dissipation to each subsystem intelligently with the present strategy, since it is very difficult to evaluate the dissipation of a coupling and it is even more difficult to share the coupling dissipation to each subsystem of a coupled structure. Also it is possible with the present method to determine the loss factor of a substructure which has a radiation loss with adjacent media of water or oil.

As a matter of fact, the measured α_{ii} should be the inverses of the diagonal elements of the inverse matrix of $[S]$. It is not difficult to proof that Eq. (6) is true with the Cramer's rule (see, for example, Ref. 24), p. 396).

One of the obvious advantages of the present strategy is when only one TLF is unknown, the accuracy only depends on the measurement of α_{ii}. However, with the method used by Bies and Hamid[18] the accuracy is influenced by all the measurements of the energies of the subsystems coupled to the considered subsystem.

If the energy from the direct power input of a subsystem is dominant, even if there may be some small errors of the predicted coupling loss factors, the TLF determined with the present strategy can give a good evaluation of the energy, since the TLF is determined based on the measurements of input power to the subsystem and resulting energy.

4. DETERMINATION OF COUPLING LOSS FACTORS

As it may be the case, some of the coupling loss factors are very difficult to be predicted with enough accuracy for built-up structures or are to be verified for the purpose of confirmation of the predicted results then it may be desirable to measure them *in situ*. Following the strategy mentioned above, if the coupling loss factors between i and j subsystems are unknown, one can inject power to subsystems i, then measure the input power and the energy of subsystems j. Therefore, the ratio of power to energy $P_i/\omega E_j$ could be determined. Let the power energy ratio $P_i/\omega E_j$ be α_{ij} and thus, following equation is readily to be established.

$$\begin{vmatrix} \eta_{11} & -\eta_{21} & \cdots\cdots & -\eta_{N1} \\ -\eta_{12} & \eta_{22} & \cdots\cdots & -\eta_{N2} \\ \cdots & \cdots & \cdots\cdots & \cdots \\ -\eta_{1i} & -\eta_{2i} & \cdots -\eta_{ji}-\alpha_{ij} \cdots & -\eta_{Ni} \\ \cdots & \cdots & \cdots\cdots & \cdots \\ -\eta_{1N} & -\eta_{2N} & \cdots\cdots & \eta_{NN} \end{vmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ \cdots \\ E_i \\ \cdots \\ E_N \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ \cdots \\ 0 \end{Bmatrix} \tag{7}$$

Therefore ([$\boldsymbol{S}(\alpha_{ij})$] denote matrix of [$\boldsymbol{S}$] with $-\eta_{ji}$ minus α_{ij}),

$$\det([\boldsymbol{S}(\alpha_{ij})])=0. \tag{8}$$

Of course, the determination of α_{ij} can be made at the same time of injecting power to i subsystem to determine α_{ii}, so as to simplify the experimental procedure. Vice versa the α_{ji} could be determined by injecting power to j subsystem, and following equations could be resulted

$$\begin{vmatrix} \eta_{11} & -\eta_{21} & \cdots\cdots & -\eta_{N1} \\ -\eta_{12} & \eta_{22} & \cdots\cdots & -\eta_{N2} \\ \cdots & \cdots & \cdots\cdots & \cdots \\ -\eta_{1j} & -\eta_{2j} & \cdots -\eta_{ij}-\alpha_{ji} \cdots & -\eta_{Nj} \\ \cdots & \cdots & \cdots\cdots & \cdots \\ -\eta_{1N} & -\eta_{2N} & \cdots\cdots & \eta_{NN} \end{vmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ \cdots \\ E_j \\ \cdots \\ E_N \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ \cdots \\ 0 \end{Bmatrix} \tag{9}$$

Thus,

$$\det([\boldsymbol{S}(\alpha_{ji})])=0. \tag{10}$$

With Eqs. (8) and (10), two additional equations are established for two unknowns. Including these equations with unknown coupling loss factors to Eq. (6), the simultaneous solving of these equations will give the results of the CLFs.

It can be seen from the strategy described above that the determination of the TLFs and the CLFs not depends on the mode count, this may effectively simplify the experimental procedure.

5. NUMERICAL EXPERIMENTS

Matrix (11) is an numerical example used by Woodhouse and Hodges[14,15] as a 'measured' inverse SEA matrix, which was assumed to be symmetrical with the application of Eq. (2).

$$\begin{pmatrix} 1.0 & 0.6 & 0.9 \\ 0.6 & 1.0 & 0.5 \\ 0.9 & 0.5 & 1.0 \end{pmatrix} \tag{11}$$

The inverse of this is:

$$\begin{pmatrix} 6.25 & -1.25 & -5.00 \\ -1.25 & 1.58 & 0.33 \\ -5.00 & 0.33 & 5.33 \end{pmatrix} \tag{12}$$

The term (2, 3) entry 0.33, which is positive, prevents this being in the correct form. With a numerical approximation method,[14] following acceptable approximate SEA matrix was resulted:

$$\begin{pmatrix} 6.28 & -1.03 & -4.97 \\ -1.03 & 1.71 & 0.00 \\ -4.97 & 0.00 & 5.34 \end{pmatrix} \tag{13}$$

which has an inverse as:

$$\begin{pmatrix} 0.97 & 0.60 & 0.89 \\ 0.60 & 0.96 & 0.56 \\ 0.89 & 0.56 & 1.01 \end{pmatrix} \tag{14}$$

In this paper, the coupling loss factors of the resulted SEA matrix (13) are supposed to be determined with theoretical prediction for a SEA system with three subsystems. The total loss factors of the system are unknown, which should be determined in built-up conditions. Thus, following matrix, [$\boldsymbol{S}$], with unknown TLFs should be dealt with:

$$[\boldsymbol{S}]=\begin{pmatrix} T_{11} & -1.030 & -4.970 \\ -1.030 & T_{22} & 0.000 \\ -4.970 & 0.000 & T_{33} \end{pmatrix} \tag{15}$$

Suppose we inject input power to each of the subsystem, and determined the power energy ratios $\bar{\alpha}_{ii}=n_iP_i/\omega E_i$ ($i=1$, 2, 3), which may be taken as the inverses of the diagonal elements of 'the measured' matrix (11), *i.e.*, $\bar{\alpha}_{11}=1.00$, $\bar{\alpha}_{22}=1.00$ and $\bar{\alpha}_{33}=1.00$. Then Eq. (6) can be used as a set of three simultaneous nonlinear equations with three unknown TLFs:

$$\det([\boldsymbol{S}(\bar{\alpha}_{11})])=0,\ \det([\boldsymbol{S}(\bar{\alpha}_{22})])=0 \text{ and } \det([\boldsymbol{S}(\bar{\alpha}_{33})])=0\,. \tag{16}$$

A simple minimization method (see Ref. 23) p. 240) for solving the nonlinear equations (16) are used, and following solutions could be obtained:

$$T_{11}=6.1400,\ T_{22}=1.6451,\ T_{33}=5.4951. \tag{17}$$

Substituting the results (17) into matrix (15), the following inverse of matrix (15) are given:

$$\begin{pmatrix} 1.0000 & 0.6261 & 0.9044 \\ 0.6261 & 1.0000 & 0.5663 \\ 0.9044 & 0.5663 & 1.0000 \end{pmatrix} \tag{18}$$

which is a very closer SEA inverse matrix to the 'measured' matrix (11) of Woodhouse.[14] The diagonal elements of matrix (18) aer the same of those of (11). It can be seen from this simple example that the determined total loss factors with the present strategy have a dominant maintenance effect for the diagonal elements of the inverse SEA matrix to be agree with experimental results. It could be expected that the measurement of the energy of a subsystem to which power is injected would be relatively more accurate than the measurement of other subsystems which do not have direct power input, since the actual complex mechanism of power transmission. Therefore, more accurate results can be expected to be achieved with the present strategy.

In case of the existence of some small error of measurement, for example, $\bar{\alpha}_{11}=0.95$, $\bar{\alpha}_{22}=1.00$ and $\bar{\alpha}_{33}=1.00$, with the present method following approximate TLFs could be determined:

$$T_{11}=6.169,\ T_{22}=1.636 \text{ and } T_{33}=5.485, \tag{19}$$

which are not much different from those of Eq. (17).

However, with the matrix fitting algorithm of Hodges *et al.*,[15] although quite good fit could be achieved from the view point of over all system, the determined individual SEA parameter may differ significantly by different algorithms. For example, for 'measured' matrix (11), the estimated internal loss factor of the first subsystem differed 4 to 5 times with the 'direct' procedure and the Lagrange multiplier approach (see Ref. 15), p. 57, matrices (14, 23)) and thus, this uncertainty may induce significant error in case of determination of SEA parameters.

Matrix (20) is another estimated SEA matrix from the 'measured' matrix of Hodges *et al.*[15]

$$\begin{pmatrix} 2.8862 & 0.0000 & -1.8505 & 0.0000 & -0.9281 \\ 0.0000 & 3.0449 & -1.1953 & -1.5015 & -0.3261 \\ -1.8505 & -1.1953 & 7.3678 & -1.0209 & -2.0779 \\ 0.0000 & -1.5015 & -1.0209 & 5.9513 & -2.6887 \\ -0.9281 & -0.3261 & -2.0779 & -2.6887 & 7.8332 \end{pmatrix} \tag{20}$$

with inverse

$$\begin{pmatrix} 0.6046 & 0.2434 & 0.2865 & 0.2152 & 0.2316 \\ 0.2434 & 0.6055 & 0.2653 & 0.3012 & 0.2278 \\ 0.2865 & 0.2653 & 0.3409 & 0.2208 & 0.2112 \\ 0.2152 & 0.3012 & 0.2208 & 0.3853 & 0.2289 \\ 0.2316 & 0.2278 & 0.2112 & 0.2289 & 0.2992 \end{pmatrix} \tag{21}$$

which is very close to the original 'measured' matrix (see Ref. 15), matrix (25)). Here the 'measured' input power and energy ratios are taken as the inverses of diagonal elements of matrix (25) of Ref. 15) as:

$$\bar{\alpha}_{11}=1/0.6046,\ \bar{\alpha}_{22}=1/0.6055,\ \bar{\alpha}_{33}=1/0.3373,$$
$$\bar{\alpha}_{44}=1/0.3853,\ \bar{\alpha}_{55}=1/0.2975 \tag{22}$$

The coupling loss factors are supposed to be determined with theoretical prediction and be the results of matrix (20), thus the following matrix with unknown TLFs should be dealt with to determine the total loss factors.

$$\begin{pmatrix} T_{11} & 0.0000 & -1.8504 & 0.0000 & -0.9281 \\ 0.0000 & T_{22} & -1.1953 & -1.5015 & -0.3261 \\ -1.8504 & -1.1953 & T_{33} & -1.0209 & -2.0779 \\ 0.0000 & -1.5015 & -1.0209 & T_{44} & -2.6887 \\ -0.9281 & -0.3261 & -2.0779 & -2.6887 & T_{55} \end{pmatrix} \tag{23}$$

With Eq. (22) and matrix (23), a set of nonlinear equations represented by Eq. (6) can be established and solved with the minimization method, therefore, following results of total loss factors can be obtained:

$$T_{11}=2.8863,\ T_{22}=3.0452,\ T_{33}=7.3504,$$
$$T_{44}=5.9463,\ T_{55}=7.8634. \tag{24}$$

Substituting the results (24) into matrix (23), the following inverse of matrix (23) are given:

$$\begin{pmatrix} 0.6046 & 0.2434 & 0.2869 & 0.2151 & 0.2308 \\ 0.2434 & 0.6044 & 0.2658 & 0.3012 & 0.2270 \\ 0.2869 & 0.2658 & 0.3412 & 0.2211 & 0.2108 \\ 0.2151 & 0.3012 & 0.2211 & 0.3853 & 0.2280 \\ 0.2308 & 0.2270 & 0.2108 & 0.2280 & 0.2975 \end{pmatrix} \tag{25}$$

It can be seen that in this case the determined TLFs as well as the inverse SEA matrix (25) with the present strategy are very close to the results of Hodges[15] determined with the Lagrange multiplier approach from the 'measured' matrix.

6. CONCLUSIONS

The main conclusion of this work is that the total loss factors as well as some coupling loss factors of a built-up structure can be determined with the described strategy, which is based on the measurements on each subsystem for injecting power and its response energy with the steady state power injection method.[18] One of the obvious advantages of the strategy is that the determined TLFs would include the dissipation of coupling, and 'automatically' share the dissipation loss of the coupling to each coupled subsystem, which is no way to be dealt with theoretically at present. Furthermore, this strategy can be used experimentally to study the coupling dissipation and in what proportion the coupling dissipation should be assigned to each of the coupled subsystem supposed that the internal loss factors would be determined in no joint dissipation conditions for comparison. Of cause, if the internal loss factors are determined in uncoupled condition, it can also be used to study the influences of the edge condition changes on the loss factors.

It could be expected that the measurement of the energy of a subsystem to which power is directly injected would be relatively more accurate than the measurement of the energies of other subsystems to which no direct power injection, since the actual complex mechanism of power transmission (some subsystems may be far apart from the power injected subsystem). Therefore, more accurate results can be expected to be achieved with the present strategy.

This strategy is supposed be to a practical effective means for the application of SEA approach to deal with the sound and vibration flow of a complex system with many subsystems. For a large practical structure, for example a ship, it is very difficult to perform experiments to determine the loss factors for each subsystem in a laboratory, not mention that the loss factors determined in uncoupled condition may differ significantly from actual values. On the other hand, the power balance equation inversion method used by Bies and Hamid[18] needs too many measurements to establish the inverse SEA matrix, which make it almost impossible to fulfill for a large system with many subsystems. It seams reasonable that some of the coupling loss factors could be theoretically predicted with sufficient accuracy, and some coupling loss factors and the total loss factors should be determined in built-up conditions, if so the present strategy provides an applicable means for determination of total loss and some coupling loss factors of built-up structures.

ACKNOWLEDGMENTS

The authors would like to acknowledge the Science and Technology Agency (STA) of Japan for the STA fellowship offered to the first author that made possible the work in this paper.

REFERENCES

1) R. H. Lyon and G. Maidanik, "Power flow between linearly coupled osilators," J. Acoust. Soc. Am. **34**, 623–639 (1962).
2) R. H. Lyon, *Statistical Energy Analysis of Dynamical Systems: Theory and Applications* (MIT Press, Cambridge MA, 1975).
3) R. H. Lyon and T. D. Scharton, "Vibrational energy transmission in a three-element structures," J. Acoust. Soc. Am. **38** (1965).
4) S. H. Crandall and R. Lotz, "On the coupling loss factors in statistical energy analysis," J. Acoust. Soc. Am. **49**, 352–356 (1971).
5) R. Lotz and S. H. Crandall, "Prediction and measurement of the proportionality constant in statistical energy analysis of structures," J. Acoust. Soc. Am. **54**, 516–524 (1971).
6) W. Wöhle, Th. Beckmann, and H. Schreckenbach, "Coupling loss factors for statistical energy analysis of sound transmission at rectangular structural slab joint, Part 1 & Part 2," J. Sound Vib. **77**, 323–334 (1981).
7) P. S. Langley, "A general derivation of the statistical energy analysis equations for coupled dynamic systems," J. Sound Vib. **135**, 499–508 (1989).
8) P. S. Langley, "A derivation of the coupling loss factors used in statistical energy analysis," J. Sound Vib. **141**, 207–219 (1990).
9) C. Simmons, "Structure-borne sound transmission through plate junctions and estimates of SEA coupling loss factors using finite element method," J. Sound Vib. **144**, 215–227 (1991).
10) J. Tratch, "Vibration transmission through machinery foundation and ship bottom structure," M. S. Diss., MIT, Cambridge MA (1985.6).
11) E. E. Ungar, "The status of engineering knowledge concerning the damping of built-up structures," J. Sound Vib. **26**, 141–154 (1973).
12) G. E. Ottesen and T. E. Vigran, "Measurement of mechanical input power with application to wall structures," Appl. Acoust. **12**, 243–252 (1979).
13) D. A. Bies and S. Hamid, "*In Situ* determination of loss and coupling loss factors by the power injection method," J. Sound Vib. **70**, 187–204 (1980).
14) J. Woodhouse, "An introduction to statistical energy analysis of structural vibration," Appl.

Acoust. **14**, 455–469 (1980).

15) C. H. Hodges, P. Nash, and J. Woodhouse, "Measurement of coupling loss factors by matrix fitting: An investigation of numerical procedure," Appl. Acoust. **22**, 47–69 (1987).

16) R. J. Craik, "Damping of building structures," Appl. Acoust. **14**, 347–359 (1981).

17) J. C. Sun and E. J. Richards, "Prediction of total loss factors of structures, I: Theory and experiments," J. Sound Vib. **103**, 109–117 (1985).

18) F. J. Fahy, "Measurement of mechanical input power to a structure," J. Sound Vib. **10**, 517–518 (1969).

19) P. B. Swift and D. A. Bies, "Steady-state measurement of loss factors," 90th Meet. Acoust. Soc. Am., San Francisco (1975.11).

20) F. J. Fahy, "Energy flow between osillator: special case of point excitation," J. Sound Vib. **11**, 481–483 (1970).

21) M. F. Ranky and B. L. Clarkson, "Frequency average loss factors of plates and shells," J. Sound Vib. **89**, 309–323 (1983).

22) B. L. Clarkson and M. F. Ranky, "On the measurement of coupling loss factors of structural connection," J. Sound Vib. **94**, 249–261 (1984).

23) J. M. Ortega and W. C. Rheinboldt, *Iterative Solution of Nonlinear Equations in Several Variables* (Academic Press, New York and London, 1970).

24) G. W. Stewart, *Introduction to Matrix Computations* (Academic Press, New York and London, 1973).