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A correction of the insertion-loss for constant sound pressure with flow

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The insertion-loss is evaluated by B parameter of the four-terminal matrix method as far as constant sound pressure source is concerned. However, the predictions using the equations in the four-terminal transmission matrix method do not reflect a practical phenomenon accurately. In this paper, the correction method to derive the insertion-loss based on the Characteristic Curve Method is presented. Correction of the four-terminal transmission matrix method was carried out by rewriting the real and imaginary parts as they depend solely on the flow velocity. Then the result was compensated for by adding the component of the temperature gradient.

Keywords: Insertion-loss, Mean flow, Resonance, Exhaust noise, Temperature gradient

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1. INTRODUCTION

Our daily life is becoming more convenient with the rapid development of traffic facilities in recent years. On the other hand, traffic noise, such as exhaust noise from an automobile or a motorbike is still prominent and noise reduction is strongly hoped for by the adoption of regulations on noise reduction year by year. Although mufflers of various forms were developed, the noise reduction characteristic has not sufficiently been conceived yet. The basic research on the noise reduction characteristic of mufflers has been advanced from both a theoretical and experimental approach. The four-terminal matrix method[1] as a representative method of theoretical research offers the most useful and convenient way for analyzing a muffler. It has many advantages, such as the computation being relatively simple and the noise reduction in each component of the muffler can be easily evaluated from the established equations. However, the prediction by means of this method does not reflect a practical phenomenon accurately. Because of such assumptions as no-flow and constant temperature distribution in the analysis, the acoustic characteristics obtained by calculations differ largely from those of measured ones.

We have already discussed about the errors ensuing from some assumptions in the four-terminal transmission matrix method by using the Characteristic Curve Method and proposed a method to correct the insertion-loss for a constant volume velocity source.[2] In this paper, the correction method to derive the insertion-loss for a constant sound pressure source is presented and we compare our results with those of Prasad.[3]

2. METHODS OF ANALYSIS

2.1 Four-Terminal Transmission Matrix Method

As far as constant sound pressure source is concerned, the insertion-loss IL is defined by[4]

$$IL=10\log\frac{W_r}{W_0}=20\log|B|-20\log R_r \qquad (1)$$

where W_r and W_0 are the radiated power at one point in space with or without the straight pipe or muffler insertion between that point and the source. R_r is the radiation impedance and B is the parameter of the four-terminal matrix method. In consideration of the attenuation constant α in the pipe, the B parameter is given by

$$B=Z\sinh(\alpha+jk)L \tag{2}$$

where

Z: characteristic impedance defined by $Z=\rho c/S$
ρ: mass density
c: wave velocity
S: cross-sectional area
k: wave number
L: pipe length.

For $\alpha L\ll 1$ in general, the B parameter can be described by the following approximated equation

$$20\log|B|\cong 10\log|\sin^2 kL+(\alpha L)^2\cos^2 kL| +20\log|Z| \tag{3}$$

The B constant is computed under the assumption in which the system would be linear, isentropic, zero mean flow, zero temperature gradient, and so on. However, these assumptions are not realized in the actual exhaust system.

2.2 Characteristic Curve Method

A strict result can be obtained only when the pressure P, velocity V and mass-density ρ at any point in the exhaust pipe satisfy the following three basic equations[5)]

$$\text{Continuity}\quad \frac{\partial\rho}{\partial t}+\rho\frac{\partial V}{\partial x}+V\frac{\partial\rho}{\partial x}=0 \tag{4}$$

$$\text{Motion}\quad \frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+\frac{1}{\rho}\frac{\partial P}{\partial x}+F=0 \tag{5}$$

$$\text{Energy}\quad \frac{\partial P}{\partial t}+V\frac{\partial P}{\partial x}-c^2\left(\frac{\partial\rho}{\partial t}+V\frac{\partial\rho}{\partial x}\right)-E=0 \tag{6}$$

where

$$E=\rho(h-1)(q+V\cdot F)$$

h: ratio of the specific heat
F: wall-friction
c: wave velocity
q: heat added per unit mass per unit time.

The Characteristic Curve Method[6)] converts the partial differential equation of Eqs. (4), (5) and (6) into six total differential equations as follows

$$C^+\begin{cases}\dfrac{dP}{\partial t}+\rho\dfrac{\partial V}{dt}-E+c\rho F=0 & (7)\\[2mm] \dfrac{dx}{dt}=V+c & (8)\end{cases}$$

$$C^-\begin{cases}\dfrac{dP}{dt}-c\rho\dfrac{dV}{dt}-E-c\rho F=0 & (9)\\[2mm] \dfrac{dx}{dt}=V-c & (10)\end{cases}$$

$$C^d\begin{cases}\dfrac{dP}{dt}-c^2\dfrac{d\rho}{dt}-E=0 & (11)\\[2mm] \dfrac{dx}{dt}=V & (12)\end{cases}$$

Equations (7), (9) and (11) are the compatible equations which are only valid along the respective characteristic lines, Eq. (8), (10) and (12). In conventional method, boundary conditions has often used,[2)] *i.e.*

[1] At the inlet pipe ($x=0$), pressure-time pattern P_1 is applied.

[2] At the outlet of pipe, the pressure is not equal to that surrounding the pipe because of the influence of flow. For the sake of simplicity, let's assume that $P_{eff}=P_0$[7)] where P_{eff} and P_0 are the pressure at the effective length of pipe and an atmospheric pressure, respectively. The effective length of pipe is defined by $L_{eff}=L+0.6\cdot a$[3)] in which a is the radius of pipe.

The numerical calculation is carried out through Eq. (7) to Eq. (12) to find the flow velocity V_1 and V_2 at the inlet and outlet pipe, respectively. The B parameter can be evaluated by $B=P_{S1}/U_2$ in the frequency domain where P_{S1} is the sound-pressure transformed from P_1 and $U_2=V_3\cdot S$. The wall-friction F and the heat added per unit mass per unit time q are defined by[4)]

$$F=\zeta\frac{L}{d}\frac{\bar{V}^2}{2g} \tag{13}$$

$$q=\frac{4\alpha_p}{d\rho}(T_0-T_G) \tag{14}$$

where

ζ: friction coefficient
d: pipe diameter
$\bar{V}$: average velocity
g: gravitational acceleration
T_G: gas temperature
T_0: external temperature.

The friction coefficient ζ and the heat transfer coefficient α_p are given by Colebrook and Kays as follows[5])

$$\frac{1}{\sqrt{\zeta}}=2\log\left[\frac{\epsilon}{3.71d}+\frac{2.51}{Re\sqrt{\zeta}}\right] \qquad (15)$$

$$\alpha_p=0.023Re^{0.8}Pr^{0.5}\lambda/d \qquad (16)$$

in which Re is Reynolds number defined by $Re=\bar{V}\cdot d/\mu$, μ is the coefficient of viscosity, ϵ is the standard roughness, Pr is Prandtl number and λ is the friction factor.

The measurements data such as the pressure-time pattern P_1 and the gas temperature characteristic are required to calculate and examine the correctness of the results obtained by the Characteristic Curve Method. The experiment was carried out with a straight pipe which was connected to an engine system (four-stroke engine, 144 cc, 3,600 rpm). The pressure sensor P_{S1} was attached at the inlet of pipe to measure the pressure waveform P_1 which is then used in calculations under boundary conditions.[1]) The one P_{S2} was located at 0.2 m from P_{S1} to measure the pressure waveform P_2 which is used to compare with the calculation result to assert the correctness of the method. The measurements of gas temperature and the average flow velocity were also done in several points in the pipe by using a thermoelectric couple and an anemometer after the engine had reached a stable state. The measured results of gas temperature are shown in Fig. 1. The increase of temperature along the pipe as well as the increase of temperature gradient can affect the acoustic propagation considerably.

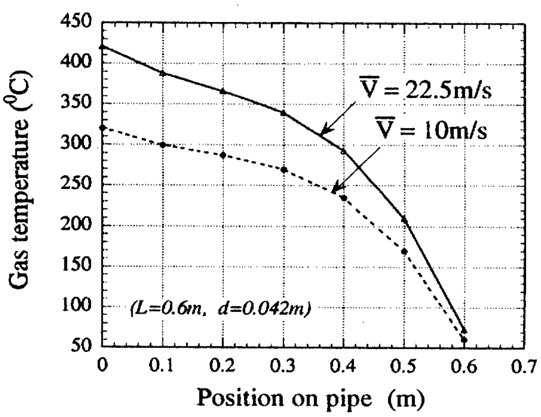

Fig. 1 Gas temperature measured with mean flow of 10 m/s and 22.5 m/s, respectively. $x=0$ is the position where the pressure sensor P_{S1} was attached.

3. RESULTS AND DISCUSSION

Figure 2 shows the B parameter calculated by the four-terminal transmission matrix method and Characteristic Curve Method of a straight pipe of 0.6 m long and 0.042 m in diameter. The upper and lower figures show the results when mean flow velocities are 10 m/s and 22.5 m/s, respectively. Our computations have been achieved with the gas temperature as shown in Fig. 1 which have the mean values of 230°C for 10 m/s case and 306°C for 22.5 m/s case. In the computation based on the four-terminal transmission matrix method, sound velocity c is defined by $c=331.5+0.61\bar{T}$ in which $\bar{T}$ is given by the mean value of the gas temperature measured in the pipe. Similarly, the mass-density ρ is also given by the mean value.

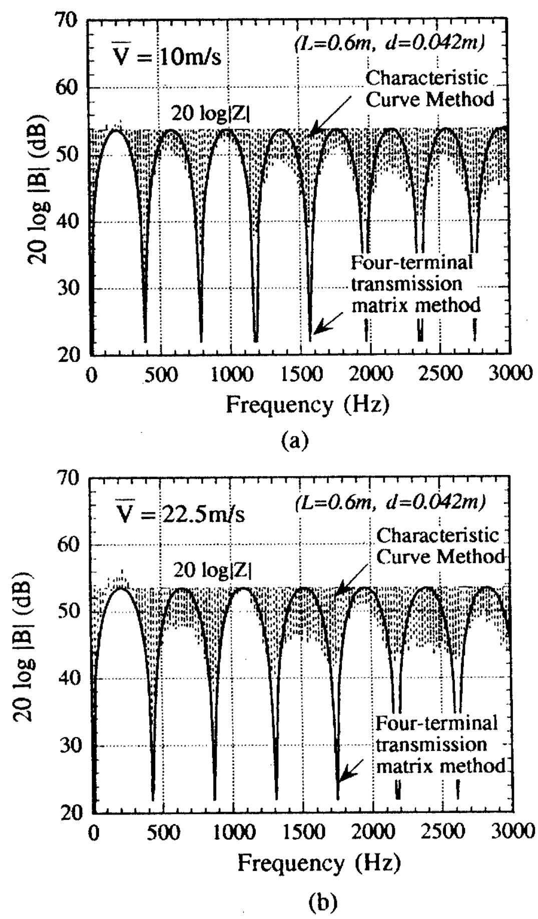

Fig. 2 Difference between Four-Terminal Transmission Matrix Method (solid line) and Characteristic Curve Method (dotted line) with mean flow of 10 m/s and 22.5 m/s, respectively. Average of gas temperatures are 230°C for 10 m/s case and 306°C for 22.5 m/s case.

The general characteristics are clearly recognized as follows

(a) The lower values of $20\log|B|$ are caused by the effect of longitudinal resonances in the pipe when $\sin^2 kL$ of Eq. (3) becomes zero. The frequencies at which the resonances occur in the two methods are slightly different and it become large with an increase of velocity. Moreover, the values at resonance and antiresonance are maintained constant in the results by the four-terminal transmission matrix method but they are not uniform in the results by Characteristic Curve Method.

(b) In the low frequency range, the results obtained from the four-terminal transmission matrix method and those from the Characteristic Curve Method are very similar. However, the difference of two results become large along with an increase of frequency.

4. CORRECTION METHOD

The correction of B parameter is performed by making the real and imaginary parts of Eq. (2) as they depend solely on the flow velocity. Then we compensate for the result by adding the contribution part of the temperature gradient. This correction method is the extension of the method proposed by Nishimura, *et al.*[2)]

To meet the condition (a) requiring the resonance frequencies to be shifted to the lower frequency range we suggest to replace k with βk ($\beta>1$) as follows

$$B=Z\sinh(\alpha+j\beta k)L \qquad (17)$$

Additionally, we propose to add a second term to the original by replacing $\sinh(\alpha+j\beta k)L$ with

$$B=Z\{\sinh(\alpha_{\mathrm{M}}+j\beta k)L+j\gamma\sin^2\beta kL\} \qquad (18)$$

The second term is to create different values at antiresonances at odd and even modes. α_{M}, a factor dependent on mean flow $\bar{V}$ should be chosen so that $\alpha_{\mathrm{M}}>\alpha$.

Before make correction for the condition (b), it is necessary to make clear the effect of temperature gradient and velocity in the results as illustrated in Fig. 2. At first, to find the effect of temperature gradient we make a simulation with T_{r} varying while the flow velocity remains the same. Data used in the simulation are the same as Fig. 2(b) and the result is shown in Fig. 3(a). $T_{\mathrm{r}}=0.0$ means that the gas temperature along the pipe is the same as the average temperature. Next, the same simulation

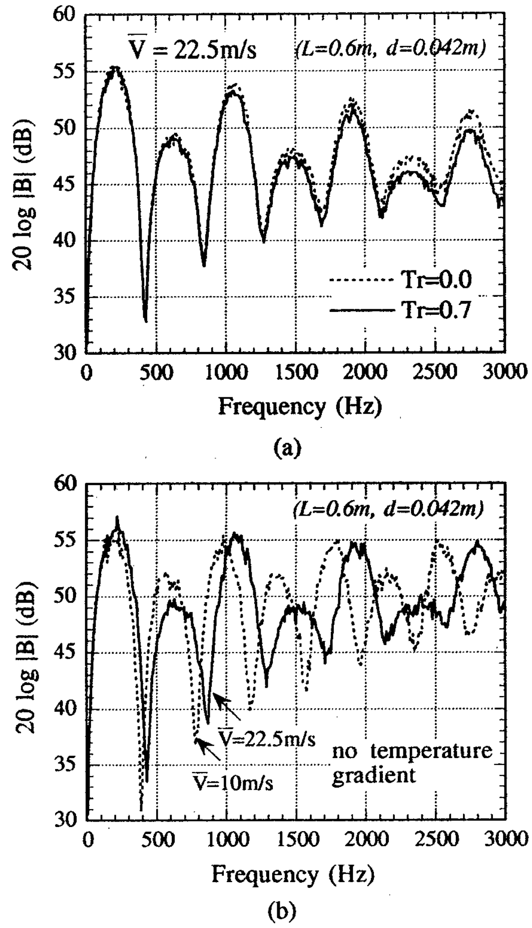

Fig. 3 Simulated results. (a) Temperature gradient characteristic of B parameter with mean flow of 22.5 m/s. (b) Mean flow characteristic of B parameter with no temperature gradient.

was performed with flow velocity varying while $T_{\mathrm{r}}=0.0$. The result is shown in Fig. 3(b). It is considered that the general spectrum characteristic does not change largely from both of the results obtained in Fig. 3. Therefore, we can correct the condition (b) as follows.

In order for B to increase exponentially with $\bar{V}$, we propose to multiply Eq. (18) with $\exp(\alpha_{\mathrm{M}}'\beta kL)$

$$B=Z\exp(\alpha_{\mathrm{M}}'\beta kL)\{\sinh(\alpha_{\mathrm{M}}+j\beta k)L+j\gamma\sin^2\beta kL\} \qquad (19)$$

where α_{M}' is a correction factor. The value of γ, α_{M} and α_{M}' could be determined by finding $|B|^2$ using Eq. (19) and applying the values of resonance and anti-resonance at some points in Fig. 3(b), namely:

$$|B|^2=Z^2\exp(2\alpha_{\mathrm{M}}'\beta kL)\{(\cosh\alpha_{\mathrm{L}}L\sin\beta kL+\gamma\sin^2\beta kL)^2+\cos^2\beta kL\sinh^2\alpha_{\mathrm{M}}L\} \qquad (20)$$

At resonance frequency ($\beta kL = n\pi$), Eq. (20) becomes

$$|B|^2 = Z^2 \exp(2\alpha_M' \beta kL) \sinh^2 \alpha_M L \qquad (21)$$

At antiresonance frequency ($\beta kL = n\pi/2$), Eq. (20) becomes

$$|B|^2 = Z^2 \exp(2\alpha_M' \beta kL)\{\gamma + (-1)^n \cosh \alpha_M L\}^2 \qquad (n=1, 2, 3, \cdots) \qquad (22)$$

Finally, to include the effect of temperature gradient as shown in Fig. 3(a), we propose to add a term $\exp(\alpha_T)$ to Eq. (19) as follows

$$B = Z \exp((\alpha_M' + \alpha_T)\beta kL) \cdot \{\sinh(\alpha_M + j\beta k)L + j\gamma \sin^2 \beta kL\} \qquad (23)$$

where α_T is the correction factor, function of temperature gradient T_r. With $\bar{V}$ varying from 8 m/s to 32 m/s, T_r varying from 0 to 0.75, using the proposed method, we obtain the following values for the factors:

$$\left.\begin{aligned} \alpha_M &= \alpha + 0.56M \\ \alpha_M' &= -0.43M \\ \alpha_T &= -0.043T_r \\ \gamma &= 8.26M \end{aligned}\right\} \qquad (24)$$

where M is Mach number defined by $M = \bar{V}/c$.

Note that Eq. (23) will reduce to Eq. (2) when no mean flow and no temperature gradient are present. Also, the four-pole parameters could be computed for the case whether the mean flow is present or not, and also for the case when the temperature gradient is present or not.

Similar study has been reported by Prasad and Crocker.[3] They derived the four-pole parameters for a straight pipe in the presence of a uniform mean flow and a linear temperature gradient by using first-order perturbation theory and a Green's function approach. No loss was assumed in their analysis, therefore the B and D parameters which they derived differ from our equation. However, it is found that the general tendency of mean flow and temperature gradient characteristic which they derived are similar to those of our results as shown in Fig. 4. Note that the calculation in Fig. 4 was performed on the same conditions as Fig. 3 and in order to satisfy a condition of linear temperature gradient, the pipe is divided into 32 segments in our calculation. The fact that Prasad did not take loss due to the friction and heat transfer in a straight pipe, leads us to believe that their equation can be used only in the limited areas of velocity and temperature gradient.

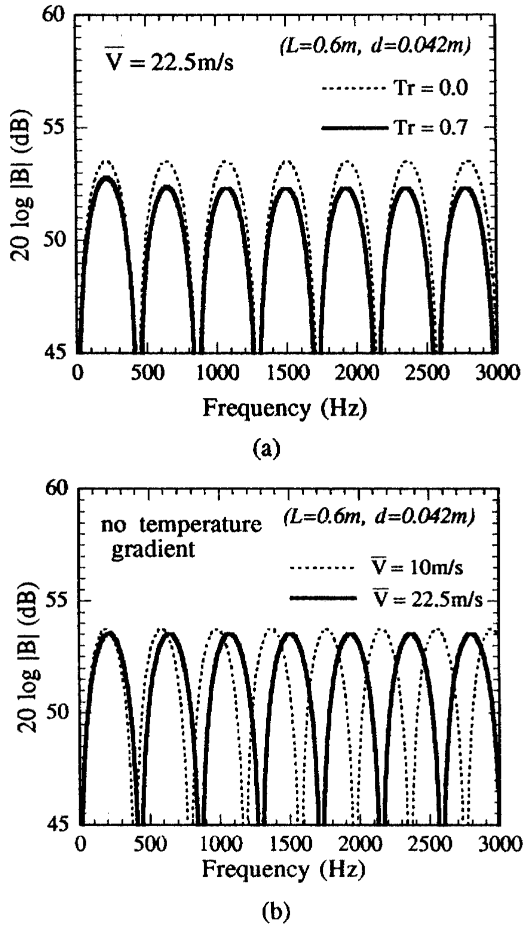

Fig. 4 B parameter based on Eq. (42) by Prasad.[3] (a) Temperature gradient characteristic of B parameter with mean flow of 22.5 m/s. (b) Mean flow characteristic of B parameter with no temperature gradient.

5. CONCLUDING REMARKS

The B parameter of the four-terminal transmission matrix method which is used to evaluated the insertion-loss was corrected based on the results of Characteristic Curve Method. The correction method was proposed by rewriting the real and imaginary parts as they depend solely on the flow velocity. Then the result was compensated for by adding the component of the temperature gradient. The B parameter with simultaneous mean-flow and temperature gradient is given by Eq. (23). The computations in this paper were carried out based on the experimental results of a single-piston, four-cycle engine running at 3,600 rpm connected with a straight pipe of length 0.6 m, and diameter of 0.042 m. Due to the fact that friction and heat transfer are non-linear, those factors in Eq. (23) can not be written in a general formula. However, under the limited conditions of the pipe size and running speed of the engine, those factors could be estimated by the proposed method.

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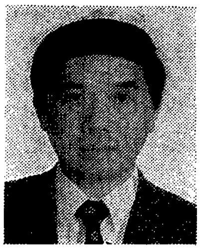

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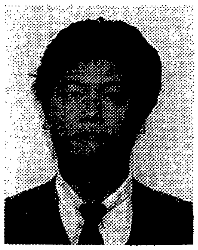

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