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Spacing of transducer-elements in transmitter and receiver arrays to collect data for ultrasonic diffraction tomography

Keinosuke Nagai,* Makoto Ohmaru,* Tomoki Yokoyama,** and Koichi Mizutani*

**Institute of Applied Physics, University of Tsukuba, Tsukuba, 305 Japan*
***Department of Applied Physics, National Defense Academy, Hashirimizu 1-1-20, Yokosuka, 239 Japan*

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We discuss the spacing of the transducer elements of transmitter and receiver arrays to sample sufficient data for ultrasonic diffraction tomography. We intend to save fairly the data acquisition time of our prior method by replacing the mechanical scanning of the pair of a transmitter and a receiver with the electrical switching of the both arrays. One of the most important parameters of the arrays is the spacing of the elements. In the previous paper we used the equation of the spacing derived from the theory of acoustical holography for convenience, though the prerequisites might not fit for the case. We now newly obtain the equation by calculating the phase of the received signal. New result proves 3 times narrower than the previous one. We confirm the feasibility of the new result by the numerical calculation and the experiment.

Keywords: Ultrasonic diffraction tomography, Spatial frequency, Compound scanning, Resolution, Transducer arrays

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1. INTRODUCTION

Ultrasonic cross-sectional images are useful for medical diagnostics, nondestructive examination, underwater imaging, exploration geophysics and so on. They are constructed by a common technique named pulse-echo method (PEM). The method has however its own drawbacks: The images do not correspond quantitatively to physical properties of the objects. Because PEM displays the magnitude of the reflected pulse wave. The reflection occurs on the interface where the acoustic impedance changes. The magnitude depends on many factors as well as the difference of the impedance: the orientation of the surface, the configuration of the transducer and objects and so on. The effects of the wave propagation such as refraction and diffraction also affect the shape of the reflected pulse wave. As the result it is difficult to interpret the physical properties of the objects from the image of PEM.

Ultrasonic diffraction tomography (UDT) has been proposed to overcome these drawbacks mainly in the field of medical diagnostics.[1-3] This technique regards the image reconstruction as the inverse problem of the wave propagation. The images are obtained as the solution of wave equations. The wave phenomenon such as diffraction, refraction and scattering are taken into account in such a way. The images reconstructed by UDT are expected to display quantitatively the physical properties of the objects. The information of the wave is so fully utilized that the higher resolution of the images may be attained by UDT of lower frequency, comparing with that of PEM.

UDT has also its drawbacks, however. The wave equations cannot always be solved. The assumption so called weak scattering is inevitably adopted in practical inverse problem. The data acquisition time is too long. We have to obtain spatially two dimensional data to reconstruct the same dimensional image. In addition the image reconstruction algorithm is very complicated and it consumes much more time. This is caused by the fact: the data acquired on a scanning line correspond to the components on the circular arc in Fourier domain of the image.[1-3] Only few experimental results have been reported[4] as the result, though a vast number of theoretical papers have been published on UDT.

We have proposed a new method of UDT[5,6] which simplify the reconstruction algorithm, though the assumption of weak scattering and the long consumed time for data acquisition remain unchanged. In the method we can get the components on a line rather than on a circular arc in Fourier domain of an image from the data obtained on a scanning line. This makes us apply the algorithm of well developed X-ray computerized tomography to image reconstruction process, though the diffraction effect is taken into account. The experiment has been easily done and good results are appreciated.[7]

In this paper we discuss the spacing of the transducer elements of transmitter and receiver arrays, that is the sampling interval of the tomographic data. In order to save fairly the data acquisition time we adopt the transmitter and the receiver arrays and replace the mechanical scanning of the pair of a transmitter and a receiver with the electrical switching of the both arrays. One of the most important parameters of the arrays is the spacing of the elements. In the previous paper[8] we used the equation of the spacing derived from the theory of acoustical holography for convenience, though the prerequisites might not fit for the case. We newly derive the equation in this paper. In Section 2 we briefly describe the method of the diffraction tomography. Then we derive the equation of the spacing in Section 3 by calculating the phase of the received signal. New result proves 3 times narrower than the previous one. In Section 4 and 5 we confirm the feasibility of the new result by the numerical calculation and the experiment. In Section 6 we make the conclusions.

2. ULTRASONIC DIFFRACTION TOMOGRAPHY WITH DATA OBTAINED BY SCANNING A PAIR OF TRANSMITTER AND RECEIVER

We have described elsewhere[5,6] the method of the ultrasonic diffraction tomography with data obtained by scanning a pair of transmitter and receiver. We present it briefly here for the discussion below.

As shown in Fig. 1 an object is embedded in a homogeneous medium. In this configuration a transmitter faces to a receiver and the object is set between them. Ultrasonic sinusoidal wave with a single frequency (a tone-burst signal in practice) is transmitted from the transmitter. It is scattered by the object and then received by the receiver. If we can assume (1) the density of the object is almost the same as that of the surrounding homogeneous medium (usually water), (2) the scattering is so weak as Born approximation holds, (3) the distance between the transmitter and the receiver is so long that the paraxial approximation is valid, the compressibility of the object relative to that of the surrounding medium is reconstructed from the data of the scattered wave in the following way.[5,6] The received signal is at first quadrature-detected and recorded as a complex datum. A pair of the transmitter and the receiver is simultaneously scanned on a line. The data are acquired at the proper interval of steps on the line, say the x-axis. Then the data are Fourier transformed with respect to the x-axis.

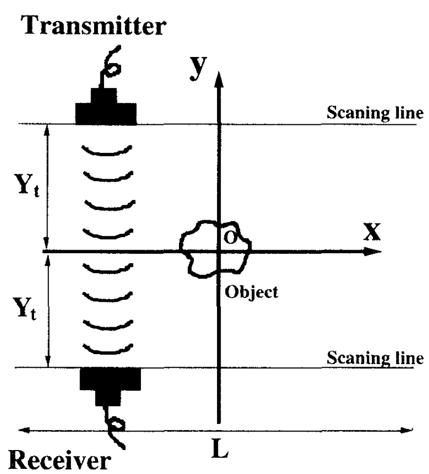


Fig. 1 Configuration of the ultrasonic diffraction tomography (UDT).

The Fourier components of the relative compressibility of the object with respect to the spatial frequency corresponding to the x-axis can be calculated from the transform. We call the relative compressibility of the object only the 'object' for the simplicity below. We can obtain whole Fourier transform components of the object if we rotate the scanning line over 180 degree and repeat the data-acquisition and Fourier transform on the line at the proper step angle. We can reconstruct the image of the object by the inverse Fourier transform.

The method consumes much more time for the data acquisition : two dimensional mechanical scanning of the pair of transmitter and the receiver. In order to reduce the time we have proposed the adoption of a transmitter array and a receiver one.[8] The block-diagram of the whole system is shown in Fig. 2. The mechanical scanning on a line is replaced with the electrical switching of a pair of elements in the both arrays. This may save plenty of time, though the arrays are still rotated mechanically. We examine in this paper the spacing of the elements of the arrays, which is one of the most crucial parameters.

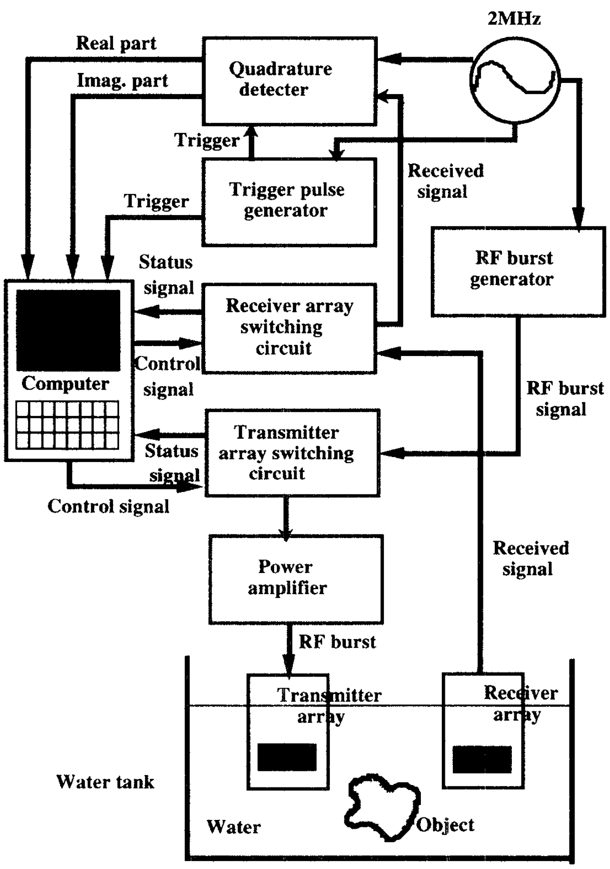


Fig. 2 Imaging system of UDT including transmitter and receiver arrays.

3. SPACING OF TRANSDUCER-ARRAY-ELEMENTS

Spacing of the elements corresponds to the sampling interval of the data. We intuitively think from the sampling theorem that the interval to sample acoustic field may be a half of the wavelength. It is, however, too short to sample the field practically over the wide range. The cost of the data sampling and data processing becomes enormous because the number of data increases so greatly for the short sampling interval. Tabei *et al.*[9] have derived the condition of the interval $\Delta\xi$ to sample acoustic holographic data after the elaborate consideration

$$\Delta\xi < \frac{\lambda}{2}\sqrt{1+4\left\{\frac{l_y}{l_x}\right\}^2}, \qquad (1)$$

where λ is the wavelength, l_x the lateral width of an object and l_y the distance from the observation plane to the reconstructed image plane. Usually $\Delta\xi$ is much larger than $\lambda/2$, because we can use fairly large l_y/l_x.

In the previous paper we apply Eq. (1) to our system, letting l_y be $2Y_t$: the distance from the transmitter to the receiver :

$$l_y = 2Y_t, \qquad (2)$$

because we have no other appropriate equation, though the prerequisites might not be satisfied in the situation. It is supposed in derivation of Eq. (1) that an object which is reconstructed is a wave source and it has no thickness in the direction of the wave propagation, say the y-axis. These conditions do not fit for the case of the tomography. Now we have to derive the equation of the sampling interval which satisfies our system.

The geometry of the data acquisition system is shown in Fig. 3. The widths of the transmitter and the receiver elements are so thin that the wave transmitted might be cylindrical (see Appendix). As the property of an object is supposed to be almost constant in the direction perpendicular to this figure plane, we consider the configuration two-dimensional. Only a pair of the elements in the both arrays facing each other is active at a time. Let the coordinates of the active transmitter and receiver be (x_t, Y_t) and $(x_t, -Y_t)$, respectively. Now we suppose the wave transmitted from the

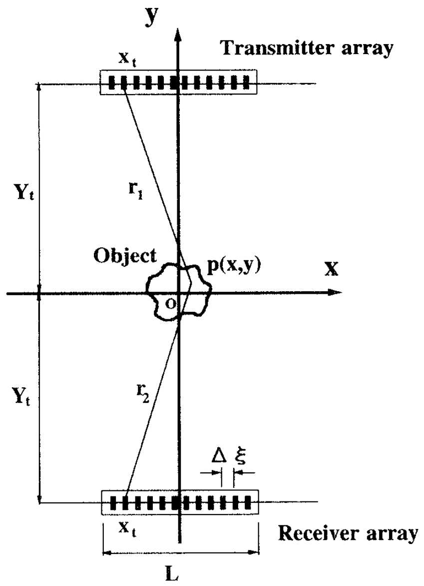


Fig. 3 Geometry of the array system.

transmitter element (x_t, Y_t) is scattered at a point $p(x, y)$ and received at the receiver element $(x_t, -Y_t)$. As an amplitude does not vary abruptly with the change of the position x_t, we have only to examine the phase shift to decide the spacing of the elements. A phase shift ϕ of the wave by the propagation is

$$\phi=\frac{2\pi}{\lambda}(r_1+r_2), \qquad (3)$$

where

$$\left.\begin{array}{l} r_1=\sqrt{(x-x_t)^2+(y-Y_t)^2}\,, \\ r_2=\sqrt{(x-x_t)^2+(y+Y_t)^2}\,. \end{array}\right\} \qquad (4)$$

The local spatial frequency u_x in the direction of the x-axis is

$$u_x=\frac{1}{2\pi}\frac{\partial\phi}{\partial x}. \qquad (5)$$

As we need the simple measure of the spacing rather than the accurate value, we use the paraxial approximation before substituting Eqs. (3) and (4) into Eq. (5): comparing with the distance $2Y_t$ between the both arrays, the length of the arrays L and the dimension of the object are much smaller, that is

$$|x_t|, |x|, |y| \ll Y_t\,.$$

Then we have

$$r_1 \simeq Y_t+\frac{(x-x_t)^2-2Y_t y+y^2}{2Y_t},$$

$$r_2 \simeq Y_t+\frac{(x-x_t)^2+2Y_t y+y^2}{2Y_t}.$$

So,

$$r_1+r_2 \simeq 2Y_t+\frac{(x-x_t)^2+y^2}{Y_t}. \qquad (6)$$

Substitution of Eq. (6) into Eq. (3) and then further substitution of the result into Eq. (5) yields,

$$u_x=\frac{2}{\lambda Y_t}(x_t-x). \qquad (7)$$

Now supposing the object is small and it is set at around the origin (0, 0): $x=0$, the local frequency becomes the maximum u_{max} at $x_t=L/2$, where

$$u_{max}=\frac{L}{\lambda Y_t}. \qquad (8)$$

The sampling interval $\Delta\xi$, at which the signal of the frequency u_{max} can be sampled without aliasing error, is given by

$$\Delta\xi<\frac{1}{2u_{max}}. \qquad (9)$$

Substituting Eq. (8) into Eq. (9) we obtain

$$\Delta\xi<\frac{\lambda Y_t}{2L} \qquad (10)$$

The sampling interval $\Delta\xi$, that is the spacing of the elements, is much greater than $\lambda/2$, because $Y_t \gg L$.

4. NUMERICAL CALCULATIONS

We execute numerical calculation to examine the equation of the spacing just derived in the preceding section. The parameters are shown in Table 1. Substituting these parameters into Eq. (1) we have[8]

$$\Delta\xi<2.7\text{ mm}. \qquad (11)$$

And similarly we get from Eq. (10),

$$\Delta\xi<0.88\text{ mm}. \qquad (12)$$

Table 1 Parameters.

Acoustic velocity c	1,500 m/s
Frequency f	2.0 MHz
Wave length λ	0.75×10^{-3} m
Distance from the transmitter array to the receiver array	3.0×10^{-1} m
Length of array	6.4×10^{-2} m

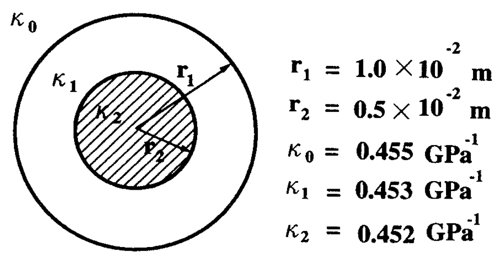


Fig. 4 An object for the numerical simulation.

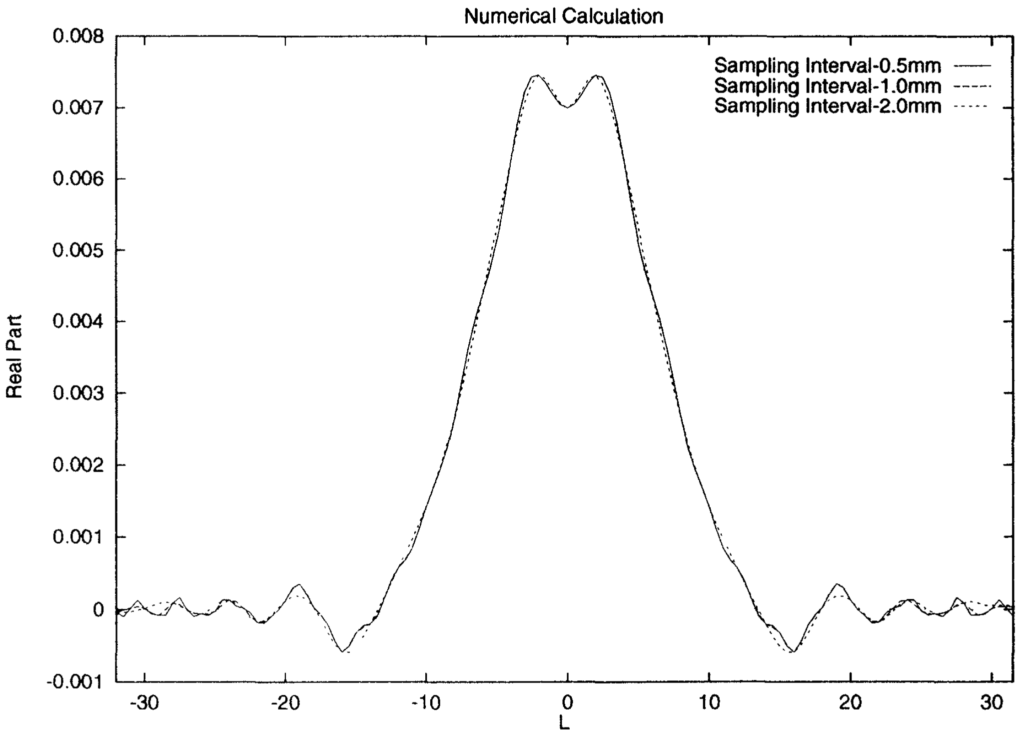


Fig. 5 Diffraction field by the object sampled at different intervals.

Our new result reduces to 1/3 smaller than the prior one. In order to decide which one is appropriate we numerically simulate the data acquisition of the diffraction tomography. The geometry is shown in Fig. 3. In this case we also use the parameters in Table 1. The object is assumed to be two-layer concentric cylinders as shown in Fig. 4. The equation of the diffraction field has been exactly obtained. Using the equation the acoustic pressure field is calculated at the sampling intervals of 0.5 mm, 1.0 mm and 2.0 mm. Only the real part of the results are shown in Fig. 5, although the imaginary part is calculated too. This is because the both are qualitatively almost the same. We can see 2.0 mm sampling interval cannot follow the fluctuation of the field near the both edges of the data acquisition range, 1.0 mm interval can almost follow the fluctuation except at just the edges and 0.5 mm interval can sufficiently follow the fluctuation even at the edges. These results show Eq. (12) that has been derived just now is more appropriate than Eq. (11) which was derived previously.

5. EXPERIMENTS

We have made the experiment to conclude which one of the two Eqs. (11) and (12) is better. We adopt again the configuration shown in Fig. 3 and parameters in Table 1 in the experiment. We use the hollow phantom as the object, shown in Fig. 6 and made from 5% agar purified powder. The rubber musk with the slit of the 1 mm width are attached on the transmitter and the receiver. The data are acquired by mechanically scanning a pair of them at the sampling interval of 0.4 mm. From the data we have made at first the set of the sampling intervals of 0.8 mm and that of 2.0 mm. And using interpolation we have also made the set of the interval of 1.5 mm. Each set are then interpolated by

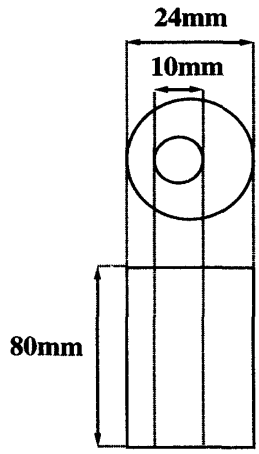
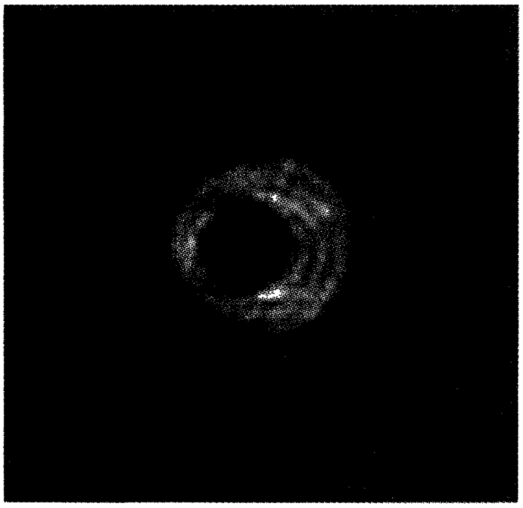
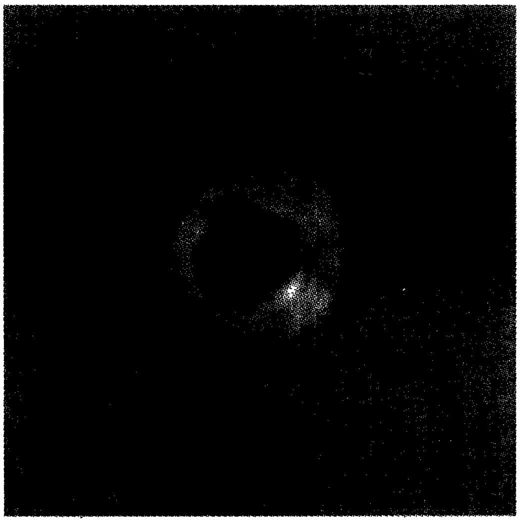


Fig. 6 An agar phantom used in the experiment.



(a)

(b)



(c)

Fig. 7 Reconstructed images from data of (a) 0.8 mm, (b) 1.5 mm and (c) 2.0 mm sampling intervals.

the use of the spline function to be 128 data of 0.5 mm sampling interval for the image reconstruction program. The data acquisition range of each set is, therefore, 64 mm long, which corresponds to the array length. The rotation step is 6 degrees and 30 scanning lines are used for all the three cases. The image is reconstructed over the range of 64 mm $\times$ 64 mm and 128 $\times$ 128 pixels. The reconstructed images with different sampling intervals are respectively shown in Fig. 7 (a), (b) and (c).

The concentration of the agar is not homogeneous. The brightness of the image corresponds to the compressibility.[10] The resolution of the image from data with the interval of 0.8 mm is the best. The quality of the image from data with the interval of 1.5 mm is slightly degraded. That of the image from data with 2.0 mm interval is the worst. These experimental results also show Eq. (12) is more feasible than Eq. (11).

6. CONCLUSIONS

We have proposed the use of the transmitter and the receiver arrays to reduce the data acquisition time for the diffraction tomography and examine the spacing of the array elements, that is the sampling interval of the data. We newly derive the equation of the spacing as the prerequisite of the equation previously obtained did not fit for our system. The feasibility of the equation just derived is verified by both of the numerical simulation and the experiments.

We are going to make the system of the diffraction tomography including the transmitter and the receiver arrays. We have to complete the system in the near future by solving practical problems.

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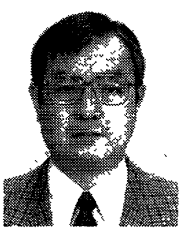
APPENDIX: RADIATED FILED FROM THIN TRANSDUCER

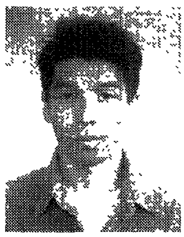
A. Macovski[A1] has shown that the acoustic field from a transducer is thought to be in the far-field region if the distance $d > D^2/\lambda$, where D the width of the transducer and λ the wavelength. The beam width in this region is approximately given by $\lambda d/D$.[A1]

$\lambda = 0.75$ mm (see Table 1) and D of our array's element is 1 mm. Using these figure we have the far field region at $d > 1.3$ mm in our geometry. Now we are interested in the field around $d = Y_t$, which is 150 mm (see Table 1) and fairly satisfies the condition: $d > 1.3$ mm. The beam width is almost 112 mm at $d = Y_t$ which is considered much wider than the size of our object. The radiated field is therefore regarded as the cylindrical wave.

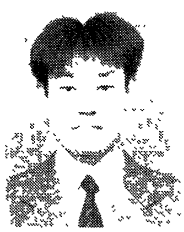
A1) A. Macovski, "Ultrasonic imaging using arrays," Proc. IEEE **67**, 484–495 (1979).



Keinosuke Nagai was born in 1942. He is a professor of the Institute of Applied Physics, University of Tsukuba. He received the B.E. degree in electronic engineering from Waseda University in 1965, and M.E. and Ph.D. degrees from Hokkaido University in 1970 and 1973, respectively. His interests are ultrasonic imaging as the inverse problem and music acoustics as the vibration analysis. He was a guest researcher of Mayo clinic in Rochester Minnesota USA in 1987. He is a member of ASJ, IEICE and IEEE.



Makoto Ohmaru received B.E. degree in electronic engineering from Chiba University in 1995. He graduated from University of Tsukuba with M. E. degree and joined Texas Instruments Japan in 1997. Now he works to develop and design HDD products in the company.



Tomoki Yokoyama was born in Hyogo, Japan, on November 21, 1968. He received the B.E., M.E., and Dr. Eng. degrees from University of Tsukuba, Ibaraki, Japan, in 1991, 1993 and 1996, respectively. Since 1996 he has been in National Defense Academy and now he is a research assistant. His research interest includes under water acoustics. Dr. Yokoyama is a member of ASJ, ASA and MASJ.

Koichi Mizutani was born in Fukuoka, Japan, on July 6, 1955. He graduated in Electrical Engineering of the National Defense Academy in 1979, and received the Dr. Eng. degrees in 1990 from the University of Kyoto. He was a researcher with Department of Electrical Engineering of the National Defense Academy from 1984 to 1988, and a Researcher with Department of Research of the Communication and Intelligence School of the Ground Self Defense Force from 1988 to 1990. He was Deputy Director of the Secretariat of Director General of Defense Agency from 1991 to 1992. Since 1992 he has been an Assistant Professor in the Institute of Applied Physics of the University of Tsukuba. He has been engaged in researches on Applied Acoustics, Acoustical Engineering and Architectural Electronics including optical CT, sound CT, Acoustical Imaging, SCaM, Sensor Probing, and so on. He is a member of the Acoustical Society of Japan (ASJ) and the Institute of Electrical Engineering (IEE) of Japan.