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Acoustic diffraction near a penetrable strip

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The diffraction of an acoustic wave by a penetrable strip introducing the Kutta-Joukowski condition is investigated. Mathematical problem which is solved is an approximate model for a noise barrier which is not perfectly rigid and therefore transmits sound. The problem is solved using integral transforms, the Wiener-Hopf technique and asymptotic methods. It is found that the field produced by the Kutta-Joukowski condition will be substantially in excess of that in its absence when the source is near the edge. Finally, physical interpretation of the result is discussed.

Keywords: Integral transforms, Wiener-Hopf technique, Diffraction theory, Asymptotic methods, Kutta-Joukowski condition

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1. INTRODUCTION

Unwanted noise from motorways, railways and airports can be shielded by a barrier which intercepts the line of sight from the noise source to a receiver. To design and performance of noise barriers, particularly, for the reduction of traffic noise, has received considerable attention in recent years.[1,2] In most of the calculations with noise barriers, the field in the shadow region of the barrier is assumed to be solely due to diffraction at the edge. This assumption supposes that the barrier is perfectly rigid and therefore does not transmit sound. However, most practical barriers are made of wood or plastic and will consequently transmit some of the noise through the barriers. Rawlins[3] presented a theoretical work on this model by considering diffraction of a sound wave from an acoustically penetrable half plane.

In the Lighthill's theory[4] for flow generated sound, regions of turbulence are modeled by spatial distributions of acoustic quadruples. According to this theory the intensity of the sound radiated by a compact turbulent eddy is proportional to M^8 (M is the Mach number). Ffowcs-Williams and Hall[5] demonstrated that, if a compact turbulent eddy is situated within an acoustic wavelength of the sharp edge of a rigid half plane, the radiated sound intensity is increased over its free field value by the large factor M^{-3}. Thus the edge is likely to be the dominant sound source especially when the source is very close to the edge. Their findings were however based upon the assumption of a potential flow near the sharp edge with velocity becoming infinite there. Therefore their inferences could no longer be regarded as valid if a Kutta-Joukowski condition were imposed at the edge.

If one wishes to prescribe that the velocity is finite at the edge then there are two possible points of view one could adopt. One way is to abandon Lighthill's theory[4] and use linearized Navier-Stoke's equation with source term as employed by Alblas.[6] The other point of view is to retain the equation of small amplitude sound waves and attempt to apply a Kutta-Joukowski condition at the edge. This cannot be done without giving up some property of the field such as continuity and one method of doing this is to introduce a vortex sheet.

Jones[7] adopted this approach and introduce the wake condition to examine the effect of Kutta-

Joukowski condition at the edge of the half plane. He calculated the field scattered from a line source parallel to a semi infinite rigid plane attached to a wake. It was observed by him that the imposition of the Kutta-Joukowski condition does not have much influence on the scattered field away from the diffracting plane. This condition produces a much stronger field near the wake than elsewhere even when the source is not near the edge. Thus the wake acts as a convenient transmission channel for carrying intense sound away from the source. This problem was further extended to a point source by Balasubramanyam[8] and to the diffraction of a cylindrical pulse by Rienstra.[9] Later on Rawlins[10] addressed the diffraction of a cylindrical acoustic wave by an absorbing half-plane in a moving fluid. The theory assumes that the acoustic sources are fixed in position and that their only time variation is harmonic. In comparisons between the cases when Kutta-Joukowski condition is applied and when it is not, the excitation of the sources is taken to be the same in both cases. If the application of the Kutta-Joukowski condition were to alter the distribution of sources, as it might in turbulent flow, our deductions would need modifications. Nevertheless, it would seem reasonable to conclude that, in general, the effect of the Kutta-Joukowski condition is to produce a beam of sound in the neighborhood of the wake and to scatter a field elsewhere which is approximately that given by Ffowcs-Williams and Hall.[5] The aim of the present paper is to analyze the diffraction of a cylindrical wave by a penetrable strip introducing the wake condition to examine the effect of the Kutta-Joukowski condition. The mathematical method used to solve the problem is Jone's method. The diffracted far field is calculated using asymptotic approximations.

2. FORMULATION OF THE PROBLEM

We shall consider small amplitude sound waves

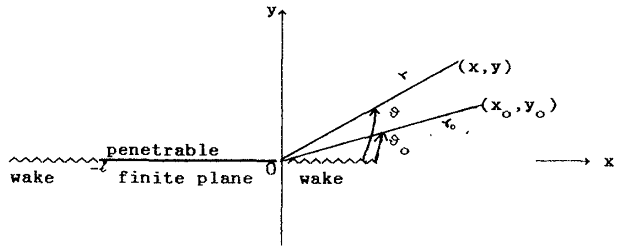

Fig. 1 Geometry of the problem.

diffracted by a strip. A penetrable strip is assumed to occupy $y=0$, $-l \le x \le 0$ as shown in the Fig. 1. The penetrable strip is assumed to be of negligible thickness and satisfying the penetrable boundary conditions on both sides of the surfaces. The time dependence is assumed to be of harmonic nature $\exp(-i\omega t)$ (ω is low angular frequency), with the free space wave number of the form

$$k=\frac{\omega}{c}=k_1+ik_2\,, \tag{1}$$

where c is the speed of sound. In Eq. (1), k has a small positive imaginary part which has been introduced to ensure the convergence (regularity) of the Fourier transform integrals defined subsequently (Eq. 10 b). The primary source is taken to be a line source which is located at the position (x_0, y_0), $y_0>0$. On suppressing the time harmonic factor, the wave equation satisfied by the total velocity potential ϕ_t in the presence of line source is

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k^2\right)\phi_t=\delta(x-x_0)\delta(y-y_0)\,. \tag{2}$$

The approximate boundary conditions for a penetrable medium of width $2h$ are given by Ref. 3)

$$\pm\frac{\partial}{\partial y}\phi_t(x,\pm h)+ika\phi_t(x,\pm h)+ik\beta\phi_t(x,\mp h)=0\,, \tag{3}$$

where

$$\alpha=\left(\frac{T^2\exp(2ikh\sin\vartheta_0)+[\exp(-2ikh\sin\vartheta_0)-R^2\exp(2ikh\sin\vartheta_0)]}{[\exp(-ikh\sin\vartheta_0)+R\exp(ikh\sin\vartheta_0)]^2-T^2\exp(2ikh\sin\vartheta_0)}\right)\sin\vartheta_0\,, \tag{3 a}$$

$$\beta=\frac{-2T\sin\vartheta_0}{[\exp(-ikh\sin\vartheta_0)+R\exp(ikh\sin\vartheta_0)]^2-T^2\exp(2ikh\sin\vartheta_0)}\,. \tag{3 b}$$

In Eqs. (3 a, b), the reflection and transmission coefficients R and T respectively are given by Ref. 3)

$$R=\frac{(1-N^2)\sin 2K_1h\exp(-2ikh\sin\vartheta_0)}{(1-N^2)\sin 2K_1h+2iN\cos 2K_1h}\,, \tag{3 c}$$

$$T=\frac{2iN\exp(-2ikh\sin\vartheta_0)}{(1-N^2)\sin 2K_1h+2iN\cos 2K_1h}, \quad (3\,\mathrm{d})$$

$$N=\frac{K_1\rho}{k\rho_1\sin\vartheta_0}, \quad n=\frac{c}{c_1},$$
$$K_1=\sqrt{(n^2-\cos^2\vartheta_0)},$$

and ρ, c and ρ_1, c_1 are the density and sonic velocity of the media $|y|>h$ and $|y|<h$ respectively. For the penetrable strip of negligible thickness ($2kh\ll 1$), Eq. (3) for $-l<x<0$, $y=0$ take the form

$$\left.\begin{aligned}&\frac{\partial}{\partial y}\phi_t(x,0^+)+ik\alpha\phi_t(x,0^+)+ik\beta\phi_t(x,0^-)=0,\\&\frac{\partial}{\partial y}\phi_t(x,0^-)-ik\alpha\phi_t(x,0^-)-ik\beta\phi_t(x,0^+)=0.\end{aligned}\right\} \quad (4)$$

We also require that the field shall be radiating outwards at infinity.

If we now ask that the field be continuous and possess finite local energy we are led to the two dimensional analogue of the field determined by Ffowcs-Williams and Hall.[5] Under the conditions stated this field is unique[11] and does not satisfy the Kutta-Joukowski condition of finite velocity at the edge. Therefore, to find a solution of Eq. (2) which satisfies the Kutta-Joukowski condition, we must abandon some of the other conditions imposed. We cannot dispose of Eq. (4) and it would seem desirable to retain the requirements that the field has finite local energy. Therefore, the only possibility left is to discard the continuity of the field.

The way to introduce a discontinuity in the field which seems most natural is to postulate a wake occupying $x<-l$, $x>0$, $y=0$. The form of this wake should be similar to that in steady flow but modified to allow for the oscillatory nature of the field. In spite of ϕ_t not being continuous across the wake we shall assume that the normal velocity $\partial\phi_t/\partial y$ is. Consequently we take the boundary condition as

$$\frac{\partial}{\partial y}\phi_t(x,y^+)=\frac{\partial}{\partial y}\phi_t(x,y^-), \quad (x<-l,\ x>0,\ y=0), \quad (5)$$

$$\left.\begin{aligned}&\phi_t(x,y^+)-\phi_t(x,y^-)=a\exp(i\mu x),\\&\qquad (x>0,\ y=0),\\&\phi_t(x,y^+)-\phi_t(x,y^-)=a\exp(-i\mu x),\\&\qquad (x<-l,\ y=0).\end{aligned}\right\} \quad (6)$$

In Eq. (6) a and μ are constants. The constant μ is regarded as known, *i.e.*

$$\mu=k\cos\vartheta_1, \quad (7)$$

where $0\leq \mathcal{R}e\,\vartheta_1<\pi$, $\mathcal{I}m\,\vartheta_1\geq 0$. While k has a positive imaginary part we shall take $0<\mathcal{R}e\,\vartheta_1<\pi$ and $\mathcal{I}m\,\vartheta_1>0$; eventually we shall be concerned primarily with the case $\mathcal{R}e\,\vartheta_1=0$, $\mathcal{I}m\,\vartheta_1>0$. In Eq. (6), '$a$' is as yet unknown and has to be found; its value depends upon the conditions imposed at the edge. We note that 'a'$=0$ corresponds to a no wake situation. It is appropriate to split ϕ_t as

$$\phi_t(x,y)=\phi_0(x,y)+\phi(x,y), \quad (8)$$

where ϕ_0 is the incident wave corresponding to the source term and ϕ is the solution of homogeneous wave Eq. (2) that corresponds to the diffracted potential. Thus ϕ_0 and ϕ satisfy the following equations

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k^2\right)\phi_0(x,y)=\delta(x-x_0)\delta(y-y_0), \quad (9)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k^2\right)\phi(x,y)=0. \quad (10)$$

3. SOLUTION OF THE PROBLEM

We define the Fourier transform pair by

$$\left.\begin{aligned}&\bar{\phi}(\nu,y)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\phi(x,y)\exp(i\nu x)dx,\\&\phi(x,y)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\bar{\phi}(\nu,y)\exp(-i\nu x)d\nu,\end{aligned}\right\} \quad (10\,\mathrm{a})$$

where ν is a complex variable. In order to accommodate three part boundary conditions on $y=0$, we split $\bar{\phi}(\nu,y)$ as

$$\bar{\phi}(\nu,y)=\bar{\phi}_+(\nu,y)+\exp(-i\nu l)\bar{\phi}_-(\nu,y)+\bar{\phi}_1(\nu,y), \quad (10\,\mathrm{b})$$

where

$$\bar{\phi}_+(\nu,y)=\frac{1}{\sqrt{2\pi}}\int_0^{\infty}\phi(x,y)\exp(i\nu x)dx,$$

$$\bar{\phi}_-(\nu,y)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-l}\phi(x,y)\exp(i\nu)(x+l)dx,$$

and

$$\bar{\phi}_1(\nu,y)=\frac{1}{\sqrt{2\pi}}\int_{-l}^{0}\phi(x,y)\exp(i\nu x)dx.$$

In Eq. (10 b), $\bar{\phi}_+$ is regular for $\mathcal{I}m\,\nu>-\mathcal{I}m\,k$, $\bar{\phi}_-$ is regular for $\mathcal{I}m\,\nu<\mathcal{I}m\,k$ and $\bar{\phi}_1(\nu,y)$ is an integral function and is therefore analytic in $-\mathcal{I}m\,k<\mathcal{I}m\,\nu<\mathcal{I}m\,k$. The solution of Eq. (9) is

$$\phi_0(x,y) = \frac{-1}{4i} H_0^{(1)}[k((x-x_0)^2+(y-y_0)^2)^{1/2}] = \frac{1}{4\pi i}\int_{-\infty}^{\infty} \frac{\exp[-i\nu(x-x_0)+i(k^2-\nu^2)^{1/2}|y-y_0|]}{\sqrt{k^2-\nu^2}} d\nu. \tag{11}$$

Making change of variables

$$x_0 = r_0 \cos \vartheta_0, \quad y_0 = r_0 \sin \vartheta_0, \quad (0<\vartheta_0<\pi),$$

in Eq. (11) and letting $r_0 \to \infty$ we obtain using the asymptotic form for the Hankel function

$$\phi_0 = b \exp(-ik(x \cos \vartheta_0 + y \sin \vartheta_0)), \tag{12}$$

where

$$b = \frac{-1}{4i}\sqrt{\frac{2}{\pi k r_0}} \exp(i(kr_0 - \pi/4)), \tag{13}$$

and ϑ_0 is the angle measured from the x-axis. Now taking Fourier transform of Eq. (10), we obtain

$$\left(\frac{d^2}{dy^2}+\gamma^2\right)\bar{\phi}(\nu,y)=0, \tag{14}$$

where $\gamma=\sqrt{k^2-\nu^2}$ and the ν-plane is cut such that $\mathcal{I}m\gamma>0$. The solution of Eq. (14) which satisfies the radiation condition is

$$\bar{\phi}(\nu,y) = \begin{cases} A_1(\nu)\exp(i\gamma y), & (y>0), \\ A_2(\nu)\exp(-i\gamma y), & (y<0). \end{cases} \tag{15}$$

Transforming the boundary conditions (4) to (6), we have

$$\bar{\phi}_1'(\nu,0^\pm) = \mp ik[\alpha\bar{\phi}_1(\nu,0^\pm)+\beta\bar{\phi}_1(\nu,0^\pm)] \mp ik(\alpha+\beta)\bar{\phi}_0(\nu,0) - \bar{\phi}_0'(\nu,0), \tag{16 a, b}$$

$$\bar{\phi}_\pm'(\nu,0^+) = \bar{\phi}_\pm'(\nu,0^-) = \bar{\phi}_\pm'(\nu,0), \tag{17 a, b}$$

$$\bar{\phi}_+(\nu,0^+) - \bar{\phi}_+(\nu,0^-) = \frac{ia}{\sqrt{2\pi}(\nu+\mu)}, \tag{18 a}$$

$$\bar{\phi}_-(\nu,0^+) - \bar{\phi}_-(\nu,0^-) = \frac{-ia\exp(i\mu l)}{\sqrt{2\pi}(\nu-\mu)}, \tag{18 b}$$

where "′" denotes differentiation with respect to "y". From Eqs. (10 b), (15) and (17), we can write

$$\bar{\phi}_+'(\nu,0)+\bar{\phi}_-'(\nu,0)\exp(-i\nu l)+\bar{\phi}_1'(\nu,0^+) = i\gamma[\bar{\phi}_+(\nu,0^+)+\bar{\phi}_-(\nu,0^+)\exp(-i\nu l) + \bar{\phi}_1(\nu,0^+)], \tag{19 a}$$

$$\bar{\phi}_+'(\nu,0)+\bar{\phi}_-'(\nu,0)\exp(-i\nu l)+\bar{\phi}_1'(\nu,0^-) = -i\gamma[\bar{\phi}_+(\nu,0^-)+\bar{\phi}_-(\nu,0^-)\exp(-i\nu l) + \bar{\phi}_1(\nu,0^-)]. \tag{19 b}$$

After eliminating $\bar{\phi}_1'(\nu,0^+)$ from (16 a) and (19 a), $\bar{\phi}_1'(\nu,0^-)$ from Eqs. (16 b) and (19 b) and adding the resulting expressions, we arrive at

$$\bar{\phi}_+'(\nu,0)+\bar{\phi}_-'(\nu,0)\exp(-i\nu l)-i\gamma N(\nu)J_1(\nu,0) = \bar{\phi}_0'(\nu,0) - \frac{a\gamma}{2\sqrt{2\pi}}\left(\frac{1}{(\nu+\mu)} - \frac{\exp(-i(\nu-\mu)l)}{(\nu-\mu)}\right), \tag{20}$$

where

$$N(\nu) = 1+\frac{k(\alpha-\beta)}{\gamma},$$

$$J_1(\nu,0) = \frac{1}{2}[\bar{\phi}_1(\nu,0^+)-\bar{\phi}_1(\nu,0^-)].$$

In a similar way by eliminating $\bar{\phi}_1(\nu,0^+)$ from Eqs. (16 a) and (19 a), $\bar{\phi}_1(\nu,0^-)$ from (16 b) and (19 b), and subtracting the resulting expressions, we obtain

$$\bar{\phi}_+(\nu,0^+)+\bar{\phi}_-(\nu,0^+)\exp(-i\nu l) - \frac{L(\nu)J_1'(\nu,0)}{ik\alpha} = (1+\beta\alpha^{-1})\bar{\phi}_0(\nu,0) + \frac{ia}{2\sqrt{2\pi}}\left[\frac{1}{\nu+\mu} - \frac{\exp(-i(\nu-\mu)l)}{\nu-\mu}\right] + \frac{\beta}{2\alpha}[\bar{\phi}_1(\nu,0^+)+\bar{\phi}_1(\nu,0^-)], \tag{21}$$

where

$$L(\nu) = 1+\frac{k\alpha}{\gamma}, \quad J_1'(\nu,0) = \frac{1}{2}[\bar{\phi}_1'(\nu,0^+)-\bar{\phi}_1'(\nu,0^-)].$$

From Eqs. (12) and (20), we have

$$\bar{\phi}_+'(\nu,0)+\bar{\phi}_-'(\nu,0)\exp(-i\nu l)-i\gamma N(\nu)J_1(\nu,0) + \frac{a\gamma N(\nu)}{2\sqrt{2\pi}}\left[\frac{1}{\nu+\mu} - \frac{\exp(-i(\nu-\mu)l)}{\nu-\mu}\right] - \frac{ak(\alpha-\beta)}{2\sqrt{2\pi}}\left[\frac{1}{\nu+\mu} - \frac{\exp(-i(\nu-\mu)l)}{\nu-\mu}\right] = \frac{-kb\sin\vartheta_0}{\sqrt{2\pi}(\nu-k\cos\vartheta_0)} \times [1-\exp(-i(\nu-k\cos\vartheta_0)l)]. \tag{22}$$

For the solution of the Wiener-Hopf functional equations, we make the following factorizations:

$$\gamma = (k+\nu)^{1/2}(k-\nu)^{1/2} = K_+(\nu)K_-(\nu), \tag{23}$$

and

$$\begin{aligned} N(\nu) &= N_+(\nu)N_-(\nu), \\ L(\nu) &= L_+(\nu)L_-(\nu), \end{aligned} \tag{24 a, b}$$

where $N_+(\nu)$, $L_+(\nu)$ and $K_+(\nu)$ are regular for $\mathcal{I}m\nu > -\mathcal{I}mk$ and $N_-(\nu)$, $L_-(\nu)$ and $K_-(\nu)$ are regular for $\mathcal{I}m\nu < \mathcal{I}mk$. Using the method as discussed by Noble [Ref. 12), p. 164] the factorizations (24) are given by

$$N_{\pm}(\nu)=1-\frac{i(\alpha-\beta)}{\pi}((\nu/k)^2-1)^{-1/2}\cos^{-1}(\pm\nu/k), \tag{25 a}$$

$$L_{\pm}(\nu)=1-\frac{i\alpha}{\pi}((\nu/k)^2-1)^{-1/2}\cos^{-1}(\pm\nu/k). \tag{25 b}$$

Thus, substitution of Eqs. (23) and (24) in Eq. (22) yields

$$\begin{aligned}\bar{\phi}_+'(\nu,0)&+\bar{\phi}_-'(\nu,0)\exp(-i\nu l)+S_+(\nu)S_-(\nu)J_1(\nu,0)\\&+\frac{i\alpha S_+(\nu)S_-(\nu)}{2\sqrt{2\pi}}\left[\frac{1}{\nu+\mu}-\frac{\exp(-i(\nu-\mu)l)}{\nu-\mu}\right]\\&-\frac{\alpha k(\alpha-\beta)}{2\sqrt{2\pi}}\left[\frac{1}{\nu+\mu}-\frac{\exp(-i(\nu-\mu)l)}{\nu-\mu}\right]\\&=\frac{-kb\sin\vartheta_0}{\sqrt{2\pi}(\nu-k\sin\vartheta_0)}\\&\quad\times[1-\exp(-i(\nu-k\cos\vartheta_0)l)].\end{aligned} \tag{26}$$

In Eq. (26), $S_+(\nu)[=K_+(\nu)N_+(\nu)]$ is regular for $\mathcal{I}m\,\nu>-\mathcal{I}m\,k$ and $S_-(\nu)[=K_-(\nu)N_-(\nu)]$ is regular for $\mathcal{I}m\,\nu<\mathcal{I}m\,k$. The unknown functions $\bar{\phi}_+'(\nu,0)$ and $\bar{\phi}_-'(\nu,0)$ in Eq. (26) have been determined using the procedure discussed by Noble [Ref. 12), p. 196] and are given by

$$\begin{aligned}\bar{\phi}_+'(\nu,0)=&\frac{-kb\sin\vartheta_0}{\sqrt{2\pi}}(S_+(\nu)G_1(\nu)+T(\nu)S_+(\nu)C_1)\\&+\frac{\alpha}{2\sqrt{2\pi}}\left(\frac{k(\alpha-\beta)}{(\nu+\mu)}-\frac{iS_+(\mu)S_+(\nu)}{(\nu+\mu)}\right.\\&\left.+\frac{T(\nu)S_+(\nu)}{(k+\mu)}C_3\right),\end{aligned} \tag{27 a}$$

$$\begin{aligned}\bar{\phi}_-'(\nu,0)=&\frac{-kb\sin\vartheta_0}{\sqrt{2\pi}}(S_-(\nu)G_2(-\nu)\\&+T(-\nu)S_-(\nu)C_2)\\&+\frac{\alpha}{2\sqrt{2\pi}}\left(\frac{k(\alpha-\beta)}{(\mu-\nu)}-\frac{iS_+(\mu)S_-(\nu)}{(\mu-\nu)}\right.\\&\left.+\frac{T(-\nu)S_-(\nu)}{(k+\mu)}C_3\right).\end{aligned} \tag{27 b}$$

In Eqs. (27 a, b),

$$S_+(\nu)=\sqrt{k+\nu}\,N_+(\nu),$$
$$S_-(\nu)=\exp(i\pi/2)\sqrt{\nu-k}\,N_-(\nu),$$

$$C_1=\frac{S_+(k)}{[1-T^2(k)S_+^2(k)]}[G_2(k)+G_1(k)T(k)S_+(k)],$$

$$C_2=\frac{S_+(k)}{[1-T^2(k)S_+^2(k)]}[G_1(k)+G_2(k)T(k)S_+(k)],$$

$$C_3=\frac{-iS_+(\mu)S_+(k)}{[1-T^2(k)S_+^2(k)]}[T(k)S_+(k)-\exp(i\mu l)],$$

$$\begin{aligned}G_1(\nu)=&\frac{1}{\nu-k\cos\vartheta_0}\left[\frac{1}{S_+(\nu)}\right.\\&\left.-\frac{1}{S_+(k\cos\vartheta_0)}\right]\\&-R_1(\nu)\exp(ikl\cos\vartheta_0),\end{aligned} \tag{28 a}$$

$$\begin{aligned}G_2(\nu)=&\frac{1}{\nu+k\cos\vartheta_0}\left[\frac{1}{S_+(\nu)}\right.\\&\left.-\frac{1}{S_+(-k\cos\vartheta_0)}\right]\\&\times\exp(ikl\cos\vartheta_0)-R_2(\nu),\end{aligned} \tag{28 b}$$

$$R_{1,2}(\nu)=\frac{E_{-1}[W_{-1}\{-i(k\pm k\cos\vartheta_0)l\}-W_{-1}\{-i(k+\nu)l\}]}{2\pi i(\nu\mp k\cos\vartheta_0)},$$

$$T(\nu)=\frac{1}{2\pi i}E_{-1}W_{-1}\{-i(k+\nu)l\},$$

$$E_{-1}=2\sqrt{l}\exp(ikl-3i\pi/4),$$

$$W_{-1}(m)=\Gamma\left(\frac{1}{2}\right)\exp(m/2)(m)^{-3/4}W_{-1/4,-1/4}(m),$$

$[m=-i(k+\nu)l$ and $W_{i,j}$ is a Whittaker function]. Now from Eqs. (10 b) and (15), we obtain

$$\begin{aligned}A_1(\nu)-A_2(\nu)=&\exp(-i\nu l)[\bar{\phi}_-(\nu,0^+)-\bar{\phi}_-(\nu,0^-)]\\&+[\bar{\phi}_1(\nu,0^+)-\bar{\phi}_1(\nu,0^-)]\\&+[\bar{\phi}_+(\nu,0^+)-\bar{\phi}_+(\nu,0^-)],\end{aligned} \tag{29 a}$$

$$\begin{aligned}A_1(\nu)+A_2(\nu)=&\frac{1}{i\gamma}\Big([\bar{\phi}_1'(\nu,0^+)-\bar{\phi}_1'(\nu,0^-)]\\&+[\bar{\phi}_+'(\nu,0^+)-\bar{\phi}_+'(\nu,0^-)]\\&+\exp(-i\nu l)[\bar{\phi}_-'(\nu,0^+)\\&-\bar{\phi}_-'(\nu,0^-)]\Big).\end{aligned} \tag{29 b}$$

Using Eqs. (17) and (18) in Eqs. (29) and then adding and subtracting the resulting expressions we get

$$\begin{aligned}A_1(\nu)=&\frac{i\alpha}{2\sqrt{2\pi}}\left[\frac{1}{\nu+\mu}-\frac{\exp(-i(\nu-\mu)l)}{\nu-\mu}\right]\\&+J_1(\nu,0)+\frac{J_1'(\nu,0)}{i\gamma},\end{aligned} \tag{30}$$

$$\begin{aligned}A_2(\nu)=&\frac{-i\alpha}{2\sqrt{2\pi}}\left[\frac{1}{\nu+\mu}-\frac{\exp(-i(\nu-\mu)l)}{\nu-\mu}\right]\\&-J_1(\nu,0)+\frac{J_1'(\nu,0)}{i\gamma}.\end{aligned} \tag{31}$$

Substituting the values of $J_1(\nu,0)$ and $J_1'(\nu,0)$ from Eqs. (20) and (21) into Eqs. (30) and (31), we obtain

$$\begin{aligned}A_1(\nu)=&\frac{i\alpha}{2\sqrt{2\pi}}\left[\frac{1}{\nu+\mu}-\frac{\exp(-i(\nu-\mu)l)}{\nu-\mu}\right]\\&+\frac{1}{i\gamma N(\nu)}\Big[\bar{\phi}_+'(\nu,0)+\bar{\phi}_-'(\nu,0)\exp(-i\nu l)\\&-\bar{\phi}_0'(\nu,0)+\frac{\alpha\gamma}{2\sqrt{2\pi}}\Big\{\frac{1}{\nu+\mu}\end{aligned}$$

$$-\frac{\exp(-i(\nu-\mu)l)}{\nu-\mu}\Big\}\Big]+\frac{ik\alpha}{i\gamma L(\nu)}\Big[\bar{\phi}_+(\nu,0^+)$$
$$+\bar{\phi}_-(\nu,0^+)\exp(-i\nu l)-(1+\beta\alpha^{-1})\bar{\phi}_0(\nu,0)$$
$$-\frac{ia}{2\sqrt{2\pi}}\Big\{\frac{1}{\nu+\mu}-\frac{\exp(-i(\nu-\mu)l)}{\nu-\mu}\Big\}$$
$$-\frac{\beta}{2\alpha}\{\bar{\phi}_1(\nu,0^+)+\bar{\phi}_1(\nu,0^-)\}\Big], \quad (32)$$

$$A_2(\nu)=\frac{-ia}{2\sqrt{2\pi}}\Big[\frac{1}{\nu+\mu}-\frac{\exp(-i(\nu-\mu)l)}{\nu-\mu}\Big]$$
$$-\frac{1}{i\gamma N(\nu)}\Big[\bar{\phi}_+'(\nu,0)+\bar{\phi}_-'(\nu,0)\exp(-i\nu l)$$
$$-\bar{\phi}_0'(\nu,0)+\frac{a\gamma}{2\sqrt{2\pi}}\Big\{\frac{1}{\nu+\mu}$$
$$-\frac{\exp(-i(\nu-\mu)l)}{\nu-\mu}\Big\}\Big]+\frac{ik\dot{\alpha}}{i\gamma L(\nu)}\Big[\bar{\phi}_+(\nu,0^+)$$
$$+\bar{\phi}_-(\nu,0^+)\exp(-i\nu l)-(1+\beta\alpha^{-1})\bar{\phi}_0(\nu,0)$$
$$-\frac{ia}{2\sqrt{2\pi}}\Big\{\frac{1}{\nu+\mu}-\frac{\exp(-i(\nu-\mu)l)}{\nu-\mu}\Big\}$$
$$-\frac{\beta}{2\alpha}\{\bar{\phi}_1(\nu,0^+)+\bar{\phi}_1(\nu,0^-)\}\Big]. \quad (33)$$

We note that

$$N(\nu)\approx 1+O(\alpha,\beta), \quad L(\nu)\approx 1+O(\alpha),$$

and assert that $(k\alpha/\gamma)$ and $(k\beta/\gamma)$ are very very small provided that $|\nu/k|$ is not too near 1. This can be justified under small parameters α, β and low frequency of the acoustic wave. Thus using this Eqs. (25), (32) and (33) give

$$N_\pm(\nu)\approx 1\mp\frac{\nu(\alpha-\beta)}{\pi\gamma}, \quad (34\,\text{a})$$

$$L_\pm(\nu)\approx 1\mp\frac{\nu\alpha}{\pi\gamma}, \quad (34\,\text{b})$$

$$A_1(\nu)=-A_2(\nu)$$
$$=\frac{1}{i\gamma}\Big(\bar{\phi}_+'(\nu,0)+\bar{\phi}_-'(\nu,0)\exp(-i\nu l)$$
$$-\bar{\phi}_0'(\nu,0)\Big). \quad (34\,\text{c})$$

Note that in writing Eqs. (34), we have retained the terms of order $O[(\alpha,\beta)/\gamma]$ and neglected the terms of $O[k(\alpha,\beta)/\gamma]$. Substitution of Eqs. (12) and (27 a, b) in Eq. (34 c) yield

$$A_1(\nu)=-A_2(\nu)$$
$$=\frac{kb\sin\vartheta_0}{\sqrt{2\pi}\,i\gamma(\nu-k\cos\vartheta_0)}\Big\{\frac{S_+(\nu)}{S_+(k\cos\vartheta_0)}$$
$$-\frac{S_+(-\nu)\exp(-i(\nu-k\cos\vartheta_0)l)}{S_+(-k\cos\vartheta_0)}\Big\}$$
$$-\frac{kb\sin\vartheta_0}{\sqrt{2\pi}\,i\gamma}\{S_+(\nu)T(\nu)C_1-S_+(\nu)R_1(\nu)$$
$$\times\exp(ikl\cos\vartheta_0)+S_+(-\nu)R_2(-\nu)$$
$$\times\exp(-i\nu l)+C_2T(-\nu)S_+(-\nu)$$
$$\times\exp(-i\nu l)\}+\frac{a}{2\sqrt{2\pi}\,i\gamma}\Big\{k(\alpha-\beta)\Big[\frac{1}{\nu+\mu}$$
$$+\frac{\exp(-i\nu l)}{\mu-\nu}\Big]-iS_+(\mu)\Big[\frac{S_+(\nu)}{(\nu+\mu)}$$
$$+\frac{\exp(-i\nu l)S_+(-\nu)}{\mu-\nu}\Big]+\frac{C_3}{(k+\mu)}\Big[T(\nu)S_+(\nu)$$
$$+\exp(-i\nu l)T(-\nu)S_+(-\nu)\Big]\Big\}. \quad (35)$$

Now putting the values of $A_1(\nu)$ in Eq. (15) and taking inverse Fourier transform the field $\phi(x,y)$ can be written as

$$\phi(x,y)=\phi^{\text{sep}}(x,y)+\phi^{\text{int}}(x,y), \quad (36)$$

where

$$\phi^{\text{sep}}(x,y)=\frac{kb\sin\vartheta_0}{2\pi}\int_{-\infty}^{\infty}\frac{S_+(\nu)\exp(i\gamma y-i\nu x)}{i\gamma(\nu-k\cos\vartheta_0)S_+(k\cos\vartheta_0)}d\nu$$
$$-\frac{kb\sin\vartheta_0}{2\pi}\int_{-\infty}^{\infty}\frac{\exp(-i(\nu-k\cos\vartheta_0)l)S_+(-\nu)\exp(i\gamma y-i\nu x)}{i\gamma(\nu-k\cos\vartheta_0)S_+(-k\cos\vartheta_0)}d\nu$$
$$+\frac{a}{4\pi}\Big[k(\alpha-\beta)\int_{-\infty}^{\infty}\frac{1}{i\gamma}\Big\{\frac{1}{\nu+\mu}+\frac{\exp(-i\nu l)}{\mu-\nu}\Big\}-iS_+(\mu)\int_{-\infty}^{\infty}\frac{1}{i\gamma}\Big\{\frac{S_+(\nu)}{\nu+\mu}+\frac{\exp(-i\nu l)S_+(-\nu)}{\mu-\nu}\Big\}\Big]$$
$$\times\exp(i\gamma y-i\nu x)d\nu, \quad (37)$$

$$\phi^{\text{int}}(x,y)$$
$$=\frac{kb\sin\vartheta_0}{2\pi}\int_{-\infty}^{\infty}\frac{1}{i\gamma}[S_+(\nu)R_1(\nu)\exp(ikl\cos\vartheta_0)$$
$$-S_+(-\nu)R_2(-\nu)\exp(-i\nu l)-S_+(\nu)T(\nu)C_1$$
$$-T(-\nu)S_+(-\nu)\exp(-i\nu l)C_2]\exp(i\gamma y-i\nu x)d\nu$$
$$+\frac{aC_3}{4\pi(k+\mu)}\int_{-\infty}^{\infty}\frac{1}{i\gamma}[T(\nu)S_+(\nu)$$
$$+\exp(-i\nu l)T(-\nu)S_+(-\nu)]\exp(i\gamma y-i\nu x)d\nu. \quad (38)$$

In order to solve the integrals appearing in Eqs. (37) and (38), we put $x=r\cos\vartheta$, $y=r\sin\vartheta$ and deform the contour by the transformation $\nu=-k\cos(\vartheta+i\xi)$. Hence after using Eqs. (7) and (13), we have for large kr

$$\phi^{\text{sep}}(x, y) = \frac{i\exp(ik(r+r_0))}{4\pi k(\cos\vartheta+\cos\vartheta_0)(rr_0)^{1/2}} f_1(-k\cos\vartheta) + \frac{a}{2(2\pi kr)^{1/2}}\Bigg[(\alpha-\beta)\exp(i\pi/4) \times\left\{\frac{1}{(\cos\vartheta_1-\cos\vartheta)}+\frac{\exp(ikl\cos\vartheta)}{(\cos\vartheta_1+\cos\vartheta)}\right\} + \frac{\exp(-i\pi/4)S_+(k\cos\vartheta_1)}{k}\left\{\frac{S_+(-k\cos\vartheta)}{(\cos\vartheta_1-\cos\vartheta)} + \frac{\exp(ikl\cos\vartheta)S_+(k\cos\vartheta)}{(\cos\vartheta_1+\cos\vartheta)}\right\}\Bigg]\exp(ikr), \quad (39)$$

$$\phi^{\text{int}}(x, y) = \frac{i\exp(ik(r+r_0))}{4\pi(rr_0)^{1/2}} f_2(-k\cos\vartheta) + \frac{a\exp(i(kr+\pi/4))}{2(2\pi kr)^{1/2}} f_3(-k\cos\vartheta). \quad (40)$$

In Eqs. (39) and (40),

$$f_1(-k\cos\vartheta) = -\sin\vartheta_0\left[\frac{S_+(-k\cos\vartheta)}{S_+(k\cos\vartheta_0)} - \frac{S_+(k\cos\vartheta)\exp(ikl(\cos\vartheta+\cos\vartheta_0))}{S_+(-k\cos\vartheta_0)}\right],$$

$$f_2(-k\cos\vartheta) = \sin\vartheta[S_+(-k\cos\vartheta)R_1(-k\cos\vartheta)\exp(ikl\cos\vartheta_0) - S_+(k\cos\vartheta)R_2(k\cos\vartheta)\exp(ikl\cos\vartheta) - S_+(-k\cos\vartheta)T(-k\cos\vartheta)C_1 - S_+(k\cos\vartheta)T(k\cos\vartheta)C_2\exp(ikl\cos\vartheta)],$$

$$f_3(-k\cos\vartheta) = \frac{C_3}{(k+k\cos\vartheta_1)}[T(-k\cos\vartheta)S_+(-k\cos\vartheta) + \exp(ikl\cos\vartheta)T(k\cos\vartheta)S_+(k\cos\vartheta)].$$

From Eqs. (36), (39) and (40), we obtain

$$\phi(x, y) = \frac{i\exp(ik(r+r_0))}{4\pi(rr_0)^{1/2}k}\left[\frac{f_1(-k\cos\vartheta)}{(\cos\vartheta+\cos\vartheta_0)} + kf_2(-k\cos\vartheta)\right] + \frac{a\exp(i(kr+\pi/4))}{2(2\pi kr)^{1/2}} \times\Bigg[(\alpha-\beta)\left\{\frac{1}{(\cos\vartheta_1-\cos\vartheta)} + \frac{\exp(ikl\cos\vartheta)}{(\cos\vartheta_1+\cos\vartheta)}\right\} + f_3(-k\cos\vartheta) - \frac{iS_+(k\cos\vartheta_1)}{k}\left\{\frac{S_+(-k\cos\vartheta)}{(\cos\vartheta_1-\cos\vartheta)} + \frac{\exp(ikl\cos\vartheta)S_+(k\cos\vartheta)}{(\cos\vartheta_1+\cos\vartheta)}\right\}\Bigg]. \quad (41)$$

In the limit $r\to 0$, Eq. (41) shows that

$$\phi(x, y) \approx 2r^{1/2}\Bigg[-\frac{\exp(ikr_0)}{4\pi(r_0)^{1/2}}\left\{f_1(-k\cos\vartheta) + \frac{k}{2}f_2(-k\cos\vartheta)\right\} + \frac{ia\exp(i\pi/4)k}{(2\pi k)^{1/2}}\Bigg\{(\alpha-\beta)(1+\exp(ikl\cos\vartheta)) - \frac{iS_+(k\cos\vartheta_1)}{k}(S_+(-k\cos\vartheta) + \exp(ikl\cos\vartheta)S_+(k\cos\vartheta)) + \frac{f_3(-k\cos\vartheta)}{2}\Bigg\}\Bigg],$$

where we have neglected the terms which are constant and $O(r)$. Therefore, the velocity will remain bounded at the edge if and only if the co-efficient of $r^{1/2}$ vanishes. Hence the Kutta-Joukowski condition requires that

$$a = \frac{\exp(ikr_0-3i\pi/4)}{(2\pi kr_0)^{1/2}} g_1(-k\cos\vartheta), \quad (42)$$

where

$$g_1(-k\cos\vartheta) = \left\{f_1(-k\cos\vartheta) + \frac{k}{2}f_2(-k\cos\vartheta)\right\} \times\Bigg\{(\alpha-\beta)(1+\exp(ikl\cos\vartheta)) + \frac{f_3(-k\cos\vartheta)}{2} - \frac{iS_+(k\cos\vartheta_1)}{k}(S_+(-k\cos\vartheta) + \exp(ikl\cos\vartheta)S_+(k\cos\vartheta))\Bigg\}^{-1}.$$

Using Eq. (42) in Eq. (41), the far field is given by

$$\phi = \phi_A + \phi_W, \quad (43)$$

where ϕ_A denotes that part of ϕ that arises when there is no wake and ϕ_W the part that arises when there is a wake. They are explicitly given by

$$\phi_A = \frac{i\exp(ik(r+r_0))}{4\pi(rr_0)^{1/2}k} g_2(-k\cos\vartheta), \quad (44)$$

$$\phi_W = \frac{i\exp(ik(r+r_0))}{4\pi(rr_0)^{1/2}k} g_3(-k\cos\vartheta), \quad (45)$$

In Eqs. (44) and (45)

$$g_2 = (-k\cos\vartheta) = \left[\frac{f_1(-k\cos\vartheta)}{(\cos\vartheta+\cos\vartheta_0} + kf_2(-k\cos\vartheta)\right],$$

$$g_3(-k\cos\vartheta) = -g_1(-k\cos\vartheta)\Bigg[f_3(-k\cos\vartheta) + (\alpha-\beta)\left\{\frac{1}{(\cos\vartheta_1-\cos\vartheta)} + \frac{\exp(ikl\cos\vartheta)}{(\cos\vartheta_1+\cos\vartheta)}\right\} - \frac{iS_+(k\cos\vartheta_1)}{k}\left\{\frac{S_+(-k\cos\vartheta)}{(\cos\vartheta_1-\cos\vartheta)} + \frac{\exp(ikl\cos\vartheta)S_+(k\cos\vartheta)}{(\cos\vartheta_1+\cos\vartheta)}\right\}\Bigg].$$

4. CONCLUSIONS

The problem solved in this paper takes into account the material properties and thickness of the finite plane. It may be that in practice it is more convenient to measure the reflection coefficient R and the transmission coefficient T for a finite plane (rather than determining the material properties), in which case the Eq. (43) can still be represented in terms of theses quantities via the expressions (3 a) and (3 b). It is also worth looking that the approximate boundary condition (3) is insensitive to the variation of the angle of incidence when the finite plane is dense ($K_1 h \to \infty$). This is because the factors α and β in (3 a) and (3 b) become independent of the incident angle. In addition, the diffracted field is found to be strongly dependent upon the frequency. The high frequency sound is diffracted into the shadow of the barrier. Therefore, a noise barrier should be designed to make the transmission as small as possible, to reduce the low frequency transmitted sound, and the edges should be treated to reduce to a minimum the high frequency diffracted noise. In the illuminated region sound can be reduced by making the reflection as small as possible. It is found from Eqs. (39) and (40) that ϕ^{sep} consists of two parts each representing the diffracted field produced by the edges at $x=0$ and $x=-l$, respectively, as though the other edges were absent while ϕ^{int} gives the interaction of one edge upon the other. It is also of interest to note how the parameter $(\alpha-\beta)$ enters the calculation. The parameter $(\alpha-\beta)$ represents the absorption of the barrier and is intimately included in the calculation through its role in the terms $N_{\pm}$ and N.

Some simple physically interesting features of Eq. (43) are also noted. First, it is observed that the imposition or otherwise of the Kutta-Joukowski condition does not have much influence on the diffracted field away from the diffracting plane. On the other hand, near the wake the field is strengthened and weakend elsewhere even when the source is not near the edge. Second, the results for no wake situation can be obtained by taking 'a'$=0$. Third, the field corresponds to a rigid barrier if we put $\alpha=0=\beta$. This situation occurs if the material comprising the finite plane becomes very dense, *i.e.* $\mathcal{I}m(n)>0$, $|n|\to\infty$ ($K_1 h\to\infty$). Fourth, the results for an absorbing finite plane in presence of a wake can be obtained by taking $\beta=0$ and $\alpha=\rho_0 c/z_1$ (ρ_0 is the density of the undisturbed stream, c is the speed of sound and z_1 is the acoustic impedance of the surface). Thus, the consideration of the penetrable finite plane with wake present a more generalized model in the theory of diffraction and quite a few interesting situations can be obtained as a special case by choosing suitable parameters.

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