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Finite element simulation of coupled vibration modes in an ultrasonic cleaning tub: Effect of the presence of a washing object

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1. Introduction

Ultrasonic cleaning tubs have been used for several decades, but not much is yet known about the distribution of the wave field in a tub. New tubs are designed on the basis of experience and by trial and error. The reason for this is that there is intense coupling between the vibration of the water and the vibration of the tub wall, making prediction a very difficult task.

A numerical analysis of this situation has been published in [1], but its emphasis is on an entire tub with a vibrator; there is no description of all the modes occurring in the tank. [2] describes the modes occurringin a tank in terms of the tank size, wall thickness and wall material.

This report discusses the effect on the acoustic modes of the tub water and the vibration modes of the tub walls due to a washing object immersed in an ultrasonic cleaning tub.

2. Formulation of finite element equations

A clear and comprehensive treatment of the formulation of finite element equations, E and EZ acoustic modes can be found in [1]. In sections 2 and 3 we will briefly review the formulation of finite element equations, E and EZ acoustic modes based on the materials in [1].

The cleansing effect of ultrasonic cleaning tanks is due to cavitation. In this paper, however, a linear vibratory system will be assumed. Analysis will be performed using a three-dimensional finite element model composed of a lineartank wall vibratory system and an acoustic system. The following simultaneous equations are obtained by finite analysis theory [1].

$$\left.\begin{aligned}&([S]-\omega^2[M])\{U\}-[\Gamma]\{P\}=\{0\}\\&([\hat{S}]/(\hat{\rho}\omega^2)-[\hat{M}])/(\hat{\rho}c^2)\{P\}-[\Gamma]^{\mathrm{T}}\{U\}=\{W\}\end{aligned}\right\}\qquad(1)$$

Here, $\{U\}$ is the system nodal displacement vector, $\{P\}$ is the system modal sound pressure vector, $[S]$ and $[M]$ are the stiffness and mass matrices of the vibratory system, $[\hat{S}]$ and $[\hat{M}]$ are the inertance and elastance matrices of the acoustic system, $[\Gamma]$ is the coupling matrix, ω is an angular frequency, $\hat{\rho}$ is the density of water, c is the acoustic speed in water and $\{W\}$ is the acoustic forcing vector.

Figure 1 shows cross sections of the cleaning tub model used in this study and how it was divided into elements. Since the tub was symmetrical, only a one-quarter model was needed for the finite element analysis. The planes of symmetry are the x-z and y-z planes. The stainless steel wall is 1.2 mm thick, which is simulated as a single element in the thickness direction. The dimensions of the cavity are (5,4,7) cm in the (x,y,z) directions, and the divisions are (10,8,16) elements. The physical constants for the stainless steel used here are the following: Young's modulus, $15\times10^{10}\,\mathrm{N/m^2}$; Poisson's ratio, 0.295; and density, $7{,}800\,\mathrm{kg/m^3}$. The upper face of the water is bounded by air, so the acoustic pressure here was set at zero. A force was applied at an acoustical element with velocity drive.

3. E and EZ acoustic modes

The tub analysis was performed for the following four combinations of acoustical boundary conditions for the x-z and the y-z symmetry planes: (zero pressure, zero pressure), (zero pressure, rigid), (rigid, zero pressure), and (rigid, rigid). Modes with high sound pressure, which are effective for cleaning, are called acoustic modes. Figure 2 shows the acoustic modes for the range 0–33 kHz under the four combinations of boundary conditions. The views show the isobars for sound pressure, and, as stated, either the x-z cross section or the y-z cross section is the plane of symmetry. The views are designated $\mathrm{E}_{l,m,n}$, where $l,m,n=1,2,3,\ldots$ When the outer boundary is assumed to have zero acoustic pressure, the designation is $\mathrm{EZ}_{l,m,n}$. If L_X, L_Y, L_Z are the tub dimensions in the x,y,z directions and the x-z and y-z planes of symmetry pass through the origin, the resonance frequency is given by the following expression [1]:

$$f(l,m,n)=\frac{c}{2}\sqrt{\left(\frac{l}{2L_X}\right)^2+\left(\frac{m}{2L_Y}\right)^2+\left(\frac{n}{L_Z}\right)^2}\qquad(2)$$

Here, $c=1{,}500\,\mathrm{m/s}$ and $l,m,n=1,2,3,\ldots$ represent the number of oval modes in the x, y and z directions, respectively, in the entire tank. When l is an even number, the y-z plane ($x=0$) is a zero-pressure boundary, when m is an even number, the x-z plane ($y=0$) is a zero-pressure boundary, and when l and m are odd numbers, these two planes are rigid walls. Table 1 shows the resonance frequencies of the $\mathrm{EZ}_{l,m,n}$ and $\mathrm{E}_{l,m,n}$ modes. Each $\mathrm{E}_{l,m,n}$ mode corresponds to an $\mathrm{EZ}_{l,m,n}$ mode and has a resonance frequency 3–4 kHz lower than the $\mathrm{EZ}_{l,m,n}$ mode.

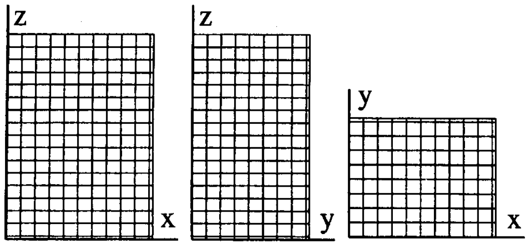


Fig. 1 Cross-sectional diagrams showing elements of ultrasonic cleaning tub for *x-z*, *y-z*, and *x-y* cross sections.

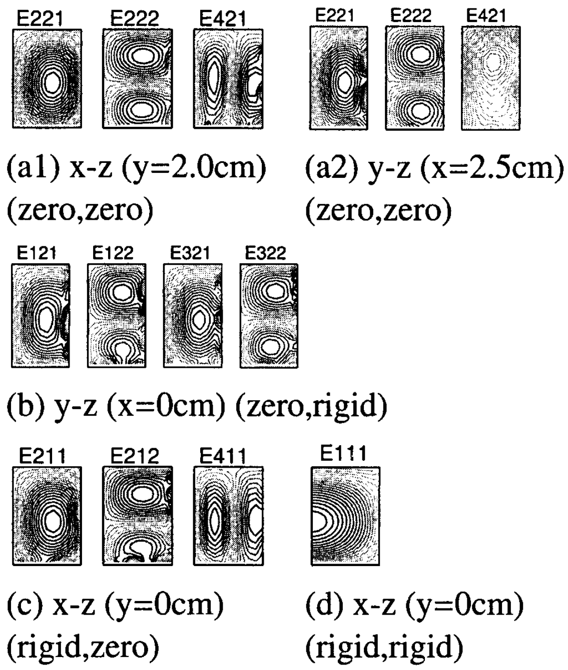


Fig. 2 Diagram of acoustic modes under 4 sets of acoustic boundary conditions (Each figure shows acoustic isobars on *x-z*, *y-z* cross sections, under (zero,zero), (zero,rigid), (rigid,zero), (rigid,rigid) boundary conditions).

Table 1 $EZ_{l,m,n}$ mode and $E_{l,m,n}$ mode [kHz].

mode	1,1,1	2,1,1	1,2,1	1,1,2	2,2,1	3,1,1	2,1,2	1,2,2
EZ	16.09	20.68	22.86	24.56	26.29	26.63	27.79	29.45
E	13.79	17.59	19.52	21.74	22.30	22.94	24.60	25.77
mode	1,3,1	3,2,1	2,2,2	3,1,2	4,1,1	1,1,3	3,2,2	4,2,1
EZ	31.02	31.19	32.18	32.46	33.21	34.31	36.29	36.96
E	26.40	27.63	28.10	28.58	29.33	31.05	31.84	32.34

4. Observations of phenomena in the presence of a washing object in the tub

It was observed how the acoustic modes and resonance modes changed when there was a washing object in the tub. The acoustic boundary condition of (rigid, rigid) walls was used. Since the mechanical boundary conditions of the washing object were limited to symmetrical conditions, displacement of the object was restricted to up or down on the central axis (z axis) or within the planes of symmetry (x-z plane, y-z plane). The washing object was free to move in any direction when in any other location. The position of the washing object was expressed in terms of elements in the x, y, z directions (see Fig. 1 for the element model).

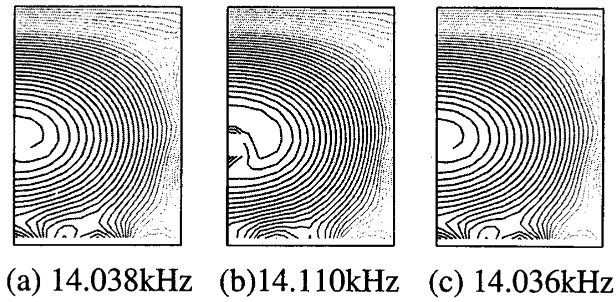


Fig. 3 Acoustic modes, varying with mechanical restraint conditions of washing object. (a) Fixed, rigid object (b) Unconstrained (c) Constrained to translate along central axis of washing object.

4.1. Mechanical boundary conditions of the washing object

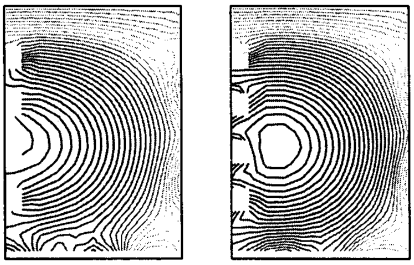
Let us begin with an examination of the changes in acoustic modes due to the presence in the tub of a one-element, solid washing object made of stainless steel, especially with reference to the $E_{1,1,1}$ mode (frequency is 13.792 kHz when no washing object is present).

Figure 3(a), (b), (c) shows the acoustic modes when a single, rectangular, parallelepiped element (a block at position (1,1,8)) is in the tub, under these three fixed conditions: fixed, unconstrained, or allowed to move only along its central axis. All parts of the figure show the first acoustic mode with the washing object on the x-z cross section. No great disturbance appeared in the acoustic mode in comparison to the $E_{1,1,1}$ mode with no washing object. However, the resonance frequencies ($14.038 = 13.792 + 0.246$ kHz (a), $14.110 = 13.792 + 0.318$ kHz (b), $14.036 = 13.792 + 0.244$ kHz (c)) of the $E_{1,1,1}$ mode in all these cases shifted by no less than 244 Hz higher compared to the frequency of the empty-tub value (13.792 kHz). Thus, there was a rather large effect due to the washing object. The mechanical boundary conditions of the object was also a large influence, since the resonance frequency was 72 Hz higher under the free boundary condition (b) than under the fixed condition (a).

Next, Fig. 4(a), (b) shows the acoustic modes in the presence of four, rigid, rectangular parallelepiped washing objects, at positions (1,1,5), (1,1,8), (1,1,11) and (1,1,14). This figure shows the acoustic modes when the objects were fixed in place and when they were mechanically unconstrained (that is, the first acoustic mode on the x-z cross section); the resonance frequencies were 14.044 kHz (a) and 14.346 kHz (b), respectively. Compared with the previous results, the added washing objects had little effect on the resonance frequency when the objects were fixed, but when they were unconstrained, the added washing objects caused a rather large change in the resonance frequency.

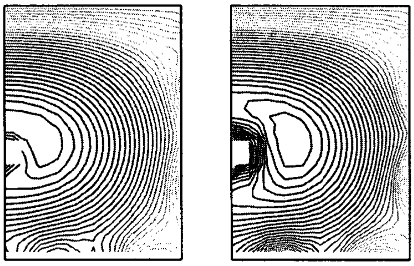
4.2. Young's modulus and density of washing objects

Let us turn to changes in the acoustic mode due to reductions in either Young's modulus or the density of the washing objects. A single, rectangular parallelepiped element



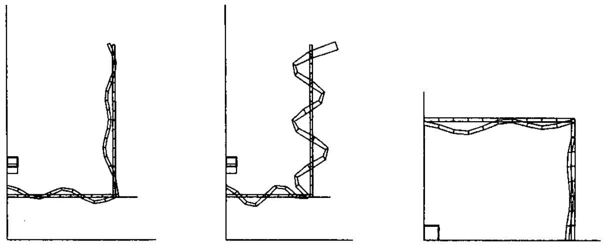
(a) 14.044kHz (b) 14.346kHz

Fig. 4 Acoustic modes when there are four washing objects (a) Fixed, rigid object (b) Unconstrained.

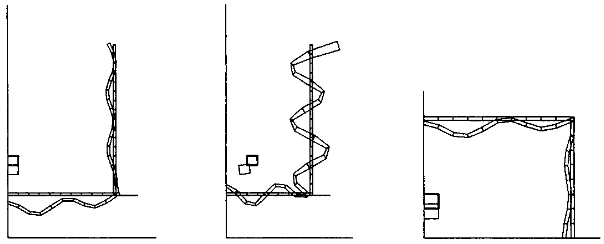


(a) 14.107kHz (b) 14.489kHz

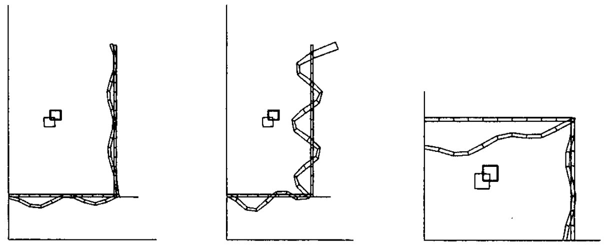
Fig. 5 Acoustic modes, varying with Young's modulus and density of solid washing object. (a) Young's modulus = $15 \times 10^9\,\mathrm{N/m^2}$ (b) Density = $1{,}000\,\mathrm{kg/m^3}$.



(a) Washing object at (1,1,5), resonance frequency = 14.072 kHz



(b) Washing object at (1,3,5), resonance frequency = 14.099 kHz



(c) Washing object at (5,5,10), resonance frequency = 14.094 kHz

Fig. 6 Vibration modes on *x-z* cross section, *y-z* section, *x-y* section when a single washing object is immersed (all mechanical conditions are unconstrained).

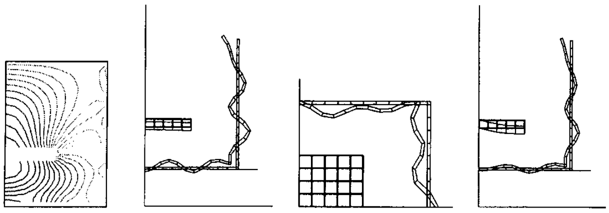
(a block at (1,1,8)) was used as the washing object under unconstrained mechanical conditions. Figure 5(a), (b) shows the acoustic modes when Young's modulus was reduced to $15 \times 10^9\,\mathrm{N/m^2}$ (density = $7{,}800\,\mathrm{kg/m^3}$) and the density was reduced to $1{,}000\,\mathrm{kg/m^3}$ (Young's modulus = $15 \times 10^{10}\,\mathrm{N/m^2}$). The resonance frequencies were 14.107 (=14.110 − 0.003) kHz (a) and 14.489 (=14.110 + 0.379) kHz (b), respectively; both graphs show the acoustic modes on the *x-z* cross sections. In comparison with the $E_{1,1,1}$ mode in the case of Fig. 3(b) (14.110 kHz), the resonance frequency was lowered by just 3 Hz and the acoustic mode showed no change after the reduction of Young's modulus. Changing the density, however, caused a much greater concentration of sound pressure in the vicinity of the washing object, and the resonance frequency rose 379 Hz. The washing object density was, therefore, found to have a large influence on the acoustic parameters of the ultrasonic cleaning tub.

4.3. Location changes of solid washing object and tub vibration

Here, the results were observed when a single, mechanically unconstrained washing object was placed on the *y-z* plane at (1,1,5) and (1,3,5) and at (5,5,10), off the plane of symmetry. The changes were observed in the resonance mode in the center of the cavity relative to the $E_{1,1,1}$ mode (for the empty tub, 13.792 kHz), due to the addition of a one-element, solid washing object. These resonance modes are shown in Fig. 6. The bold solid lines depict the tub walls before excitation, and the dashed lines, after excitation. These are cross sections of the tub in the first mode of vibration on the *x-z* cross section, the *y-z* cross section and the *x-y* cross section. The vibration amplitudes are magnified. It is also clear that, although the washing object shows no shape change, its position shifted. All the results indicate a downward shift of the object. In the case shown in Fig. 6(a), the object appeared to oscillate about its original position during each cycle. If the Young's modulus of the object is reduced, the vibration amplitude increases. But, for a material with the density of stainless steel, the variation in the maximal value of the vibrating motion is only about 2%.

4.4. Large washing objects

The changes in acoustic and vibration modes due to immersion of a rather large washing object in the tub were observed. This object measured 5 elements (2.5 cm) 4 elements (2.0 cm) 1 unit (0.4875 cm) in the *x*, *y* and *z* directions, respectively. It was placed at the 7th element above the tub bottom with an unconstrained mechanical boundary condition. Three different types of acoustic modes appeared when the modes were observed at 17 kHz–21 kHz; these were at 18.943 kHz, 19.397 kHz and 19.680 kHz. Figures 7(a1)–(a3) show the acoustic and vibration modes at 19.680 kHz; the acoustic mode was the first mode, as seen on the *x-z* cross section, and included the washing object. The vibration mode was also the first, as is shown on the *x-z* and *y-z* cross sections. The blank portion of the acoustic mode in the figure is the washing object. The acoustic modes approached zero pressure in the vicinity of the walls and assumed oval



(a1) 19.680kHz (a2) 19.680kHz (a3) 19.680kHz (b) 19.713kHz
(a1-a3)Mechanically unconstrained central axis of washing object (b)Constrained to translate along

Fig. 7 Acoustic pressure and vibration modes when a large washing object is immersed in tub.

shapes surrounding the washing object; high-intensity acoustic waves are seen over a wide region. The waves were of opposing phases in the upper and lower portions of the tub. On the other hand, the vibration mode was in the vertical direction along the z axis with the washing object. The larger the washing object, the more it approached the condition of acting as a part of the tub, and the more difficult it became to predict the sound pressure distribution.

The same three modes appeared when the axis of the washing object was fixed at 18.961 kHz, 19.420 kHz and 19.713 kHz. There were no dramatic changes in the acoustic modes due to the change in the mechanical boundary conditions, but the frequencies did rise. However, the vibration which is characteristic of a beam cantilevered at the y axis was seen. Figure 7(b) shows the vibration mode on an x-z cross section at 19.713 kHz.

5. Summary

The acoustic modes in an ultrasonic cleaning tub containing washing objects vary with the objects' locations, mechanical restraint conditions, Young's modulus, and density. If small washing objects are inserted into regions of high sound pressure, the resonance frequency of the tub increases, but the mode shape does not change remarkably. The resonance frequencies change due to the fixed or free mechanical boundary conditions of the objects. It was found that this is due to the oscillation of the washing objects, although the objects do not change their shape. Also, high sound-pressure regions are concentrated around lower-density washing objects. Acoustic modes must be predicted numerically when the washing objects are large.

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