Acoust. Sci. & Tech. **25**, 6 (2004)

INVITED REVIEW

Recent research on the acoustics of pianos

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Abstract: Recent research on the acoustics of the piano are reviewed focusing on the topics which were presented at the International Symposium on Musical Acoustics in Nara (ISMA2004) and the International Conference on Acoustics in Kyoto (ICA2004) which were held in Japan from late March to the beginning of April in 2004. The topics include the secondary partials in piano tones, string excitation by the hammer, and the coupling between the strings, the bridge and the soundboard. The existence of the secondary partials was known since late 1970s and called 'phantom partials' in a paper published in 1997.

Keywords: Piano, Piano string, Phantom partials, Hammer felt, Soundboard

PACS number: 43.75.Mn [DOI: 10.1250/ast.25.413]

1. INTRODUCTION

The International Symposium on Musical Acoustics in Nara (ISMA2004) and the International Conference on Acoustics in Kyoto (ICA2004) were held in Japan from late March to early April in 2004. A purpose of this article is to review recent studies on the acoustics of the piano focusing on topics which were presented in these conferences. Topics include the secondary partials in piano tones, string excitation by the hammer, and the coupling between the strings, the bridge and the soundboard. The existence of the secondary partials was known since late 1970s and called 'phantom partials' in a paper published in 1997.

2. SECONDARY PARTIALS

It is well known that a frequency of an upper partial of an ideal string is an integer multiple of its fundamental [1,2]:

$$f_n^i = \frac{n}{2L}\sqrt{\frac{T}{\rho A}} \tag{1}$$

where n is the partial number, T is the tension, ρ is the mass density, A is the section area of the string and L is the speaking length. In the piano, the speaking length is the distance between an agraffe (or bearing) to a bridge pin as shown in Fig. 1. Equation (1) is based on the assumption that a string is perfectly flexible. Practically, a real piano string has elastic stiffness which resists to bending and the frequencies of the partials increase. In this case, the frequency of the n-th partial is given by

$$f_n^e = f_n^i\sqrt{1 + Bn^2} \approx f_n^i\left(1 + \frac{1}{2}Bn^2\right) \tag{2}$$

where B is a constant given by

$$B = \frac{\pi^2 EI}{TL^2} \tag{3}$$

where E is the Young's modulus, I is the area moment of inertia given by

$$I = \frac{\pi d^4}{64} \tag{4}$$

where d is the diameter of the string [3].

This deviation from the harmonics is called inharmonicity and this characteristic has been known for a long time and the description can be found in the historical treatise by Rayleigh [4].

Another series of partials, however, was observed in the piano tone since late 1970's in which the degree of inharmonicity is about a quarter of the normal inharmonicity [5–8]. An example of the acoustical spectrum is shown in Fig. 2. Though the investigation on this secondary inharmonicity continued for considerable time, the cause remained undetermined. In 1997, Conklin [9] rediscovered this series of partials and named them 'phantom partials' in his paper.

Conklin wrote in his 1999 paper [10] that phantom partials are those that appear at frequencies exactly harmonic to normal inharmonic string partials, and at frequencies equal to the sums of the frequencies of normal inharmonic partials. He also paid attention to the tension variation during vibration and tried to explain the mechanism of the phantom partials. He assumed the end force of a string, $F(t)$, to be proportional to the square of the displacement at the center, D.

He pointed out that when two frequencies, ω_1 and ω_2 are put into the center displacement, harmonics and sum and difference frequencies generated as shown in the following equation:

$$\begin{aligned} F(t) = kD^2 &= k(A_1 \sin\omega_1 t + A_2 \sin\omega_2 t)^2 \\ &= a_0 + a_1 \cos 2\omega_1 t + a_2 \cos 2\omega_2 t \\ &\quad + a_3 \cos(\omega_1 + \omega_2)t + a_4 \cos(\omega_1 - \omega_2)t \end{aligned} \tag{5}$$

where k, A_i, and a_i are constants.

For example, when the center displacement is forced to vibrate at a frequency $f_{n/2}{}^e$ with n to be even, it is obvious from Eq. (5) that $F(t)$ has a frequency of $2f_{n/2}$. By replacing n by $n/2$ in Eq. (2), we can obtain

$$F_n^F \equiv 2f_{n/2}{}^e \approx f_n^i\left(1 + \frac{1}{2}\frac{B}{4}n^2\right) \tag{6}$$

showing that the degree of inharmonicity is about a quarter of the normal one.

This model is too simplified and does not reflect the actual loading condition of a piano string which is struck by a hammer. He could not arrive at a governing equation of a string to reproduce the secondary partials, either. However, this paper attracted researchers to the tension of a string.

As far as I know, a successful governing equation for the transverse displacement which can reproduce the secondary partials was first found in 2000 through the investigation by several research groups in Japan. Naganuma *et al.* [11,12,17] and Takasawa *et al.* [13,15,18] examined several formulas to incorporate the effect of the axial force of a string. Finally, it was made clear that the secondary/phantom partials can be reproduced by adding a term which represents local variation of tension along the string. The derivation is summarized below.

When the tension T depends on the axial location x, the differential equation for the transverse displacement v is given by

$$\rho A \frac{\partial^2 v}{\partial t^2} = \frac{\partial}{\partial x}\left(T\frac{\partial v}{\partial x}\right) - EI\frac{\partial^4 v}{\partial x^4} \tag{7}$$

where the second term in the right hand side corresponds to the elastic stiffness.

If we decompose T into a constant tension T_0 and fluctuation ΔT caused by local elongation, the tension T may be expressed by

$$T = T_0 + \Delta T = T_0 + (\Delta\sigma)A = T_0 + E(\Delta\varepsilon)A \tag{8}$$

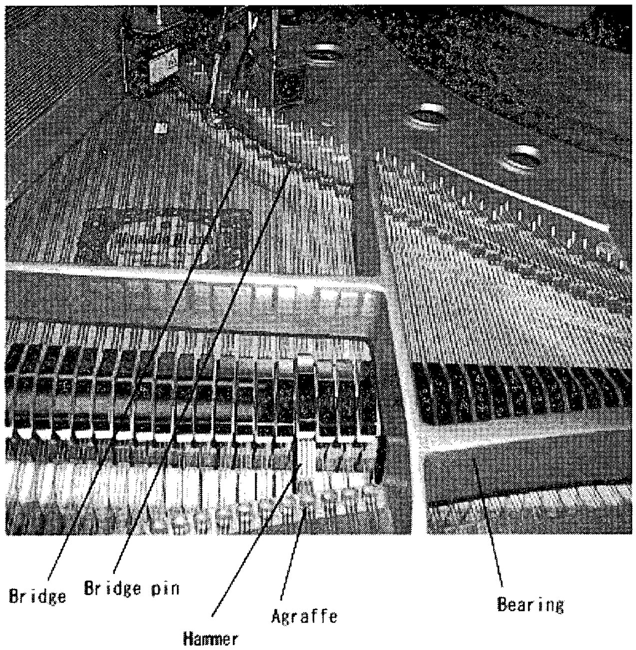

Fig. 1 Agraffes, bearing, bridge and hammers of the grand piano.

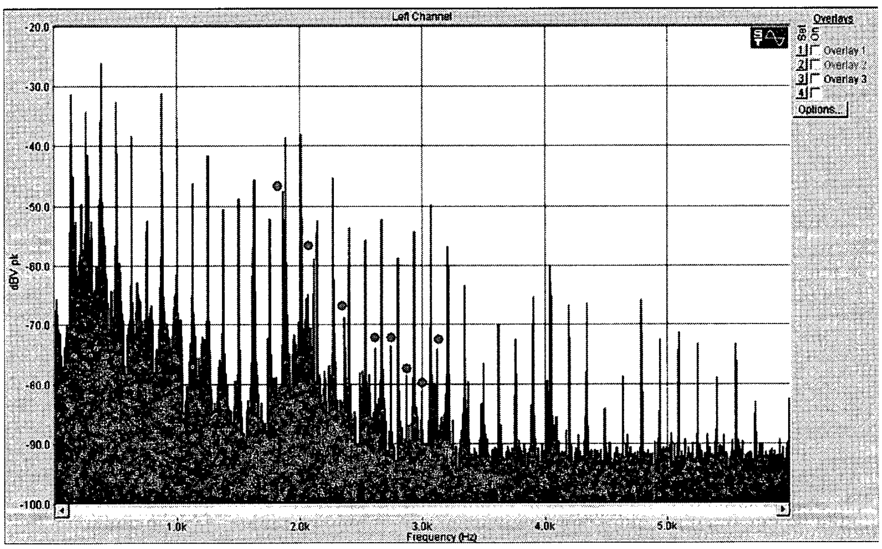

Fig. 2 Acoustical spectrum of B3 note. Peaks of the secondary/phantom partials are shown by ●.

where $\Delta\sigma$ is the increment of the axial stress due to the local elongation of the string, $\Delta\varepsilon$ is the axial strain of the string which may be given by

$$\Delta\varepsilon = \sqrt{1 + \left(\frac{\partial v}{\partial x}\right)^2} - 1 \approx \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 \tag{9}$$

Substituting Eq. (9) into Eq. (8) and combining with (7) give

$$\rho A \frac{\partial^2 v}{\partial t^2} = T_0 \frac{\partial^2 v}{\partial x^2} + \frac{3}{2} EA \left(\frac{\partial v}{\partial x}\right)^2 \frac{\partial^2 v}{\partial x^2} - EI \frac{\partial^4 v}{\partial x^4} \tag{10}$$

The second term in the right hand side is generated by the local elongation of the string. It is pointed out in [19] that this term was employed by Lee in a 1957 paper [22].

The above mentioned investigation has been performed by employing a one dimensional differential equation in which the transverse displacement is the only unknown. A different approach has been employed by the present author in which spatial movement of a string is simulated by the finite element method (FEM) based on the large deformation theory [14]. In the large deformation theory, an undeformed configuration and a deformed configuration are strictly distinguished. The momentum balance of a body is preserved in the course of the deformation and the changes of direction and amplitude of axial force due to deformation are taken into account. This is not the case for the small strain theory in which the equilibrium equation of a body is solved with respect to the undeformed configuration. Using the method, it was shown that the secondary partials appear in the spectrum of the velocity component in the axial direction [20] as well as transverse direction [16]. One of the characteristics of the secondary partials is its dependency on the amplitude of loading. It is also shown in the simulation that the secondary partials become notable as the loading increases [20,21].

Though the generation mechanism of secondary/phantom partials has been made clear, it is not certain whether these partials contribute significantly to the perceived quality of piano tones [10]. Bensa *et al.* wrote in his ISMA2004 paper [23] that those partials contribute to the "warmth" factor of the timbre, especially for low-pitched notes. However, it is difficult to say that it is a well accepted theory.

3. STRING EXCITATION BY HAMMER

Hammers are key components in the piano and they have direct influence on a piano tone. Though it may sound hard to believe at first, it is a common practice for piano tuners to adjust the tone quality of a piano by needling the hammer felt in a procedure called *hammer voicing*. The change can be found clearly in the sound spectra [24]. Accordingly, how to deal with the dynamics of hammers is one of the key issues in the construction of a physical model for the piano. Firstly, a hammer is covered by one or two layers of felt and the material property of the felt is complicated. The relationship between the applied force and the compression of the hammer felt shows hysteretic behavior [25,27,31,32]. Furthermore, very complex process occurs when a hammer strikes a string. The contact time between a hammer and a string varies depending on the hammer velocity. When the contact time exceeds the round-trip time from the hammer to the agraffe, the returning wave reaches the hammer at the striking point and the hammer re-reflects the wave as a nonrigid support. Therefore, there can be multiple reflections between hammer and agraffe [26,28–30].

In the ISMA/ICA2004, these issues on hammer were discussed in several papers.

Stulov [33] showed an analytical solution of a hammer motion, in which a hammer is modeled by a point mass and a linear elastic spring interacting with a long flexible string. The condition in which no reflection wave is needed for a hammer to rebound is discussed. He also performed numerical simulation of the lowest ten notes in a grand piano, in which a nonlinear spring model is employed. It showed that a hammer leaves a string without the aid of a reflected wave. In this model, the relationship between the applied force F and the spring compression $u(t)$ is given by

$$F(u(t)) = t\left[u^p + \alpha \frac{d(u^p)}{dt}\right] \tag{11}$$

where k, α and p are constants, which are determined for each hammer experimentally.

We note that the second term in the brackets is responsible for the hysteretic behavior of $F(u)$. Stulov [34] also used this model to evaluate the spectra of two consecutive notes where the number of strings changes or where strings are connected to different bridges. The effect of the position of the striking point is also discussed.

Giordano *et al.* [35], on the other hand, evaluated the hammer model shown in Eq. (11) based on the force-compression relations obtained by their experiments. They made an assertion that Eq. (11) could not reproduce the experimental results satisfactorily and they proposed the following function as an alternative which correlates their experiments well.

$$F(u(t)) = k_\infty \left\{ 1 + \frac{\beta}{1 + \exp\left[\gamma \int_0^t u(\xi) d\xi\right]} \right\} u^p \tag{12}$$

where k_∞, β and γ are constants.

According to Eq. (12), the coefficient of u^p decreases and approaches k_∞ as the compression of a hammer felt accumulates.

Other ISMA/ICA2004 papers which dealt with the interaction between a hammer and a string was [36] and [21]. In [36], the movement of a string is visualized based on the hammer-string interaction model, in which a hammer is expressed by a point mass with a nonlinear spring. The relationship between the force $F(u)$ and the compression u is assumed to be

$$F(u) = k_1 u^2 + k_2 u^3 + k_3 u^4 \tag{13}$$

where k_1, k_2 and k_3 are constants.

In the presented calculation, only the last term is employed. The deformation of a hammer shank is also visualized in which the hammer head strikes a rigid string.

In [21], the FEM described in the previous section was used to simulate the hammer-string interaction. A hammer was modeled by a point mass with a spring and a dashpot connected in parallel. A gap element was employed to simulate the contact and detachment between the hammer model and a string. It is shown that multiple reflections between hammer and agraffe can be reproduced by the method.

4. INTERACTION AMONG STRINGS, BRIDGE AND SOUNDBOARD

A familiar characteristic of piano tones is *double decay* or *compound decay* [37–42]. That is, the sound amplitude decreases with two distinct rates, breaking from an original fast decay called prompt-sound to a later slow decay called after-sound. It is known that the movement of a piano string is not restricted in a plane determined by the axis of string and the direction of the applied force. It is considered that this is a cause of the double decay. Another mechanism which may contribute to this compound decay is the coupling between unison strings. In the following, these phenomena are discussed.

As for the experimental investigation of the spatial movement of a string, Tanaka *et al.* measured the movement of a E1 (the lowest E in the piano) string using a pair of photo transistors [43,44]. In the ISMA/ICA2004, Mori [46] also measured the movement of a E1 string using a photonic displacement sensor and a pair of accelerometers. According to his experiment, horizontal vibration begins when the vertical wave generated by a hammer strike reaches the bridge pin for the first time. It is estimated that about 1/5 of the amplitude of vertical velocity is transformed into horizontal velocity at bridge pin. The movement of a string obtained by a high-speed camera and a mirror was also reported for A♯2 and A3 [14,21]. It is shown that the decay of the vertical displacement is faster than that of horizontal one.

Concerning the numerical investigation of the spatial movement of a string, Naganuma *et al.* [47] proposed a model in which a string moving in both vertical and horizontal directions is mounted on a soundboard. They employed an equivalent electrical circuit using a mobility analogy in which force and velocity correspond to current and voltage, respectively. In the model, the soundboard is represented by a spring and a dashpot connected in series and the movement of a string in each direction is modeled by a mass connected to a spring and a dashpot. Coupling between vertical movement and horizontal movement is represented by an ideal transformer with a turns ratio of n. By means of this model, the discrepancy between the vertical frequency and the horizontal frequency which was observed by Tanaka *et al.* [43,44] is explained.

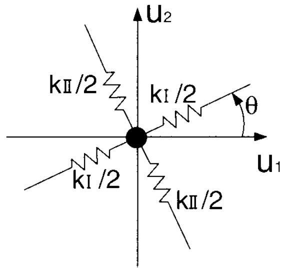

Fig. 3 Three-parameter modeling at a bridge pin.

Another numerical modeling for the coupling between vertical movement and horizontal movement at bridge was proposed in [21,45]. In this model, three parameters, a pair of spring constants of orthogonally oriented springs, k_I, k_{II}, and the rotation angle, θ, are employed as shown in Fig. 3. In this model, the direction of displacement does not coincide with the direction of the imposed force, except for two principle axes under the condition $k_I \neq k_{II}$, and coupling between the vertical motion and horizon motion occurs. This modeling can be used to connect a string and a soundboard or as an equivalent support which includes the effect of the soundboard. In [21], FEM analyses of a string on an soundboard is also shown. The soundboard is a 1 m-by-1.5 m rectangular plate with thickness of 10 mm. It can be seen that the motion of the soundboard is considerably different from the motion of the string suggesting the importance of including the soundboard into the simulation.

5. FINAL REMARKS

Finally, I would like to add some words on goals of the research on the piano acoustics. One of them may be to construct a numerical model which can reproduce the actual vibration and the wave propagation in and around a piano based on the Newtonian mechanics and measured material properties. If such a physical model were realized, it would have an important meaning on the design of the piano. Seen from this view point, it may be safe to say that the extent of various factors which contribute to the tone of the piano has become clear significantly through longstanding research. I believe, however, it is too early to conclude that the generation mechanism of the piano sound has been clarified *qualitatively*. In my view, we could hardly say that we understand the piano tone unless we can simulate the tone of the piano faithfully. In other words, it may be said that there is no *qualitative* understanding without *quantitative* understanding. Seen in this light, it will take some time to understand the complicated physical phenomena behind a piano tone.

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