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Evaluation of variances of ultrasonically measured strain tensor components

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1. Introduction

It is remarkable that the pathological state of human soft tissues highly correlates with their static and low-frequency mechanical properties, particularly shear elasticity. Accordingly, many researchers, including us, have been developing strain or displacement-measurement-based shear modulus reconstruction methods [1–3] and using various ultrasonic (US) strain/displacement measurement methods (e.g., Doppler method and autocorrelation method [4,5], cross-correlation method [6–8], spectrum cross-correlation method [9], sum-squared difference (SSD) method [10], multidimensional cross-spectrum phase gradient method (MCSPGM), i.e., 3-dimensional (3D) or 2D CSPGM [11,12], multidimensional autocorrelation method (MAM) [13,14] and multidimensional Doppler method (MDM) [13,14]).

Previously, we proposed the use of the regularization method [15,16] for strain tensor measurement [17–19] and shear modulus reconstruction [17,19,20], for which the respective effective setting methods of the regularization parameters, i.e., methods using the variances of the measured displacement vector and strain tensor components have been reported. That is, for strain measurement and shear modulus reconstruction, the respective measurement accuracies of the displacement vector components and strain tensor components are properly used at each position in the region of interest (spatially variant regularizations). In strain measurement, regularization is properly applied to the respective displacement components (i.e., displacement-component-dependent regularization). For instance, in a previous study, only the lateral displacement was regularized [21]. Through such regularizations, spatially uniform stabilities of strain measurement and shear modulus reconstruction can be realized.

As shown in this report, the respective strain variances can be experimentally evaluated using plural displacement field measurements, i.e., using variances and covariances of displacement components and differential filter coefficients [22]. Otherwise, as shown in ref. [18], single field measurement is performed. For instance, when using the cross-correlation method, the upper bounds of the axial [23–25], lateral/elevational [26] and shear [27] strain variances can be theoretically derived on the basis of the Ziv-Zakai Lower Bound (ZZLB) [28,29] and the finite difference approximation of partial derivatives. Because the ZZLB can be used for all cross-correlation-based displacement measurements, the theoretically derived strain variances [23–27] can also be used when employing MCSPGM. In this report, we also derive the strain variances using the ZZLB and differential filter coefficients. Single field measurement is suitable for practical *in vivo* measurement/reconstruction [17–20], whereas plural field measurements can be used to cope with the decreases in the accuracy and stability of the measurement/reconstruction inherent to each US system and the US parameters used (US frequencies, bandwidth).

In Sect. 2, the respective strain variances are expressed by variances and covariances of displacement components and differential filter coefficients, after which the strain variances are experimentally evaluated using plural displacement field measurements.

2. Variances of strain tensor components estimated using differential filter coefficients and variances/covariances of displacement vector components [22]

In practical applications of digital differential filters, the bandwidths are limited in all directions using cutoff frequencies. Such 2D and 1D differential filters are described in ref. [30]. Here, the 3D differential filters $hx(x, y, z)$ and $hy(x, y, z)$ are respectively used for the differentiations in the x (i.e., axial)- and y (i.e., lateral)-directions of the axial or lateral displacement distributions, i.e., $ux(x, y, z)$ or $uy(x, y, z)$. Using an expectation operator E[ ] and a covariance operator cov[,], the means and variances of the measured axial (or lateral) and shear strains are estimated using the means and covariances of the measured axial and lateral displacements [22], i.e., for axial strain $e_{xx}(x, y, z)$,
the mean is

$$E[e_{xx}(x,y,z)] = \sum_k \sum_l \sum_m \{hx(k,l,m)E[ux(x-k,y-l,z-m)]\} \quad (1)$$

and the variance is

$$\begin{aligned} var[e_{xx}(x,y,z)] &= E[(e_{xx}(x,y,z) - E[e_{xx}(x,y,z)])^2] \\ &= \sum_k \sum_l \sum_m \sum_{k'} \sum_{l'} \sum_{m'} \{hx(k,l,m)hx(k',l',m') \\ &\quad \times cov[ux(x-k,y-l,z-m),\ ux(x-k',y-l',z-m')]\}, \end{aligned} \quad (2)$$

and for shear strain $e_{xy}(x, y, z)$,

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the mean is

$$E[e_{xy}(x,y,z)] = \frac{1}{2}\sum_k\sum_l\sum_m\{hy(k,l,m)E[ux(x-k,y-l,z-m)] + hx(k,l,m)E[uy(x-k,y-l,z-m)]\} \quad (3)$$

and the variance is

$$\begin{aligned} var[e_{xy}(x,y,z)] &= \frac{1}{4}\sum_k\sum_l\sum_m\sum_{k'}\sum_{l'}\sum_{m'} \\ &\quad \{hy(k,l,m)hy(k',l',m')cov[ux(x-k,y-l,z-m),\ ux(x-k',y-l',z-m')] \\ &\quad + hx(k,l,m)hx(k',l',m')cov[uy(x-k,y-l,z-m),\ uy(x-k',y-l',z-m')] \\ &\quad + 2hy(k,l,m)hx(k',l',m')cov[ux(x-k,y-l,z-m),\ uy(x-k',y-l',z-m')]\} \\ &= \frac{1}{4}\sum_k\sum_l\sum_m\sum_{k'}\sum_{l'}\sum_{m'} \\ &\quad \{hy(k,l,m)hy(k',l',m')cov[ux(x-k,y-l,z-m),\ ux(x-k',y-l',z-m')] \\ &\quad + hx(k,l,m)hx(k',l',m')cov[uy(x-k,y-l,z-m),\ uy(x-k',y-l',z-m')]\}, \end{aligned} \quad (4)$$

when $cov[ux, uy] = 0$.

In practical applications, the variances and covariances of the displacement components can be evaluated using the spatial or temporal average, or ensemble average, i.e., plural measurements, statistically. All the means and variances of strains can also be evaluated after obtaining the strain data (samples) statistically.

Otherwise, as the variances of the measured axial and lateral displacements (i.e., the axial and lateral time delay estimates $\hat{\tau}_a$ [31] and $\hat{\tau}_l$ [26]), those expressed using the Cramer-Rao Lower Bound (CRLB) σ_{CRLB} can be used instead.

$$\sigma_{CRLB}{}^2(\hat{\tau}_a) = \frac{3}{2\pi^2 T(B_a{}^3 + 12B_a f_{0a}{}^2)}\left\{\left(1+\frac{1}{SNR_c}\right)^2 - 1\right\} \quad (5)$$

$$\sigma_{CRLB}{}^2(\hat{\tau}_l) = \frac{B_a{}^2 + 3f_{0a}{}^2}{B_l^2}\sigma_{CRLB}{}^2(\hat{\tau}_a) \quad (6)$$

Here, T is the local region length used for MCSPGM, f_{0a} is the US frequency, B_a and B_l are the axial and lateral bandwidths, respectively, and SNR_c [32] is the combined SNR defined as

$$SNR_c = \frac{SNR_\rho SNR_s}{1 + SNR_\rho + SNR_s} \quad (7)$$

using the echo SNR (i.e., SNR_s) and the correlation SNR (i.e., SNR_ρ) defined as

$$SNR_\rho = \frac{\rho}{1-\rho}. \quad (8)$$

The correlation SNR exhibits an effect on the measurement accuracy of the decorrelation of the local echo data induced by rigid motion and deformation. Thus, the variances of the displacements can also be estimated by a single measurement [18,20], i.e., by using the actual US parameters (frequencies and bandwidths), local region length, echo SNR and correlation coefficient (peak value of cross-correlation function).

Thus, for instance, the variance of the measured axial strain component can also be estimated as Eq. (2) using the theoretically derived axial displacement variance (5) and a differential filter [30]. That is, the axial strain variance can be estimated as in ref. [23] under the assumption that the variances of the estimated axial time delays $\hat{\tau}_a$ within a filtered region are the same as that of the point of interest and that each covariance has a linear dependence on the variance in conjunction with the overlap of the two local regions used for the displacement measurements (i.e., the covariances are proportional to the variance). In this case, the variance of the axial strain component is expressed as being proportional to CRLB [Eq. (5)]. This assumption can also be used for the evaluation of the variances of the lateral/elevational and shear strains. The axial strain variances evaluated using Eqs. (2) and (5) were previously used [18,20] to realize spatially uniform stability in shear modulus reconstruction.

In the next section, the evaluated ensemble axial [Eq. (2)], lateral [Eq. (2)] and shear [Eq. (4)] strain variances are shown using plural displacement field measurements for an agar phantom that is spatially fixed. That is, the measurement accuracies are determined using only the echo SNR, the parameters (US frequency, bandwidths, f-number, sampling frequency, beam pitch, local region size, cutoff frequency of the differential filter) and the displacement measurement method, i.e., except for echo decorrelation due to rigid motion and deformation.

3. Phantom experiment

We produced a uniform agar phantom of 200 mm (height) × 100 mm (lateral width) × 50 mm (elevational width). The concentration of the agar was 3.5 percent. To control US attenuation, graphite powder was added (concentration, 3.0 percent). The linear array transducer with a nominal frequency of 5 MHz (LNR5539, Aloka Co., Ltd., Tokyo, Japan) was used together with US imaging equipment (SSD5500, Aloka Co., Ltd., Tokyo, Japan). Rf-echo data were digitized with 12-bit resolution at a sampling rate of 20 MHz

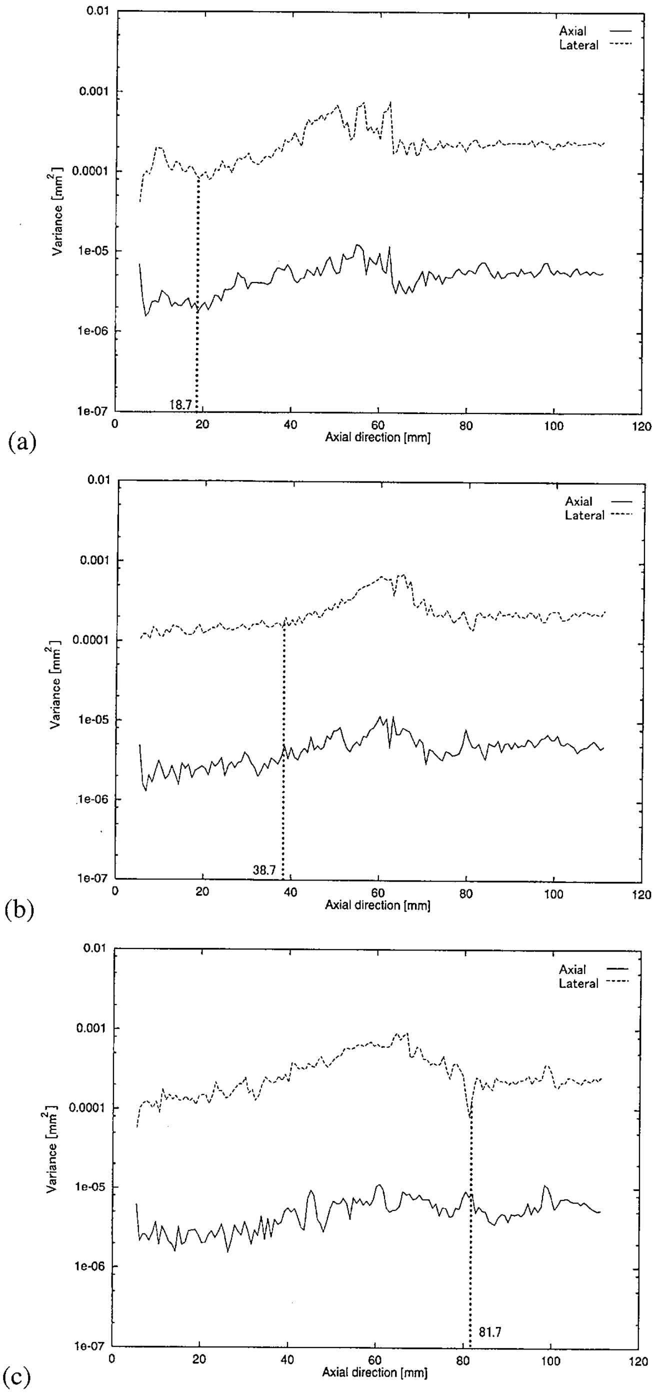

Fig. 1 Variances of displacement vector components evaluated using plural field measurements vs axial position for focus depths of (a) 18.7, (b) 38.7 and (c) 81.7 mm.

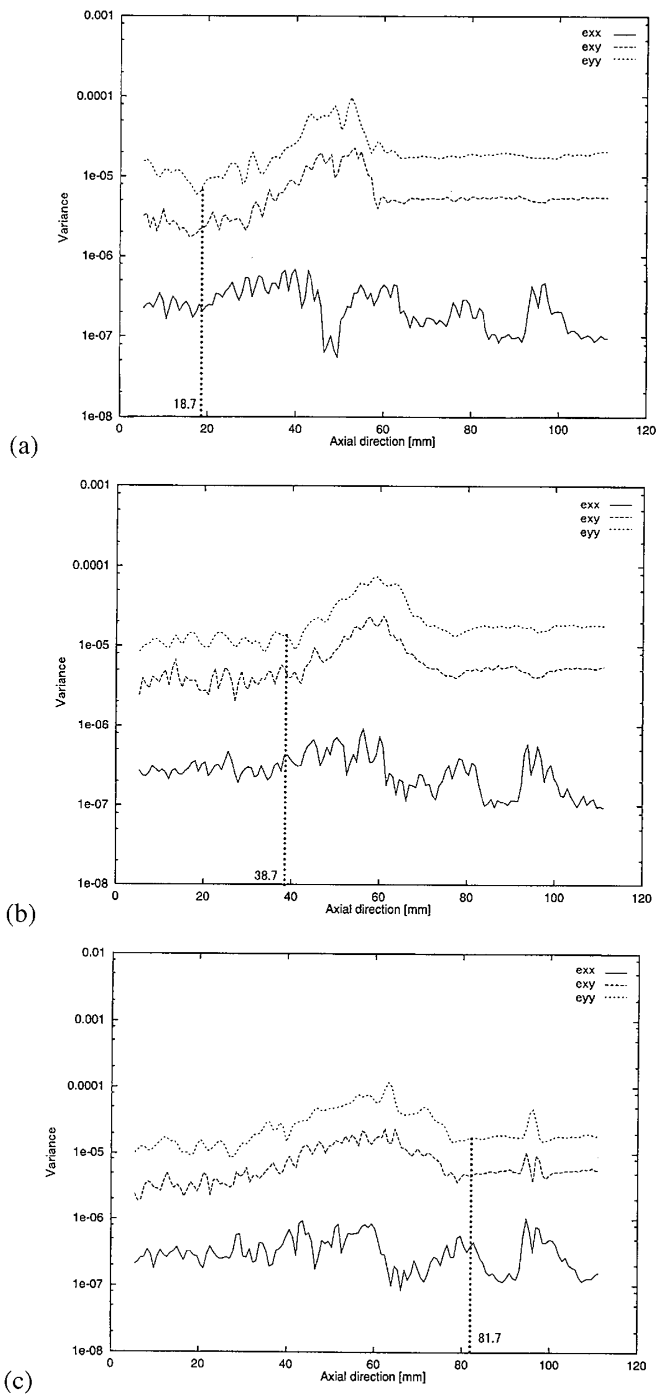

Fig. 2 Variances of strain tensor components evaluated using plural field measurements and Eqs. (2) and (4) vs axial position for focus depths of (a) 18.7, (b) 38.7 and (c) 81.7 mm.

(0.2 mm beam pitch). The f-number was 1.0. The focusing positions were set at the depths of 18.7, 23.7 and 81.7 mm.

The 2D CSPGM [11,12] was used (local region size, 2.4 mm $\times$ 3.2 mm). The cutoff frequency of the differential filter [30] was 0.128 mm^{-1}. The 1,050 paired rf-echo data frames were acquired at different positions without compression or stretching. Because two samples were obtained at each depth from paired frames, the variances of the 2D displacement vector components were evaluated using 2,100 samples at each depth. In Figs. 1(a) to 1(c), the evaluated variances of the measured displacement components are respectively shown for the focus depths of 18.7, 23.7 and 81.7 mm. From the variances and covariances of the measured displacement components, the variances of the strain tensor components were evaluated using Eqs. (2) and (4). These are shown for the focus depths of 18.7, 23.7 and 81.7 mm in Fig. 2(a) to 2(c).

The respective variances of the axial displacements [Figs. 1(a)–1(c)] are smaller than those of the lateral displacements. That is, the accuracy of the axial displacement is higher than that of the lateral displacement. Because of the US attenuation, the variances increase in the deep region. However, note that the measurement accuracy of the axial displacement is fairly stable compared with that of the lateral displacement [Figs. 1(a)–1(c): log-scale graphs]. Moreover, the focusing makes both variances small, particularly that of lateral displacement, even in a deep region [see Fig. 1(c)]. The variance of the axial strain (e_{xx}) is the smallest and those of shear (e_{xy}) and lateral (e_{yy}) strains are larger [Figs. 2(a)–2(c)] because an inaccurate lateral displacement is used.

4. Discussions and conclusions

In our regularization of the displacement (strain) measurements [17–19] and shear modulus reconstruction [17,18,20], the evaluated variances of the displacements and strains were respectively used. In this report, the variances of the displacements experimentally evaluated by plural field measurements were used for the evaluations of strain variances. The applications of the variances of displacements and strains will be shown elsewhere, together with those obtained by a single field measurement [e.g., 18–20]. Here, note that such evaluated variances are large at the boundary of tissue (tumor, organ etc.). In such cases, since the measurement targets may not be smooth, the evaluated variances should not be directly used in the regularizations to prevent the measurement targets becoming overly smoothed.

Because the variances exhibit the limitations of the US system, the parameters (US frequency, bandwidths, f-number, sampling frequency, beam pitch, local region size, cutoff frequency of the differential filter) and the displacement measurement methods used, such evaluation will also be employed to adjust them for increasing the accuracies of the displacement/strain measurement and shear modulus reconstruction.

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