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PAPER

Vibrational analysis of glass harp and its tone control

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Abstract: In this paper, we focus on the control of the tone of a glass harp during the process of manufacturing, and on the analysis of the vibration of the glass harp using a finite element method. First, the effect of various factors on results is studied, such as the element type that enables a good geometries approximation and the number of divisions on the accuracy of mode frequencies. Second, we examine how each mode frequency changes with the bulge of the glass harp. The results demonstrate that the use of a bulge is effective in controlling the pitch and the timbre. Third, the pitch can be varied over a wide range by changing the bulge of the cup. In particular, it can be finely adjusted by cutting the inside bottom of the cup so as to form of a small circular groove instead of using a conventional method, which involves cutting the outside of the cup.

Keywords: Glass harp, Glass harmonica, Wineglass, Finite element method, Pitch tuning

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1. INTRODUCTION

In recent years, several experimental and analytical studies have been carried out on glass harps [1–4]. However, the analytical studies have been focused on the tuning of a commercially available wineglass by filling it with water and/or by cutting the outside of the bottom of the cup. We have four glass harps for use in concerts, which were developed for the Tsukuba International Exposition held in 1990 [5], and since then we have started analyzing them by a finite element method (FEM). However, large discrepancies in the mode frequencies were found between theory and experiment. We thus need to make the error as small as possible to permit quantitative discussion. In this study, we also aim to use the glass harp as a percussion instrument, in addition to as a conventional instrument, which is to rub the rim to produce only harmonic tones. A glass harp can be used as a percussion instrument in a performance, and its tapping tone is also clear. Therefore, by studying the possibility of whether/how we can control the pitch and tone quality by changing the bulge, we intend to design a glass harp as a percussion instrument.

2. MEASUREMENT OF GEOMETRY AND MATERIAL CONSTANTS

We could not obtain the dimensions of the glass harp with satisfactory precision, even though we tried various methods such as using a photograph and a nondestructive technique based on the use of a contour measuring machine. The geometry of a glass harp is too complex to measure by conventional nondestructive techniques.

Following Ref. [2], we also cut the cup in half after dividing a glass harp into a cup part and a stem part, and then the precise cross section of the cup was obtained using a copy machine (Fig. 1).

We cut off two strips ($50 \times 10.1 \times 2$ mm) from the stem to calculate the density by measuring their weight and Young's modulus. The frequencies of the fundamental bending mode were obtained from tapping tones recorded on a digital audio tape recorder (DAT) in an anechoic room and using a fast Fourier transform (FFT); the frequency resolution is 2.7 Hz.

The densities and Young's moduli of the two strips are given in Table 1. The material is crystal glass containing 24% PbO [5]. We used the average values of the two samples in this study (Table 2) because the discrepancy between the two sets of values obtained above is small. The value of Poisson's ratio in Table 2 was measured in our laboratory using another larger specimen. The values of Young's modulus and Poisson's ratio listed in Table 2 are almost the same as those in Ref. [2].

3. SPECTRA OF TONES OF GLASS HARPS

The rubbing and tapping tones were detected using a 1/2 inch condenser microphone (B&K, type 4190) about 1

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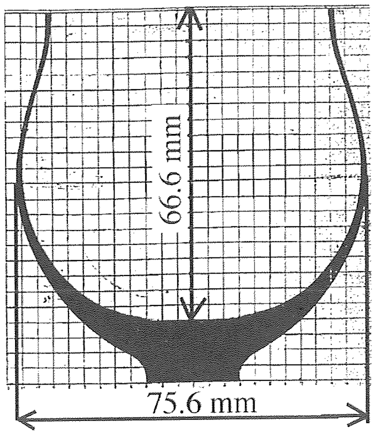

Fig. 1 Cross section of cup of glass harp.

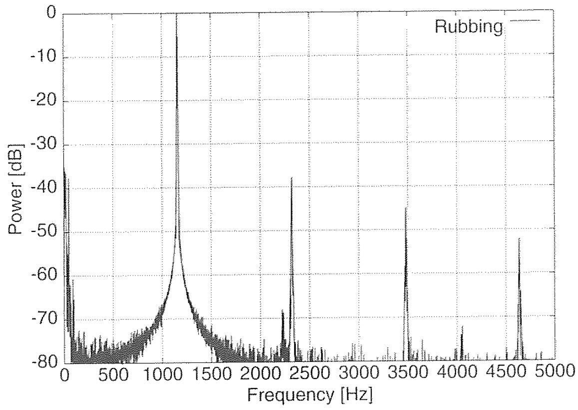

Fig. 2 Example of spectrum of rubbing sound.

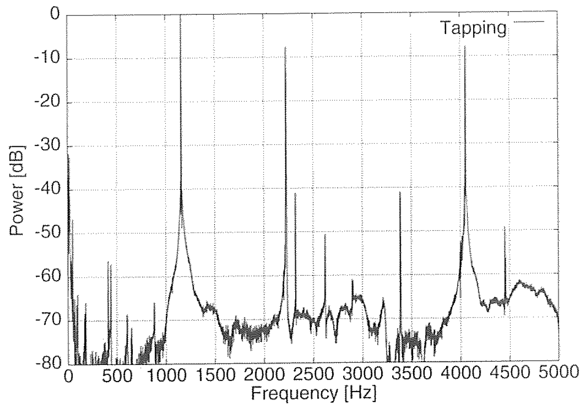

Fig. 3 Example of spectrum of tapping sound.

Table 1 Densities and Young's moduli of two strips.

Sample	Density [kg/m^3]	Young's modulus [GPa]
1	3010	61.27
2	3004	61.15

Table 2 Material constants used for FEM analysis.

Young's modulus E	61.2 [GPa]
Poisson's ratio σ	0.262
Density ρ	3010 [kg/m^3]

meter from the stem of the glass harps in an anechoic room and recorded on a DAT (SONY, type DTC-ZA5ES). Glass harps were played by rubbing the rim with a wet finger and by tapping the rim with a wooden mallet. Examples of the spectra of the rubbing and tapping tones of the glass harp in Fig. 1 are shown in Figs. 2 and 3, respectively. The spectra were obtained by FFT analysis of the tones recorded on the DAT. The duration of the tone analyzed was 2 s. From Fig. 2, we can see that the rubbing tone has only small overtones and thus produces almost sinusoidal tones [1,2]. On the other hand, the tapping tone has rather large inharmonic components and the affect the timbre, although the pitch is the same as that of the rubbing tone. Some important dimensions of our four glass harps are given in Table 3, and the frequencies of the components of the spectra, their ratios to the frequency of the first component (in parentheses), and the nearest note to each frequency are given in Table 4.

Table 3 Some important dimensions of four glass harps (A, B, C, D) [mm].

	A	B	C	D
Diameter at top	61.0	62.0	72.0	81.4
Maximum diameter	71.0	75.6	106.0	123.2
Height of cup	65.0	66.6	102.0	114.5
Thickness of rim	1.4	1.3	1.3	1.7
Length of stem	55	55	60	60
Diameter of stem	20.4	20.0	20.9	20.5

From Table 4, we can see that not only the pitch but also the timbre of the glass harp can be controlled by changing its dimensions and geometry.

Table 4 Frequencies of components, their ratios (in parenthesis), and the nearest note to the tapping tones of the four glass harps.

Component no.	A	Note	B	Note	C	Note	D	Note
1	1,160(f)	D^3	585(f)	D^2	1,271(f)	D^3#	630(f)	D^2#
2	2,226(1.92f)	C^4#	1,532(2.62f)	G^3	3,099(2.44f)	G^4	1,652(2.62f)	G^3#
3	4,056(3.50f)	H^4	2,950(5.04f)	f^4#	5,759(4.53f)	f^5#	3,120(4.95f)	G^4
4	8,995(7.75f)	—	4,335(7.41f)	C^5#	8,988(7.07f)	—	4,753(7.54f)	D^5

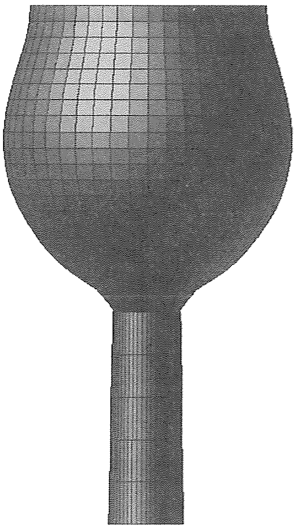

Fig. 4 Subdivision of whole glass harp into 1,152 parabolic isoparametric elements.

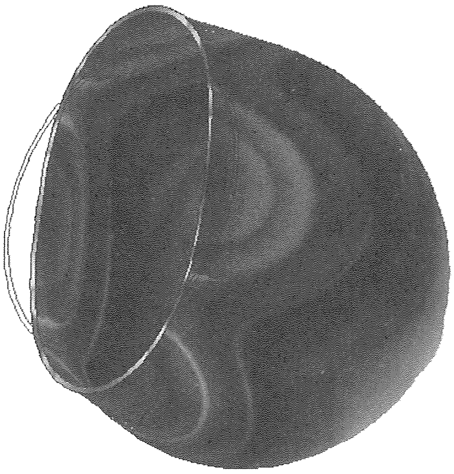

Fig. 5 (2,0) mode shape.

4. FINITE ELEMENT ANALYSIS

In this study, a general-purpose finite element code MSC/NASTRAN [6,7] was used to obtain mode shapes and frequencies, and a graphic pre/postprocessor program MSC/PATRAN was used to visualize mode shapes.

The cross-sectional geometry (the right-half portion of Fig. 1) is inputted and rotated 360 degrees, thereby forming the shape of a glass harp for the finite element method. Five-sided and six-sided parabolic isoparametric elements were used for the model. The cross section was subdivided into 48 elements in the circumferential direction, 24 in the height direction, and 1 in the thickness direction. A glass harp model completely subdivided into 1,152 elements is shown with a grayscale pattern in Fig. 4. The main mode shapes obtained by FEM analysis are shown in Figs. 5–8 [10].

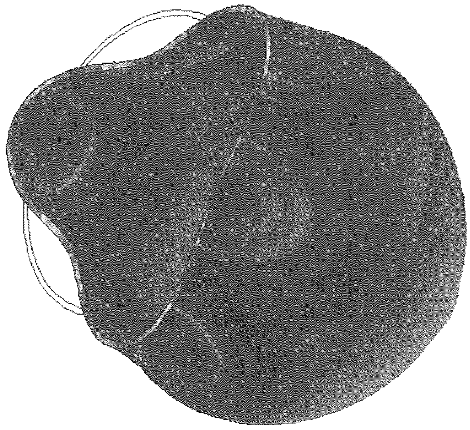

Fig. 6 (3,0) mode shape.

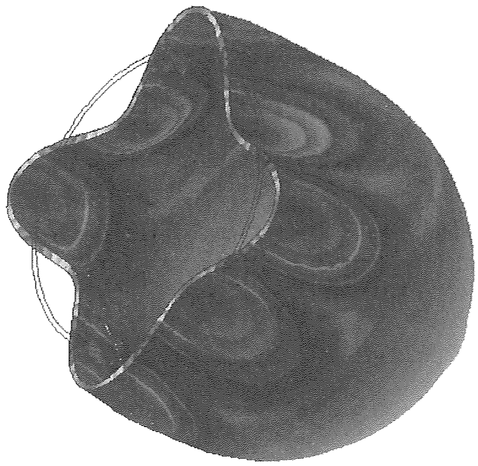

Fig. 7 (4,0) mode shape.

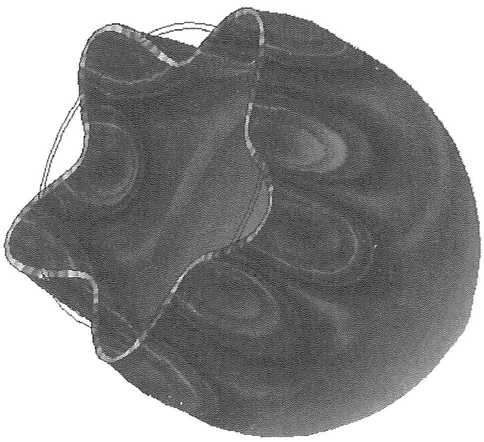

Fig. 8 (5,0) mode shape.

The mode frequencies and shapes obtained here correspond to those in other Refs. [1,2]; thus, the lowest four modes are denoted as (2,0), (3,0), (4,0), and (5,0) following Ref. [1]. To examine the accuracy of the mode frequencies, the mode frequencies obtained by FEM and FFT analyses of its tapping tone were compared. The results are given in Table 5 for material A in Table 3.

Table 5 Mode frequencies obtained by experiment and FEM analysis.

Mode	Experiment [Hz]	FEM [Hz]	Deviation [%]
(2,0)	1,161	1,169	0.68
(3,0)	2,227	2,295	3.0
(4,0)	4,056	4,161	2.5
(5,0)	6,350	6,491	2.2

Table 7 Mode frequencies for various boundary conditions and length of stem.

Boundary condition Stem length [mm]	Free 3	Free 55	Fixed 55	Free 100	Fixed 100
(2,0) mode frequency [Hz]	1,169	1,168	1,168	1,168	1,168
(3,0) mode frequency	2,293	2,294	2,294	2,294	2,294
(4,0) mode frequency	4,158	4,158	4,158	4,158	4,158
(5,0) mode frequency	6,487	6,486	6,487	6,487	6,487

From Table 5, we can see that the deviation is less than 3% (1/2 semitone) for every mode and, in particular, the deviation for the most important mode (2,0) is less than 0.7%. Therefore, we can consider that the results obtained by the FEM analysis are sufficient in accuracy for quantitative discussion.

5. COMPARISON OF TWO TYPES OF ELEMENT

We examined the difference between the mode frequencies obtained by FEM using 20-noded (parabolic) and 8-noded (linear) isoparametric elements.

First, we tried to analyze the model divided into 20-noded isoparametric elements, which has 40 divisions in the circumferential direction, 20 in the height direction and one in the thickness direction. Next, we analyzed the model divided into 8-noded isoparametric elements, which has 80 divisions in the circumferential direction, 40 in the height direction and 2 in the thickness direction. Therefore, the latter model has 8 times the number of elements than the former; that is, the total number of analytical nodes is slightly greater (the calculation time is roughly proportional to the square of the total number of nodes) than that of the former. The mode frequencies are given in Table 6, and the differences between the mode frequencies are also given in the fifth column of the table. From Table 6, it is clear that all the mode frequencies of the 20-noded model are markedly lower and closer to those obtained in the experiment than those of the 8-noded model. Therefore, it can be said that the difference originates from errors in the geometrical approximation (the smoothness of the shape is greater for the 20-noded model) and/or the intrinsic accuracy of the elements used [8,9].

We conjecture that sufficient accuracy can be obtained by FEM analysis using the parabolic isoparametric elements even when the FEM is applied to a vibrator with curved surfaces such as a glass harp. The quantitative differences between the mode frequencies in Figs. 8 and 10 in Ref. [2] may originate from the difference between the cross-sectional shapes of real glasses and those of the analyzed glasses using 8-noded elements.

Table 6 Mode frequencies obtained by using 20-noded and 8-noded isoparametric elements.

Mode	20-noded [Hz]	8-noded [Hz]	Difference [%]
(2,0)	1,169 (1.00)	1,483 (1.00)	26.9
(3,0)	2,299 (1.97)	2,810 (1.89)	22.2
(4,0)	4,169 (3.57)	4,738 (3.19)	13.6
(5,0)	6,505 (5.56)	7,094 (4.78)	9.05

6. CHANGE IN MODE FREQUENCY DUE TO BOUNDARY CONDITIONS

As shown in Fig. 1, a glass harp used for a concert is different from a commercially available wineglass. The glass harp has a thinner rim and a thicker stem than a wineglass [5].

The change in the mode frequency with the length of the stem and the manner of fixing was examined. In the boundary conditions, "Fixed" indicates that all the nodes on the cylinderlike handled stem are perfectly fixed three dimensionally. The result is given in Table 7.

From the Table 7, we can see that the length of the stem and the boundary condition (free or fixed) do not change the mode frequencies at all. This result implies a great practical advantage, because we need not consider how to fix the handle of the glass harp. We also confirmed the result experimentally by measuring the spectral peaks of the tones of the four glass harps held by hand and by a thick (20 mm) brass plate with holes of various diameters. The diameters of the top holes were made slightly larger than those of the bottom holes to fit the slightly tapered handles of the glass harps.

Thereafter, a stem of 3 mm length (hereafter called the standard geometry) was used to save on calculation time.

7. CHANGE IN MODE FREQUENCY CAUSED BY BULGE OF CUP

A change in the frequency of components may be possible using a cup with a bulge (which is characterized by the diameter at the height of the middle of the cup). The bulge was examined by keeping both the height of the cup and the diameter of the rim the same as those in the standard geometry. The bulge was changed substantially, as shown in Fig. 9, to examine by how much fundamental and higher mode frequencies and their ratios can be

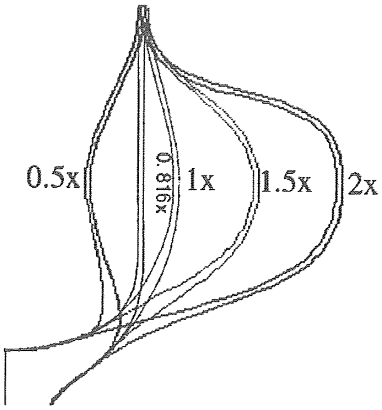

Fig. 9 Cross sections of overdrawn glass harp with various bulge coefficients.

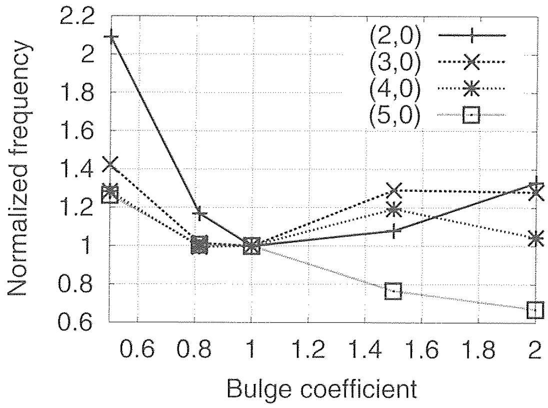

Fig. 10 Effect of bulge coefficient on mode frequencies normalized by respective mode frequencies whose bulge coefficient is equal to unity.

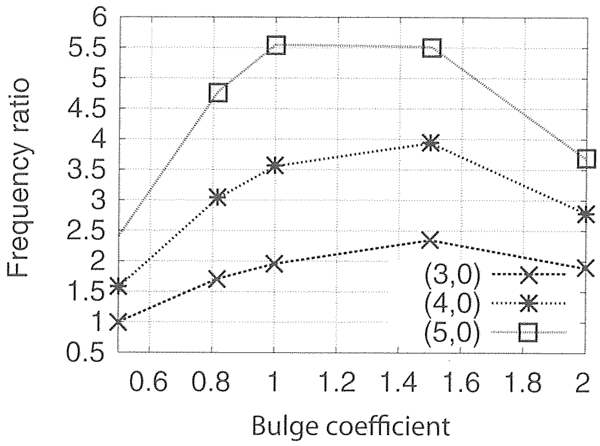

Fig. 11 Effect of bulge coefficient on frequency ratios of modes higher than the respective fundamental mode.

changed by the bulge. The intermediate maximum diameters of the cup are 0.5, 0.816 (cylindrical cup), 1.5, and 2.0 times that of the standard geometry. We set the bulge coefficients to be 0.5, 0.816, 1.5, and 2.0 (see Fig. 9).

The results are shown in Figs. 10 and 11. The horizontal axis is the bulge coefficient. The vertical axis in Fig. 10 indicates the mode frequency normalized by the respective standard mode frequency (see Table 5), and the vertical axis in Fig. 11 indicates the mode frequency normalized by each (2,0) mode frequency. The fundamental mode frequencies increase by making the bulge coefficient both large and small. For example, the fundamental frequency for a bulge coefficient of 0.5 is more than twice the frequency for the standard geometry.

From Fig. 10, it can be seen that the (2,0) mode frequency is increased to values greater than unity with either smaller or larger bulge coefficients in the range of 0.5–2.0; the change in the high mode frequency is smaller than that in the (2,0) mode frequency for the smaller bulge coefficient.

Therefore, it may be possible to tune the frequency of the 2nd or 3rd partial to a small integer ratio by reducing the bulge coefficient slightly.

Figure 11 shows the higher mode frequencies of the partials normalized by the respective foundamental (2,0) mode in the case of five bulge coefficients. These are an effective shapes when we think about the timbre of the tapping sound. We can see from Fig. 11 that the frequency ratio can be controlled by adjusting the bulge coefficient. Furthermore, we can obtain a normalized frequency of the 2nd or 3rd component of 1.5, 2.0, 2.4, 2.5, or 3.0. In particular, for the standard glass harp, the frequency of the second mode is approximately an octave higher than that of the fundamental mode and a fifth higher for cylindrical cup (0.816×).

Having consulted a manufacturer, we found that there is no additional difficulty in making a glass harp with the various cross sections shown in Fig. 9 compared with making one with the standard (1×) cross section.

8. FINE TUNING REALIZED BY CUTTING BOTTOM OF CUP

To make a musical instrument, it is important to tune the pitch of a glass harp exactly. In Refs. [2,5], the pitch is tuned slightly by cutting out portions from the outside of the bottom of the cup, which weakens the mechanical strength.

We propose a new technique of slightly cutting the inside of the bottom in an annular manner. We studied the change in mode frequencies with the depth of cutting. Although the cutting curve cannot be exactly determined quantitatively in this study, the depth of cutting is approximately 1 mm–4 mm. Half of the cross section of the cutting model is shown in Fig. 12, and the results are shown in Figs. 13 and 14.

From Fig. 13, it can be seen that the pitch decreases almost in proportion to the amount of cutting; the rate is

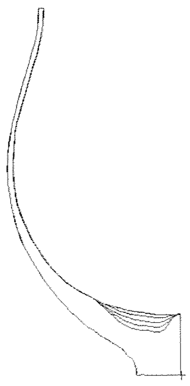

Fig. 12 Cross sections of glass harp with bottom cut out to various depths.

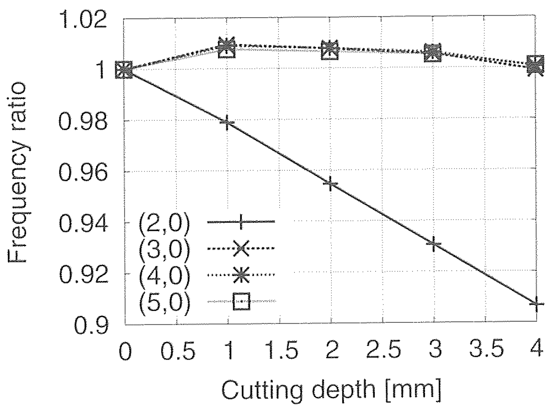

Fig. 13 Effect of cutting depth on mode frequencies normalized by respective mode frequencies, where cutting depth at inside bottom is equal to unity.

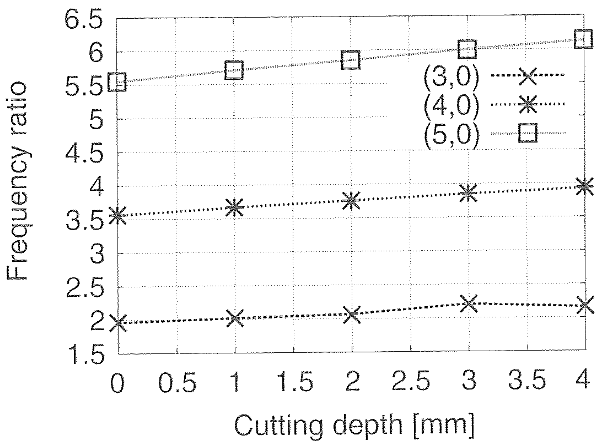

Fig. 14 Effect of cutting depth on frequency ratios of modes higher than respective fundamental mode.

slightly larger than 2% per 1 mm, and the frequencies of higher modes are almost unchanged (less than 1%). From Fig. 14, we can also see that the frequency relations between the (2,0) mode and the higher modes, therefore the timbre, change little.

We consider the reason for the frequency decrease to be as follows. The bottom of the cup undergoes great stress, although the vibration is small. Therefore the rigidity acts as a spring constant. Cutting the bottom of the cup reduse the thickness of the slight cylindrical groove and the spring constant equivalently. The effect is larger for the fundamental mode because the vibrating part for the lower mode is close to the bottom of the cup [1].

In practice, we can cut gradually using a milling machine and a jig with a cylindrical tip covered by carborundum or diamond powder, keeping the handle fixed, and checking the pitch by rubbing or tapping lightly while cutting.

9. CONCLUSION

The mode frequencies and mode shapes of a glass harp for concert use were studied by FEM and by experiments using the glass harp.

The important results obtained in this study are as follows: (1) The vibration of the glass harp is determined by the cup alone; the length and boundary conditions of the stem do not matter. (2) The pitch and timbre of a glass harp can be changed by changing the bulge of the cup. (3) Pitch can be finely adjusted by cutting the bottom of the cup.

In a future study, we intend to examine the effect of the bulge of the cup and other parameters more precisely. We also want to make a new percussion instrument from crystal glass.

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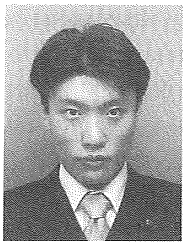
[10] An example of the animation of the (2.0) mode is available at "http://swallow.ee.uec.ac.jp/glshrp.html" on the Internet.



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