Reliability of Inspected Structure

— A Bayesian Approach —

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This paper discusses how to formulate the reliability analysis of an inspected structure to make the best use of the experience, taking into account the uncertain factors in inspection, proof-load test and statistical characteristics of random variables involved.

Two typical cases are taken as examples; one in which basic information is avairable for the analysis including detailed output of NDI and another case where simple visual inspection is only source of data. For both cases,Bayesian method is applied. Monte Carlo simulation demonstrates the use of present method.

1. Introduction

For a structural reliability analysis to be performed with a high degree of engineering, if not mathematical, confidence, the analytical model that describes the physical mechanism of structural failure must obviously be known reasonably well. In addition, those load and resistance variables that apper in such an analytical model must be specified in termes of probability density functions (PDF's) or, more generally, in termes of a joint probability density function. Most works on the reliability of ship structures have been done with this postulate. (1), (2), (3) Unfortunately, the prevailing paucity of pertinent data obscures the credibility not only of the analytical forms of these PDF's, but also of thier parameter values, established or estimated on the basis of such data.

Under these circumstances, the type of information to be gathered and the method to be implemented for its utilization must be carefully examined, since such an examination is indeed an integral part of the procedure for structural reliability assessment. For example, the sizes of the initial cracks in various structural detailes, the detectability of the initial and propagating cracks by non-destructive inspections (NDI's), fracture toughness of the material used and the mechanism of fatigue crack propagation are all part of such information to be gathered, and they are also all significant factors that influence the structural reliability (4), (5). In particular, the knowledge of their statistical characteristics constitutes an essential part of the data base that is needed for the reliability analysis. These statistical characteristics are usually determined on the basis of an initial preservice inspection, proof-load testing and regularly or irregularly scheduled in-service inspections. Since such information as a whole can hardly be supplied in actual cases, it is considered an unrealistic approach and sometimes it leads to denial of reliability analysis. One must thereby rely upon some other methods.

One of the solutions would be the Bayesian approach where the probability laws are assumed initially on the basis of engineering judgement and then properly upgraded according to the result of each procedure.

Application of a Bayesian reliability analysis to the problem of inspection of structures was first made by Itagaki, et al. (6) in which a method for the selection of the most suitable function among those possible assumed on the distributions of defects and detectability of the defects, etc.was discussed. In this analysis, it has been assumed that the inspections are of a considerably high accuracy. However, in the case of ordinary visual inspections of ship structures, the accuracy of the measurement of crack length is relatively low and thus the distribution functions involved are assumed to contain parameters of high degree of uncertainty. Thus, a sophisticated application of fracture mechanics is almost meaningless.

In the following, a breif discussion is made on the reliability analysis taking the uncertainties of inspection into account, according with a result of US-Japan joint research between prof.Shinozuka of Columbia University and the present author (8) and also, following the recent study by Y.Akita, A.Nitta and the author. (9)

2. Procedure of Reliability Analysis with Several Uncertain Factors

In the following analysis, those factors considered to be intrinsically random are the initial crack size, detectability of NDI's, fracture toughness and rate of fatigue crack growth. On the other hand, those factors interpreted as probabilistic due to underlying uncertainties are the parameters of PDF's, detectability and the intensity level of the proof-load test and the stress level of the operational cyclic loads.

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2.1 Estimation of PDF of Undetected Initial Crack Size

One of the most significant factors that influences the structural reliability is the PDF of the size of the cracks that remain undetected by an initial inspection. It is well known that such a PDF can be obtained as the product of the detectability $F_D(x|given\ parameters)$, defined as the probabiliity of detecting a crack of a given size x, and the PDF f_0 $(x_0|given\ parameters)$ of the initial crack size x_0. The detectability depends, for example, on the NDI technique used and the inspector's competence, while the PDF of x_0 on the microscopic material property, geometry of the structural details, etc.

For those dependencies to be quantitatively identified, the data base appears to be woefully inadequate at present and unfortunately is likely to remain so, unless major research and development efforts are made, disregarding economic and time constraints. Therefore, for the purpose of developing the PDF of an undetected crack size, the present paper follows an approach in which appropriate analytical forms are first postulated for the detectability and the PDF of x_0 and then the unkown parameters appearing in these analytical expression are estimated with the aid of the Bayesian technique.

Specifically, the following forms are postulated for the detectability and PDF of x_0, respectively.

$$E_D(x|a,\ b)=[1-exp\{-a(x-b)\}]H(x-b) \quad \cdots\cdots\cdots (1)$$

$$f_0(x_0|c,\ d)=c\cdot exp(-c(x_0-d))H(x_0-d) \cdots\cdots\cdots\cdots\cdots (2)$$

where H(•) is the Heaviside unit step function. Eqs. (1) and (2) involve four paramerters a, b, c and d, in which a and c are the unknowns to be estimated, while b and d are assumed to be constant.

Consider then a structure consisting of N structural details to be subjected to an initial inspection, starting from detail 1 to detail N. The following two events will result from such an inspection at each detail:

event E_1= detection of crack of size x_0, and

event E_2= no crack detected.

When E_1 occures, however, a repair is made to remove the detected crack. Applying Bayes' theorem to the results of the inspection, the following recurrence formulas can be obtained for $p^i(a, c)$ which indicates the posterior joint PDF of a and c estimated on the basis of the inspection up to and including structural detail i (= 1, 2, ..., N); if the inspection at detail i results in E_1,

$$p^i(a,\ c)=\frac{\{f_0(x_0|a)F_D(x_0|c)\}p^{i-1}(a,\ c)}{\iint(Numerator)\,dadc} \cdots\cdots\cdots\cdots\cdots (3)$$

while, if the same inspection produces E_2,

$$p^i(a,\ c)=\frac{\{\int_0^\infty f_0(x_0|a)\overline{F}_D(x_0|c)\,dx_0\}p^{i-1}(a,\ c)}{\iint(Numerator)\,dadc} \cdots (4)$$

where $\overline{F}_D(x_0|c)=1-F_D(x_0|c)$ and $p^0(a,\ c)$ is the prior joint PDF of the unknown parameters a and c.

For a large N, the posterior joint PDF tends to concentrate at the point $(a_M,\ c_M)$. Thus, the PDF, $f_0{}^*(x_0)$, of the crack size remaining in each structural detail after the initial inspection is completed, may be approximated as

$$f_0{}^*(x_0)=f_0(x_0|a_M)\overline{F}_D(x_0|c_M)/\int_0^\infty(Numerator)\,dx_0 \cdots\cdot (5)$$

The use of eq. (5) in the following analysis implies that the PDF of the undetected crack size at each structural detail is given by the same function $f_0{}^*(x_0)$ above, regardless of the inspection having resulted in either event E_1 (with detected crack repaired) or in event E_2 at that detail.

2.2 Effect of Proof Load Test

The structure is subjected to a proof load test subsequent to the initial inspection but prior to its actual use under service conditions. The proof load test is performed to eliminate those structures that do not have a specific minimum load carrying capacity. On the other hand, the structure that has survived the proof load test will have a greater performance reliability than those placed in service without the proof load test, provided that the proof load test introduces no damaging effect on the structure (an assumption to be used in this analysis). On the other hand, there is the probability that the structure will fail under the proof load. Indeed, the probability P_{pl} that a structural detail will fail due to unstable crack growth under the proof load can be written as

$$P_{p1}=\int_0^\infty[\int_0^\infty\{\int_0^{K_0(x_0,s)}f_{K_C}(y)f_0{}^*(x_0)f_S(s)\,dy\}dx_0]ds \cdots (6)$$

where $f_{K_C}(y)$ and $f_S(s)$ are the PDF's of the critical stress intensity factor K_C and of the stress level s produced by the proof load at the structural detail, respectively, and $K_0(x_0,\ s)=s\sqrt{\pi x_0/2}$ is the stress intensity factor. The density function $f_{K_C}(\cdot)$ rpresents the intrinsic randomness of K_C while $f_S(\cdot)$ reflects the modeling and other uncertainties involved in the analysis to be performed in order to estimate the value of the stress level s corresponding to the proof load L.

The available data suggest that the use of a two-parameter Weibull density is appropriate for the PDF of K_C. In this study the shape parameter α is considered to be known while the scale parameter β to be unknown;

$$f_{K_C}(y|\beta)=(\alpha/\beta)\cdot(y/\beta)^{\alpha-1}\cdot exp\{-(y/\beta)^\alpha\} \cdots\cdots\cdots\cdots (7)$$

As to the stress level s corresponding to the proof load, the PDF is assumed to be normally distributed with known parameters, i.e., mean μ_S and coefficient of variation COV_S;

$$f_S(s)=1/(\sqrt{2\pi}\sigma_S)\cdot exp\{-(s-\mu_S)^2/(2\sigma_S^2)\} \quad \cdots\cdots (8)$$

where $\sigma_S=COV_S\cdot\mu_S$ is the standard deviation of s.

With the only unknown parameter being β in eqs. (7) and (8), the probability of failure P_{p1} becomes dependent solely on β once the integration indicated in eq. (6) is completed: $P_{p1}=P_{p1}(\beta)$. It then follows that the joint PDF of the critical stress intensity factor K_C, initial (undetected) crack size x_0 and stress level s associated with the proof load for the structural detail that survived the proof load test can be given by

$$f_{p1}(y,\ x_0,\ s|\beta)=f_{K_C}(y|\beta)f_0^*(x_0)f_S(s)/\{1-P_{p1}(\beta)\} \cdots (9)$$

over the $y-x_0-s$ domain defined by
$y>K_D(x_0,\ s)=s\sqrt{\pi x_0/2}$.

2.3 Probability of Fatigue Failure in Service

The cracks remaining in the structure subsequent to the proof load propagate under service load. In reality, the service load is usually represented by a random function of time. In the present study, however, it is assumed that the service load is cyclic with a constant load range ΔL and further that the crack grows in accordance with

$$dx/dn=\varepsilon(\Delta K)^l \quad \cdots\cdots (10)$$

where x is the crack size, n represents the number of load cycles, l and ε are both material constants which also depend on the geometry of the structural detail, and $\Delta K=\Delta s\sqrt{\pi x/2}$ is the range of the stress intensity factor. The quantity Δs is the stress range corresponding to the load range ΔL and is given by

$$\Delta s=(\Delta L)s/L \quad \cdots\cdots (11)$$

If the constant l is asumed to be l=2.0, the fatigue crack size x_n immediately after n load cycles, can be shown to be

$$x_n=x_0 exp\{\pi n\varepsilon(\Delta s)^2/2\} \quad \cdots\cdots (12)$$

Experimental evidence suggests that, even if deterministic values are assigned to x_0 and Δs, the crack size x_n will exhibit a wide statistical scatter and should be treated as a random variable. In order to reproduce such a randomness in the growth model considered in eq. (10), an assumption is introduced here that ε is a normally distributed random variable with mean μ_ε and coefficient of variation COV_ε or standard deviation $\sigma_\varepsilon=COV_\varepsilon\cdot\mu_\varepsilon$. In the present analysis, COV_ε is considered to be known while μ_ε is assumed to be unknown parameter. Hence, the PDF of ε becomes

$$f_\varepsilon(\varepsilon|\mu_\varepsilon)=1/(\sqrt{2\pi}\sigma_\varepsilon)\cdot exp\{-(\varepsilon-\mu_\varepsilon)^2/(2\sigma^2_\varepsilon)\} \cdots (13)$$

The joint PDF of the critical stress intensity factor K_c, fatigue crack size x_n, stress range Δs and constant ε in the fatigue crack growth law can be derived from eqs. (9), (11), (12) and (13) as follows:

$$f_{s1}(y,\ x_n,\ \Delta s,\ \varepsilon|\beta,\ \mu_\varepsilon)=f'_{p1}(y,\ x_n,\ \Delta s|\beta,\ \varepsilon)f_\varepsilon(\varepsilon|\mu_\varepsilon) \cdots (14)$$

in which

$$f'_{p1}(y,\ x_n,\ \Delta s|\beta,\ \varepsilon)=|J|f_{p1}[y,\ x_n exp\{-\pi n\varepsilon(\Delta s)^2/2\},\ L(\Delta s)/(\Delta L)|\beta,\ \varepsilon] \quad \cdots\cdots (15)$$

with $|J|$ being the Jacobian of transformation of x_0 and s into x_n and Δs, respectively. It follows from eqs. (11) and (12) that

$$|J|=(L/\Delta L)\,exp\{-\pi n\varepsilon(\Delta s)^2/2\} \quad \cdots\cdots (16)$$

Integrating eq. (14) over the domain of y, x_n, Δs and ε in which these random variables satisfy the condition of survival (see Fig.1), the survivability probability of a

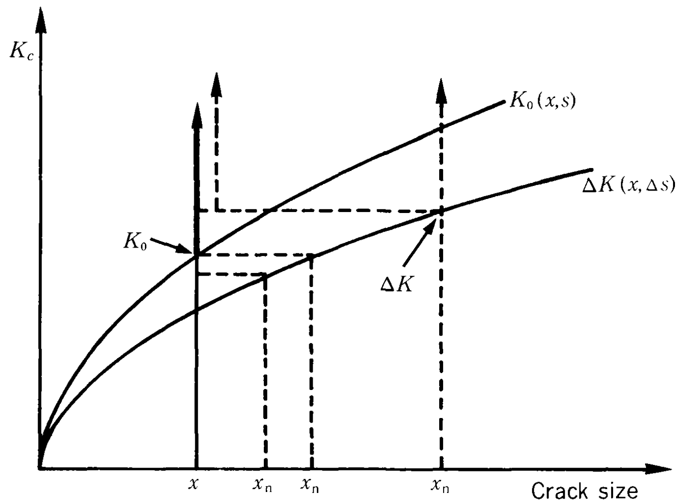

Fig.1 Domain of Integration

structural detail immediately subsequent to n load cycles, is obtained as

$$R_{s1}(n|\beta,\ \mu_\varepsilon)=R_{s11}(n|\beta,\ \mu_\varepsilon)+R_{s12}(n|\beta,\ \mu_\varepsilon) \cdots\cdots (17)$$

with

$$R_{s11}(n|\beta,\ \mu_\varepsilon)=\int_0^\infty\{\int_0^\infty\{\int_0^\eta\{\int_\lambda^\infty f_{s1}(y,\ x_n,\ \Delta s,\ \varepsilon|\beta,\ \mu_\varepsilon)\,dy\}\,d(\Delta s)\}\,d\varepsilon\}\,dx_n \quad \cdots\cdots (18)$$

and

$$R_{s12}(n|\beta,\ \mu_\varepsilon)=\int_0^\infty\{\int_0^\infty\{\int_\eta^\infty\{\int_\xi^\infty f_{s1}(y,\ x_n,\ \Delta s,\ \varepsilon|\beta,\ \mu_\varepsilon)\,dy\}\,d(\Delta s)\}\,d\varepsilon\}\,dx_n \quad \cdots\cdots (19)$$

where

$$\eta=\{4/(\pi n\varepsilon)\cdot\log(L/\Delta L)\}^{1/2},\quad \lambda=s(\pi x_0/2)^{1/2},\quad \xi=\Delta s(\pi x_n/2)^{1/2} \quad \cdots\cdots (20)$$

Eq.(18) indicates the survivability probability of a structural detail when $K_0(x_0, s) > \Delta K(x_n, \Delta s)$ or $\eta > s$. The integration with respect to y over the interval from λ to ∞ in the same equation reflects the fact that the structural detail has survived the proof load test and therefore $K_C > K_0(x_0, s)$ or $y > \lambda$. On the other hand, eq. (19) indicates the survivability probability when $K_0(x_0, s) \leqq \Delta K(x_n, \Delta s)$ or $\Delta s \geqq \eta$, and the integration with respect to y is carried out from ξ to ∞, since, in this case, the critical stress intensity factor must be larger than $\Delta K(x_n, \Delta s) = \Delta s\sqrt{\pi x_n/2}$ for the structural detail to survive. The failure probability P_{s1} of the structural detail during the first n cycles is obviously given by

$$P_{s1}(n|\beta, \mu_\varepsilon) = 1 - R_{s1}(n|\beta, \mu_\varepsilon) \cdots\cdots (21)$$

If the structural detail survives the first n load cycles, the joint PDF of K_c, x, Δs and ε can be written as

$$f_{s2}(y, x_n, \Delta s, \varepsilon|\beta, \mu_\varepsilon) = \frac{f_{s1}(y, x_n, \Delta s, \varepsilon|\beta, \mu_\varepsilon)}{1 - P_{s1}(n|\beta, \mu_\varepsilon)} \cdots\cdots (22)$$

while the PDF of x_n is given by

$$f_n(x_n|\beta, \mu_\varepsilon) = \int_0^\infty [\int_0^\eta \{\int_\lambda^\infty f_{s2}(y, x_n, \Delta s, \varepsilon|\beta, \mu_\varepsilon)\,dy\}\,d\Delta s + \int_\eta^\infty \{\int_\xi^\infty f_{s2}(y, x_n, \Delta s, \varepsilon|\beta, \mu_\varepsilon)\,dy\}\,d\Delta s]\,d\varepsilon \cdots\cdots (23)$$

2.4 Estimation of Uncertain Parameters by Between-service Inspections

The unknown parameters β and μ_ε introduced in the preceding sections can be estimated from a between-service inspection with the aid of the Bayesian method in exactly the same way as the unknown parameters a and c were estimated in Section 2.1 on the basis of an initial inspection. Indeed, the between-service inspection performed at the end of the first n load cycles will also result either in event E_1 = detection of (and repair of) crack of size x_n or in event E_2 = no crack detected. Then, similarly to eqs. (3) and (4), one obtains the following recurrence formulas for the posterior joint PDF, $p^i(\beta, \mu_\varepsilon)$, of β and μ_ε ($i=1, 2, \ldots, N$);

$$p^i(\beta, \mu_\varepsilon) = \frac{\{f_n(x_n|\beta, \mu_\varepsilon)F_D(x_n{:}c_M)\}p^{i-1}(\beta, \mu_\varepsilon)}{\iint (Numerator)\,d\beta d\mu_\varepsilon} \cdots\cdots (24)$$

and

$$p^i(\beta, \mu_\varepsilon) = \frac{\{\int_0^\infty f_n(x_n|\beta, \mu_\varepsilon)\bar{F}_D(x_n{:}c_M)\,dx_n\}p^{i-1}(\beta, \mu_\varepsilon)}{\iint (Numerator)\,d\beta d\mu_\varepsilon} \cdots (25)$$

Eq.(24) is for the case where the between-service inspection results in event E_1 at structural detail i, while eq. (25) is for the case in which the inspection results in event E_2. In both cases, $p^0(\beta, \mu_\varepsilon)$ represents the prior joint PDF of β and μ_ε. It is expected, as in the case of parameters a and c, that the joint PDF of β and μ_ε tend to concentrate at the point (β_M and $\mu_{\varepsilon M}$) for a large N. These modal values will be used for β and μ_ε in the reliability analysis of the structure that is placed back in service afrer between-service inspection.

2.5 Numerical Example····1

A numerical example is presented to demonstrate how the proposed method can be applied. The true, but in reality unknown, values of four parameters, a, c, β and μ_ε are listed in Table 1 together with the values of the other (known) parameters. These unknown parameters are treated as if they were independent random variables and their (a prior) density functions are assumed to be uniform over the ranges also indicated in Table 1. The Monte Carlo technique is used to generate the results of the initial inspection, proof load test and first between-service inspection, all performed in simulation.

Table 1 Parameter of Numerical Example

Parameter	True	Assumed
a (1/mm)	0.5	0.3 ~0.7
b (mm)	1	known
c (1/mm)	0.1	0.005~0.2
d	0	kown
α	3	known
β (kg/mm$^{3/2}$)	500	400~580
$\mu_s/\mu_{\Delta s}$ (kg/mm^2)	60/40	known
$COV_s/COV_{\Delta s}$	0.05	known
m	2	known
μ_ε (10^{-8})	8.0	6.0~9.6
COV_ε	0.03	known

Figure 2 depicts the posterior joint PDF of parameters a and c estimated on the basis of a (simulated) initial inspection performed on three hundred structural details. The modal values $a_M = 0.48$ and $c_M = 0.095$, which are satisfactorily close to the true values $a=0.5$ and $c=0.1$, are obtained from Fig.2. The COV's of a_M and c_M are found to be 0.183 and 0.052, while 75.4% of the initial

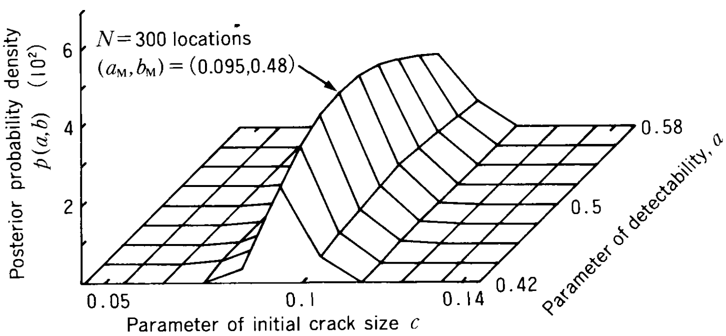

Fig.2 Posterior Joint Probability Density of a and c during Initial Inspection ($N=300$ inspected locations)

cracks are detected during the (simulated) initial inspection. The fact that the COV of c_M is much smaller than the COV of a_M indicates that a is much easier to estimate than c. The PDF of initial crack size and the detectability are shown in Fig.3 using both true and estimated values of a and c. Simirarly, the PDF's of the remaining crack size are then plotted in Fig.4 again using both true and estimated values of a and c. In addition, in both Figs.3 and 4, the simulated histograms for x_0 are also shown.

On the basis of the simulated between-service inspection which results in either event E_1 or event E_2 at each of the three hundred structural details, the posterior joint PDF of β and μ_ε is estimated and plotted in Fig.5. The modal values β_M and $\mu_{\varepsilon M}$ of β and μ_ε, respectively, are then obtained from Fig.5: $\beta_M = 480\,\text{kg/mm}^{3/2}$ and $\mu_{\varepsilon M} = 8\times10^{-8}$. As anticipated, these modal values are very close to their corresponding true values; Indeed, $\mu_{\varepsilon M}$ exactly coincides with its corresponding true value. The COV's of β_M and $\mu_{\varepsilon M}$ turn out to be approximately 0.121 and 0.051, respectively. This implies that the estimation of $\mu_{\varepsilon M}$ is more reliable than that of the simulated between-service inspection resulted in 66% of the fatigue cracks being detected.

The PDF $f_n(x_n|\beta, \mu_\varepsilon)$ of the fatigue crack size is plotted in Fig.6 for $\beta = \beta_M$ and $\mu_\varepsilon = \mu_{\varepsilon M}$. The same PDF for two other sets of values of β and μ_ε is also plotted in Fig.6.

The above results demonstrated the use of Bayes' theorem in estimating the unknown parameters in the probability density functions of an initial crack size and critical stress intensity factor, in the crack propagation law and in the expression for the detectability associated with a nondestructive inspection. The estimation proceeds with a pre-service initial inspection and proof load test and the first between-service inspection in that sequence. This cycle of an estimation process may be

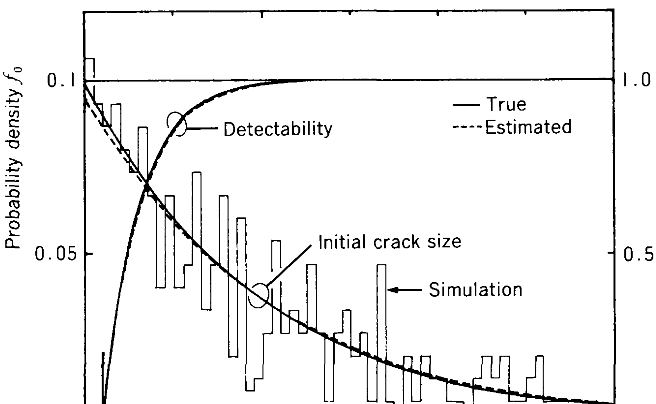

Fig.3 Probability Density of Initial Crack size and Detectability of NDI

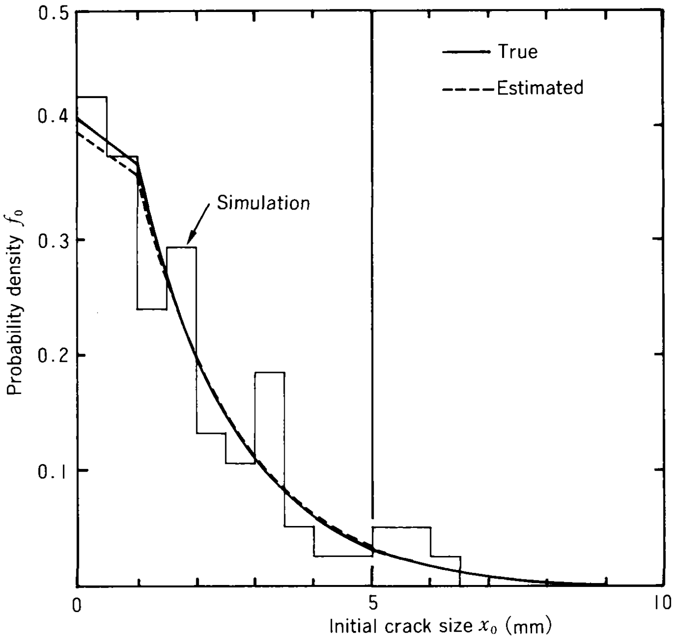

Fig.4 Probability Density of Remaining Crack Size after Initial Inspection

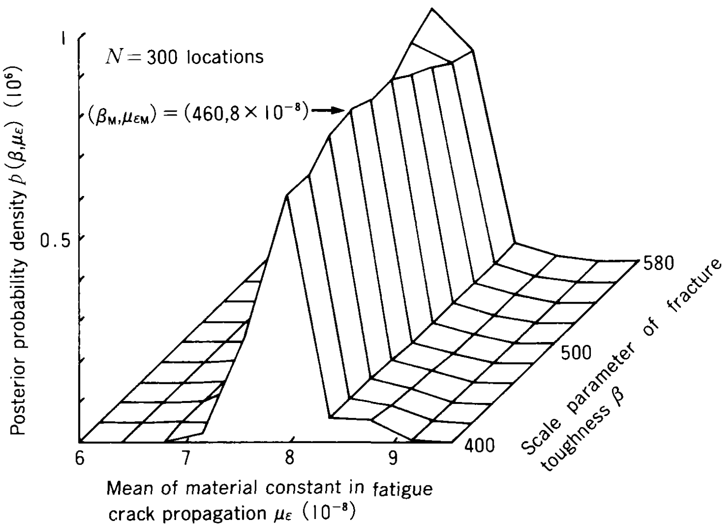

Fig.5 Posterior Joint probability Density of β and μ_ε during the First Regular Inspection (n=6000 load cycles, N=300 inspected locations)

followed by a second cycle of proof-load test and between-service inspection, ans so on.

3. Case of visual inspection

The above analysis was performed under the pragmatic assumption that the analytical forms of the probability density function, crack propagation law and detectability are known a priori. Furthermore, it is assumed that reliable data are obtained during each cycle to perform such an analysis. Consequently, the method was straightforward in computing the reliability and also its interpretation can be as usual. On the

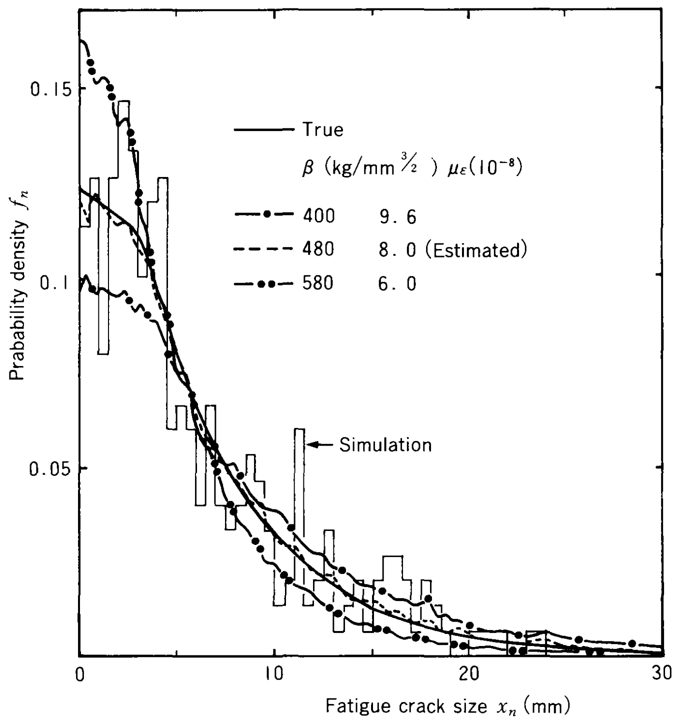

Fig.6 Probability Density of Fatigue Crack Size (n=6000 load cycles)

contrary, as to the ship structural members which are visually inspected according to the rule, such data are realy obtained. Operational conditions are not clear and the results of inspections are sometimes classified into very few categories giving very rough quantitative information, for exsample detected cracks are devided into two groups, ones larger and the others smaller than 200mm.

Under these circumstances, very detailed analysis of inspection results considering fracture mechanics, statistics and so on seems no good. Even so, as long as the inspections are perfomed orderly, the results should be properly interpreted and used to improve the relability of the structures.

As an example, take the case of regular inspections where fatigue cracks of a significant size are visually detected and counted as "one event of member failure." It should be noted that the purpose of the inspections is not only to find the cracks to be repaired but also to gain additional knowledge on the state of ship structural integrity.

3.1 Formulation of Problem

The fatigue life of the structural member under consideration, T, is between the start of its service and the time at which the discovery of a crack is made by visual inspection. The probability distribution function of T is assumed to be a Weibull distribution of two parameters:

$$F_T(t|\gamma,\ \delta) = 1 - exp\{-(t/\delta)^{\gamma}\} \cdots\cdots (26)$$

Even though a crack is found in an inspection, it is extremely difficult to determine whether the crack originates from a latent flaw or it initiated at and propagated from a location of stress concentration. Therefore, if a crack is discovered while the ship is in service, then the fatigue life of the structural member is assumed to be less than the elapsed time up to the inspection.

In view of the lack of accuracy in the evaluation of the cracks, eq. (26) is interpreted as follows: When a crack is found as a result of an inspection performed at time t_0, then t_0 is regarded as a realization of T given by eq. (26).

In general, structural members should be designed under the principle that the probability of failure, in a prescribed period of service years, t_0,

$$p_f(t_0) = F_T(t_0|\gamma,\ \delta) \cdots\cdots (27)$$

is sufficiently small.

However, in the usual design of ship structures, a probabilistic analysis is seldom performed on individual members for evaluating the failure probability, and current rules or design codes are not always formulated on the basis of a probabilistic concept for the reliability of ships, even if considerations on the ship's life given by eq. (26) are made at the design stage, a great deal of uncertainties would be involved in the determination of γ and/or δ, and therefore little practical significance may be expected from such considerations. While the only possible meaning of design procedures would be the fact that scantlings of the structural menbers could have been determined to ensure a sufficiently small probability of failure (although the value of p remains unknown) through judgment on the basis of past experience.

In the following discussion, a simple assumption is made that a certain definite value for γ corresponding to a suitable coefficient of variation is asigned, while for δ, several estimated values are considered in the analysis. When estimating the characteristic value of life time service, δ, a subjective guess will be made on the basis of past experience by those persons responsible for the design, fabrication, inspection and maintenance of ships. The degree of belief or uncertainty is measured in termes of the probability that a true value of δ exists around a prescribed value u ($\delta_1 \leqq u \leqq \delta_2$). Then, it folows that the reliability of a structural member is between $\exp\{-(t_0/\delta_1)^{\gamma}\}$ and $\exp\{-(t_0/\delta_2)^{\gamma}\}$.

In other words, the expected prior Bayesian reliability can be written by the following formula:

$$\bar{R}^0(t_0) = \int_{\delta_1}^{\delta_2} \{1 - F_T(t_0|\gamma,\ u)\} f^0(u)\, du \cdots\cdots (28)$$

It is assumed that periodical inspections are performed evry t_0 years after the ship enters into service, and detectability is defined as before by eq. (1). The detectability depends upon the kind of defects as well as on the method of inspection, and various approximated formulae have been proposed (2). In this section, a visual inspection is considered, for general aspect of the analysis. In this formula, b is the minimun size of cracks detectable through inspection. This quantity is also considered to be one of the uncertain factors but it is a safe-side estimation used to assume the minimum size of cracks ever detected in past inspections. On the other hand, for parameter a, the prior density is assumed as before.

A crack will exist in a structural member of which the fatigue life has already been reached at the time of inspection. The crack size C is a random variable and follows a time-dependent probabilistic distribution, $F_C(x|c,\ d,\ t>T)$. This represents the probability that the crack size C is smaller than x, under the condition of given parameter values c and d as well as of time, t, exceeding the fatigue life T.

It should be noted here that no essential change will be caused in the course of the analysis if any functions are assumed for F_C other than eq.(2). In this distribution function, the parameters c and d are of course, of uncertain value and are perhaps more complicated than those of the fatigue life, the detectabililty and other characteristics. In this discussion, the value of the parameter d is assumed to be approximately given, while for c, a uniform prior density is assumed.

Formulation of these prior densities should naturally be made by experienced engineers on the basis of their technical knowledge concerning the degree of brief in the parameter values. When the number of samples for inspection is relatively large, it would be advisable to assume the prior densities on the very safe side, since the approximation of reliability will be successively improved by the feedback of the results of the inspection, that is, by the properly modified posterior densities.

When inspections consist of a procedure wherein no information on the crack size is available from the results of the inspections, and therefore, the results are either

event E_{B1} = detect a crack; or

event E_{B2} = detect no crack.

The probability of each event is obtained by using the previous assumptions:

$$\begin{aligned} P_r[E_{B1}] &= P_r[\,(T\leqq t_0)\cap(d\leqq c<\infty)\cap(detect)\,] \\ &= F_T(t_0|\gamma,\ \delta)\tilde{D}(t_0) \\ &= p_{B1} \end{aligned} \quad \cdots\cdots (29)$$

$$P_r[E_{B2}] = 1-P_r[E_{B1}] = 1-p_{B1} = p_{B2} \quad \cdots\cdots (30)$$

where

$$\tilde{D}(t_0) = F_T(t_0|\gamma,\ \delta)\int_0^\infty f_C(x|c,\ d,\ t_0)\,F_D(x|a,\ b)\,dx$$

As can be seen from the above equations, the uncertainties of inspection and crack size are expressed in terms of $\tilde{D}$, where it is not possible to consider the uncertainties individually of a and c. This is not necessarily an unfavorable aspect, because proper estimations of the distribution of defects and of the probability of detection seem to be extremely difficult as compared with that of the life. In fact, sometimes it is almost impossible to select suitable parameters for these distributions, while it is usually the case that the average detectability $\tilde{D}$ can be estimated with relative ease.

Consider that M nominally identical members of a structure are inspected and that cracks are found in m of these members and no cracks in the remaining $M-m$ members. Then the posterior joint density with respect to δ, a and c is given, according to Bayes' theorem, by the following formula:

$$f^1(u,\ v,\ w|M,\ m,\ t_0) = \frac{P_r[DATA]\cdot f_\delta{}^0(u)\cdot f_a{}^0(v)\cdot f_c{}^0(w)}{\iiint [Numerator]\cdot du\cdot dv\cdot dw} \quad \cdots\cdots (31)$$

where

$$P_r[DATA] = (p_{B2})^{M-m}(p_{B1})^m \quad \cdots\cdots (32)$$

3.2 Use of Inspection Results

Assume that inspections are made on M structural members arbitrarily selected out of N nominally identecal members and that crack failures are detected in m members and are all repaired back to their original shape.Then, the reliability $R_N(t)$ for service years t after the inspection is given by the following

$$R_N(t) = R_1{}^{N-M}\cdot R_2{}^{M-m}\cdot R_3{}^m \quad \cdots\cdots (33)$$

where, R_1: reliability of a member not inspected;
R_2: reliability of a member inspected where no crack was detected;
R_3: reliability of a repaired member.

When γ and δ in eq. (26) are given, then R_1 and R_3 can be expressed as follows:

$$R_1(t|\gamma,\delta) = 1-F_T(t_0+t|\gamma,\delta) = exp\left\{-\left(\frac{t_n+t}{\delta}\right)^\gamma\right\} \quad (34)$$

$$R_3(t|\gamma,\ \delta) = 1-F_T(t|\gamma,\ \delta) = exp\{-(t/\delta)^\gamma\} \quad \cdots\cdots (35)$$

Furthermore, R_2 will be obtained in the following manner.

The event that a member passed an inspection at t_0, but was found to be cracked during the next t years is identical to the event either

(1) that the member had developed a crack but was overlooked in the inspection; or

(2) that the fatigue life of the member is between t_0 and t_0+t.

The probability of these events is given by

$$p=F_T(t_0|\gamma,\ \delta)\int_0^{\infty}f_C(x|\lambda,\ u,\ t_0)\{1-F_D(x|a,\ b)\}dx$$
$$+\{F_T(t_0+t|\gamma,\ \delta)-F_T(t_0|\gamma,\ \delta)\}$$
$$=F_T(t_0+t|\gamma,\ \delta)-F_T(t_0|\gamma,\ \delta)\tilde{D}(t_0)$$

On the other hand, the probability of the event that the member passes the inspection at t_0 is

$$P_r[pass]=F_T(t_0|\gamma,\ \delta)\int_0^{\infty}f_C(x|c,\ d,\ t_0)\{1-F_D(x|a,\ b)\}dx$$
$$+\{1-F_T(t_0|\gamma,\ \delta)\}$$
$$=1-F_T(t_0|\gamma,\ \delta)\tilde{D}(t_0)$$

Therefore, the probability of failure during the subsequent service years t, under the condition that it has passes the inspection, is as follows:

$$p_f=p/Pr[pass]=\frac{F_T(t_0+t|\gamma,\ \delta)-F_T(t_0|\gamma,\ \delta)\tilde{D}(t_0)}{1-F_T(t_0|\gamma,\ \delta)\tilde{D}\ (t_0)}$$

Hence

$$R_2(t|\gamma,\ \delta)=1-p_f=\frac{1-F_T(t_0+t|\gamma,\ \delta)}{1-F_T(t_0|\gamma,\ \delta)\tilde{D}(t_0)}\cdots\cdots\cdots(36)$$

Note that

$$R_2(0)=\{1-F_T(t_0|\gamma,\ \delta)\}/\{1-F_T(t_0|\gamma,\ \delta)\tilde{D}(t_0)\}\leqq 1$$

This means that there is always the possibility that a member has been crack failured even if it passes the inspection. When the inspection is perfect (i.e., $\tilde{D}=1$), then $R_2(0)=1$; whereas if $\tilde{D}=0$ then $R_2=R_1$, which implies that the inspection was not particularly useful. It is also noted that the improvement of the reliability R_2, being increased from R_1 is regarded as the effect of the inspection.

By using eqs. (33) to (36), and by taking into account the uncertainties of the parameters in the posterior reliability, the following measure of reliability is obtained:

$$\bar{R}_N{}^1(t)=\iiint[R_1(t|\gamma,\ u)]^{N-M}\cdot[R_2(t|\gamma,\ u)]^{M-m}$$
$$\times[R_3(t|\gamma,\ u)]^m\cdot f^1(u,\ v,\ w)\cdot du\cdot dv\cdot dw$$
$$\cdots\cdots\cdots\cdots(37)$$

where the integration is to be performed over a certain given range assumed for each parameter, and $\tilde{D}(t_0)$ is a function of v and w.

Since the reliability of the N structural members at the time of inspection is given by the following equation,

$$\bar{R}_N{}^0(t_0)=\int_{\delta_1}^{\delta_2}exp\{-N(t_0/u)^{\gamma}\}f_{\delta}{}^0(u)\,du\cdots\cdots\cdots(38)$$

The time interval t_1 upto the subsequent inspection should be so selected as to at least satisfy the following relation:

$$\bar{R}_N{}^1(t_1)\geqq\bar{R}_N{}^0(t_0)\cdots\cdots\cdots(39)$$

In other words, since it has been considered that the expected value of the number of failures to be detected during the first t_0 years out of N members is at most limited to

$$E[m]=N\{1-\bar{R}^0(t_0)\}\cdots\cdots\cdots(40)$$

the following relation must be satisfied:

$$E[m]\geqq E[m'(t_1)]\cdots\cdots\cdots(41)$$

where,

$$E[m'(t_1)]=(N-M)\{1-\bar{R}_1{}^1(t_1)\}+(M-m)\{1-\bar{R}_2{}^1(t_1)\}$$
$$+m\{1-\bar{R}_3{}^1(t_1)\}$$
$$\cdots\cdots\cdots(42)$$

It should be mentioned here that, in the discussions based on the ordinary classic theory of reliability, the reliability of structures after being subjected to an inspection can always be expected to be improved, whereas in the case of Bayesian analysis this is not necessarily true, depending on the initially assumed parameter values. This is mainly due to the fact that if these initial values are excessively unconservative, namely, $\bar{R}_N^0(t_0)$ is assumed to be too large, then the posterior density will move towards such a direction as to correct the value into a suitable one in accordance with the results of the inspection. In particular, if the number M of the members actually subjected to inspection is too small as compared with the total number N of the structural members, then it is possible that even the reliability immediately after inspection $\bar{R}_N^1(0)$ becomes less than $\bar{R}^0(t_0)$. In such a case, it would be necessary to adopt a proper measure such as increasing the frequency of he inspections or reinforcing the structures, etc.

3.3 Numerical Example⋯⋯ 2

In order to demonstrate the use of a Bayesian approach developed in the previous sections and to examine the influence of the number M of structural members subjected to inspection in relation to the total number N as well as of the parameters for uncertain factors on the results of the Bayesian reliability analysis, numerical calculations have been carried out by using the parameter values given in Table 2. The values of δ_0 in Table 2 are so chosen that on the average, five and fifteen fatigue failures will occur out of N=200 members in twelve years' service, respectively. On the other hand, the assumption for the value of a for case (ii) corresponds to a safe-side assumption for detectability as compared with that for case (i). These values of the parameters are not generally applicable to structural members of ships, but are assumed for the purpose of discussing the quantitative influence of these parameters on the reliability of structures. The assumption that δ_1= 20.0 and δ_2=50.0, respectively, corresponds to the fact that, if the unknown true value of δ $(=\delta_0)$ is about forty, it gives a fairly good estimation, while if the true value is about thirty, it is an assumption of too large a value for δ in the fatigue life of the members.

Table 2 Values of parameters in numerical ealuulation

Assumed certain factors

shape parameter for fatigue life, γ	3.0
minimum size of detectable cracks, b	30.0 (mm)
crack size parameter, c	5.9×10^{-3} (1/mm)
minimum crack size, d	30.0 (mm)
number of members to be inspected, N	200
prescribed service year, t_0	12 (year)

Prior estimations for uncertain factors

Scale parameter for fatigue life, δ

true value (unknown), δ	28.1	40.9
prior distribution of δ		
lower limit, δ_1	20.0	
upper limit, δ_2	50.0	

Detectability parameter, a

true value (unknown), a	2.4×10^{-2}	
prior distribution of a	case (i)	case (ii)
lower limit, a_1	1.0×10^{-2}	5.0×10^{-3}
upper limit, a_2	5.0×10^{-2}	3.0×10^{-2}

In this calculation, only δ and a are chosen as the uncertain factors for which the prior densities are given by a uniform density as follows, respectively,

$$\left.\begin{array}{ll} f^0(u)=1/(\delta_2-\delta_1)\ , & \delta_1\leqq u\leqq\delta_2 \\ f_a^0(v)=1/(a_2-a_1)\ , & a_1\leqq v\leqq a_2 \end{array}\right\}\cdots\cdots(43)$$

The average detectability, $\tilde{D}$ is given by

$$\tilde{D}(t_0)=\frac{a}{a+c}exp\{-c(b-d)\}\cdots\cdots(44)$$

where, an assumption is made that $b=d$, and d is as given in Table 2.

Let the true value of the average detectability be D, (namely, the value of $\tilde{D}(t_0)$ for a true value of a $(=a_0)$, and the true reliability be $R_T(t_0)$; then the expected value of the number of failures detectable by inspecting M members is given as follows:

$$\bar{m}=M\{1-R_T(t_0)\}\tilde{D}\cdots\cdots(45)$$

Figure 7 illustrates a result of the calculations, where a relation is shown between the sample ratio (M/N) and the reliability after inspection, $\bar{R}^1(0)$ (taking $t_0=0$ in eq. (37)), as a ratio to the expected value of reliability, $\bar{R}^0(t_0)$ for a service year of t_0 (=12). This figure shows the effect of the prior assumption for the uncertain factors (δ and a) on the results of the reliabillity $\bar{R}^1(0)$. Curve 1 corresponds to the case where the prior assumption of δ is fairly appropriate, while Curve 2 represents the case of the prior assumption which is on the unsafe side.

Curve No.	prior estimations uncertain factors	
	δ	a
1	conservative	case (i)
2	unconsevative	case (i)
3-1	conservative	case (ii)
3-2	unconsevative	case (ii)

	true value	true value
4-1	$\beta_0=40.9$	$a_0=2.4\times10^{-2}$
4-2	$\beta_0=28.1$	

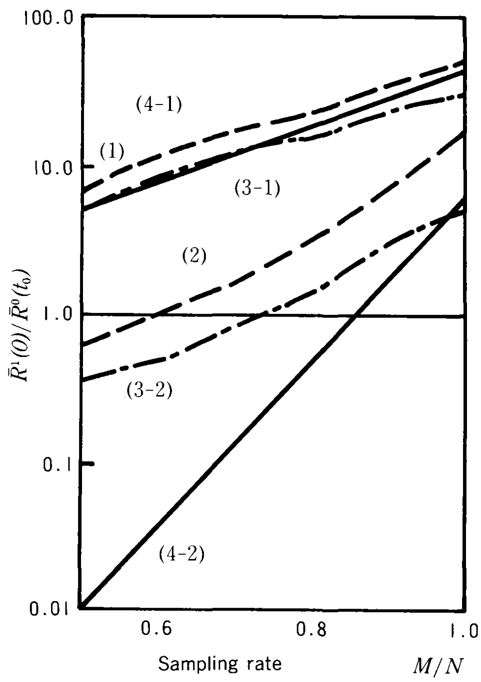

Fig.7 The effect of prior estimations on the uncertain factors δ and a

Curves 3-1 and 3-2 indicate the results of the calculations for the case where the prior density of a is more conservative than that of the above cases. Theoretically speaking, it is not possible to obtain the true reliability. However, Curves 4-1 and 4-2 for R(0) are shown in the same figure being obtained from the following formula by using $R_T(t_0)$ and $\bar{D}$:

$$R(0)=\{R_T(t_0)\}^{N-M}\left\{\frac{R_T(t_0)}{1-[1-R_T(t_0)]\bar{D}}\right\}^{M-m}\cdots\cdots(46)$$

As can be clearly seen from the figure, there is a definite

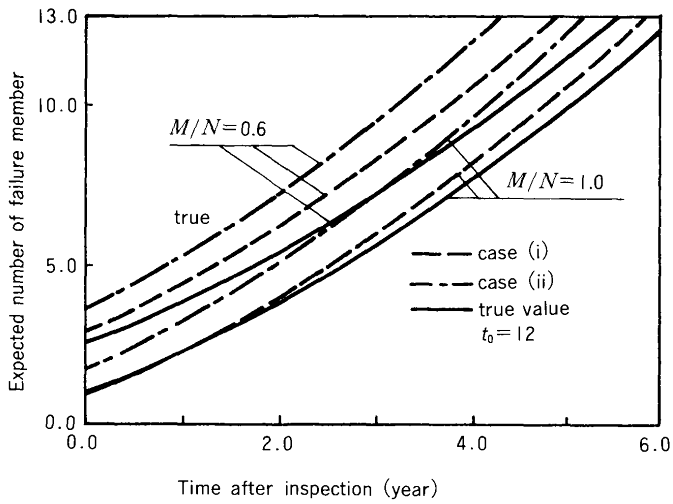

Fig.8 The relationship between time elapsed after inspection and the Bayesian expected number of failed members (conservative estimate for fatigue life)

trend that, as the value of the sample ratio M/N increases , the reliability of the structure is raised, and the expected value approaches more closely the true reliability of the structure. Indeed, it is rather obvious to expect a higher reliability of the structures by detecting and repairing the failed members.

Fig.8 plots the results of the calculations based on a reasonable estimation of the prior density of fatigue life with M/N=0.6 and 1.0, respectively, indicating the relationship between the time elapsed after the inspection performed for the first time after twelve year's service and the Bayesian expected number of failed numbers (eq.(42)).

In this example, the expected number of failed members out of N members during the first t_0 years given by eq. (40) is about 12 and therefore the interval to the subsequent inspection t_1 is to be so determined as to at least satisfy the relationship of eq.(41) with $E[m]$=12.

4. Conclusion

This paper describes a reliability analysis based on a Bayesian method which incorporates the results of inspections and/or of proof-load test performed on nominally identical fatigue-sensitive structural members. The inspections are performed for the detection of fatigue cracks regularly or at such intervals as dictated by the result of each inspection; a unique feature of the Bayesian approach.

Introducing a general concept of uncertain factors into PDF's and detectability of defects, a practical method has been developed for obtaining the posterior joint density of the uncertain factors on the basis of the results of the inspection. Using the posterior joint density function thus developed, one can estimate not only the posterior reliability of the structural members but also the expected number of failed members within each inspection interval. Numerical examples based on hypothetical data indicate, as anticipated, that this method is useful for the reliability analysis of a sample of structural members of reasonably large size. In order to put this method to practical use, it is necessary to determine the prior density functions of the uncertain parameteres on the basis of engineering experience and judgment.

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