Non-Linearity in Sagging Moment and Shear Force of Fine Ships

by Hideomi Ohtsubo and Takao Kuroiwa***

Rational estimation of ship's longitudinal strength should be based on accurate evalution of bending moment and shear force. The non-linear strip theory and model experiments reveal that the external force by waves in rough seas differ from those obtained by the linear theory. Rules of classification societies, based on the long term prediction of structural responses, assume linearity in response and use the linear superposition methods. According to the recent studies, the effect of non-linearity needs to be incorporated in the rules of classification societies.

In the present paper, calculations are performed for a fine container ship using the nonlinear strip theory. Numerical results are obtained for regular waves of various wave heights, wave lengths and wave encountering angles and also for irregular waves of ISSC spectra by the non-linear strip theory. These results are compared with the result of O.S.M. (linear theory). Results obtained are as follows :

1) *Sagging moment increases with the increase of wave height due to non-linear response, and the moment is about 1.7 times than that of O.S.M. at the wave height 8m in regular head sea for the container ship.*

2) *On the contrary, the effect of non-linearity in hogging moment is not so significant as that in sagging moment. Therefore, long term prediction of hogging moment is obtained approximately by linear theory.*

3) *Approximate prediction of long term response of bending moment is performed taking into account the effect of the non-linearity and compared with the present classification rules.*

4) *Positive shear force distribution in the for'd part shows non-linearity in response.*

1. Introduction

At present to predict the external force acting on the ship structure, linear superposition technique [1][2] is used in general and classification rules for ship structure are mainly based on experience and experiments [3]-[5]. These linear superposition rules are derived from existing O.S.M. (ordinary strip method) . However, for fine ships in high waves there is non-linearity in external forces acting on the ship hull, as mentioned in ref. (6) and (7). Hence, there is a tendency to include the effect of non-linearity in classification rules also.

In this paper we will focus on the non-linearity of vertical bending moment of a fast container ship with very low C_b. At first, the effects of wave height, wave length, encountering angle and ship's speed on the degree of non-linearity of response amplitude are investigated. Then the degree of non-linearity in irregular waves is found out using simulation for short term waves .
Finally, taking the effect of non-linearity of wave loads, new rationalised guide lines for the standards of ship structural design are proposed. The simulation is based on the theory developed by Yamamoto et al, (8). In this paper, however, the calculations is based on rigid body motions without considering the elastic vibration of the ship hull caused by slamming and we focused our attention on the variation of ship response in waves with time periods same as that of the encountering waves.

It is quite obvious that in case of slamming, apart from hydrostatic force there exists hydrodynamic force such as impact load on the hull at the time of contact and the force due the elastic vibration of the hull as a whole. To determine the stochastic characteristics of hull vibration, we have to consider the number of immergence, the slamming impact pressure amplitude and the duration of impact.

2. Response in Regular Waves

2.1 Particulars of Ship

Since the degree of non-linearity of the ship hull response due to change of hull shape with the change of draft is expected to be more for slender ships, we chose a container ship with $C_b = 0.581$.

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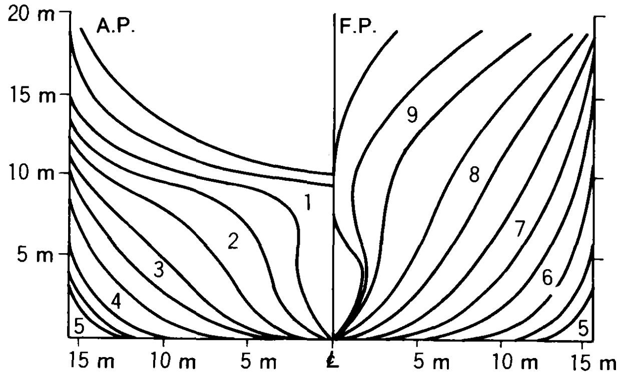

Fig. 1 Body Plan of Container Ship

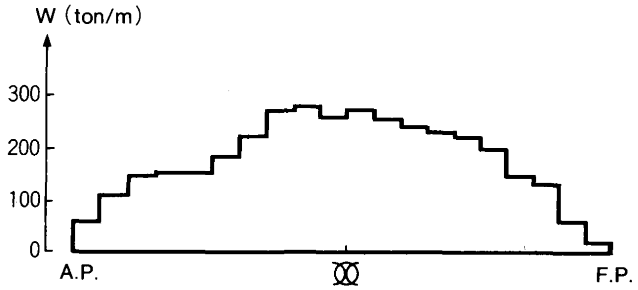

Fig. 2 Weight Distribution of Container Ship

Table 1 Particulars of Containre Ship

L_{pp}	Length between Perp.	200.0 m
L	Length	211.4 m
B	Breadth Moulded	32.2 m
D	Depth Moulded	21.5 m
Δ	Full Load Displacement	40,290 ton
C_b	Block Coeff.	0.581

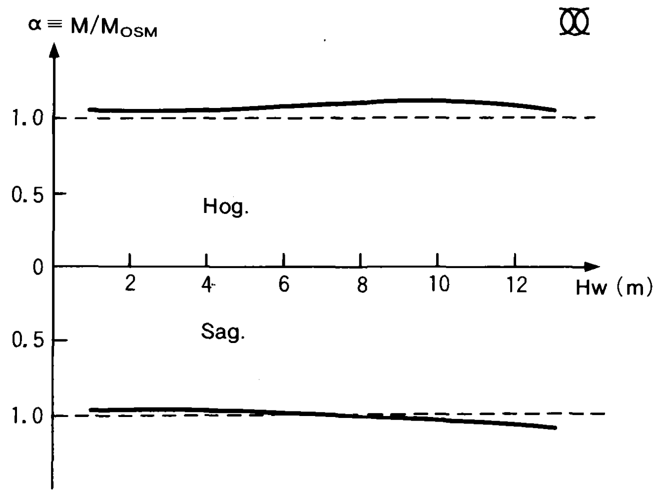

Fig. 3 Response Amplitude of Bending Moment at Midship
(Bulk Carrier, $F_n=0.1$, $\lambda/L=1.0$, $\kappa=180°$)

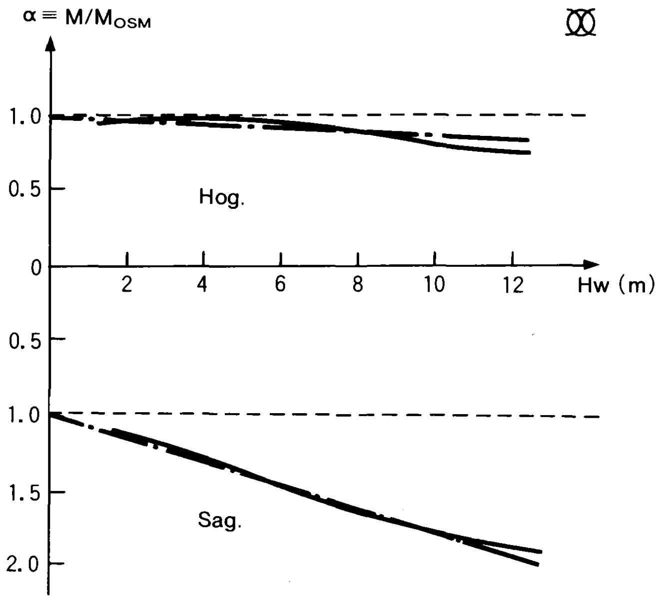

Fig. 4 Response Amplitude of Bending Moment at Midship
(Container Ship, $F_n=0.237$, $\lambda/L=1.0$, $\kappa=180°$)

The body plan, weight distribution and the main particulars are given in Fig. 1, Fig. 2 and Table 1. As the classification rules for wave load is mainly based on the block coefficient, we compared the simulated response of the above container ship with that of a bulk carrier with $C_b=0.85$ [8].

2.2 Numerical Results

To determine the degree of non-linearity of the ship response with wave height, wave length, encountering angle and the ship speed as parameters, we compare the midship bending moment values determined by non-linear theory and O.S.M. denoted by M and M_{osm}. Later on we refer the ratio of M and M_{osm} by α.

(a) Effect of wave height

It is clear from Fig.3 that there is hardly any non-linearity in the bending moment response of a bulk carrier. This can be well explained by the fact that the assumptions of O.S.M. i.e. the wall sidedness of the ship hull at the water level, the direct proportionality of the bouyancy force on the draft and almost constant value of structural damping coefficient over a wide range of draft variation, are valid and applicable. For more than 10m of wave height, the effect of slamming on hull response is neglected due to the reasons given at the end of introduction.

Compare to bulk carrier, there is high non-linearity in the sagging moment than hogging mo-

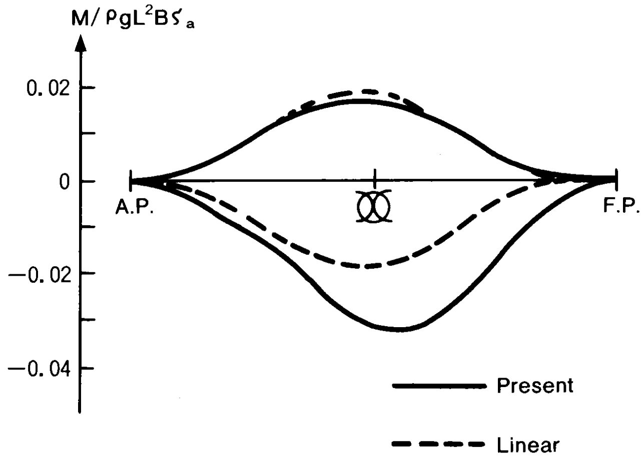

Fig. 5 Distribution of Maximum Bending Moment in Regular Wave
(Container Ship, F_n=0.237, λ/L=1.0, κ=180°, H_w=8 m)

ment for finer ships. For example, in 8m wave (H_w= 8m) it is 1.7 times the linear estimation by O.S.M.. Fig. 5 shows the longitudinal distribution of bending moment. Sagging moment shows high non-linearity from midship onwards. At S.S. 7.5, it is 3.7 times that of O.S.M.. High bow flare causes a significant change in bouyancy force and added mass, leading to a non-linear response. The above discussion proves that the response of ships with smaller C_b is more non-linear so far as the external forces is concerned.

(b) Effect of wave length

As shown in Fig.6 the wave length hardly causes any significant non-linear hull response.

For $\frac{\lambda}{L}>0.8$, $\alpha = M/M_{osm}$ is almost constant.

(c) Effect of encountering angle

The response is more in the region $\kappa < 30°$ and $\kappa > 150°$ as shown in Fig. 7. The ratio $\alpha = M/M_{osm}$ is less than 1 for hogging moment and greater than 1 for sagging moment (refer Fig. 8).

(d) Effect of ship speed

The bending moment is almost constant in case of hogging. For sagging it increases from F_n=0.12. So at higher speed non-linearity in sagging moment response increases with the increase of ship speed. However, for simplicity we neglected the reduction ship's speed in high waves.

From the above result we see that due to non-linearity of external load, sagging moment increases and hogging moment decreases irrespective of wave length, encountering angle, and ship speed. However, the decrease of hogging moment is very much less

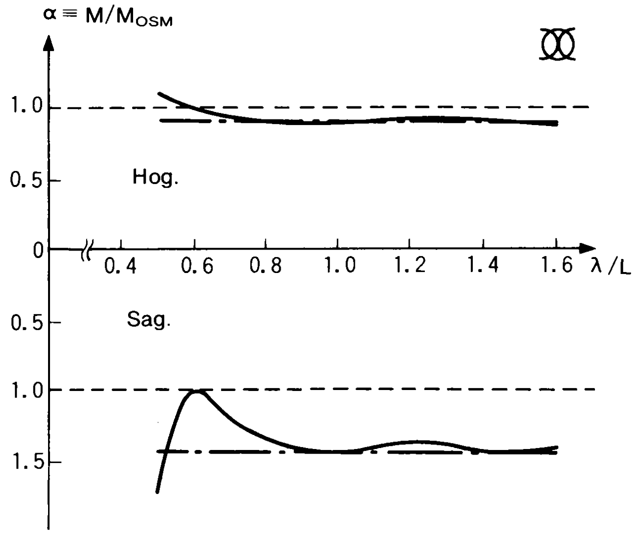

Fig. 6 Response Amplitude of Bending Moment at Midship
(Container Ship, F_n=0.154, κ=180°, H_w=8 m)

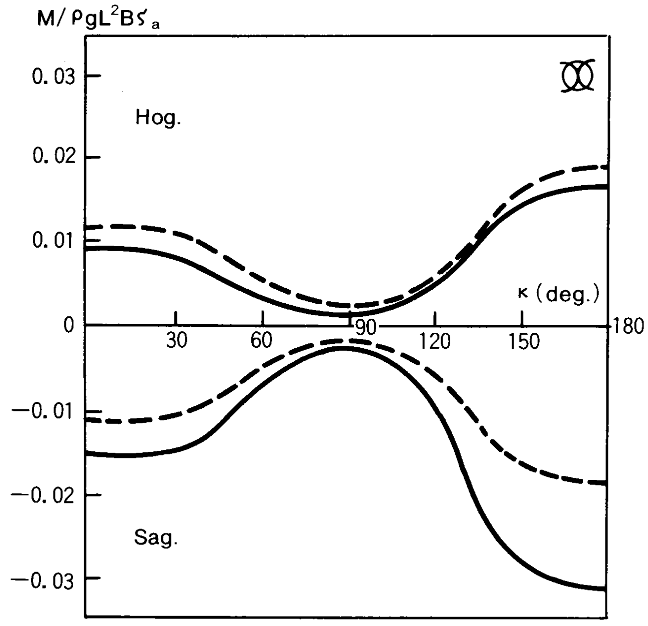

Fig. 7 Response Amplitude of Bending Moment at Midship
(Container Ship, F_n=0.237, λ/L=1.0, H_w=8 m)

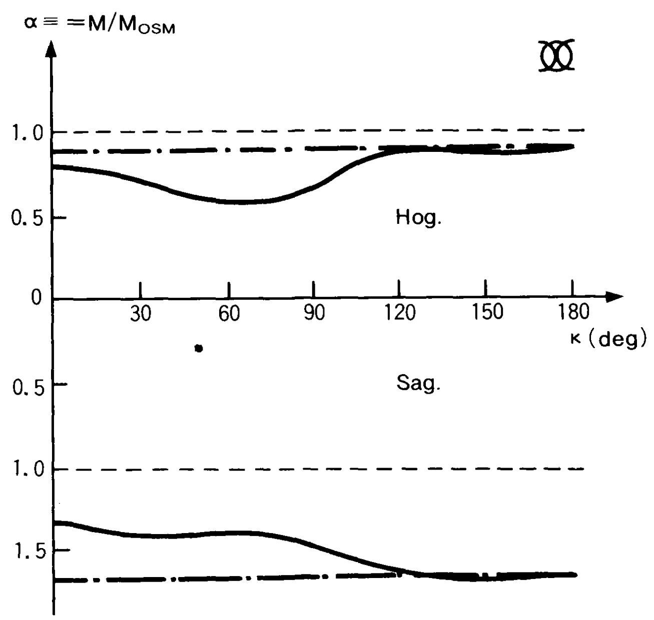

Fig. 8 Response Amplitude of Bending Moment at Midship
(Container Ship, F_n=0.237, λ/L=1.0, H_w=8 m)

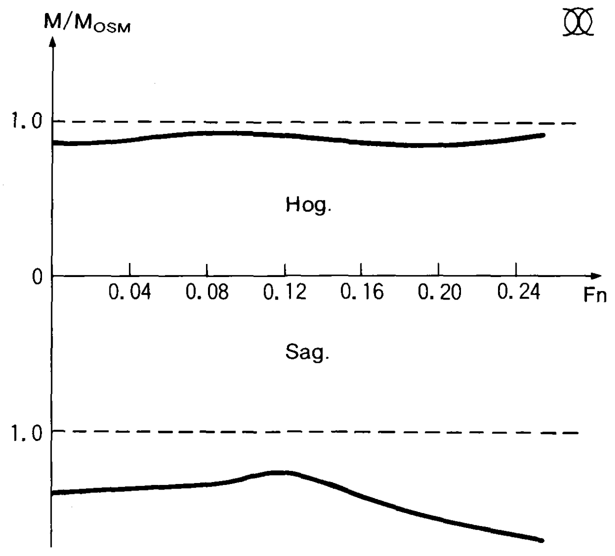

Fig. 9 Response Amplitude of Bending Moment at Midship
(Container Ship, κ=180°, λ/L=1.0, H_w=8 m)

than the increase of sagging moment. Similar results which are not included here due to lack of space are obtained for other sections also.

3. Simulation in Irregular Waves

In this section we will discuss the results of ship response in stationary short-term irregular waves.

3.1 Simulation Results

Our discussion will be based on ISSC spectrum waves of significant wave height $H_{1/3}$=8 m, average time period T=11.642 sec., F_n=0.237, κ=180 deg. (heading seas) and λ/L=1.0. Total time of simulation is 49 minutes.

Fig. 10 shows a part of the simulation for 240 sec. From Fig. 11 it is obvious that the standard deviation of hogging moment is same for both the non-linear theory and O.S.M.. However, for sagging moment σ_M of non-linear analysis is 1.1 times at S.S. 2.5, 1.2 times at midship and 1.8 times at S.S. 7.5 of O.S.M. values. The values of standard deviation σ_M is determined from M_- and M_+ values of response time history for hogging and sagging respectively in accordance with the following formula.

$$\sigma_M\,(sag)=\sqrt{\Sigma M_-^2/N};\ \sigma_M\,(hog)=\sqrt{\Sigma M_+^2/N} \quad (1)$$

where N is the total no. of peaks. Similar plot of σ_{SF} is shown in Fig. 11. Positive shear force at S.S. 7.5 and negetive shear force at S.S. 2.5 are more than those of O.S.M..

3.2 Determination of Probability of Exceedence

In Fig. 12 and Fig. 13 the probability distribution of maximal value of simulated input wave and output (bending moment) response at midship are shown. Since the chosen ISSC wave spectrum is not a narrow band one, it doesn't tally with the Rayleigh distribution. It coincides with an extreme value distribution[10] of band width ε=0.59. The output hogging moment distribution has almost similar distribution. This can be explained from the fact that hogging moment response doesn't show any non-linearity. On the other hand the sagging moment response is stretched forward due to non-linearity. The peak distribution of bending moment at midship and shear force at S.S. 7.5 are shown in Fig. 13 (a) (b). The continuous line (+ve peak; refer Fig. 10) of output response indicates the distribution of those peaks where the response changes from increasing values to decreasing ones and dotted line (−ve peak; refer Fig. 10) denotes those peaks where the response changes in the reverse fashion. It is obvious from the above two figures that +ve peak distribution of shear force at S.S. 7.5 and −ve peak distribution of mid-ship bending moment shift towards the maximum value of the corresponding distributions.

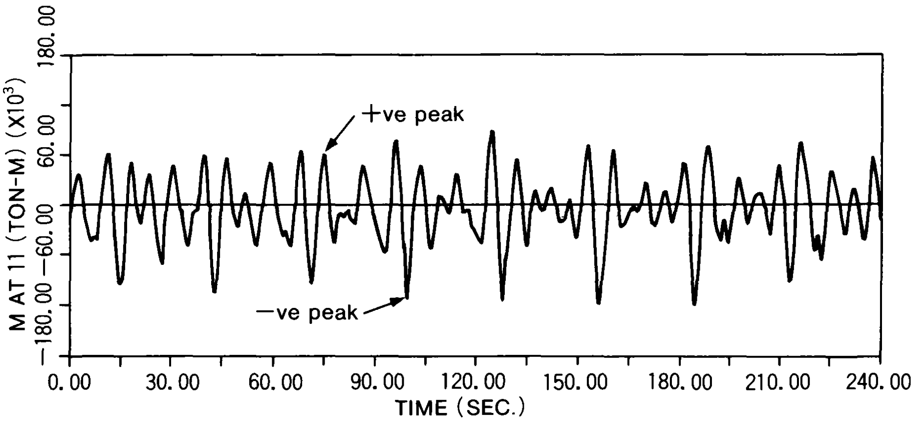

Fig. 10 Time History of Bending Moment at Midship in Irregular Wave (Container Ship, $F_n=0.237$, $\kappa=180°$, $T=11.642$ sec, $H_{1/3}=8$ m)

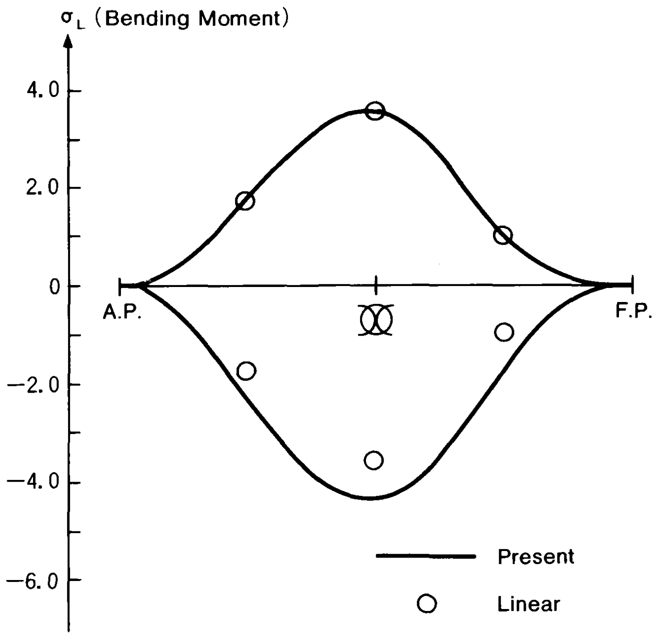

Fig. 11 Standard Deviation of Bending Moment in Irregular Wave (Container Ship, $F_n=0.237$, $\kappa=180°$, $T=11.642$ sec, $H_{1/3}=8$ m)

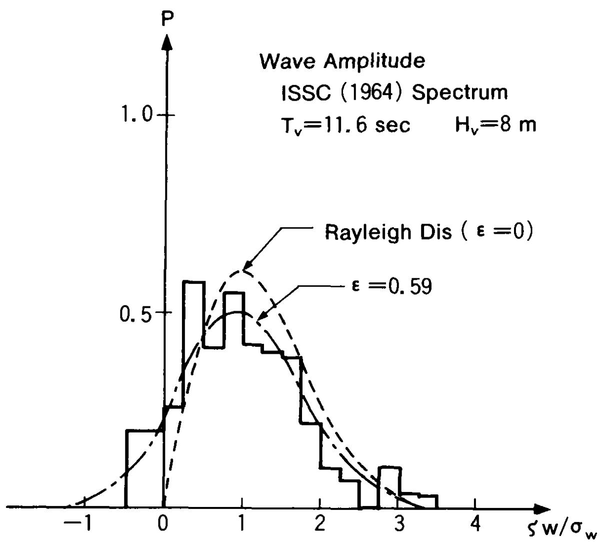

Fig. 12 Probability Distribution of Maximal Value of Simulated Wave

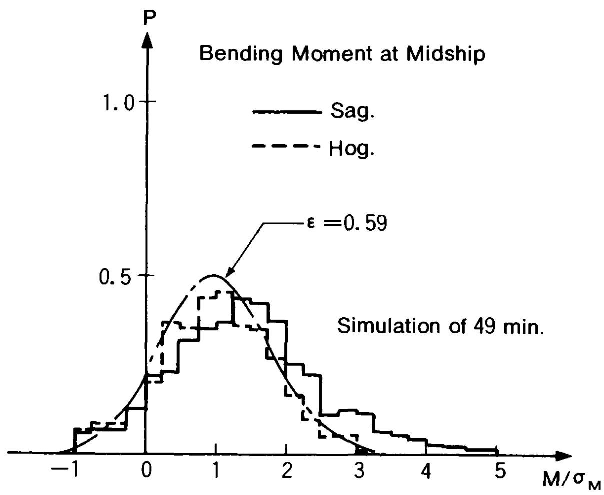

Fig. 13 Probability Distribution of Maximal Value of Bending Moment at Midship

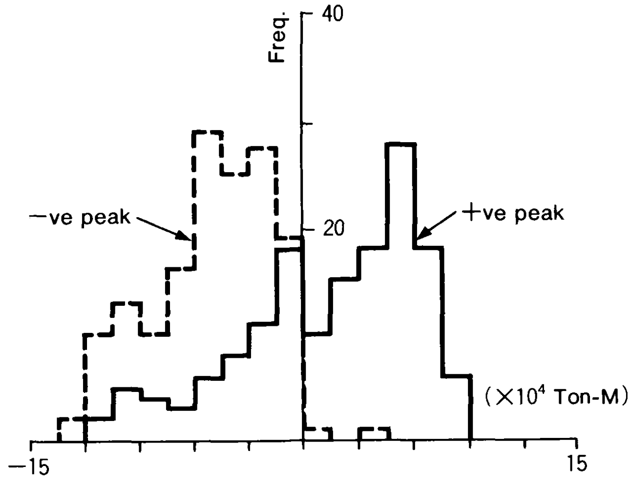

Fig. 13(a) Peak Distribution of Midship Bending Moment

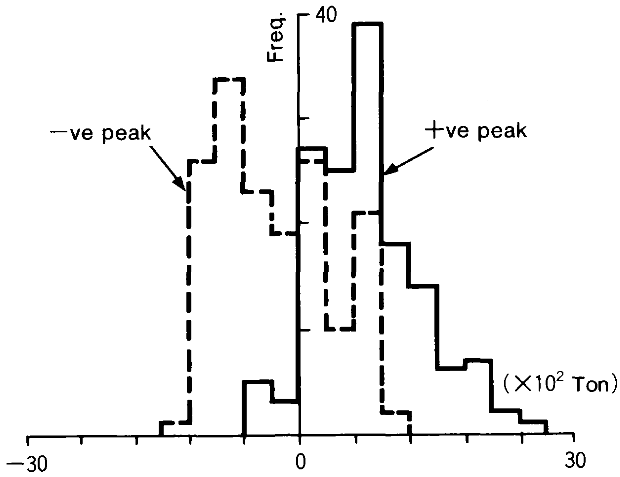

Fig. 13(b) Peak Distribution of Shear Force at S.S 7.5

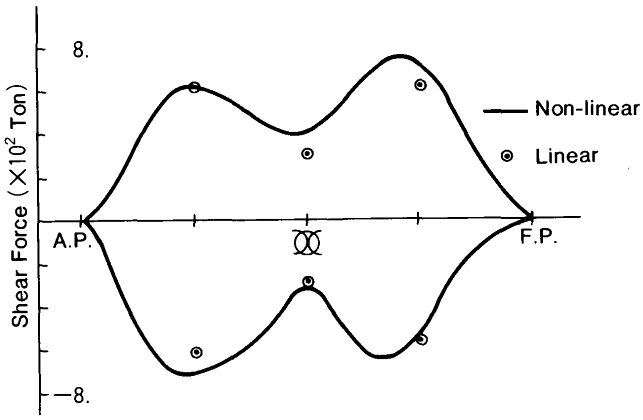

Fig. 13(c) Distribution of Standard Diviation of Shear Force in Irregular Waves with 90 Components

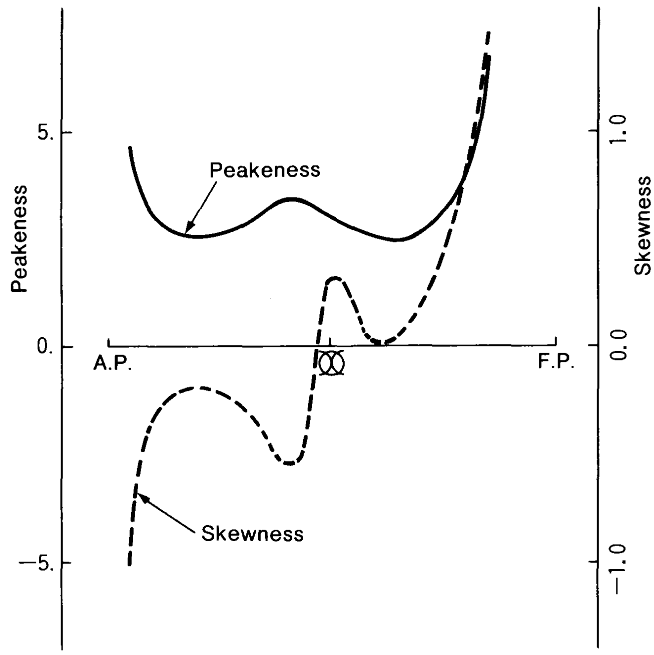

Fig. 13(d) Distribution of Peakedness and Skewness of Shear Force in Irregular Waves with 90 Components

The skewness S and the peakedness P are the two measures of degree of non-linearity of a distribution [10]. The values of P and S are given in Fig. 13(c)(d). The value of peakedness of a Gaussian distribution is 3. The value of P at midship is almost equal to 3. The fullness (i.e, wall sidedness) of section at and near mid-ship justifies the above value since the variation of added mass and change of bouyancy is linear with the change of draft. On the other hand as we go away from midship the fineness of end sections (particularly at bow region) causes a drastic change of P and S values due to non-linearity of external load.

Based on the above results, we will now propose a method to determime the probability of exceedence $q(M^*)$ of a certain value M^* for non-linear response. As seen in sec. 2.2, the non-linear parameter α can be approximated by the following eqn.

$$\alpha(\omega, \kappa, H_w) = 1 + \beta(\omega, \kappa, H_w) \tag{2}$$

where the coeff. $\beta(\omega, \kappa)$ can be determined from Fig. 6 and Fig. 8. For example with $\omega = 0.540$ Hz, $\lambda/L = 1.0$, $\kappa = 180$ deg.

$$\beta(\omega, \kappa) = \begin{cases} 0.081 & (\textit{for sagging}) \\ -0.013 & (\textit{for hogging}) \end{cases} \tag{3}$$

Assume non-linear response M^* is α times the linear response M' and the corresponding probability of occurance $q(M^*)$, $q(M')$ are equal, as shown in Fig. 14. Hence, the relation of M^* and M' can be approximated as a function H_m by the follwing relation.

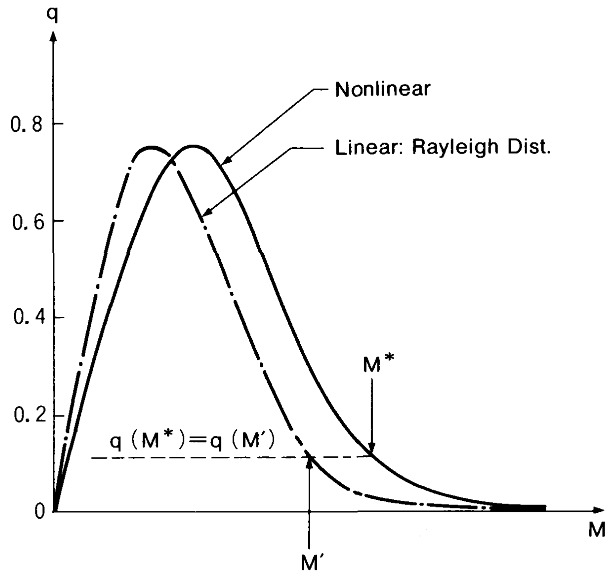

Fig. 14 Frequency Distribution of Bending Moment

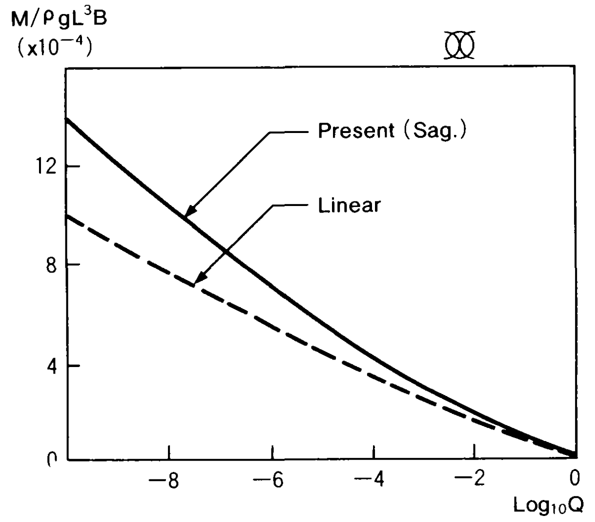

Fig. 15 Long Term Prediction of Bending Moment at Midship (Container Ship, F_n=0.237)

$$M^* = M' \cdot \alpha(\omega, \kappa, H_w) = M' \cdot (1 + \beta(\omega, \kappa, H_w)) \tag{4}$$

where wave height H_m corresponding to linear response M' can be determined from the standard deviation σ_M in short term waves of significant wave height H_v.

$$\frac{H_m}{H_v} = \frac{M'}{\sigma_M} \tag{5}$$

Assuming M' follows Rayleigh distribution, the probability of exceedence $q(M')$ becomes

$$q(M') = exp\left(\frac{M'^2}{2\,\sigma_M^2}\right) \tag{6}$$

With the above assumption, the probability of exceedence $q(M^*)$ of non-linear response M^* is determined by first calculating the corresponding value of M' from eqn. (4) & (5) and then substituting that value of M' in eqn. (6).

4. Application in Ship Structural Design

In this section we calculated the long term prediction of hull response considering non-linear wave load and compared that with the existing classification rules based on linear theory (O.S.M).

4.1 Results of Long Term Prediction

Using the values of short term prediction $q(M^*)$ as proposed in sec. 3.2, long term prediction can be done by eqn. (7) and the results are given in Fig. 15 and Fig. 16.

$$Q(M^*) = \iint p(H, T) \cdot q(M^*, H, T)\, dH dT \tag{7}$$

where observed wave height is H, average time

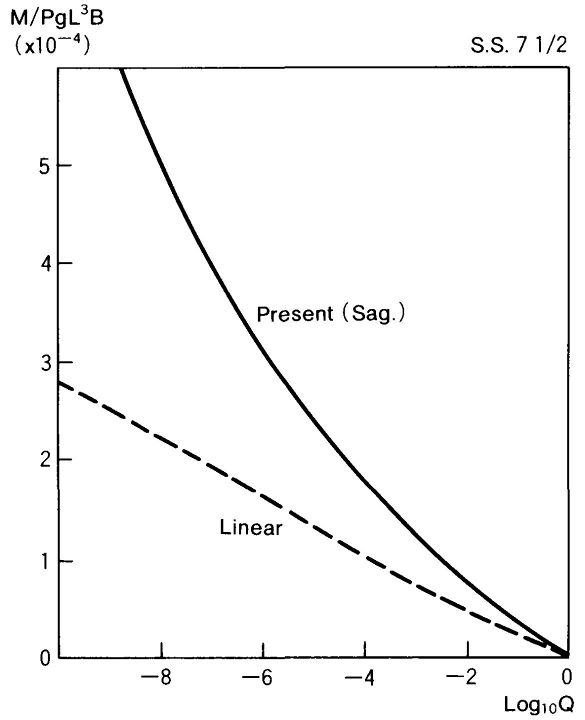

Fig. 16 Long T· ·m Prediction of Bending Moment at S.S. 7.5 (Container Ship, F_n=0.237)

period is T and the prob. of occurence for long term sea state is given by Walden[1) 1] for North Atlantic Sea. The present calculation is based on F_n =0.237 with all heading angles. The long term predictions ($\log_{10}Q$ = − 10) of sagging bending moment with non-linear estimation of wave force and O.S.M. are

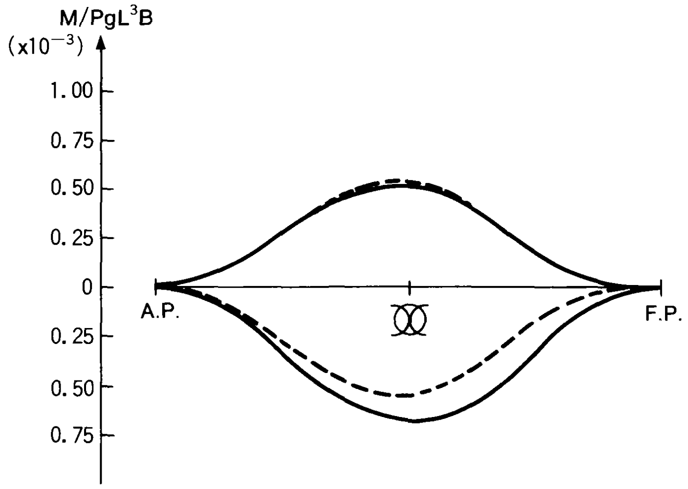

Fig. 17 Long Term Prediction of Bending Moment ($\mathrm{Log}_{10}\, Q=-6$)

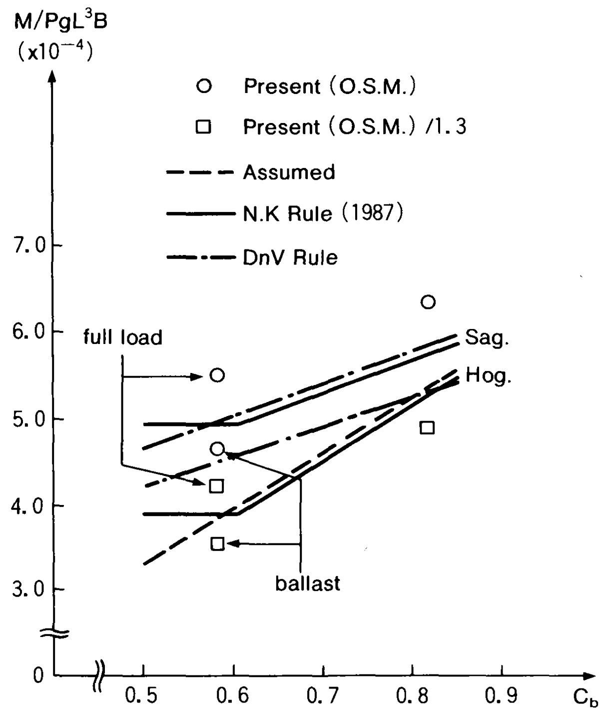

Fig. 18 Comparison of Rule and Long Term Prediction of Bending Moment at Midship

plotted in Fig. 15 and Fig. 16 with continuous and dotted line. At mid-ship and at S.S. 7.5 the nonlinear estimated values are 1.2 and 2 times of O.S.M. values. Fig. 17 gives the distribution of long term prediction of bending moment for $\log_{10}Q=-6$. Compared to O.S.M., the predicted bending moments are 1.2 and 2.0 times more for non-linear estimation at S.S. 2.5 and at S.S. 7.5. Hence long term prediction must include non-linear estimation of wave force acting on the hull.

4.2 Investigation of Classification Rules

The difference among the rules of different classification standards arises from the fact that they are based on different sea state, probability of exceedence of external force, percentage of safety level and allowable stress level. For example, in D.N.V rules existing standard for external wave load is calculated by dividng the rule values corresponding to ($Q=10^{-8}$) with 1.70.

In Fig. 18 we have plotted the existing rules of D.N.V. and N.K. (1987) on the same probability of exceedence level of $Q=10^{-8}$. In order to do that we multiplied the existing rule values by 1.7 and divided by a factor 6/8 to fit into $Q=10^{-8}$ basis.

It is obvious that the proposed hogging and sagging moments lie on opposite sides of the existing rules indicating the difference of expected values with respect to block coeff. C_b. The following conclusion can be drawn from the above Fig. 18 :

1) The more finer the ship (i.e. the smaller the C_b), the greater is the non-linearity in sagging moment response.

2) The present calculation is for $C_b=0.587$ and hogging moment is 30 % less than sagging moment. However, in the existing rule the hogging moment is fixed as 10 % less than sagging moment. To avoid the over estimation of hogging moment, N.K. proposed new rules in 1987. One alternative is that use O.S.M. values as the basis for hogging moment estimation since it is almost linear with repsect to external wave force.

4.3 Expected Bending Moment including Non-Linear Effect

4.3.1 Hogging moment

We have also plotted the values of bulk carrier in Fig. 18 ; the extreme right ones are the values for sagging and hogging conditions. Though these values differs from that of new N.K. (1987) rules, we tried to fit the hogging values by using a factor of 1.3 and it is shown by the dotted line in Fig. 18. The above dotted line can be approximated as

$$M=0.137C_w L^2BC_b[kN\text{-}m] \quad (\mathit{for\ hog}) \tag{8}$$

C_w is a parameter with respect to the length of the ship. As these interpolation is based on the simulated results of only 2 ships with 3 conditions in total, we need to verify it with a large no. of ships.

4.3.2 Sagging Moment

The sagging moment response for ships with large block coeff. (such as $C_b=0.85$) does not show much non-linearity. For ships with small C_b values, it can be approximated as

$$M=0.075C_w L^2B\,(C_b+0.70)[kN\text{-}m] \quad (\mathit{for\ sag}) \tag{9}$$

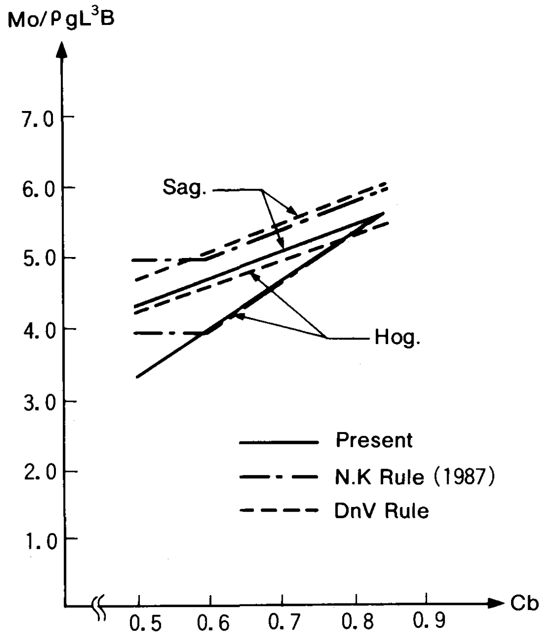

Fig. 19 Proposal of Bending Moment at Midship

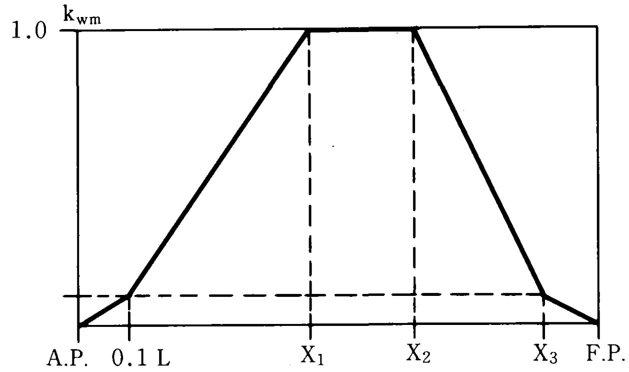

Fig. 20 Proposal of Distribution Factor K_m

In the present proposal eqn. (8) & (9) we assume that hogging and sagging moment are proportional to C_b and $C_b+0.7$ as suggested in the new D.N.V. proposal. The proposed eqns. (8) & (9) does not include the effect of elastic vibration of ship hull.

4.3.3 Longitudinal Distribution

The coeff. K_m of longitudinal bending moment distribution given in Fig. 20 is based on the long term prediction of a container ship and a bulk-carrier.

$$x_1/L=0.25+C_b/4 \; ; \; x_2/L=0.25+C_b/2 \; ; \; x_3/L=0.70+C_b/4 \; (\text{for } hog)$$

$$x_1/L=0.25+C_b/4 \; ; \; x_2/L=0.45+C_b/4 \; ; \; x_3/L=0.90 \; (\text{for } sag) \qquad (10)$$

Compare to above distribution of K_m existing rules for the sagging moment in the for'd part might come out to be very small.

5. Conclusion

(1) The sagging bending moment and the shear force for slender ships show non-linearity in regular wave. On the other hand for ships with large block coeff. the non-linearity of wave force is hardly visible. The hogging moment can be fairly estimated by O.S.M. irrespective of the block coeff. of a ship.

(2) The above conclusion can be drawn for long term prediction in long waves. Specially, the effect of non-leariy of external force is significant at fore and aft parts.

(3) For container ships with non-linear wave load, the simulated results show that the sagging moment at midship and at S.S. 7.5 are more than that of O.S.M. values by 30 % and 20 % respectively. Slenderness of fore and aft sections leads to non-linearity in shear force response.

(4) As stated above in this paper we have suggested a more rationalised formula to determine the ship response due to non-linear wave load. Comparing the existing rule with the proposed one we can conclude that the former might over estimates the hogging moment at mid-ship and under estimates the sagging moment at for'd. The proposed rules (10) are based on the simulation of 2 ships only. Hence we need to carry out simulation for a large no. of ships to check the validity of the proposed eqn. (10).

In this paper we have focussed our attention on the non-linearity aspect of the ship response due external wave force without considering slamming and flexural vibration of hull. Further research is necessary in this aspect.

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