コンクリート工学年次論文報告集, Vol. 20, No. 3, 1998

# 論文 Concrete Cover Effect on Tension Stiffness of Cracked Reinforced Concrete

Hamed M. SALEM [1], Bernhard HAUKE[2], and Koichi MAEKAWA [3]

**ABSTRACT:** The aim of the present study is to check the effect of non-sufficient concrete cover on the tension stiffness of reinforced concrete. Splitting cracks are predicted by solving equilibrium among radial bond stresses, softening tensile stresses of splitting concrete planes and transverse stress on reinforcement. The bond behavior after splitting cracks is the point of study The analytical model is derived from the micro-bond characteristics. An experimental program was carried out to verify the analysis. The analysis fairly agrees with the reality.

**KEY WORDS:** Bond-slip-strain, tension stiffening, crack spacing, confining pressure, splitting

## 1. INTRODUCTION

When the concrete cover is not sufficient, longitudinal cracks, named as splitting cracks, are formed parallel to reinforcing bars. The occurrence of these cracks is a result of the three dimensional bond transfer mechanisms. The deformed bars' lugs induce bearing stresses in the concrete, resulting in conical compressive struts. The conical bond forces between bar and concrete can be resolved into radial and tangential components. Usually, the tangential one is called bond stress, whereas the radial one is called confining stress. The radial stresses can be analogues to hydraulic pressure acting on a thick-walled concrete ring. When the tangential ring stresses exceed the cracking strength, the splitting crack is formed. The bond behavior for concrete having such cracks was studied by Gambarova et al.[5]. He tested many specimens with artificial splitting crack. Changing the splitting crack width and the confining pressure on the bars, an empirical formula was proposed for bond stresses after cover splitting. Abrishami and Mitchell [9] studied the splitting cracks' effect on tension stiffening. Specimens with shallow depth were targeted. Here, the concrete cover was insufficient from both sides. The common members of civil structures are deep and the cover problem is that of one side cover. Therefore, a less effect of splitting crack would exist. Salem and Maekawa [10] derived tension stiffening from local bond stress development by assuming thick covers. The aim of this study is to derive smeared model for reinforced concrete in tension from microscopic behavior, taking into account the possible reduction in bond stresses due to non-sufficient cover accompanying longitudinal splitting cracks.

## 2. SPLITTING BOND STRESS

### 2.1 MEMBERS WITHOUT TRANSVERSE REINFORCEMENT

The principal direction of bond forces between deformed reinforcing bar and surrounding

[1] M. Sc. Graduate Student, The University of Tokyo, Member of JCI
[2] Dipl.-Ing. Graduate Student, The University of Tokyo, Member of JCI
[3] Dr. Eng. Professor, The University of Tokyo, Member of JCI

concrete makes an angle with the bar axis. The bond forces can be resolved into radial and tangential components. Usually, the tangential one is called bond stress, whereas the radial one is called confining stress or pressure. The angle of inclination denoted by $\alpha$ ranges from 45 to 80 degrees as reported by Goto [2]. The radial stresses due to bond action act like hydraulic pressure acting on a thick-walled concrete ring. An elastic solution for the stresses in a thick-walled cylinder subjected to internal pressure is given by Timoshenko[1], and Avalle et al.[8] as,

$$\sigma_r = p\, R^2_{cr}\left(\frac{1-\frac{R^2_{max}}{r^2}}{R^2_{max}-R^2_{cr}}\right),\quad \sigma_t = p\, R^2_{cr}\left(\frac{1+\frac{R^2_{max}}{r^2}}{R^2_{max}-R^2_{cr}}\right) \tag{1}$$

where, $\sigma_r$, $\sigma_t$: radial and tangential stresses at radial distance r from the centre of the bar, p: radial pressure, $R_{cr}$: radius of cracked concrete zone, $R_{max}$: cover of concrete + $\Phi/2$ and $\Phi$: bar diameter.

These equations are valid for the non-cracked concrete. However, in cracked concrete, the tension fracturing develops as illustrated in Fig. 1.

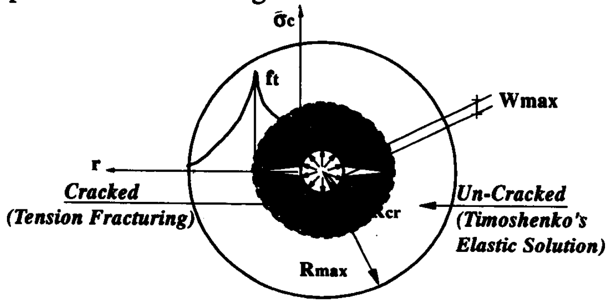


**Fig.** 1 Tangential Stress in Cracked and Non-Cracked Concrete

According to Avalle et al.[8], the bond pressure which causes a splitting crack of radius $R_{cr}$ can be computed by equilibrating the bond pressure p with the tangential stresses in both the cracked and non-cracked concrete as,

$$p = \frac{2f_t}{\Phi}\left(R_{cr}\left(\frac{R_{max}^2 - R_{cr}^2}{R_{max}^2 + R_{cr}^2}\right) + \int_{\Phi/2}^{R_{cr}}\left(\frac{\sigma_c(w(r))}{f_t}\right)dr\right) \tag{2}$$

where, w(r), is the splitting crack width at radius r and, $\sigma_c(w(r))$, is the residual tensile stresses corresponding to crack width equal to w(r). The tension softening model adopted here is given by Uchida et al.[7] as,

$$\sigma_c(w(r)) = f_t\left(1+0.5\left(\frac{f_t}{G_f}\right)w(r)\right)^{-3} \tag{3}$$

where $G_f$ is the fracture energy ranging from 0.1 to 0.15 kgf/cm for plain concrete.

In Equation (2), Avalle assumed two propagating splitting cracks. This assumption agrees with the experimental observation of Morita and Kaku [3] who reported that two or three splitting cracks propagate to surface of a concrete cylinders in pull-out tests. Moreover, in structural members, this is usually the case where splitting cracks propagate towards the side of less cover. Avalle also assumed tangential strain compatibility by equating the circumferential elongation at r equal to $\Phi/2$ and r equal to $R_{cr}$ with the concrete elasticity denoted by $E_c$ as,

$$2\pi R_{cr}\frac{f_t}{E_c} = 2w_{max} + \left(2\pi\frac{\Phi}{2} - 2w_{max}\right)\frac{\sigma_c(w_{max})}{E_c} \tag{4}$$

Using Equation (3), the splitting crack width at the reinforcing bar's face $w_{max}$ is computed.

However, the crack width distribution has to be assumed in order to integrate the second part in the right hand side of Equation (2). The authors assumes the splitting crack width distribution to be linear, ranging from $w_{max}$ at r equal to Φ/2 to zero at r equal to $R_{cr}$ as follows.

$$w(r) = w_{max}\left(1 - \frac{r - \frac{\Phi}{2}}{R_{max} - \frac{\Phi}{2}}\right) \tag{5}$$

The previous equations assume that concrete is an elastic-damaging material in tension. But in reality, the rapid relaxation of tensile stress at the higher level is observed in concrete as a time dependency. Therefore, concrete plasticity is simply introduced as a yielding plateau equal to twice of cracking strain, as proposed by Okamura and Maekawa [6]. Fig. 2 illustrates the idealized concrete plasticity in computing confining pressure.

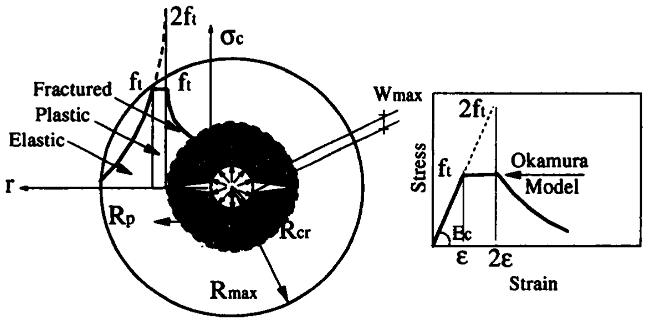


**Fig.** 2 Elasto-Plastic and Fracturing Concrete Model in Tension

The solution is derived by considering an exact elastic solution and determining the position of the point with a tangential stress equal to $f_t$ relative to the position of a point with tangential stress equal to 2 $f_t$. At the location $r = R_{cr}$, tangential stress equals to $2f_t$ and, for $r = R_p$, tangential stress equals to $f_t$, then substituting in Equation (1), we have,

$$R_p = R_{cr}\, R_{max}\sqrt{\frac{2}{R_{max}^2 - R_{cr}^2}} \tag{6}$$

Thus, the radial pressure p is computed as,

$$p = \frac{2f_t}{\Phi}\left(R_p\left(\frac{R_{max}^2 - R_p^2}{R_{max}^2 + R_p^2}\right) + \int_{\Phi/2}^{R_{cr}}\left(\frac{\sigma_c(w(r))}{f_t}\right)dr + (R_p - R_{cr})\right) \tag{7}$$

The ultimate splitting pressure, which is the one when the splitting crack reaches the concrete surface, is the same in both cases as shown in Fig. 3.

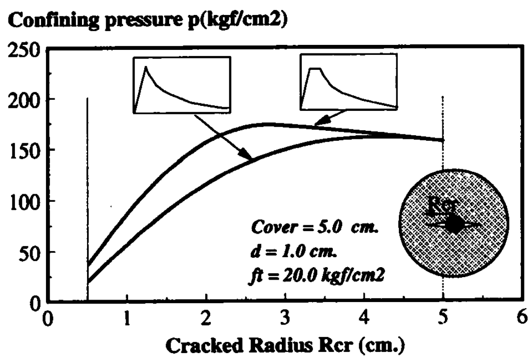


**Fig.** 3 Effect of Concrete Plasticity Prior to Cracking on Splitting Crack Radius

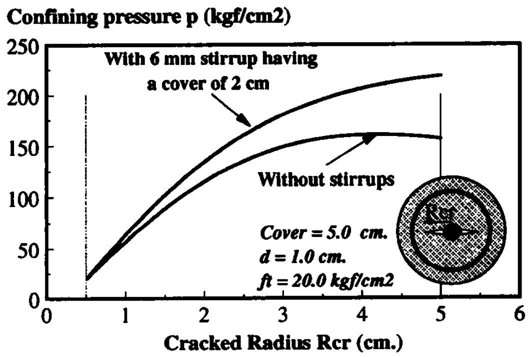


**Fig.** 4 Effect of Stirrups Confining on Splitting Crack Radius

## 2.2 MEMBERS WITH TRANSVERSE REINFORCEMENT

If transverse reinforcement is used, the resistance to splitting cracks increases and the confining pressure on bars is increased. To consider the effect of stirrups, the same analysis, adopted in previous section, is used, and the confining stress produced by stirrups is added. The splitting crack width at the location of stirrup is computed. This width is equal to the slip of stirrups. Knowing the slip of stirrups, the stress in the stirrup can be computed as showed by Okamura and Maekawa [6]. Hence, the stirrups' confining can be estimated as shown in Fig. 4.

## 3. SIZE EFFECT SIMULATION

The present model can successfully simulate the size effect of splitting pressure. Since the splitting crack width is proportional to the size of specimen, tension softening and hence splitting pressure of large-scale specimens is reduced. Fig.5 shows the computed size effect on splitting for geometrically similar specimens.

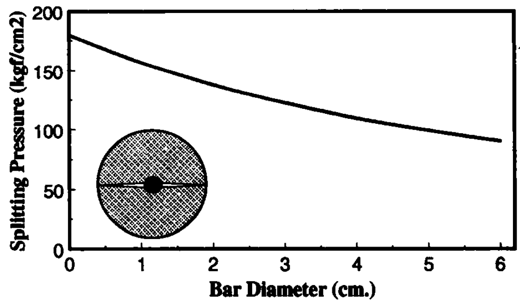


Fig. 5 Size Effect on Splitting Pressure

## 4. BOND BEHAVIOR AFTER SPLITTING CRACKS OCCURENCE

After Splitting cracks occur, the bond stress becomes sensitive to confinement of reinforcing bar. This confining action could be provided by the residual stresses transmitted between the faces of the splitted concrete and by transverse reinforcement distributed along the main bar as illustrated in Fig. 6.

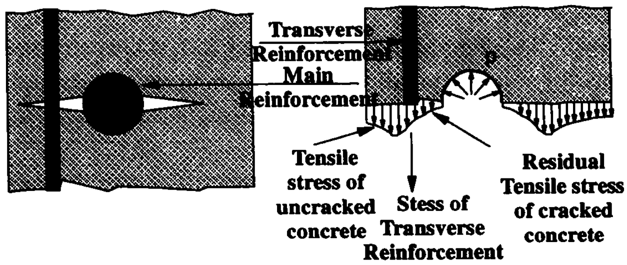


Fig. 6 Confining Pressure Acting on Reinforcement

Gambarova [5] developed an empirical model for bond stress after formation of splitting cracks. The model represents the bond stress as a function of splitting crack width and bar confining as,

$$\tau = f_c' \left(0.042 - 0.288 (w_{max} / \Phi)\right) + \left( \frac{0.258}{\left((w_{max} / \Phi) + 0.11\right)} - 1.018 \right) p \qquad (8)$$

However, the bond-slip-strain model of Shima et al. [4], which is used in the analysis, does not take into account the effect of splitting cracks. Therefore, the model of Shima is modified by changing the slip function as following,

$$\tau(\varepsilon, s) = \tau_0(s) / (1 + 10^5 \varepsilon) \qquad (9)$$

$$\tau_0(s) = \tau_l \qquad (10)$$

where , $s_l$ : Slip at splitting and $\tau_l$ : $\tau_0(s)$ at splitting = $\tau_0(s_l)$.

However, when the bond stress computed from Gambarova's model exceeds the original Shima's model, in case of very small crack width, the original model of Shima is used.

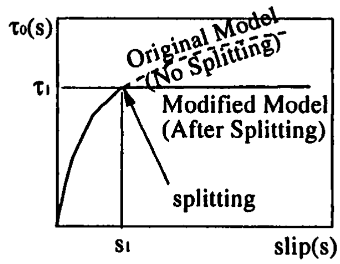


Fig. 7 Extension of Shima Model

## 5. ANALYSIS

Based on microscopic bond behavior, Salem and Maekawa [10] computed the macroscopic behavior of reinforced concrete in tension as illustrated in Fig. 8. In the analysis, local stresses of both concrete and reinforcement are evaluated. Hence, the average strains and stresses are computed.

However, when the concrete cover is not sufficient, splitting of concrete cover may occur and the possible reduction of bond stresses has to be checked. Here, both Gambarova's model and Shima's modified model are used with coupling.

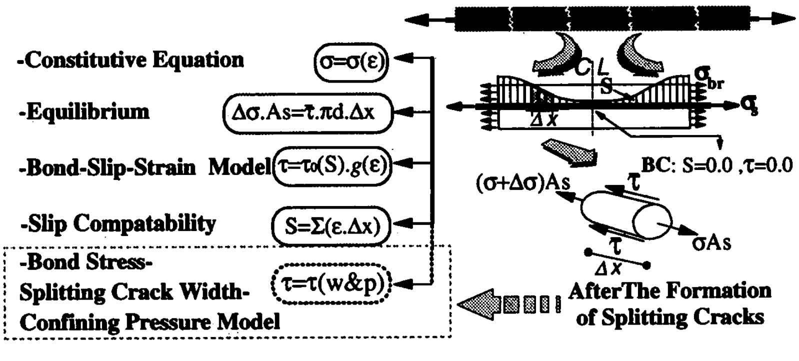


**Fig.**8 Scheme of Solving Bond Governing Equations with Finite Discretization

## 6. EXPERIMANTAL VERIFICATION

As a verification of analysis, two specimens of two meters length were tested. The specimens' details are shown in Fig. 9. The specimens' cross sections, reinforcement and concrete cover are identical. But, one of them has no stirrups, while the other is transversely reinforced with 6 mm stirrups. The ratio of cover to bar diameter in both specimens is 1.0, which would give no tension stiffening and no transverse cracks according to Abrishami and Mitchell [9]. The authors deemed that Abrishami's model might be valid primarily for his experiments where the tested specimens are of shallow depth and the cover is insufficient from both sides. The tested specimens in this study represent the more common case in civil structures. The behavior is expected to be deviant from Abrishami's model since different confining and different bond properties are expected.

Fig. 11 and 12 show the analytical and experimental results. The analysis predicted splitting load of 4.9 ton in specimen (1) and no splitting in specimen (2). The observed splitting load of specimen (1) was 4.5 ton with a deviation of 8%, while no splitting cracks were observed in specimen (2) reinforced with transverse reinforcement as shown in Fig. 10. Also, the predicted crack spacing was close to the experiment with deviation of 12% and 19%, respectively. In analysis of the two specimens, the bond stresses were not affected by splitting cracks. This is due to the large confining of bars even after splitting cracks' occurrence. Fig.13 illustrates the confining pressure on bars of specimen (1). It can be seen that, the confinement of the inner side of concrete is the predominant one. In Abrishami's experiment, this confining action does not exist, leading to the great reduction of bond stresses.

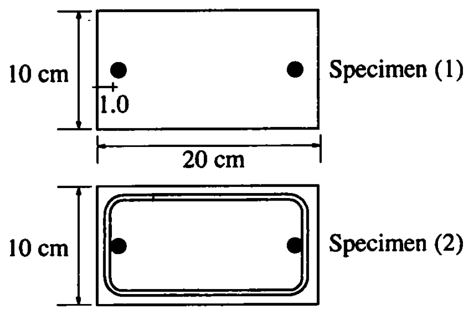


**Fig.** 9 Test Specimens

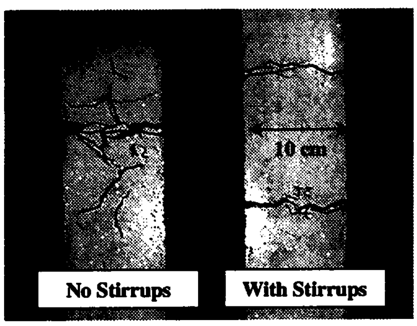


**Fig.** 10 Observed Crack Pattern

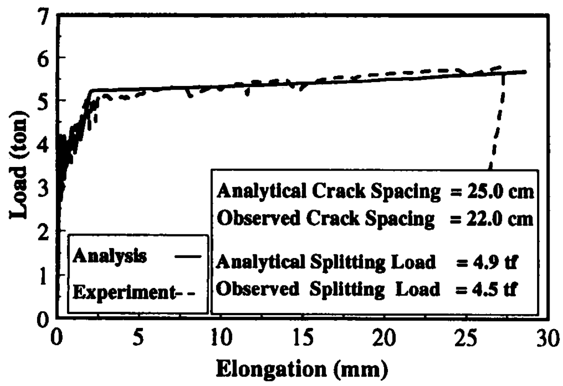


Fig. 11 Results of Specimen (1)

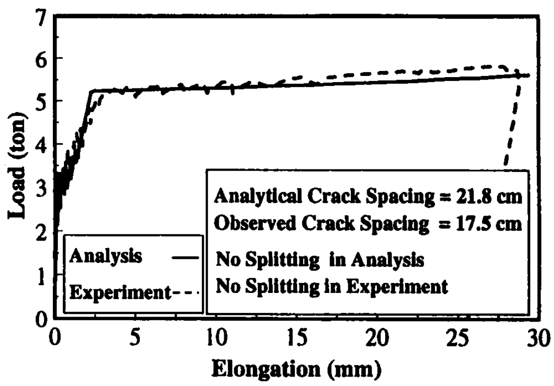


Fig. 12 Results of Specimen (2)

## 7. CONCLUSIONS

1. Based upon the tension softening of plain concrete, the radial bond stresses, and hence the splitting load of tension members with non-sufficient cover can be predicted.
2. The effect of splitting cracks on the bond properties, and tension stiffening is huge for structural members with shallow thickness, like thin shells, where the concrete cover is not sufficient from all sides. However, this effect is negligible for deep structural members, like beams, even if the concrete cover is not sufficient. This is due to the effect of confinement action of the deep side of concrete.

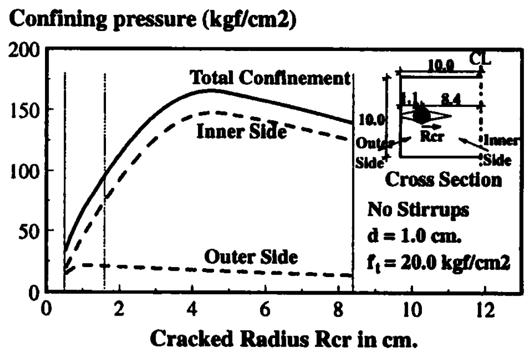


Fig. 13 Confining of Bars of Specimen (1)

## REFERENCES


1. Timoshenko, S.," Strength of Materials, Part II: Advanced Theory and Problems", Princeton, N. J., D. Van Nostrand Company Inc., 1956, pp. 205-210.
2. Goto, Y., " Cracks Formed in Concrete Around Deformed Tension Bars" ACI Journal, Proceedings Vol. 68, No. 4, 1971, pp 244-251.
3. Morita, S. and Kaku, T., " Splitting Bond Failure of Large Deformed Reinforcing Bars" ACI Journal, Proceedings, Vol. 76, 1979, pp 93-110.
4. Shima, H., Chou, L. and Okamura, H., " Micro and Macro Models for Bond in Reinforced Concrete" Journal of The Faculty of Engineering, The University of Tokyo (B), Vol.XXXIX, No 2, 1987.
5. Gambarova, P., Rosati, G. and Zasso, B., " Steel-to-Concrete Bond after Concrete Splitting: Constitutive Laws and Interface Deterioration" Materials and Structures, Vol. 22 ,1989, pp 347-356
6. Okamura, H. and Maekawa, K., "Nonlinear Analysis and Constitutive Models of Reinforced Concrete", Gihodo, Tokyo, 1991.
7. Uchida, Y., Rokugo, K. and Koyanagi, W. " Determination of Tension Softening Diagrams of Concrete by Means of Bending Tests" Proc. of JSCE, Vol. 14, No. 426, 1991.
8. Avalle, M., Iori, I. and Vallini, P., "Analisi Numerica Del Comportamento Di Elementi In Conglomerato Armato Semplicemente Tesi", Studi E Ricerche, Vol. 14,1993, pp 29-54. ( *In Italian* )
9. Abrishami, H. and Mitchell, D., " Influence of Splitting Cracks on Tension Stiffening" ACI Structural Journal, Vol.93, No.6 , 1996, pp 703-710
10. M. Salem, H. and Maekawa, K., " Tension Stiffness for Cracked Reinforced Concrete Derived from Micro-bond Characteristics" Proc. of JCI, Vol.19, No.2 , 1997, pp 549-554