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Review paper

LOCAL APPROACH OF FRACTURE BASED ON CONTINUUM DAMAGE MECHANICS AND THE RELATED PROBLEMS

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Abstract: Recent development of the local approach to fracture based continuum damage mechanics and finite element method together with the related numerical problems is reviewed. After brief description of the concept, procedure and applicability of the local approach, the essential features and the causes of the mesh-dependence of numerical results are discussed. As regularization methods to avoid or to improve the mesh-dependence in time-independent materials, schemes based on localization limiters and nonlocal damage theory are discussed in some detail. The mesh-dependence problem in time-dependent damage, i.e., creep damage, are also discussed.

Key words: *Continuum damage mechanics, Local approach, Fracture, Mesh dependence, Strain softening, Bifurcation, Nonlocal damage, Creep damage*

1 INTRODUCTION

Recent development of continuum damage mechanics (CDM) has provided an important framework for the failure analyses of components in engineering structures [1-9]. By use of the proper constitutive and evolution equations taking account of material damage, CDM is capable of describing the whole process of failures ranging from damage development, crack initiation, crack (or failure zone) growth to final fracture. Namely, if we characterize a crack by an aggregate of material elements where the damage variable attains to a critical value, we can analyze the process of initiation and growth of cracks, and this scheme is usually called a *Local Approach to Fracture* [2, 7, 10-12]. Because of its potential capability of engineering applications, local approach based on CDM combined with finite element method (FEM) has been extensively investigated in various fields including creep, elastic-plastic and fatigue fracture of traditional metallic materials, damage problems of composite materials, and fracture problems of various brittle materials (concrete, ice, rock, bone, cement mortar, ceramics, etc.).

While the FEM-based local approach has proved its significant applicability, it has been often found to be subject to *mesh-dependence* of its numerical results [13-20], and leads to non-objective or no convergent results with respect to the mesh refinement. Since this mesh dependence is a crucial problem to the practical applications of CDM, it has received considerable attentions in recent papers [19, 21-31].

So far most investigations on mesh dependence are related to strain softening of materials. In many time-independent damage models, especially those applied to brittle materials like concrete and rock, the strain softening inevitably leads to a loss of uniqueness or stability of solution in the sense of continuum mechanics. Thus, mesh-dependence has been often discussed in relation to the material bifurcation or instability [32-41], and several regularization methods (such as nonlocal formulation, the use of higher order gradient, etc.) were proposed to overcome the mesh-dependence. The mesh-dependence problems occur also in the local approach in materials without strain-softening and material instability [20, 28, 42, 43]. Though these problems have drawn less attention, they also are very important in the applications of the local approach, especially in time-dependent damage model (like creep and fatigue).

After a brief review of the notion and procedure of local approach based on CDM and FEM, the state of recent application of the approach will be addressed in Chapter 2. Characteristic feature of mesh-dependence, their relation to strain-softening and bifurcation together with several schemes of the regularization will be discussed in Chapter 3. In Chapter 4 the nonlocal model as one of the most promising method to regularize the mesh-dependence will be presented. Finally, mesh dependent problems in time dependent damage models, as well as its causes and the relevant regularization methods will be discussed in Chapter 5.

2 LOCAL APPROACH AND ITS RECENT APPLICATIONS

2.1 Continuum Damage Mechanics and Local Approach

According to the notion of *Continuum Damage Me-*

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chanics (CDM), we take a body B as shown in Fig. 1 with distributed microscopic cavities, and suppose a *Representative Volume Element* (RVE) in the body B around a material point P at $\boldsymbol{x}$, sufficiently smaller than B. Then, if the damage state of the element V can be described by a properly defined *Damage Variable* $\boldsymbol{D}(\boldsymbol{x})$ ($0 \leq |\boldsymbol{D}| \leq D_{cr}$), the process of damage and fracture caused by the development of distributed microscopic cavities can be analyzed in the framework of continuum mechanics by the following procedure [7, 10, 12, 17]:

(a) representation of the mechanical effects of distributed microscopic defects by a proper damage variable $\boldsymbol{D}$;
(b) formulation of the evolution equation of the damage variable;
(c) formulation of the constitutive equations to describe the mechanical behavior of the damaged material;
(d) solution to the initial-boundary value problem governed by the above coupled evolution and constitutive equations.

In the usual procedure of CDM described above, the damage state at a point $\boldsymbol{x}$ in the material is described by a damage field $\boldsymbol{D}(\boldsymbol{x})$. Thus, if a crack is characterized by an aggregate of material elements where the damage has attained to its critical state $|\boldsymbol{D}| = D_{cr}$ as shown in Fig. 2, the process of damage development and crack growth can be analyzed directly by calculating the states of stress, strain and damage. This scheme is usually called *Local Approach of Fracture* [2, 7, 10-12]. In particular, the local approach of fracture based on CDM and *Finite Element Method* (FEM) was proposed for the first time to the analysis of creep fracture by Hayhurst, Dimmer and Chernuka [11], and thereafter has been applied to various problems of ductile fracture, brittle fracture, low cycle fatigue, etc.

In general, methods of fracture analysis pursuing the local process of crack extension are also called local approach. Namely, a method of crack growth analysis without recourse to damage mechanics, in which one may assume that a crack grows by a critical length when a certain physical quantity attains to its critical value in an element apart from the crack tip by the critical length [2, 44, 45], or a method of deriving an analytical solution of the damage field by use of some simplified damage law [46, 47] may be classified into the local approach. However, the following discussions will be exclusively concerned with the local approach of fracture based on the combined method of CDM and FEM.

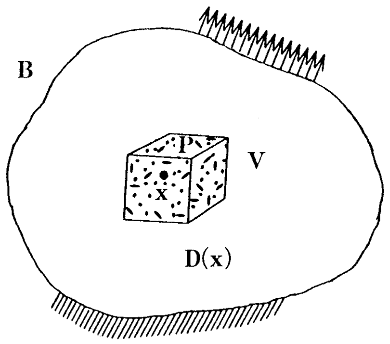

Fig. 1. Reference volume element and damage variable.

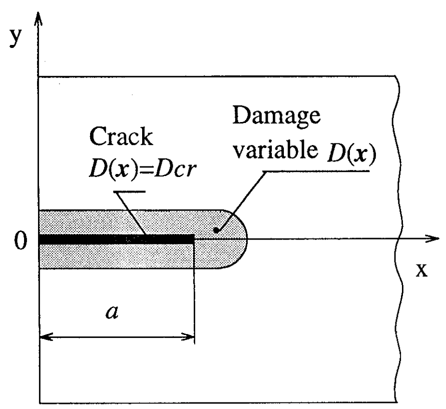

Fig. 2. Damage field and local approach to fracture.

2.2 Recent Applications of Local Approach

After Hayhurst et al. [48] analyzed creep crack growth in aluminum and copper plates by a local approach based on CDM and FEM, a great number of similar analyses were performed for creep fracture [11, 17, 49-56] and ductile fracture [57-65], low cycle fatigue and creep-fatigue interaction in metals [7, 66-72], as well as brittle fracture in various materials including metals, composites [73-80], concrete [13, 22, 81-85], ice [86-89], rock [90-94] and other materials [95-97]. Among others, some of recent applications will be mentioned as follows:

- creep crack growth under neutron irradiation [98, 99];
- failure history of a pressure vessel weldment under long-term creep condition [53];
- prediction of crack initiation of a center-cracked alu-

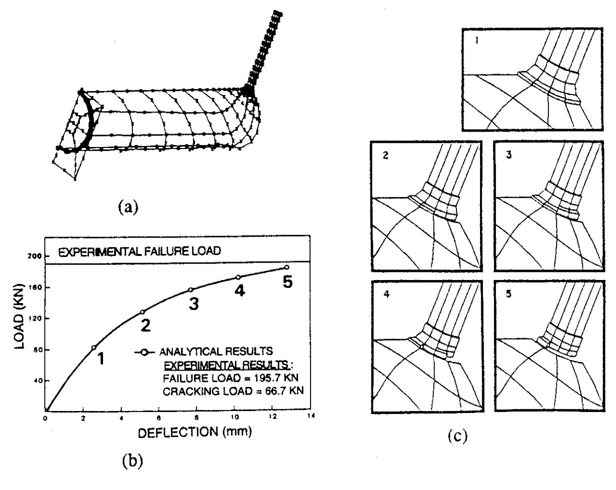

Fig. 3. CDM simulation of the failure of welded tubular T-joint in a pressure vessel [100, 101].

minum plate by several anisotropic elastic-plastic damage models [62];
- simulation and prediction of the fracture process of a welded tubular T-joint [100, 101] (see Fig. 3);
- prediction of failure load of an industrial pressure vessel [63];
- fatigue damage under thermal cycle loading for a 68 I/O ceramic leader chip carrier [102];
- damage initiation and propagation in a slag tap component under 60000 thermal cycles [69];
- damage of a thin-walled pressure vessel made of metal matrix composite [78];
- progressive damage calculation of a notched composite specimen under tension-tension fatigue [80];
- examination of the anomalous behavior of a ceramic projectile in hyper-velocity (>1 km/s) impact [96];
- mechanical behavior of cancellous bone in canine proximale femur [97];
- crash analysis of aluminum bumper [103];
- prediction and assessment of nuclear safety tests conducted for large structures [64, 65];
- seismic analysis of concrete gravity dams [103, 104] (see Fig. 4).

3 MESH-DEPENDENCE IN TIME-INDEPENDENT MATERIALS

In the applications of the CDM-based local approach, it has been observed that the numerical results of the local approach are often very sensitive to the finite element mesh, and this feature gives rise to a crucial problem for engineering application of the local approach. As the causes of the mesh-dependence of the local approach based on CDM and FEM, we can count the following factors in general [28, 39]:

(a) stress singularity;
(b) bifurcation and strain localization due to material instability;
(c) localization of damage field;
(d) dependence of crack zone on the mesh size;
(e) errors in numerical calculation.

In the case of time-independent materials, in particular, the mesh-dependence is observed most frequently in relation to the above cause (b), and most of recent papers are concerned with the effects of the strain-softening induced by damage [14-16, 21, 22, 31, 36].

3.1 Mesh-Dependent Phenomena

Figure 5 shows a typical damage-induced strain-softening behavior of quasi-brittle materials (concrete, rock, etc.) under uniaxial tension [105]. In such strain-softening material, salient mesh-dependence is usually observed in damage analysis by the conventional local approach. An example is shown in Fig. 6 [29], where a plate is subject to displacement on the upper boundary,

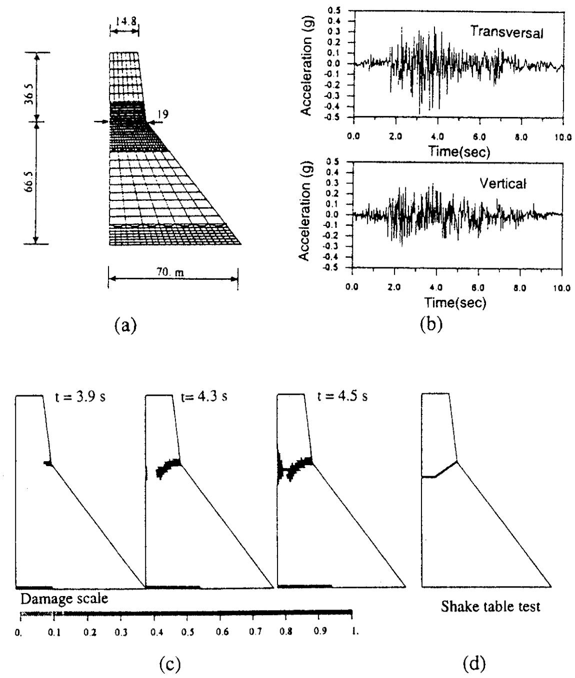

Fig. 4. Local approach to failure of a dam subjected to an earthquake of magnitude 6.5 [104]: (a) geometry and FEM mesh; (b) ground accelerations due to the earthquake; (c) damage evolution in the dam; (d) results of shake table test.

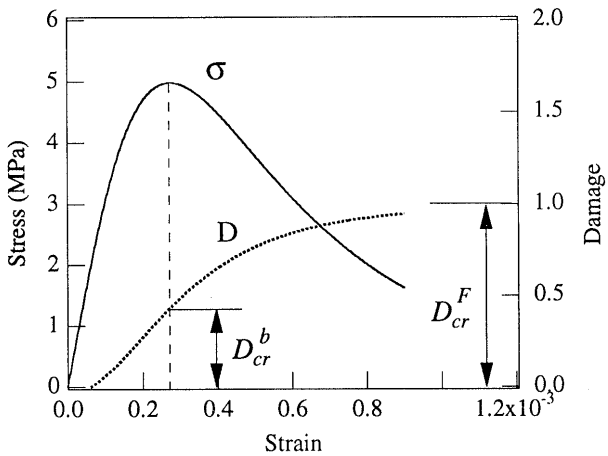

Fig. 5. A typical strain softening constitutive relation for quasi-brittle materials [105].

and five FEM meshes with different sizes shown in the figure are employed. The resulting traction on the upper boundary is shown in Fig. 6 as a function of the displacement in *y*-direction. Salient mesh sensitivity is observed.

The most important features observed in the mesh-dependent behavior may be the appearance of localized intense deformation bands which have a width approximately equal to the mesh size. Namely, the energy dissipation before fracture will occur only in the band, and

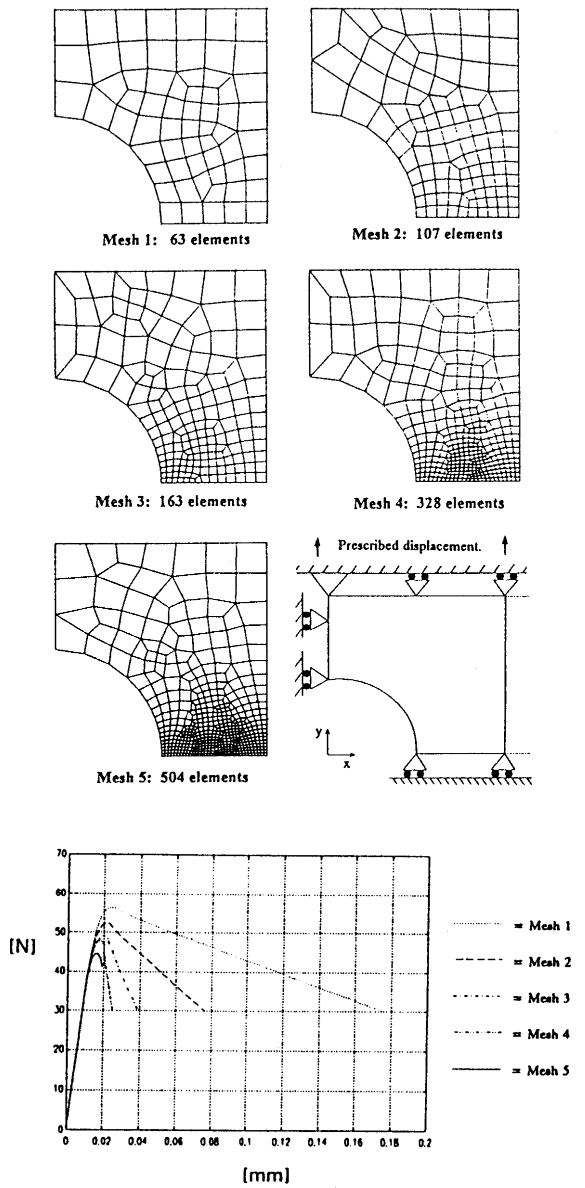

Fig. 6. Mesh-dependence of damage analysis of strain softening material [29].

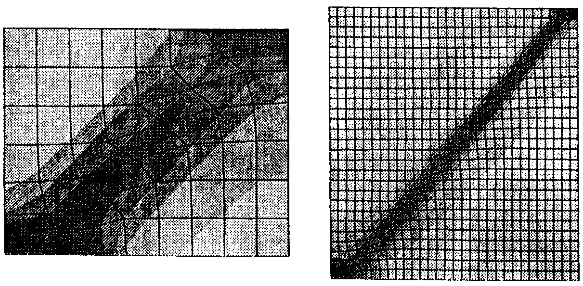

Fig. 7. Mesh-dependent strain localization bands [30].

thus is governed by the mesh size. Figure 7 shows such an example where damage analysis is conducted for a material with a doubly periodic array of soft spots by use of a porous plastic damage model [30]. The mesh-dependent localization deformation bands are clearly observed.

3.2 Bifurcation and Mesh-Dependence

The cause of the above mentioned mesh-dependence has been discussed in many papers [13, 34-36, 39, 40]. The material softening shown in Fig. 5 can be described by the following stress rate-strain rate relation coupled with damage:

$$\dot{\sigma}_{ij} = E_{ijkl}(D)\dot{\varepsilon}_{kl}, \tag{1}$$

where D is the isotropic damage variable, and $E_{ijkl}(D)$ is the damage-dependent elastic stiffness tensor. Because of the material degradation, material stiffness $\|E_{ijkl}\|$ always decreases as damage D increases.

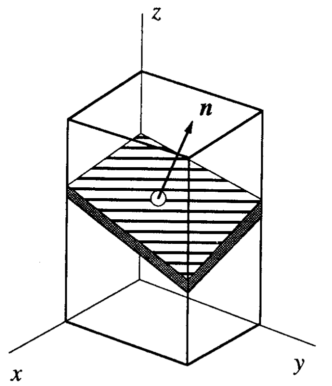

Fig. 8. Material element with a discontinuous deformation band.

In the general framework of continuum mechanics, the strain softening behavior may lead to bifurcation or material instability, which indicates the emergence of a discontinuous deformation band as shown in Fig. 8. The general bifurcation condition [34-36, 39] for material of Eq. (1) requires the decrease of the stiffness $\|E_{ijkl}\|$ to some critical value so that

$$\det(n_j E_{ijkl} n_l) = 0, \tag{2}$$

where n_i is the normal vector of the discontinuity band shown in Fig. 8. In the case of an axial tension or compression, the bifurcation condition becomes simply that the tangent modulus $E_t = 0$, which corresponds to the peak point of the stress-strain curve in Fig. 5. For time-independent damage models used for modeling of strain-softening materials, condition (2) can be satisfied at a rather small critical value of damage, denoted as D_{cr}^b. Usually D_{cr}^b is much smaller than the critical value of damage at failure, denoted by D_{cr}^F, i. e.,

$$D_{cr}^b << D_{cr}^F, \tag{3}$$

This implies that, for these materials, there exists a considerable strain softening range as shown in Fig. 5.

It should be mentioned that not all the damage models have such obvious strain softening features. Especially, it is found that, for time-dependent damage models, the bifurcation condition can not be satisfied except at failure where $D = D_{cr}^F$ [20]. Since the stress will be reduced to approximately zero in the failure zone of material, the bifurcation, if occurs, will have little effects on the results of calculation [20]. The mesh-dependence and the relevant improvements in the local approach of time-dependent materials will be discussed in Chapter 5.

It is well known that, after the condition (2) is satisfied, the boundary value problem will lose ellipticity and well-posedness. When bifurcation occurs, the numerical analysis exhibits emergence of an intense deformation band. Since the damage zone tends to localize into a band as narrow as possible, the eventual band width will be governed by the mesh size in the numerical analysis [13, 36, 106]; the mesh-governed band width or band orientation will bring about obvious mesh-dependence in numerical analyses as shown in Fig. 6.

3.3 Limitation of Localization

In order to overcome the difficulties of mesh-dependence in local approach in strain-softening conditions, different regularization methods have been proposed for numerical calculations in both deformation and damage analyses. Some excellent reviews on these methods are available in literature [31, 36, 39, 106, 107]. Here, these methods will be discussed briefly with special emphasis on their applicability to the local approach based on CDM and FEM.

1) Mesh Size Limitation [106]

Obviously, the simplest scheme to limit the localization is to impose a minimum admissible mesh size. This is called also as *crack band model* when applied for concrete materials [108]. Because of its easiness to apply, the method has been often employed in the practical engineering calculations; some recent applications include the simulation of ductile crack growth in several large structural tests by use of local approach [63, 64, 109]. Besides its simplicity as a localization limiter, mesh size limitation method can describe also the mechanical effects of structure size in concrete materials [108].

The main disadvantage of this method is that, the reasonable precision of the solutions in the calculation can not be obtained because mesh refinement is prohibited. Another problem of difficulty is how to determine the minimum mesh size which conforms to the actual material properties.

2) Nonlocal Damage [15, 22, 106]

A more sophisticated localization limiter is provided by the concept of nonlocal continuum [110, 111]. Directly based on this concept, a nonlocal damage formulation in which all the state variables are nonlocal was proposed by Bazant et al. [112]. However, it was found that the formulation led to a nonstandard boundary-value problem with additional boundary and interface conditions. This prevents it from the practical application because the conventional FEM method can not be used.

Therefore, a new formulation was proposed [15, 21, 22], in which only damage variable or its thermodynamic conjugate force was assumed to be nonlocal. This simplification in the formulation has greatly reduced the complexity of calculation, while the essential features of the nonlocal concept are still preserved.

This later nonlocal formulation has been demonstrated to be very effective to mitigate the mesh-dependence in various damage calculations [23, 25, 29, 30, 41, 65, 84, 105, 113, 114]. In recent years, the nonlocal formulation has become one of the most important approaches to damage and failure analyses based on CDM. Some detailed discussions about this method will be given in Chapter 4.

3) Gradient-Dependent Material Models [115, 116]

Another method of localization limiter is to include the gradient of field variables in the constitutive equation. In gradient-dependent plastic theory, stress becomes a function not only of the plastic strain but also of the first or second spatial derivatives of the plastic strain.

The gradient-dependent formulation is in fact an alternative form of the nonlocal formulation, since it has been found that the formulation can be derived from nonlocal one [116-118]. Its main advantage over the nonlocal model is that it leads to a simpler equation for consistency condition in plastic problem, rather than an implicit integration equation in nonlocal formulation. This permits a formulation by a variational principal [39, 116], which enables proper formulation of the additional, non-standard boundary conditions. A dis-advantage of the formulation is the necessity of an additional variable to be solved at a global level [39].

4) Cosserat Continua [119, 120]

The use of Cosserat continua incorporating the couple-stress and micro-curvature provides an internal length scale, and thus may give naturally a characteristic length for the limitation of the localization [120]. It was found that the rate boundary value problem remains elliptic after the onset of shear banding due to the existence of a length scale in Cosserat continua [120].

While this method has several advantages from a numerical point of view, its distinct disadvantage is that the localization limiter provided by the model is only effective for pure shear deformation [39].

5) Mesh Size Dependent Softening Property [107, 121]

This method imposes a limitation on damage dissipation, rather than on the size of localization band. In order to maintain the identical dissipation for mesh-dependent localization bands (width), a mesh-dependent softening modulus has been employed [107]. This leads to a very simple algorithm which can be easily implemented into a standard finite element code.

The effectiveness of this method has been demonstrated by several authors [103, 104, 121, 122]. Especially, by use of this method, a seismic analysis of concrete gravity dams based on CDM has been successfully conducted [104], see Fig. 4.

The disadvantage of this regularization is obvious. From a physical point of view, it is highly questionable to use a mesh-dependent material property. Moreover, the different meshes will give different and even unreal local deformation, although the overall fracture energy can be correctly predicted.

Other regularization methods include the introduction of *artificial viscoplastic* [16, 107] into the constitutive equation, and the use of a *stochastic description* of the damage threshold [123, 124].

4 NONLOCAL DAMAGE THEORIES

Because of the increasing use of the nonlocal damage formulation in damage and fracture simulation based on CDM [23, 25, 29, 30, 41, 65, 84, 105, 113, 114], the procedure, possibilities, limitations, and the related problems of the nonlocal method will be discussed in more detail in this Chapter.

4.1 Formulation

In the nonlocal damage model, an averaged damage variable is introduced by means of a spatial averaging procedure [15, 22, 41]. For a scalar damage variable, the nonlocal damage variable $\overline{D}$ is defined as

$$\overline{D}(x) = \int_\Omega D(\zeta)\phi(x,\zeta)d\Omega(\zeta) \Big/ \int_\Omega \phi(x,\zeta)d\Omega(\zeta), \qquad (4)$$

or

$$\dot{\overline{D}}(x) = \int_\Omega \dot{D}(\zeta)\phi(x,\zeta)d\Omega(\zeta) \Big/ \int_\Omega \phi(x,\zeta)d\Omega(\zeta), \qquad (5)$$

where D is a local damage variable which may be specified by different local damage theories, and $(\dot{\ })$ represents the derivative with respect to time t, x denotes a characteristic material point, and ζ an arbitrary material point in solid body Ω.

In the nonlocal definition of Eqs. (4)-(5), a weighing function $\phi(x,\zeta)$ is used to determine the influences of surrounding material points on $\overline{D}$. An often employed form of the weighing function is the Gaussian (normal) distribution function [22, 23], i. e.

$$\phi(x,\zeta) = \exp\left\{-\left[d(x,\zeta)/d^*\right]^2\right\}, \qquad (6)$$

where $d(x,\zeta)$ is the distance between x and ζ and d^* is a characteristic distance, a new material constant which specifies the range of averaging of D.

A similar formulation by use of the thermodynamic conjugate force of damage has been also proposed [21, 31]. It was found that these two formulations gave almost the same results [31]. While the second one was suggested to be easier to be implemented into finite element code for brittle damage problem, the nonlocal formulation of Eqs. (4)-(6) for damage variable has been widely accepted. The nonlocal formulation has been proposed also for creep damage [15, 23, 114] and ductile damage [30, 65].

An alternative nonlocal formulation to be mentioned is so called cell model [25, 29, 65]. This model is characterized by use of a fixed cell mesh as the range of nonlocal averaging, and choose of the δ-function as $\phi(x,\zeta)$ in Eq. (6). These simplifications result in some computational advantages, but the resulting nonlocal damage becomes discontinuous over the boundaries of adjacent cells [29].

Although the above mentioned nonlocal damage formulations have been mainly employed to scalar damage models, the nonlocal damage concept can be applied also to the anisotropic damage problems without specific difficulties [31, 125].

4.2 Effects of Nonlocal Damage Formulation

The nonlocal formulations described above have been applied to a number of problems [23, 25, 26, 29, 30, 65, 84, 105, 106, 114, 124, 126, 127], and have been shown very effective to mitigate mesh-dependence in local approach. This can be seen in Fig. 9 where the damage distributions have been obtained by nonlocal damage model for the same boundary value problem shown in Fig. 6. Finite and almost identical width of the damage zone can be observed for two different meshes. This results in the well convergent and almost mesh-independent curve of force vs. displacement as shown in Fig. 10.

Delocalization effect of the nonlocal formulation has been elucidated by some theoretical analyses [31, 40, 41]. Based on the concept of damage loading function or damage surface, the local damage evolution can be given

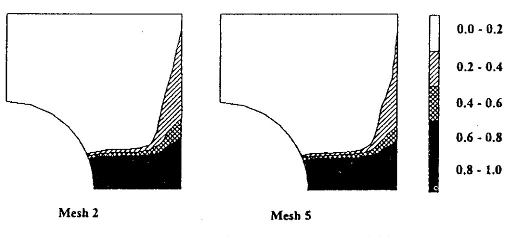

Fig. 9. Damage distribution for two different meshes shown in Fig. 6, obtained by nonlocal damage formulation [29].

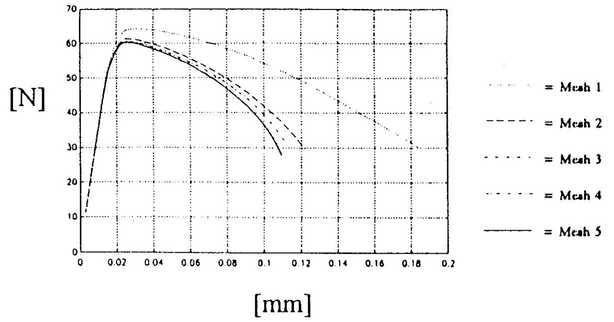

Fig. 10. Improvement of mesh-dependence shown in Fig. 6 by use of nonlocal damage formulation [29].

as follows [31]:

$$D = f(Y) \ , \tag{7}$$

$$Y = \frac{1}{2} E_{ijkl} \varepsilon_{ij} \varepsilon_{kl}, \tag{8}$$

or

$$\dot{D} = \frac{df(Y)}{dY} E_{ijkl} \varepsilon_{ij} \dot{\varepsilon}_{kl}, \tag{9}$$

where $f(Y)$ is a material function. In view of Eq. (4), we have a constitutive equation of nonlocal damage model as follows:

$$\sigma_{ij} = (1 - \overline{D}) E_{ijkl} \varepsilon_{kl}, \tag{10}$$

Differentiation of this equation furnishes

$$\dot{\sigma}_{ij} = (1 - \overline{D}) E_{ijkl} \dot{\varepsilon}_{kl} - E_{ijpq} \varepsilon_{pq} \dot{\overline{D}}, \tag{11}$$

where $\dot{\overline{D}}$ is given by Eqs. (5) and (9) as follows:

$$\dot{\overline{D}} = \frac{\int_{\Omega} \frac{df(Y)}{dY} E_{rskl} \varepsilon_{rs} \dot{\varepsilon}_{kl} \phi(x, \zeta) d\Omega(\zeta)}{\int_{\Omega} \phi(x, \zeta) d\Omega(\zeta)}. \tag{12}$$

Because $\dot{\overline{D}}$ is determined from the integration of strain rate $\dot{\varepsilon}_{kl}$, $\dot{\overline{D}}$ and $\overline{D}$ are always continuous even if $\dot{\varepsilon}_{kl}$ is discontinuous. This condition implies that the strain localization due to the onset of a discontinuity of the velocity fields is suppressed by the nonlocal model of Eq. (11). This may explain why nonlocal model can prevent the vanishing width of damage localization band.

Furthermore, more complicated theoretical analysis can show that strain softening will produce a loss of uniqueness of the solution of a boundary value problem [40]. This may be used to explain why there still exists damage localization in the numerical calculations using nonlocal damage model, although the width of the localization band becomes finite.

For the nonlocal form of the well known Gurson model of ductile damage, similar conclusions concerning the delocalization effects have also been obtained by recent theoretical investigations [41]. The numerical calculations using this nonlocal Gurson model [30] show that the inherent mesh sensitivity of the numerical failure predictions can be removed.

Besides the bifurcation behavior, the nonlocal formulation of Eqs. (4)-(5) has effects to reduce the damage, stress or strain concentrations directly [28]. This may be less important for the strain softening materials where bifurcation behavior is essential for improvement of mesh-dependence. However, for time-dependent problem such as creep damage where no bifurcation occurs, this regularization effect to damage concentration become more important, and will be discussed in Chapter 5.

4.3 Limitations and Further Development

One of the main difficulties in the application of nonlocal model is how to identify the new material constant d^* in Eq. (6). This internal characteristic length is related to the range of the nonlocal effects, and it can not be identified directly from the conventional uniaxial tests like other constants in constitutive equations.

At present time, the determination of d^* usually needs the aid of the inverse analysis concept, e.g., d^* is adjusted in the calculations so as to obtain results that are in good agreement with test ones. To elaborate the process, inverse analysis technique using nonlinear optimization was sometime employed [31]. However, in some cases, the values of d^* required to fit experimental data for every different geometry may differ significantly, and therefore it cannot be regarded as a true material constant [27]. Thus, the choice of the nonlocal modeling as well as the length d^* have been made rather arbitrarily or for reason of computational convenience [30].

Physically, the nonlocal damage formulation should be based on the interaction of micro-defects in materials. Some researches in this direction have been conducted in recent years [27, 113, 128, 129].

Another problem of the nonlocal damage formulation is related to its application to time independent plastic (ductile) damage analysis. In this case, the incremental form of consistency condition will become an implicit integral equation which is difficult to solve [30, 31, 39, 105]. Moreover, the nonlocal formulation will leads to a non-symmetrical tangent matrix that requires much more computational efforts. However, these numerical difficulties can be avoided if a total stress-strain relation rather than a incremental one can be employed in the analysis. The elastic-damage material described by Eqs. (7)-(12) is such an example. The difficulty does not occur also in the nonlocal viscoplastic (creep) damage models

[15, 23, 114] where tangent modulus is an elastic matrix coupled with damage.

5 MESH-DEPENDENCE IN TIME-DEPENDENT MATERIALS

Since bifurcation does not occur due to time-dependent deformation [16, 20], the mesh-dependence of local approach in time-dependent materials should be accounted for by other causes than the strain-softening due to damage [28]; i.e. the causes (a), (c) through (e) mentioned in Chapter 3. Some recent results on mesh-dependence problem in time-dependent materials will be discussed briefly in this Chapter.

5.1 Mesh-Dependent Phenomena and Their Causes

A typical example of time-dependent problem is the creep (viscoplastic) damage analysis by use of local approach based on well established Kachanov-Rabotnov creep damage theory [1, 130]. Though the theoretical analysis shows that the bifurcation behavior is ruled out except at failure [20], creep damage analysis exhibits serious mesh-dependence of numerical results [15, 17, 20, 23, 43, 126]. An example of mesh-dependence in creep crack growth analysis is given in Fig. 11 [23].

In these numerical calculations, it is observed that the failure zone, or crack, is always localized in a single row of Gaussian points (or element) and thus the width of crack is determined by the mesh size employed. However, there is no localized deformation bands as those shown in Fig. 7. We will now review some other causes discussed in recent paper [28], i.e, causes (a) (c) through (e) mentioned above:

1) Initial Stress Singularity at Crack-tip of Sharp Crack

For creep damage models governed by stresses, the essential cause of the mesh-dependence of damage analysis in the specimens containing sharp cracks has been shown to be the stress singularity at the crack tip [20].

In the numerical calculation of an initially sharp crack, as finite element size Δe in front of crack-tip decreases, the stress in this element (or at some Gaussian points in the element) will increases approximately according to asymptotic singularity solution [20], i.e.,

$$\sigma \propto (\Delta e)^{-\alpha}, \quad (13)$$

where α is the order of singularity, equal to 1/2 for elastic material and $1/(n+1)$ for n-power law material, respectively. According to the above equation, the mesh size governs the stress appearing in the local zone in front of the crack-tip, and thus governs the predicted damage development and fracture in this zone.

Figure 12 is an example of the mesh dependent behavior governed by singularity [20]. In this log-log figure, t_{ig} denotes the initiation time of crack growth, and decreases almost linearly with the decrease of mesh size Δe from 1.0 mm to 1/128 mm. Obviously, no finite convergent value of the initiation time can be expected. By use of the singular stress field (13), one obtain $t_{ig} \propto \Delta e^{-1.40}$ [20]. This is in good agreement with the calculation results shown in Fig. 12. It has been elucidated that the mesh-dependent behavior of incipient crack growth is induced by the effects of initial stress singularity, even if the singular stress field has been redistributed by the development of damage [20].

The similar singularity-induced mesh-dependent behavior has been observed also in the crack growth analyses of time-independent elastic-brittle materials [106, 131]. In the case of dynamic ductile crack growth, on the other hand, it is found that, for initially sharp cracks the initiation of crack growth is quite sensitive to the mesh size, while for initially blunt cracks the mesh sensitivity of the initiation time vanishes [19].

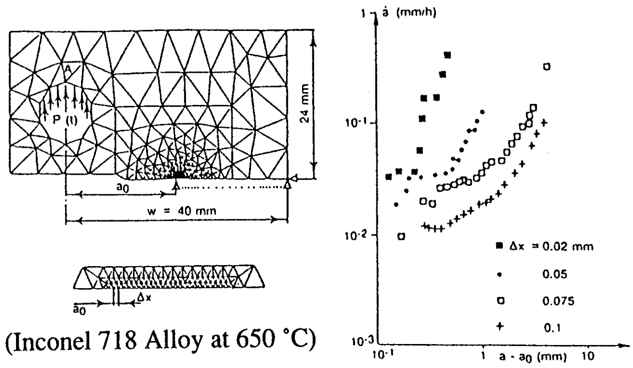

Fig. 11. Mesh-dependence in creep crack growth analysis by local approach [23].

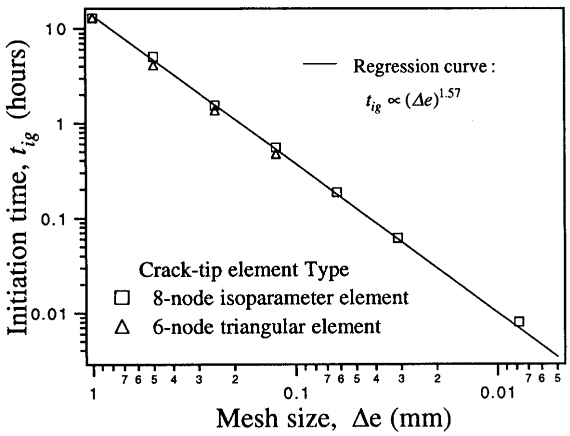

Fig. 12. Mesh-dependence of crack initiation time and stress singularity [20].

2) Damage Localization Due to Stress Sensitivity of Damage Evolution Equation

For the problems without initially sharp crack, time-dependent crack growth analysis shows that mesh-dependent results appear only after the initiation of crack growth [43, 132]. Here, the completely damaged zone (CDZ), i.e., the crack, and its size are found to have very significant influences on the subsequent results, because CDZ prescribes the newly-generated free boundary in the problem. It is observed [23, 57, 133] that the width of CDZ becomes arbitrarily narrow with regards to mesh refinement, and thus mesh-dependence occurs.

If there is no effects of strain softening, nor initial stress singularity, the narrowing of the CDZ width should be attributed to other causes. One of the possible reasons is the damage localization induced by *stress sensitivity of damage evolution equation* [28, 134]. Numerical calculations under uniform stress confirmed that this stress sensitivity will causes the *damage localization* even from numerical errors of order of 10^{-10} [28, 134]. Thus, a small stress gradient in a non-uniform stress field will lead to an extremely high damage concentration (localization) which can be resolved only by use of a mesh size close to zero. This may be the reason why CDZ width always equals to the smallest mesh size.

3) Local Fracture Criterion

Another more important factor that leads to a zero width of CDZ may be the local failure criterion employed by the present local approach [28]. The local failure implies that the failure will always occurs in a single Gaussian point (or a constant strain element) when the damage value at this Gaussian point leads to its critical value. The damage development of the neighborhood Gaussian points in the crack width direction will cease because of the unloading, and thus a CDZ (or crack) consisting of a single row of Gaussian points will be formed. It has been pointed out that a nonlocal fracture criterion may be essential to improve the mesh-dependent problems [28].

5.2 Development of Regularization Methods

For both the strain softening calculations and the time-dependent calculations, the damage zone sizes decrease when meshes are subdivided. However, their mechanisms to induce mesh-dependence are quite different. In the case of strain softening, the energy dissipation in the damage zones (localized bands) has a dominant influences on the calculation results. On the other hand, in the case of creep crack simulations, there is almost no dissipation in CDZ because CDZ is a stress free zone, and it is the stress or strain states influenced by the CDZ size that leads to mesh-dependent results.

The mesh-governed CDZ size implies that the stress and strain concentration in front of a crack will be dependent on mesh size. Thus, the mesh-dependent stress concentrations can be considered as the *immediate reason* for the mesh dependence of crack growth in the time-dependent calculations. In order to avoid the mesh-dependent stress concentration, a *stress limitation method* [43] have been proposed, in which the stress level in the front of CDZ can be limited to a value approximately equal to yield stress. The effectiveness of this method has been confirmed by a series of numerical calculations [43, 135]. It is found that the same regularization effects can be obtained by only limiting the stress used for damage calculation and leaving the elastic-creep constitutive equation intact. The later simplification greatly reduce the computational efforts [133, 135, 136]. This stress limitation method is very simple and very ease to be incorporated into a standard finite element code. However, the mesh-governed CDZ width can not be removed by use of this method. Moreover, it can not give a correct prediction to crack growth if the real stress concentration ahead of CDZ is lower than the material yield stress.

The *nonlocal formulation* of Eqs. (4)-(6) as well as the *simplified cell model* has also been employed to regularize the mesh-dependent problems in creep crack growth analyses [23, 25, 28, 126]. Good improvements of mesh-dependence have been obtained in these investigations, although sometimes different values of the characteristic length $d*$ seems necessary for different specimen geometry [23, 28].

On the other hand, more essential improvement methods to the problem lies upon the regularization of mesh-governed CDZ size. Several methods under investigations includes [28, 134]:

- modification of the damage evolution equations;
- reduction of the critical damage values;
- use of nonlocal fracture criterion;
- introduction of statistic properties of damage.

Because of the recent development in the applications of the CDM based local approach to industrial problems, further investigations in this aspect become more important and necessary.

6 CONCLUDING REMARKS

A review is made concerning the recent application status of the local approach based on continuum damage mechanics and finite element method, especially, its related mesh-dependent problems. Several concluding remarks are given as follows:

1) Industry applicability of the local approach to fracture based on CDM has received increasing attentions in recent years.
2) Mesh-dependence is a crucial problem that prevents more reliable application of CDM-based local approach.

3) There exist some other causes for mesh-dependence besides the damage-induced strain-softening often investigated so far. This is especially true for the time-dependent damage problems.
4) Although nonlocal damage formulation has been often employed as an effective regularization method to the mesh-dependence, more reasonable regularization methods are still necessary.

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