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Review paper

RECENT PROGRESS IN CONSTITUTIVE MODELING FOR RATCHETTING

Nobutada OHNO

Department of Mechanical Engineering, Nagoya University, Chikusa-ku, Nagoya 464-01, Japan

Abstract: Classical constitutive models of cyclic plasticity are very poor in predicting the progressive deformation of ratchetting, though ratchetting is an important factor in the design of structural components. Recent works done in the last decade, however, have enabled us to simulate the strain accumulation due to ratchetting with reasonable accuracy. In the present paper, first, the state of the art in constitutive modeling for ratchetting is described by criticizing the classical models and by reviewing the recent modifications introduced for ratchetting. Then, the application of recent and calssical models to ratcheting problems such as the thermal ratchetting induced by moving temperature distribution is discussed to show the effectiveness of recent models in simulating ratchetting.

Key words: *Ratchetting, Simulation, Constitutive model, Cyclic plasticity, Anisotropic hardening, Back stress, Evolution equation*

1. INTRODUCTION

When materials are subjected to cyclic plastic loading with nonzero mean stress, strain usually accumulates in the direction of mean stress with the increase of the number of cycles. This kind of strain accumulation, which is called *ratchetting*, in general occurs under uniaxial cyclic loading with nonzero mean stress, cyclic shear loading combined with constant normal stress, cyclic thermal loading superposed on steady mechanical load, and so on. Ratchetting is therefore one of the key factors in the design of structural components. However, since ratchetting is the progressive deformation accumulating cycle by cycle, it is not easy to predict the development of ratchetting accurately.

An important example in high temperature structural engineering is the thermal ratchetting induced by moving temperature distribution [1, 2]. When a hollow cylinder is subjected cyclically to axial travelling of temperature distribution, circumferential strain is forced to accumulate in the travel region with the increase of the number of cycles. This strain accumulation is now known as *liquid level induced thermal ratchetting*, because it can take place when the level of a high temperature liquid changes cyclically in the cylinder. Since the thermal ratchetting can occur even under no mechanical load, it is in contrast with the classical problem of Bree [3], in which the superposition of mechanical and thermal loads is prerequisite. The Subcommittee on Inelastic Analysis of High Temperature Materials, JSMS, has done a comparative study in the last three years for the purpose of finding appropriate constitutive models to simulate liquid level induced thermal ratchetting [1, 2].

The accuracy in simulating ratchtting is a matter concerned mainly with the modeling of anisotropic strain hardening. Ratchetting under uniaxial cyclic loading, referred to as *uniaxial ratchetting*, is brought about by the difference between tensile and compressive stress-strain curves in each cycle; the difference is related to anisotropic strain hardening. On the other hand, ratchetting under nonproportional cyclic loading, referred to as *multiaxial ratchetting*, is a result of the plastic flow under nonproportional cyclic loading; the flow is affected significantly by the anisotropy in strain hardening.

The modeling of anisotropic hardening can be traced back to the linear kinematic hardening model of Prager [4]. The model is simple but expresses only the anisotropic hardening proportional to plastic strain. Then, models capable of representing nonlinear anisotropic hardening were proposed around 1960 to 1980 by Besseling [5], Armstrong and Frederick [6], Mróz [7], Valalnis [8, 9], Backhaus [10], Krieg [11], Dafalias and Popov [12], Cernocky and Kremple [13], and so on. Most of the classical models, however, are not suitable for predicting ratchetting, as was revealed for example in the benchmark project done by the Subcommittee on Inelastic Analysis of High Temperature Materials, JSMS [14, 15]. Thus, in a review on cyclic plasticity published in 1990, the present author described the state of the art in constitutive modeling for ratchetting as follows [16]: "Constitutive modeling of cyclic plasticity and cyclic viscoplasticity has developed so markedly in the last two decades. Ratchetting is still one of the most difficult problems in modeling."

The poor capability of classical models for ratchetting has urged recent works. Especially the Armstrong and Frederick model has been brought up in most of the recent works, as will be reviewed in this paper. This is because the concept employed in the model, i.e., the nonlinear evolution of back stress due to strain hardening and dynamic recovery, is simple and physically sound. The recent works thus have enabled us

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to predict or simulate ratchetting with reasonable accuracy.

The present paper describes the current state of the art in constitutive modeling for ratchetting. After discussing in Sec. 2 the capability of classical models to simulate ratchetting, the modifications of the Armstrong and Frederick model by Burlet and Cailletaud [17], Chaboche [18], Ohno and Wang [19], and so on are reviewed and compared qualitatively in Sec. 3. Section 4 is devoted to the application of the Ohno and Wang model to mechanical ratchetting as well as to the thermal ratchetting induced by moving temperature distribution.

2. CAPABILITY OF CLASSICAL MODELS FOR RATCHETTING

To begin with, classical models representing nonlinear anisotropic hardening are reviewed with respect to the predictive capability for ratchetting. Especially the Armstrong and Frederick model is discussed in some detail, since the concept employed in the model has been used in recent works to simulate ratchetting appropriately.

2.1. Armstrong and Frederick Model (AF)

To express the transient nonlinear hardening after yielding, Armstrong and Frederick [6] introduced the concept of strain hardening and dynamic recovery to formulate the evolution equation of back stress $\boldsymbol{\alpha}$. Chaboche et al. [20, 21] then extended the Armstrong and Frederick model by decomposing $\boldsymbol{\alpha}$ into M components $\boldsymbol{\alpha}_i$ ($i = 1, 2, ..., M$). This extended model, which will be indicated as the AF model hereafter, can be written as

$$\boldsymbol{\alpha} = \sum_{i=1}^{M} \boldsymbol{\alpha}_i, \tag{1}$$

$$\dot{\boldsymbol{a}}_i = \frac{2}{3} h_i \dot{\boldsymbol{\varepsilon}}^p - \zeta_i \boldsymbol{a}_i \dot{p}, \tag{2}$$

where $\boldsymbol{a}_i$ is the deviatoric part of $\boldsymbol{\alpha}_i$, $\boldsymbol{\varepsilon}^p$ denotes plastic strain, h_i and ζ_i are material constants, and $\dot{p}$ stands for the accumulating rate of plastic strain defined as

$$\dot{p} = [(2/3)\dot{\boldsymbol{\varepsilon}}^p : \dot{\boldsymbol{\varepsilon}}^p]^{1/2}. \tag{3}$$

The first and second terms in the right hand side in Eq.(2) represent strain hardening and dynamic recovery, respectively.

The AF model mentioned above, which has been used extensively by Chaboche and his coworkers [20-24], has been recognized as a reliable model. However, it was pointed out that the AF model tends to overpredict ratchetting very seriously [14, 15, 25-28]. Figure 1 exemplifies the overprediction by the AF model with respect to uniaxial ratchetting of Modified 9Cr-1Mo steel at 550°C [28]. The experiments shown in the figure were done by Tanaka et al. [29]. As seen from the figure, the AF model predicts very significant ratchetting, which is in marked contrast to a little ratchetting observed in the experiments. The AF model overpredicts further the multiaxial ratchetting of the material subjected to constant tensile stress combined with cyclic torsional straining, as shown in Figs.2(a) and (b). Thus we can say that the AF model is very poor in predicting ratchetting.

When the dynamic recovery term in the AF model is neglected, the model is reduced to the linear kinematic hardening model of Prager [4]. It is well known that the Prager model predicts no uniaxial ratchetting. This prediction seems to be much better than the excessive ratchetting given by the AF model as far as the uniaxial ratchetting shown in Fig.1 is concerned. It is then suggested that the dynamic recovery term introduced in the AF model is too active to simulate ratchetting appropriately.

2.2. Other Classical Models

There are some models which may have the same problem in predicting ratchetting as the AF model. It was shown that the two surface model proposed by Krieg [11], in which the bounding surface is located outside the yield surface, is mathematically equivalent to the AF model if the bounding surface neither expands nor translates [21, 30]. It was also shown that the hereditary integral model called the endochronic theory formulated by Valanis [9] as well as by Backhaus [10] can be reduced to the AF model if the kernel functions in the hereditary integral are taken to be exponential functions [31]. These two models therefore may overpredict both uniaxial and multiaxial ratchetting.

The multisurface model of Mróz [7], which predicts no uniaxial ratchetting, also may have the same problem of overprediction with respect to multiaxial ratchetting as the AF model. It was shown that the translating direction of multisurfaces in the Mróz model is in effect the same as that in the multisurface version

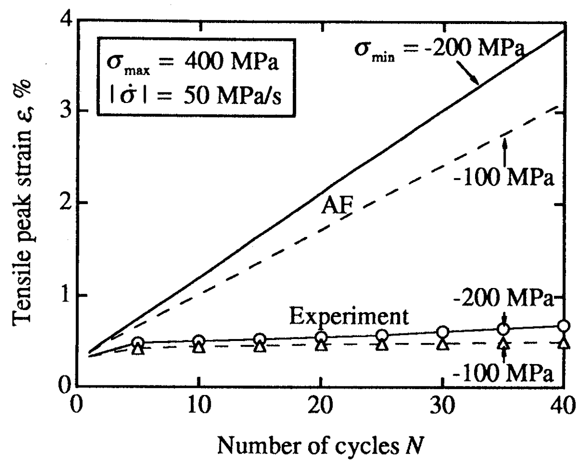

Fig.1. Uniaxial ratchetting of Modified 9Cr-1Mo steel at 550°C [28]; experiments [29] and simulations by the AF model.

of the AF model [32], suggesting that the two models have a similarity with respect to the anisotropic hardening under nonproportional loading. Hence, the Mróz model may overpredict multiaxial ratchetting, which is a consequence of the plastic flow under nonproportional cyclic loading. The translation rule of Mróz [7] was used in the two surface model by Dafalias and Popov [12], so that their model also may overpredict multiaxial ratchetting. It is noticed that the two surface model of Dafalias and Popov, in which an updated rule is adopted, is different from that of Krieg though both models are based on the multisurface model of Mróz.

The multilayer model of Besseling [5] is another well known classical model expressing nonlinear

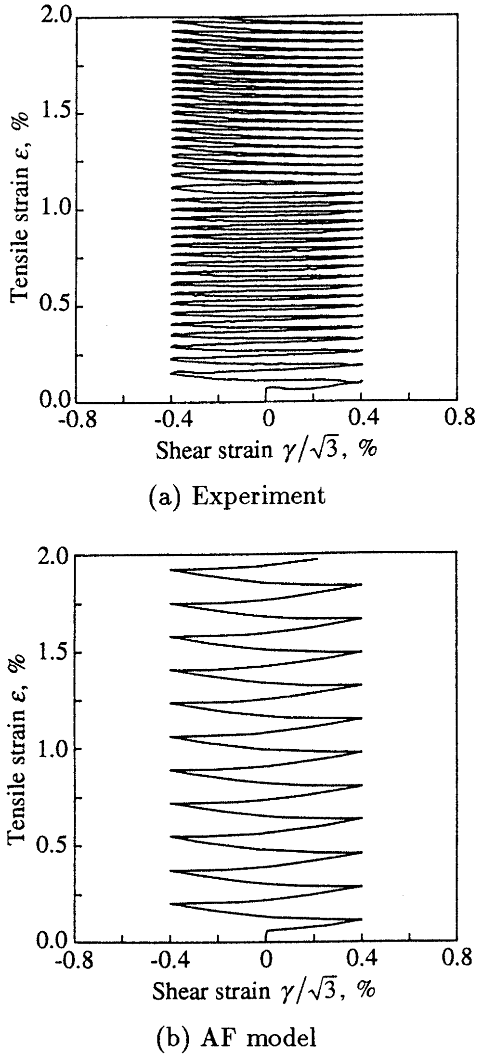

Fig.2. Multiaxial ratchetting of Modified 9Cr-1Mo steel at 550°C under constant tensile stress of $\sigma =$ 100 MPa combined with cyclic torsional straining of $\Delta\gamma/\sqrt{3} = 0.8$ % at $|\dot{\gamma}|/\sqrt{3} = 5 \times 10^{-2}$ %/s [28]; experiment [29] and simulation by the AF model.

Table 1. Tendency in simulation of ratchetting by classical nonlinear anisotropic hardening models.

Model	Uniaxail Ratchetting	Multiaxial Ratchetting
Multilayer (Besseling [5])	zero	–
Nonlinear Kinematic (Armstrong-Fredercik [6])	too large	too large
Multisurface (Mróz [7])	zero	too large
Hereditary Integral (Valanis [9], Backhaus [10])	too large	too large
Two Surface (Krieg [11])	too large	too large
Two Surface (Dafalias-Popov [12])	–	too large

anisotropic hardening, but the model does not allow any uniaxial ratchetting to occur.

Hence, we can say that most of the classical nonlinear anisotropic hardening models are not adequate to simulate ratchetting, as summarized in Table 1.

3. MODIFICATIONS OF THE AF MODEL

The AF model overpredicts ratchetting, as discussed in the preceding section. Nevertheless, the concept employed in the model, i.e., the nonlinear kinematic hardening based on strain hardening and dynamic recovery, is simple and physically sound. On the basis of this concept, therefore, modeling of anisotropic hardening has been discussed in the last decade, as reviewed here.

3.1. Modifications

Since the dynamic recovery term in the AF model is too active to simulate ratchetting, Burlet and Cailletaud [17] assumed a back stress component which has the dynamic recovery taking place in the direction of plastic strain rate:

$$\dot{\boldsymbol{a}}_i = \frac{2}{3}\left(h_i - \zeta_i \boldsymbol{a}_i : \boldsymbol{n}\right)\dot{\boldsymbol{\varepsilon}}^p, \tag{4}$$

where $\boldsymbol{n}$ denotes the direction of plastic strain rate, i.e.,

$$\boldsymbol{n} = \dot{\boldsymbol{\varepsilon}}^p/\dot{p}. \tag{5}$$

They showed that this dynamic recovery, which they called "radial evanescence", is effective for multiaxial ratchetting.

Chaboche and Nouailhas [25, 26] as well as Chaboche [18] examined several modifications of the

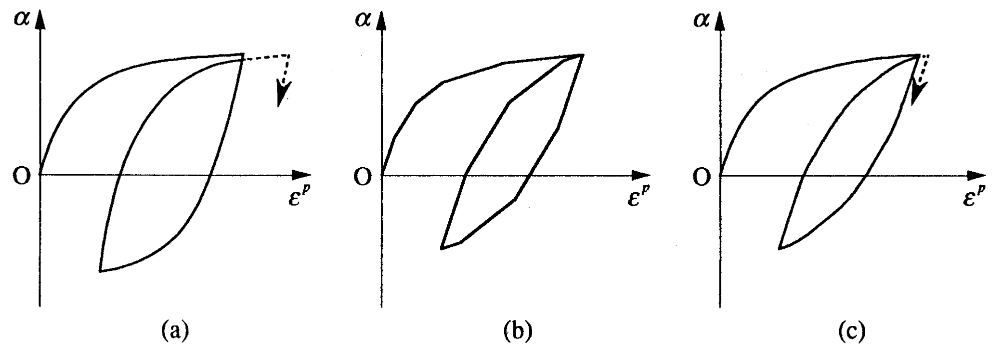

Fig.3. Change of uniaxial back stress α under tensile straining followed by reverse straining and restraining.

AF model. Chaboche [18] thus concluded that the following modification with a threshold $\overline{a}_i^*$ in the dynamic recovery term is most successful in simulating ratchetting:

$$\dot{\boldsymbol{a}}_i = \frac{2}{3}h_i\dot{\boldsymbol{\varepsilon}}^p - \zeta_i\left\langle 1 - \frac{\overline{a}_i^*}{\overline{a}_i}\right\rangle \boldsymbol{a}_i\dot{p}, \qquad (6)$$

where $\langle\ \rangle$ indicates the Macauley bracket (i.e., $\langle x\rangle = x$ if $x \geq 0$ and $\langle x\rangle = 0$ if $x < 0$), and $\overline{a}_i$ denotes the magnitude of $\boldsymbol{a}_i$ defined as

$$\overline{a}_i = \left[(3/2)\boldsymbol{a}_i : \boldsymbol{a}_i\right]^{1/2}. \qquad (7)$$

It is seen that the dynamic recovery term in Eq.(6) is active when $\overline{a}_i > \overline{a}_i^*$.

To overcome the excessive ratchetting predicted by the AF model, Ohno and Wang [19] on the other hand assumed that the dynamic recovery of $\boldsymbol{a}_i$ is *fully* activated when the magnitude $\overline{a}_i$ attains a critical value r_i, and they derived the form

$$\dot{\boldsymbol{a}}_i = \zeta_i\left\{\frac{2}{3}r_i\dot{\boldsymbol{\varepsilon}}^p - H(f_i)\left\langle \dot{\boldsymbol{\varepsilon}}^p : \frac{\boldsymbol{a}_i}{\overline{a}_i}\right\rangle \boldsymbol{a}_i\right\}, \qquad (8)$$

where H stands for the Heaviside step function, and

$$f_i = \overline{a}_i^2 - r_i^2. \qquad (9)$$

Equation (8) allows the dynamic recovery term to be active only in the critical state $f_i = 0$. They then extended it on the assumption that the dynamic recovery of $\boldsymbol{a}_i$ is activated nonlinearly as $\boldsymbol{a}_i$ approaches the critical state $f_i = 0$:

$$\dot{\boldsymbol{a}}_i = \zeta_i\left\{\frac{2}{3}r_i\dot{\boldsymbol{\varepsilon}}^p - \left(\frac{\overline{a}_i}{r_i}\right)^{m_i}\left\langle \dot{\boldsymbol{\varepsilon}}^p : \frac{\boldsymbol{a}_i}{\overline{a}_i}\right\rangle \boldsymbol{a}_i\right\}, \qquad (10)$$

which is reduced to Eq.(8) when $m_i = \infty$.

Moreover, Eq.(10) with $\langle\dot{\boldsymbol{\varepsilon}}^p : \boldsymbol{a}_i/\overline{a}_i\rangle$ replaced by $\dot{p}$, i.e.,

$$\dot{\boldsymbol{a}}_i = \zeta_i\left\{\frac{2}{3}r_i\dot{\boldsymbol{\varepsilon}}^p - \left(\frac{\overline{a}_i}{r_i}\right)^{m_i} \boldsymbol{a}_i\dot{p}\right\}, \qquad (11)$$

was discussed by Ohno and Wang [33], Chaboche [34], and Jiang et al. [35-37]. The special case of $m_i = 1$ was examined by Chaboche [18].

Other modifications of the AF model were discussed by Freed and Walker [27], Hassan et al. [38], Voyiadjis and Basuroychowdhuryin [39], and so on. Incidentally, modeling for ratchetting in other frameworks was dealt with in [40-43].

3.2. Comparison of Models

Let us compare the models described in Sec. 3.1 and the AF model. Here we discuss only uniaxial ratchetting for simplicity. Then the models may be classified roughly into the three types illustrated in Figs.3(a) to (c), which are concerned with the change of uniaxial back stress α under tensile straining to a plastic strain followed by small reverse straining and restraining to the plastic strain.

According to the AF model, α attains a smaller value at the end of the restraining than just before the reverse straining (Fig.3(a)), since the dynamic recovery term in Eq.(2) expresses the erasure of memory. Consequently, the hysteresis loop has an opening, which leads to excessive ratchetting, as indicated by the dashed line in the figure. The Buret and Cailletaud model, Eq.(4), does not improve the excessive ratchetting under uniaxial cyclic loading, because under uniaxial loading Eq.(4) is reduced to the AF model:

$$\dot{\alpha}_i = h_i\dot{\varepsilon}^p - \zeta_i\alpha_i|\dot{\varepsilon}^p|. \qquad (12)$$

Hence, the Burlet and Cailletaud model is effective only for multiaxial ratchetting.

On the other hand, the first version of the Ohno and Wang model, Eq.(8), represents the complete closure of uniaxial stress and strain hysteresis loops, resulting in no uniaxial ratchetting (Fig.3(b)). This will be understood from the fact that Eq.(8) with $\dot{\boldsymbol{\varepsilon}}^p$ replaced by deviatoric strain rate is identical to Besseling's multilayer model [19]. Equation (8) is therefore applicable to the materials which exhibit the almost perfect closure of stress and strain hysteresis loops leading to very little uniaxial ratchetting. This characterization is valid especially for 304 and 316 stainless steels at such high temperatures as 400 to 550°C [44-46].

Equations (6), (10) and (11) can describe the nearly complete closure of hysteresis loops, as illustrated in Fig.3(c), because the dynamic recovery terms in them are not so active as that in the AF model. They thus allow uniaxial ratchetting to occur slightly. It was shown that Eqs.(10) and (11) do not give very different simulations if m_i is somewhat large [33, 34]. It is however noticed that Eq.(10) is useful since it can be integrated efficiently [47].

Incidentally, Jiang et al. [35, 36] showed that the material constants ζ_i and r_i in Eqs.(8), (10) and (11) can be determined systematically using the following equations if rate-independent plasticity with no isotropic hardening is assumed:

$$\zeta_i = \frac{1}{\varepsilon^p_{(i)}}, \tag{13}$$

$$r_i = \left\{ \frac{\sigma_{(i)} - \sigma_{(i-1)}}{\varepsilon^p_{(i)} - \varepsilon^p_{(i-1)}} - \frac{\sigma_{(i+1)} - \sigma_{(i)}}{\varepsilon^p_{(i+1)} - \varepsilon^p_{(i)}} \right\} \varepsilon^p_{(i)}, \tag{14}$$

where $\sigma_{(i)}$ and $\varepsilon^p_{(i)}$ denote stress and plastic strain at the i-th point on the monotonic tensile stress versus plastic strain curve approximated multilinearly (see Fig.4). Hence, all the material constants in Eq.(8) can be determined from uniaxial tensile data. Equations (10) and (11), on the other hand, have additional material constants m_i ($i = 1, 2, ..., M$), the values of which are adjustable so that the simulations can fit ratchetting experiments.

4. SIMULATION OF RATCHETTING

This section is devoted mainly to the application of the Ohno and Wang model, Eqs.(8) and (10), to ratchetting problems. Equations (8) and (10) will be referred to as the OW I and II models hereafter, respectively.

4.1. Mechanical Ratchetting

Let us start with the uniaxial ratchetting of Modified 9Cr-1Mo steel shown in Sec. 2.1 to demonstrate the incapability of the AF model. Figure 5 compares one of the experiments with the simulations obtained using the OW I and II models as well as the AF model. As seen from the figure, the OW II model simulates accurately the experiment whereas the AF model overpredicts it too seriously. The accurate simulation of the OW II model is attributable partly to the adjustable constant m_i, the value of which was determined for the simulation to fit the experiment. It is however noticed that even the OW I model can predict the experiment fairly accurately though this model has no such material constants as m_i. The success of the OW II model, therefore, should be ascribed largely to the critical state of dynamic recovery, which was introduced in the OW I and II models so as to limit the activity of the dynamic recovery of back stress.

The OW I and II models simulate well the multiaxial ratchetting experiment exemplified in Fig.2(a), too, as seen by comparing the experiment with the corresponding simulations shown in Figs.6(a) and (b). It is noticed that the same material constants were used to simulate both the uniaxial and multiaxial ratchetting experiments [28], and that even the OW I model, which has no material constants adjustable to ratchetting experiments, simulates the multiaxial experiment with reasonable accuracy. Therefore we can conclude that the critical state of dynamic recovery assumed in the OW I and II models is effective for simulating multiaxial, as well as uniaxial, ratchetting.

The same conclusion as mentioned above was obtained by examining multiaxial ratchetting and multiaxial cyclic relaxation of IN738 LC at 850°C [48]. Incidentally, the OW II model with or without modifications was employed successfully to simulate mechanical ratchetting of other materials [49-52].

We have seen that the OW I and II models are effec-

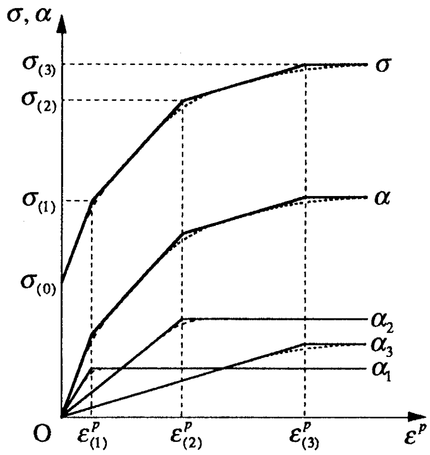

Fig.4. Changes of uniaxial tensile stress σ and back stress α; solid lines by Eq.(8) and dashed lines by Eqs.(10) and (11).

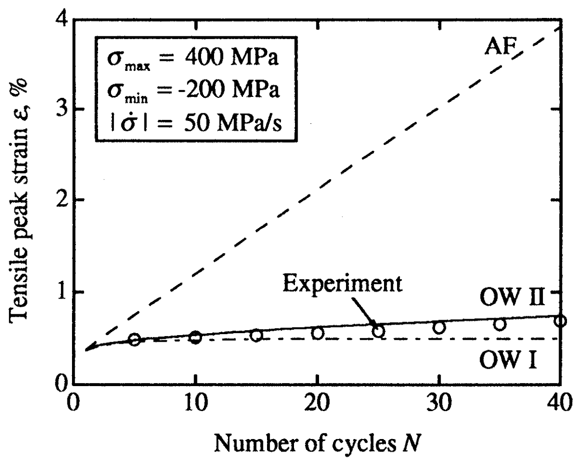

Fig.5. Uniaxial ratchetting of Modified 9Cr-1Mo steel at 550°C [28]; experiment [29] and simulations by the AF, OW I and OW II models.

tive in simulating mechanical ratchetting. It is however necessary to examine the models under other mechanical loading conditions, since it may happen that newly developed models give unexpected results especially under complicated nonproportional loading. For the OW I and II models, it was found that, if neither ratchetting nor cyclic stress relaxation is significant, they give nearly the same predictions as the AF model [28, 48].

4.2. Thermal Ratchetting

Now we are concerned with the thermal ratchetting induced by moving temperature distribution. This thermal ratchetting was analyzed first by Goodman [53] and then by Ponter et al. [54-56] and Wada et al. [57]. Their analyses were based on the elastic-perfectly plastic model, which will be indicated as the PP model henceforth. It, however, turned out that the analysis based on the PP model overestimates the thermal ratchetting experiments of 304 stainless steel cylinders [58]. Kobayashi and Ohno [59] then discussed the effect of kinematic hardening on the thermal ratchetting analysis by implementing the OW I, AF and PP models in a finite element method; thus, they found that the analysis depends significantly on the kinematic hardening models employed.

Let us present an example of the thermal ratchetting analysis performed by Kobayashi et al. [60]. In this example, a 316FR steel cylinder which had 260 mm in length, 76.5 mm in mean radius and 2 mm in thickness was subjected cyclically to the thermal loading consisting of heating up, travelling of temperature distribution by 20 mm in the axial direction, and cooling down (Fig.7). This is one of the cases examined by the Subcommittee on Inelastic Analysis of High Temperature Materials, JSMS, which has the mission to find appropriate constitutive models for analyzing the thermal ratchetting [1, 2].

Figures 8 and 9 show the distribution of residual hoop strain after 100 cycles and the development of maximum residual hoop strain with the increase of the number of cycles, respectively. The OW I, AF and PP

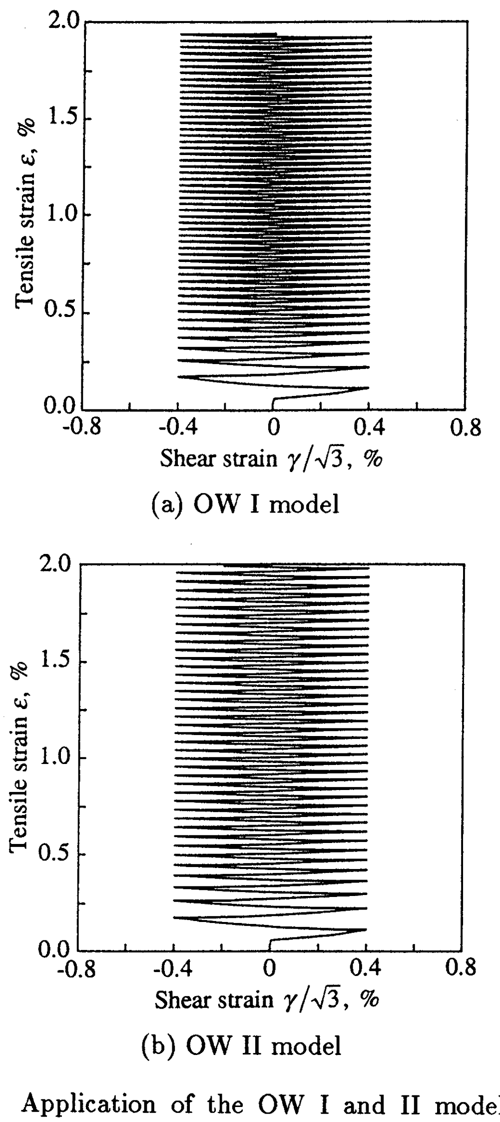

Fig.6. Application of the OW I and II models to the multiaxial ratchetting experiment shown in Fig.2(a) [28].

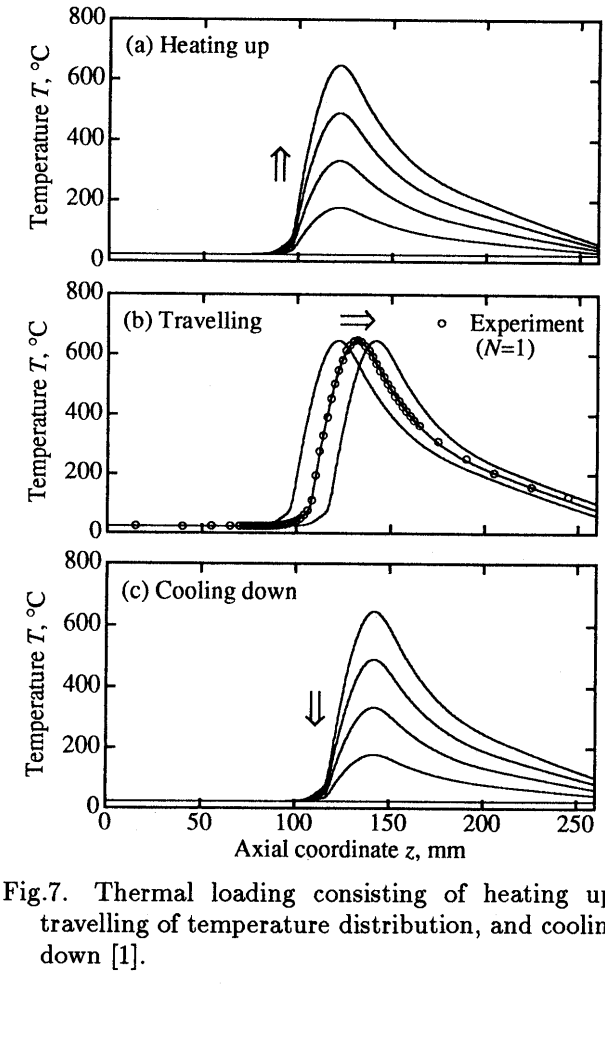

Fig.7. Thermal loading consisting of heating up, travelling of temperature distribution, and cooling down [1].

models were employed in the analysis. It is noticed that 316FR steel has the almost perfect closure of stress-strain hysteresis loops in strain-controlled uniaxial ratchetting deformation [1]. This validates the use of the OW I model rather than the OW II model. It is also noticed that for the OW I and AF models the pure kinematic and combined hardening versions were examined, and that the isotropic hardening based on the maximum plastic strain was taken into account in the combined hardening version as a result of uniaxial ratchetting experiments with strain controlled under isothermal and nonisothermal conditions [61]. It is seen from Figs.8 and 9 that the OW I model predicts fairly accurately the experiment while the AF and PP models overpredict it, and that the isotropic hardening does not have a significant effect on the thermal ratchetting analysis.

Takahashi and Tanimoto [62] also reported that the OW I model, as well as the OW II model with large m_i, is appropriate to analyze the thermal ratchetting of 316FR steel cylinders subjected to axial movement of temperature distribution.

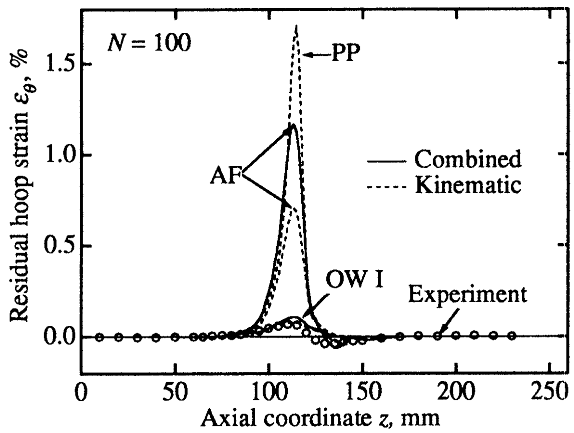

Fig.8. Distribution of residual hoop strain in 316FR cylinder after 100 cycles of axial travelling of temperature distribution [60].

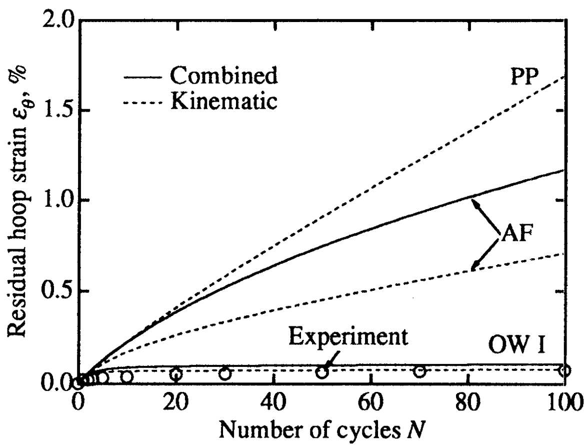

Fig.9. Increase of maximum residual hoop strain in 316FR steel cylinder subjected to axial travelling of temperature distribution [60].

Finally, let us show a result obtained in the comparative study by the Subcommittee [2]. Five constitutive models were employed to simulate eight experiments of 316 stainless and 316FR steel cylinders, and the computations were carried out up to the tenth to thirtieth cycles. As shown in Fig.10, the OW I model simulated the experiments almost within the factor of three, indicated by the dashed lines in the figure. This extent of agreements is much better than those of the other models.

It is necessary to notice the following: The thermal ratchetting caused by moving temperature distribution is *structural* in the sense that it occurs as a result of the stress and strain distributions in structures [60, 64]. This type of ratchetting should be distinguished from *material* ratchetting occurring in uniform stress and strain states, because a constitutive model which allows uniaxial ratchetting is not always necessary to analyze structural ratchetting.

5. CONCLUDING REMARKS

Classical constitutive models of nonlinear anisotropic hardening, which were proposed without paying much attention to ratchetting, are not suitable for simulating ratchetting. Recent works, however, have enabled us to simulate or predict ratchetting with reasonable accuracy; especially the Armstrong and Frederick model has been modified to be applicable to ratchetting, as was reviewed in the present paper. This progress is surprising if we notice that ratchetting

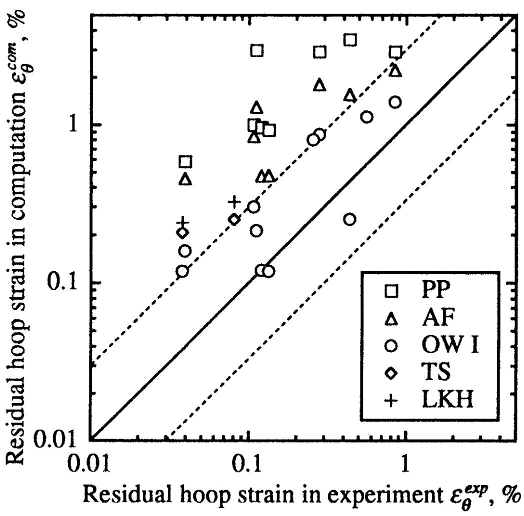

Fig.10. Comparison of experiments and computations with respect to maximum residual hoop strain in the last cycles in computation [2]; the models employed are PP (elastic-perfectly plastic), AF (Armstrong and Frederick), OW I (first version of Ohno and Wang), TS (PNC two surface [63]), and LKH (linear kinematic hardening).

is the progressive deformation occurring as a result of the accumulation of secondary deformation proceeding cycle by cycle.

The distortion of the yield surface, which have not been taken into account in the recent models reviewed in Sec. 3.1, can bring about a serious effect on multiaxial ratchetting, as was analyzed first by Shiratori et al. [65]. A recent work related to this effect was that of Corona et al. [66]: Performing biaxial ratchetting experiments by subjecting thin-walled cylinders to five histories of simultaneous cycling of internal pressure and axial strain, they reported that all the models which they examined predicted very poorly some of their experiments, and they concluded that the discrepancy is attributable to the distortion of the yield surface. This conclusion, however, is not always correct, since the OW I and II models predict fairly accurately all their experiments without taking into account the distortion [47].

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