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Review paper

RESIDUAL STRENGTH OF NOTCHED COMPOSITE LAMINATES

Theoretical Models and Experimental Results

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Abstract: Several fracture models used for prediction of notched residual strength of composite laminates are reviewed. Emphasis has been placed on analytical fracture models which are simple to utilise. Published experimental results on the notched strength of composite laminates containing a circular hole or centre crack and subjected to quasi-static axial tensile loading are also reviewed. Characteristic parameters associated with fracture models have been determined and theoretical predictions of residual strength are compared with all notched strength data sets. Applicability of the different fracture models in predicting the residual strength of composite laminates is also discussed.

Key words: *Notched strength, Notch sensitivity, Modelling, Characteristic parameters, Failure criterion, Cohesive stress*

1. INTRODUCTION

Fibre-reinforced composite materials normally have high specific strength and stiffness as well as other excellent mechanical and thermal properties over metal alloys such as steel or aluminium. This has led to increased utilisation of composite materials as major structural components in many practical applications. They have been used with success in aerospace applications as primary and secondary structural elements. A basic requirement for such use is an understanding of their mechanical behaviour and their failure response. Hence, failure of fibre-reinforced composite laminates has been one of the major issues being studied extensively in recent years. Many analyses have been developed to predict the failure strengths of composite laminates containing stress concentrations such as circular holes or cut-outs.

The residual strength of notched composite laminates has been shown to be influenced strongly by many parameters such as ply orientation, ply thickness, notch size and constituent properties, etc. Local damage can develop inside the material near the stress concentration area at an early loading stage, and it leads to catastrophic failure of the composite laminates. The knowledge of damage tolerance of laminated composites containing cut-outs is very important in practice for the design of structural components made of composite materials.

In this paper, the notch sensitivity for fibre reinforced composite laminates is discussed. Then, theoretical models used to characterise the residual strength of notched composite laminates are presented and their limitations are addressed. Finally, experimental results are reviewed and compared with the predictions from theoretical models.

2. NOTCH SENSITIVITY

The presence of a notch or hole reduces the strength of a component for two reasons. First, the load bearing area is reduced and second, stress concentrations are produced at the notch tips. Two possibilities exist for a plate with a notch or hole: (i) the strength of the component is directly proportional to the cross-sectional area, associated with notch insensitive materials (Failure load = Failure stress of unnotched component × Net section area in presence of notch/hole) or (ii) the strength reduces by a greater amount compared with the reduction in load-bearing area, associated with notch sensitive materials. These two cases are distinguished in Fig. 1 by plotting the net cross-sectional strength $\sigma_n = P/(W - 2a)t$ against the notch size, a. If the material were completely notch insensitive, then a simple net-section yield would predict failure when $\sigma_c / \sigma_o = 1 - 2a/W$. The strength of notch sensitive materials is strongly dependent on notch size, and decreases rapidly as the size increases. The rate of decrease is greater for sharp notches than for blunt notches.

Experimental data have shown that the tensile strength of composite laminates generally is severely reduced by the presence of a stress concentration in the form of a crack or a hole. This appears to be related to the brittleness of the fibres used in these materials. Metals generally yield, making the presence of a stress concentration less severe. However, the strength of composite laminates is not reduced in inverse proportion to the stress concentration factor. This is explained by the opportunity that exists to reduce the local stress concentration by local damage creation mainly as matrix micro cracking, fibre breakage, and delamination.

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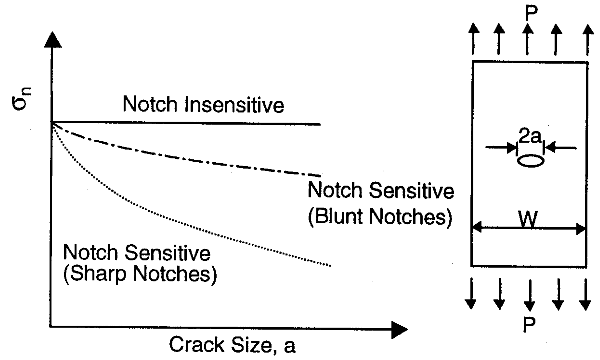

Fig. 1. Strength properties of notch sensitive and notch insensitive materials.

Notch sensitivity is related to the response of the material at the root of the notch to the high local stresses. If microcracking processes occur without the notch size increasing, the stress concentration is reduced because the tip of the notch is blunted and the load is distributed over a larger volume of material. On the other hand, if no damage occurs, the stress concentration dominates the failure process, and crack growth takes place at the root of the notch [1]. The response of composite materials to the presence of notches is complicated, and it depends on a variety of intrinsic (laminate configuration, stacking sequence, constituent properties, fibre volume fraction, fibre-matrix interface, and fabrication procedures, etc.) and extrinsic (specimen geometry, test temperature, moisture content, and loading condition, rate and history, etc.) variables.

3. THEORETICAL MODELS

Many models have been developed for failure analysis of notched composite laminates, with varying capability, accuracy, and required computational effort. The models are all basically two-dimensional and can be categorised as follows:

- Non-progressive models
- Progressive models
- Hybrid models

The non-progressive models are based on the plane stress laminate theory which converts an anisotropic inhomogeneous laminate into a homogeneous solid. With the non-progressive approaches, two lines of activities exist: (1) a fracture mechanics approach; and (2) a hybrid approach of fracture mechanics and notch theory. These models can only evaluate residual strength, and do not account for the formation and growth of damage. Consequently, no information on the extent and type of damage in the laminate during loading can be provided by these models. However, the non-progressive models compensate for this, applying a failure criterion with a "characteristic parameter" (usually a length) from the edge of the notch. The characteristic parameter, which is determined from experiments for one geometry, is used to predict failure for a specimen of a slightly different geometry. A basic problem with these models is that they may require different values of the characteristic length for various size of the notch in the same type of laminate [2-6].

The progressive models are based on a ply-by-ply failure progressive analysis using lamination theory [7, 8] to predict the failure initiation and ultimate strength of notched laminates. Certain failure criteria were utilised to determine failure and failure modes of each lamina, and some rules were employed to estimate the degradation of the mechanical properties in the damaged region. Because of the limitation of the two-dimensional nature of lamination theory, the effect of interlaminar stresses on the failure of laminates was not included in the failure analysis. In addition, these models require intensive finite element computation. The hybrid models [9, 10] combine some capabilities of the progressive type models with the low computational effort of the non-progressive models, being able to take into account the formation and growth of damage. The applicability of these models varies, and the resulting strength prediction is not always accurate. These models have frequently been refined to develop modified models.

3.1. The Inherent Flaw Criterion

A direct application of linear elastic fracture mechanics (LEFM) to through-thickness cracks has been tried repeatedly, with some success [11-13]. One of the early applications of LEFM to notched composite laminates on a macroscopic scale was given by Waddoups et al. [14].

3.1.1. Circular holes

In this approach, it is assumed that intense energy regions exist at the hole edge, which can be modelled as through-thickness cracks of a constant length a_o extending symmetrically from each side of the hole perpendicular to the applied load direction. Using Bowie's [15] solution for the stress intensity factor, K_I, for cracks emanating from a circular hole, the fracture stress σ_c for the laminate with a hole is given by:

$$\sigma_c = \frac{K_{IC}}{\sqrt{\pi a_o}\, f(a_o / R)} \tag{1}$$

where $f(a_o/R)$ is a function of a_o and R $(=D/2)$. The unnotched strength can also be calculated by:

$$\sigma_o = \frac{K_{IC}}{\sqrt{\pi a_o}} \tag{2}$$

where an inherent crack is assumed to have twice the characteristic length $2a_o$. Combining Eqs. (1) and (2) yields:

$$\frac{\sigma_o}{\sigma_c} = f(a_o / R)\,. \tag{3}$$

3.1.2. Centre cracks

Waddoups et al. [14] applied the concept of an intense energy region at the crack tip for the case of composite laminates containing a centre crack of length $2a$. Considering the isotropic stress intensity factor [16] and applying the similar concept of Irwin's plastic zone correction [17] for metals, results in the following equation at failure:

$$K_{IC} = \sigma_c \sqrt{\pi(a + a_0)} \tag{4}$$

where a_o is the crack tip damage zone size at failure. For unnotched specimens the Eq. (4) becomes:

$$K_{IC} = \sigma_0 \sqrt{\pi a_0}\ , \tag{5}$$

Combining Eqs. (4) and (5) yields:

$$\frac{\sigma_c}{\sigma_0} = \sqrt{\frac{a_0}{a + a_0}}\ . \tag{6}$$

However, several limitations of this model preclude its general use. Firstly, as discussed by Kanninen et al. [18], LEFM requires self-similar crack growth, i.e. the crack should propagate in the same plane and direction as the initial crack. But generally in composite laminates this is not the case, and also these laminates are hardly isotropic. Secondly, the stress intensity factor for cracks emanating from circular holes, developed by Bowie [15], is limited to isotropic material systems. Thirdly, the assumption that the characteristic dimension a_o is a material constant is invalid [3, 19]. In addition, this is an empirical model, which means that experimental results are approximated by the theory to determine the characteristic parameter a_o [9].

Several other researchers have used LEFM concepts to predict notched laminate strength. Cruse [20] introduced a model involving a crack whose half-length was slightly larger than the hole radius. This difference in dimensions was interpreted as the length of the crack extending from the hole. When Cruse fitted this theory to the data, he computed crack lengths that compared closely with those found by Waddoups et al. [14]. Tirosh [21] assumed that a microcrack inherent to the laminate touches the hole and lies along the line normal to the maximum tensile stress at the hole edge. Mar and Lagace [22] proposed a theory based on the assumption of a crack embedded in the matrix, with the crack tip at the matrix-fibre interface. Although all of these approaches produced good agreement with test results, they require difficult crack-tip analyses involving anisotropic material properties and complex crack configuration.

3.2. Point and Average Stress Criteria

Both point and average stress criteria were presented by Whitney and Nuismer [19]. In these approaches, the stress distribution ahead of the notch is obtained using linear elasticity.

3.2.1. Circular holes

In the Point Stress Criterion (PSC), it is assumed that the failure occurs when the stress over some distance (d_o = constant) away from the notch edge is equal to or greater than the unnotched strength, σ_o. It is further assumed that this characteristic distance is a material property, being independent of laminate geometry and stress distribution. That is,

$$\frac{\sigma_c}{\sigma_0} = \frac{2}{2 + \xi^2 + 3\xi^4 - (K_T^\infty - 3)(5\xi^6 - 7\xi^8)} \tag{7}$$

where $\xi = R/(R + d_o)$ and the stress concentration factor for the infinite plate is given by:

$$K_T^\infty = 1 + \sqrt{2\left(\sqrt{\frac{E_y}{E_x}} - \nu_{yx}\right) + \frac{E_y}{G_{xy}}}\ . \tag{8}$$

The second approach, the Average Stress Criterion (ASC), can be established by assuming that the notched strength, σ_c, is obtained when the average stress over some distance, (l_c = constant), equals the unnotched laminate strength. Hence, for an anisotropic plate the ASC is given by:

$$\frac{\sigma_c}{\sigma_0} = \frac{2(1 - \psi)}{2 - \psi^2 - \psi^4 + (K_T^\infty - 3)(\psi^6 - \psi^8)} \tag{9}$$

where $\psi = R/(R + l_c)$.

3.2.2. Centre cracks

Applying the PSC, the ratio of notched strength to unnotched strength for a composite laminate containing a centre crack is given by:

$$\frac{\sigma_c}{\sigma_0} = \sqrt{(1 - \zeta^2)} \tag{10}$$

where $\zeta = a/(a + d_o)$. The critical stress intensity factor becomes

$$K_Q = \sigma_0 \sqrt{\pi a (1 - \zeta^2)} \tag{11}$$

which is a function of crack length.

Applying the ASC with the exact stress distribution, the ratio of notched strength to unnotched strength is given by:

$$\frac{\sigma_c}{\sigma_0} = \sqrt{\frac{1 - \psi}{1 + \psi}} \tag{12}$$

where $\psi = a / (a + l_c)$. In this case, the critical stress intensity factor for the infinite plate becomes:

$$K_Q = \sigma_0 \sqrt{\pi c \frac{1-\psi}{1+\psi}} \tag{13}$$

which again is a function of crack length.

The PSC and ASC are two parameter models, using composite laminate unnotched strength, σ_o, and characteristic dimension to evaluate the residual strength of these materials. The characteristic dimensions were found to depend on the size of the notch [23], stacking sequences [24], geometry of the specimen [2, 3], loading directions [25] and they were not material constants. Modifications with effects of notch size on the characteristic dimension incorporated, have been suggested by other workers. Karlak [2] modified the PSC by introducing the concept of a variable characteristic length. He expressed d_o as a function of the hole radius and the characteristic factor K_o.

On the other hand, these models are simple to be applied and are therefore attractive to designers. Characteristic distance approaches have frequently been used for evaluating the tensile strengths of bolted composite laminates [26, 27], composite laminates containing an elliptical opening [28], notched laminates subjected to fatigue loading [29] and compressive loading [30].

Garbo and Ogonowski [31] presented a computerised application of the PSC criterion, advancing the two-parameter strength prediction technology in three significant ways. First, the procedure was formulated to handle generalised biaxial loading. Accordingly, the point stress calculations were made at intervals around the hole to determine the failure location. Second, these calculations were made on a ply-by-ply basis and the first ply failure (FPF) [32] was defined as laminate failure. The third and perhaps the most significant innovation was the use of the Tsai-Hill failure criterion [33] to predict failure at the point near the hole.

In 1983, Lo et al. [34] presented a progressive failure model based on the PSC approach. Instead of simply predicting failure at a point, they predicted a failure zone extending from the hole. Calculations were made on a ply-by-ply basis using the Tsai-Wu failure criterion [35]. Also using the concepts of PSC and ASC a new model was developed [36] on the basis of an equivalent damage size concept and stress fracture criterion. The PSC and ASC fracture models and subsequent modified models are semi-empirical in that the parameters involved have to be determined experimentally for the individual material system.

3.3. The Three-Parameter Model

A three-parameter model, proposed by Pipes et al. [3] was based on the work of Waddoups et al. [14] and Whitney and Nuismer [19].

3.3.1. Circular holes

In this model the ratio of notched strength to unnotched strength of an infinite laminate is given by:

$$\frac{\sigma_c}{\sigma_o} = 2\left\{\frac{2+f(R)^{-2}+3f(R)^{-4}-(K_T^{\infty}-3)}{\left(5f(R)^{-6}-7f(R)^{-8}\right)}\right\}^{-1} \tag{14}$$

where $f(R) = 1 + (R^{m-1} / R_o^m C)$, R_o is the reference radius, m is an exponent and C is notch sensitivity. Note that for $C = 0$, i.e. complete notch insensitivity, the notched strength is equal to the unnotched strength for all notch radii. In the case of infinite notch sensitivity ($C = \infty$), the ratio of notched strength to unnotched strength is equal to $1 / K_T^{\infty}$. The exponential parameter should be $0 \leq m < 1$ for admissible material systems. With $m = 1$, Eq. (14) will be independent of the notch radius, while for the lower bound $m = 0$, this equation reduces to a two parameter model. In general, the influence of K_T^{∞} is important for large notch radii. As the notch radius approaches infinity, notched strength converges to σ_o / K_T^{∞}

A radius superposition method was applied to composite laminates with the same stress concentration factor, to obtain a single master curve for all laminate stacking sequences. However, the restriction of the same K_T^{∞} for superposition limits the class of composite laminates in which the principle may be applied.

3.3.2. Centre cracks

For the case of orthotropic laminates containing a centre crack [37], the subsequent formulation is analogous to that for laminates containing circular holes Eq. (14),

$$\frac{\sigma_c}{\sigma_o} = \sqrt{1 - f(a)^{-2}} \tag{15}$$

where $f(a) = 1 + (a^{m-1} / a_o^m K_n)$ and a_o is a reference half-crack length [38] and K_n is defined as the crack notch sensitivity factor. Pipes et al. [3] proposed a procedure by which it is possible to superimpose the notched strength data onto a single curve, by defining a crack length shift parameter. However, it should be noted that the notched strength data of laminates containing centre cracks do not shift onto the master curve as precisely as the data for laminates containing circular holes [39]. In addition, as mentioned in previous sections, this approach is also semi-empirical.

3.4. The Mar-Lin Model

Based on experimental observations Mar and Lin [40] proposed that the major damage mechanism in a unidirectional composite containing a discontinuity (crack or hole) is splitting, i.e. cracks running parallel to the filaments, which initiate at the tip of the discontinuity They correlated this with linear elastic fracture mechanics as applied to homogeneous materials:

$$\frac{\sigma_c}{\sigma_o} = H(2a)^{-m} \tag{16}$$

H is similar to K_{IC} but has dimensions of "stress × (length)^{m}" which is different from those for K_{IC}. However, this is merely a parameter used to fit the experimental data. The exponent "-m" is the value of the stress singularity at the tip of a discontinuity lying at the fibre/matrix interface.

It has been shown that "m" is a function of the shear moduli and the Poisson ratios of fibre and matrix materials. The most likely value of the singularity for a graphite/epoxy system is approximately 0.26. However, Fenner [41] obtained 0.28 for the same material.

Since the Mar-Lin criterion requires a great number of tests for determination of the "toughness" parameter, H, for each laminate, Soriano and Almeida [42] modified this criterion and proposed an approximation method to evaluate the "toughness" parameters for [0/90] and [±45] laminates.

It should be noted that a detailed analysis of the split growth behaviour after split initiation was not given by Mar and Lin [40]. In addition, they did not explain the mechanisms controlling the splitting process [43]. Further more, the stress singularity "m" is dependent on the orientation of fibres in the laminate [42].

3.5. The Damage Zone Criterion (DZC)

Eriksson and Aronsson [44] postulated that during increased loading of a notched composite laminate, a damage zone forms and grows in the most stress-intense region at the edge of the notch. When the load is increased so that the tensile stress at the edge of the notch reaches the strength of the unnotched laminate, σ_o, a damage zone is assumed to start to form. With increased remote loading, the damage zone will grow into the laminate. In addition, it is assumed that stress relaxation occurs within the damaged zone. Micromechanically, this stress relaxation may be caused by various failure mechanisms e.g., fibre pullout due to increased displacements within the damage zone. Hence, the stress level at the notch edge is now lower than σ_o and the stress level at some point ahead of the actual damage zone is equal to σ_o.

3.5.1. Circular holes

The damage is modelled by a fictitious crack with cohesive stresses, σ_{coh}, acting on the crack surfaces. For simplicity, a constant relationship between the crack opening, v, and the cohesive stress, σ_{coh}, is assumed. The strength of the notched specimen is derived from the equilibrium condition. Furthermore, according to the Finite Element analysis, the specimen width only has a minor effect on the elastic stress distribution [44]. Therefore, no finite width correction factor was used. The ratio of notched strength to unnotched strength for an open hole specimen is:

$$\frac{\sigma_c}{\sigma_o} = \frac{W - 2R}{W - m_2 + m_3 + m_1\left[\frac{W}{2} - R - d_I^*\right]} \tag{17}$$

where d_I^* is the critical damage zone length (= constant) and m_1, m_2 and m_3 are geometry functions [44].

3.5.2. Centre cracks

Using the linear elastic stress distribution along the net-section plane for an infinite orthotropic plate containing a centred crack of length $2a$, the ratio of notched strength to unnotched strength for a composite laminate of finite width, W, is

$$\frac{\sigma_c}{\sigma_o} = \frac{W/2 - a}{W/2 - \left[\sqrt{(W/2)^2 - a^2} - \sqrt{d_I^*(2a + d_I^*)} - \frac{(a + d_I^*)\left[W/2 - (a + d_I^*)\right]}{\sqrt{d_I^*(2a + d_I^*)}}\right] Y(a/W)} \tag{18}$$

where $Y(a/W)$ is the finite width correction factor [45].

There are some notable points in this criterion. Firstly, the actual damage zone, which in reality has two-dimensional extension (three-dimensional if the thickness direction is included) is represented by a crack in the net-section plane. Secondly, the assumption of a constant relationship between the crack opening and the cohesive stress is against the fact of stress relaxation with damage growth. Finally, the prediction by the DZC is from a macroscopic point of view, so that the basic parameters of the DZC, such as critical damage zone length, must be determined for each individual laminate to be used in design.

3.6. The Effective Crack Growth Model (ECGM)

Afaghi-Khatibi et al. [10, 46] developed an effective crack growth model based on global equilibrium. For a composite laminate with a centre notch of size $2a$ and applied loads perpendicular to the notch, damage is assumed to initiate ahead of the notch tip when the local normal tensile stress reaches the unnotched strength of the composite laminate. The damage is simulated by a fictitious crack with cohesive stress acting on its surfaces. Upon further loading, the fictitious crack is assumed to open and grow step by step, simulating progressive damage in the composite laminate. The cohesive force in each step, F_{coh}, can be correlated with the crack opening displacement, $v_{(n)}$, using the apparent fracture energy, G_c^*,

$$F_{\rm coh} = \sigma_0\left(1-\frac{v_{(n)}}{v_c}\right)\Delta c\, t \qquad (19)$$

where $v_c = 2G_c^*/\sigma_0$. In addition, the total crack opening profile can be described by:

$$v_{(n)} = v_{\sigma_{\rm app}(n)} + v_{\sigma_{\rm coh}(n)} \qquad (20)$$

It is obvious that $v_{\sigma_{\rm coh}(n)}$ is of negative sign to indicate crack clouser under the cohesive stress. To evaluate the crack opening displacement, $v_{(n)}$, associated with the applied stress and the cohesive stress, an appropriate equation was proposed (see next section).

The applied load, associated with the fictitious crack growth in the notched composite laminate can be evaluated from the global equilibrium condition, i.e. the applied load is in equilibrium with the cohesive stress acting on the surface of fictitious crack and the elastic stress, σ_y acting on the undamaged section plane

$$\sum_{n=1}^{i} F_{coh} + \int_{a+c_{(i-1)}}^{W/2}(\sigma_y + \Delta\sigma_y)t\,dx = \sigma_{\rm app}(i)\frac{W}{2}t \qquad (21)$$

where $\Delta\sigma_y = \sigma_o - \sigma_y|_{x=a+c_i}$. Using an iterative technique, the applied load corresponding to any specific fictitious crack length can be evaluated from Eqs. (19)-(21). In such an approach, the residual strength of notched composite laminates is defined by the unstable point of the applied load with damage growth. The flow chart of the evaluation procedure is given in Fig. 2.

For the reason of equilibrium, stress redistribution occurs with damage growth. This point has been taken into account in evaluating the applied load from the ECGM [10, 46].

3.6.1. Circular holes

The linear elastic stress distribution, σ_y, along the net-section plane (x-axis) for an infinite orthotropic plate containing a circular hole of diameter $2R$, is given by :

$$\sigma_y = \left\{\begin{array}{l} 2+\left(\frac{R}{x}\right)^2 + 3\left(\frac{R}{x}\right)^4 - \\ (K_T^\infty - 3)\left[5\left(\frac{R}{x}\right)^6 - 7\left(\frac{R}{x}\right)^8\right]\end{array}\right\}\frac{\sigma_{\rm app}}{2} \qquad (22)$$

If the length of the fictitious crack divided by the hole radius is less then 1.8 [47], the COD formulations for the edge cracks [45], are applied to simulate the crack opening displacements of the fictitious crack:

$$v = \frac{4\sigma}{E_{11}}\sqrt{a^2 - x^2}\, g(x/a) \qquad (23)$$

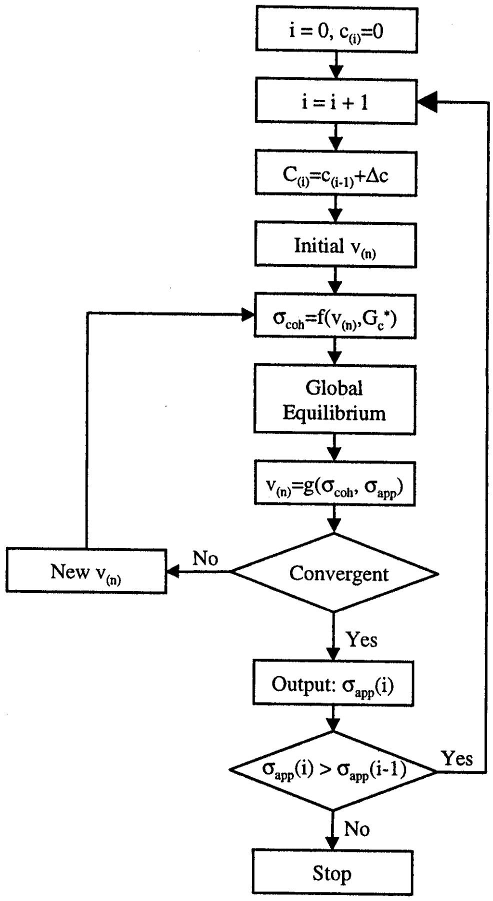

Fig. 2. Flow chart for residual strength evaluation by the ECGM.

where $g(x/a)$ is given in [45] and σ is the applied stress for $v_{\sigma\,\rm app(n)}$ or the cohesive stress for $v_{\sigma\,\rm coh(n)}$, respectively, and a the fictitious crack length.

3.6.2. Centre cracks

The linear elastic stress distribution in this case is given by:

$$\sigma_y = K_I\frac{x}{\sqrt{\pi a(x^2 - a^2)}}. \qquad (24)$$

The stress intensity factor for a plate with a finite width can be described as:

$$K_I = \sigma_{\rm app}\, Y\left(\frac{a}{W}\right)\sqrt{\pi a} \qquad (25)$$

where $Y(a/W)$ is the finite-width calibration function [47].

The crack opening profile due to the applied load for plane stress conditions is [48]:

$$v_{\sigma_{\rm app}(n)} = \frac{4\sigma_{\rm app}}{E_{11}}\sqrt{a_{\rm tot}^2 - x^2} \qquad (26)$$

where $a_{tot} = a + c_i$. For a fictious crack with cohesive stresses acting on crack surface, according to the Dugdale model the crack opening displacement is given by [50]:

$$v_{\sigma_{coh}}(n) = \frac{4\sigma_{coh}}{\pi E_{11}}\left[a \log\left|\frac{\sqrt{a_{tot}^2 - a^2} + \sqrt{a_{tot}^2 - x^2}}{\sqrt{a_{tot}^2 - a^2} - \sqrt{a_{tot}^2 - x^2}}\right| - x \log\left|\frac{a\sqrt{a_{tot}^2 - x^2} + x\sqrt{a_{tot}^2 - a^2}}{a\sqrt{a_{tot}^2 - x^2} - x\sqrt{a_{tot}^2 - a^2}}\right| - 2\sqrt{a_{tot}^2 - x^2}\arccos\left(\frac{a}{a_{tot}}\right)\right] \quad (27)$$

It should be noted that the value of G_c^* is dependent on the laminate geometry. However, a parametric study [46] shows that changing G_c^* from 50 to 150 kJ/m^2 does have an effect on residual strength evaluated by the ECGM but not remarkably.

4. EXPERIMENTAL RESULTS

In this section, the residual strength of composite laminates containing a circular hole or a centre crack is simulated using fracture models, namely Point Stress Criterion (PSC), Damage Zone Criterion (DZC) and Effective Crack Growth Model (ECGM). Twenty four groups of published experimental data for residual strength of notched carbon fibre/epoxy, plain woven glass/epoxy, satin woven glass/polyester and plain-woven carbon/epoxy composite laminates were applied (Table 1).

4.1. Residual Strength Evaluation

To evaluate the residual strength of notched composite laminates from theoretical models, characteristic parameters for the PSC, DZC and apparent fracture energy fracture for the ECGM should be determined. Using the experimental data, these parameters were determined. The value of G_c^* for each composite laminate system was evaluated based on the method suggested by Eriksson and Aronsson [44]. All these values are presented in Table 2.

4.2. Notch Sensitivity of Composite Laminates

The experimental results indicate that all composite laminates reviewed in this study are highly notch sensitive. In Fig. 3, notch sensitivity curves for glass/epoxy and glass/polyester (laminates D and E) are presented. The width of specimens was varied 10-40 mm.

Effect of notch shape on residual strength of notched graphite/epoxy $[0/\pm45/90]_{2S}$ composite laminates is shown in Fig. 4. It is clear that laminates with a centre crack exhibit more notch sensitivity than those of with a circular hole, as discussed in section 2.

Table 1. Summary of experimental material systems.

Laminate	Material	Lay Up	Ref
Circular Holes			
A_1 A_2	AS4/948A1	$[0/90]_{4S}$	[10]
B_1 B_2	AS4/948A1	$[0/\pm45/90]_{2S}$	
C_1 C_2	T300/1034	$[0/\pm45/90]_{2S}$	[9]
D_1 D_2 D_3	Glass/Epoxy	Plain Woven	[51]
E_1 E_2 E_3 E_4	Glass/Polyester	Satin Woven	
F_1 F_2	Carbon/Epoxy	Plain Woven	
G_1	Scotchply 1002	$[0/\pm45/90]_{2S}$	[52]
G_2	Scotchply 1002	$[0/90]_{4S}$	
H_1	T300/N5208	$[0/\pm45/90]_{2S}$	
H_1	T300/N5208	$[0/90]_{4S}$	
Centre Crack			
I	T300/914C	$[(\pm45/0/90)_3/0/90/\pm45]_S$	[44]
J_1	Scotchply 1002	$[0/\pm45/90]_{2S}$	[52]
J_2	Scotchply 1002	$[0/90]_{4S}$	
K_1	T300/N5208	$[0/\pm45/90]_{2S}$	
K_2	T300/N5208	$[0/90]_{4S}$	

4.3. Effect of Notch Size

The effects of the notch size on the residual strength of the composite laminates with different notch geometry are illustrated in Fig. 5. The residual strength of notched composite laminates clearly decreases when the notch size is increased. For laminates A_1 and K_1 with different notch sizes (Figs 5(a) and 5(b), respectively), the ECGM provides predictions with very good accuracy.

4.4. Effect of Specimen Width

The effect of specimen width on the residual strength of the notched composite laminates with different width is illustrated in Fig. 6. It can be seen that the residual strength raises significantly as the specimen width is increased and the hole size kept constant. Graphical comparisons of the experimental results with the predictions for laminate C are illustrated in Figs. 6(a) and 6(b), where the normalised residual strength is plotted against the specimen width (hole diameter is 10 mm and 20 mm, respectively). The agreements between the experimental data and the predictions from the PSC, DZC and ECGM are very good in all cases.

Table 2. Characteristic parameters and apparent fracture energy for all material systems.

Laminate	d_I^*, mm	d_o, mm	G_c^*, kJ/m^2
Circular Holes			
A	1.27	0.9	120
B	1.4	1.1	70
C	1.15	1.045	35
D	1.02	0.81	25
E	0.74	0.6	15
F	1.38	0.86	50
G_1	0.72	1.01	20
G_2	0.45	1.01	33
H_1	3.82	1.01	27
H_2	0.84	1.01	57
Centre Crack			
I	1.2	0.97	29
J_1	0.72	1.01	20
J_2	0.45	1.01	33
K_1	3.82	1.01	27
K_2	0.84	1.01	57

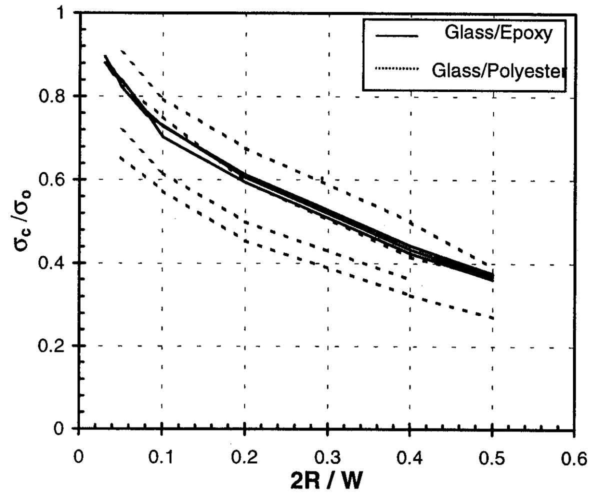

Fig. 3. Notch sensitivity of laminates D and E containing a circular hole.

5. SUMMARY

Several of the fracture models proposed to predict the notched residual strength of composite laminates have been reviewed. Experimental results were also reviewed and compared with these models. The various parameters associated with the models were determined and it was found that they depend on laminate configuration and material system. Good agreement between all fracture models and experimental notched strength can be established.

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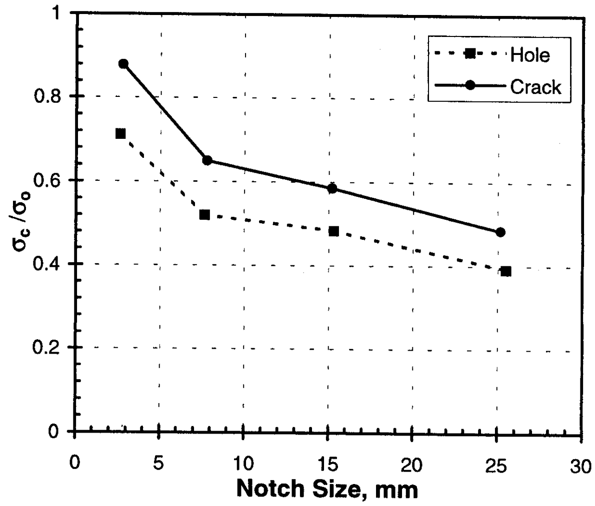

Fig. 4. Effect of notch shape on notch sensitivity of laminate K_1.

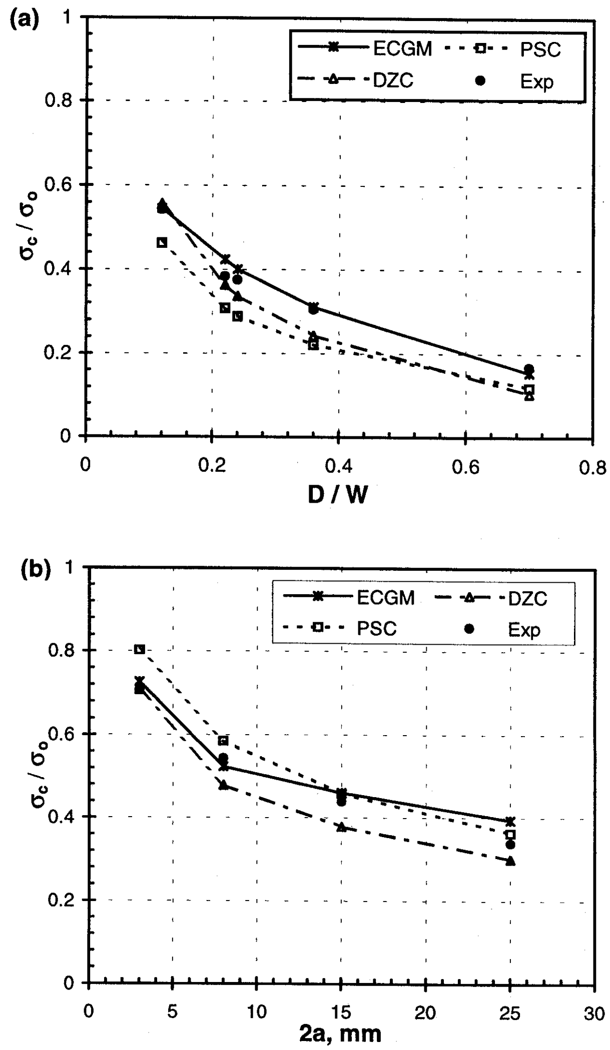

Fig. 5. Effect of notch size on residual strength (a) laminate A_1 and (b) laminate J_1.

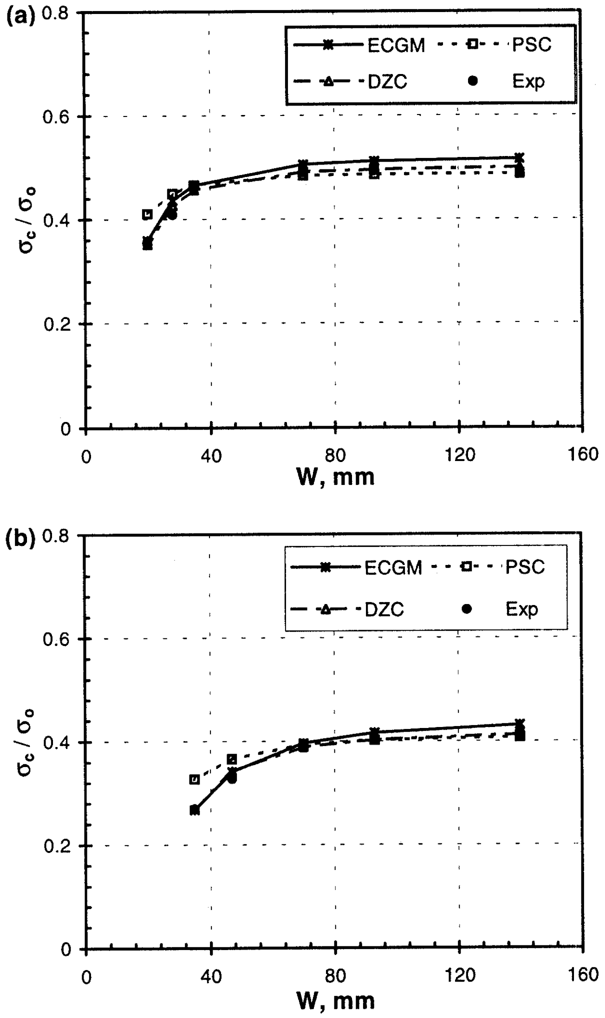

Fig. 6. Effect of specimen width on residual strength of laminate C (a) D = 10 mm and (b) D = 20 mm.

LIST OF SYMBOLS

$2a$: Crack length
a_o: Inherent crack length
a_{tot}: Total half crack length
c_I: Fictious crack length in *ith* step
D: Hole diameter
d_o: Characteristic length in PSC
d_I^*: Characteristic length in DZC
E_{11}: Longitudinal elastic modulus
E_x: Longitudinal effective elastic modulus
E_y: Transverse effective elastic modulus
G_c^*: Apparent fracture energy
G_{xy}: Shear effective elastic modulus
K_I: Mode I stress intensity factor
K_{IC}: Mode I Fracture toughness
K_Q: Critical stress intensity factor
K_T^∞: Stress concentration factor
l_c: Characteristic length in ASC
P: Applied load
R: Hole radius
t: Specimen thickness
$v(n)$: Crack opening displacement (COD)
v_c: Critical COD
$v\sigma_{app}$: COD due to applied stress
$v\sigma_{coh}$: COD due to cohessive stress
W: Specimen width
x: Distance from hole centre
σ_{app}: Applied tensile stress
σ_c: Notched residual strength
σ_n: Net cross-sectional strength
σ_o: Unnotched strength
σ_y: Elastic stress distribution

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