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*General paper*

# Upper-Bound Equation of Compressive Load-Carrying Capacity of RC Column Considering Characteristic of Material and Buckling of Primary Reinforcement

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**Abstract** : The fundamental equations estimating the compressive load-carrying capacity of reinforced concrete columns with tie and/or spiral reinforcements are used all over the world, based upon the ultimate limit state design, but the common equations include both the elastic term and the plastic one ; so, there is no unification concept of the ultimate limit state. In recent years, the high-strength type reinforcement (SBPD type) has been used frequently in the RC column and beam in Japan. Now, the common equations can not apply to the case of the high-strength primary reinforcement of the RC column. This paper describes the improvement of the concrete's sharing capacity, the application range of the common equations and the generalized practical equation for the ultimate limit state load-carrying capacity considering the buckling effect of the primary rebars.

**Key words :** *Column, Load-carrying capacity, Upper-bound, Primary rebar, Buckling, Instability failure*

## 1.INTRODUCTION

The fundamental equations estimating the compressive load-carrying capacity of RC columns are based on the ultimate limit state design method through the world. These estimated capacities give the upper limits and indicate the standard of judgment on the ultimate design so as not to exceed these values in any case ; that is, this design method is able to contribute to the integrity of the human life and property by virtue of the durability for the larger loads as the falling rock and the great earthquake, if the maximum load-carrying capacity is obtained in spite of occurrence of wider cracks than allowable crack width and larger displacement and/or deformation. Thus, the various procedures[1-2] for improving the load-carrying capacity including the ordinary or heavy confinement have been actively reported. In general expression, the design load can be determined by the load factor design which estimates the accidental large load due to multiplying the common load by a load factor. The ultimate limit state of the section failure is examined by comparing the design load with the design load-carrying capacity. In this sense, it is very important to estimate the design load-carrying capacity strictly. On the other hand, in keeping step with development of the quality of materials of RC member, the estimation equation for the compressive load-carrying capacity must be looked at again, in order to prevent a serious trouble by virtue of its misestimation. This paper deals with a proposition of the generalized equation and its theoretical background, considering the buckling of the primary rebars.

## 2. ANALSIS OF STATUS QUO

### 2.1. Common Equation

The upper limit for the design axial compressive load-carrying capacity $N'_{oud}$ is calculated by Eq.(1) where tie reinforcement is assumed, and by Eq.(1) for spiral reinforcement or by Eq.(2) if less[3-4]

$$N'_{oud} = (0.85f'_{cd}A_c + f'_{yd}A_{st}) / \gamma_b, \qquad (1)$$

$$N'_{oud} = (0.85f'_{cd}A_e + f'_{yd}A_{st} + 2.5 f_{pyd}A_{spe}) / \gamma_b, \qquad (2)$$

where, $f'_{cd}$ is the design compressive strength of concrete, $f'_{yd}$ is the design compressive yield strength of axial reinforcement, $A_c$ is the area of concrete section, $A_e$ is the area of core concrete, $A_{st}$ is the total amount of axial reinforcement, $f_{pyd}$ is the design tensile yield strength of spiral reinforcement, $A_{spe}$ is the idealize

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cross–sectional area of spiral reinforcement ($= \pi d_{sp}A_{sp}/s$), $d_{sp}$ is the diameter of core concrete, $s$ is the pitch of spiral reinforcement, $\gamma_b$ is the member factor (1.3), and 0.85 is the factor considering the strength reduction due to permanent loads, the strength difference between the test specimen and the structural concrete and so on.

### 2.2. Sharing Load-Carrying Capacity of Concrete

The meaning of the design compressive strength of concrete $f'_{cd}$ must be reconsidered from a point of view of the ultimate limit state design concept.

$$f'_{cd}=0.65f'_{ck}\ [\text{when } f'_{ck}\leq 50\text{N/mm}^2] \\ =0.57f'_{ck}\ [\text{when } f'_{ck}\geq 60\text{N/mm}^2]. \quad (3)$$

That is to say, the design compressive strength of concrete uses 57% to 65% of the characteristic compressive strength $f'_{ck}$; so, this stress level corresponds to the proportional limit of the stress-strain relation of concrete[5]. Such a procedure is not consistent with the original meaning because of the reason of the complex type consisting of the allowable stress design or the serviceable limit state design and the ultimate limit state design. Therefore, Eqs.(1) and (2) should be modified such as Eqs.(4) and (5), respectively.

$$N'_{oud} = (0.85f'_{ck}A_c+f'_{yd}A_{st})/\gamma_b, \quad (4)$$

$$N'_{oud}=(0.85f'_{ck}A_e+f'_{yd}A_{st}+2.5f_{pyd}A_{spe})/\gamma_b. \quad (5)$$

## 3. UPPER-BOUND OF AXIAL COMPRESSIVE LOAD-CARRYING IN "SHORT COLUMN"

### 3.1. In Case of Ordinary Structural Reinforcement

Hitherto, the general types of SR235, SR295, SD295, SD345 and SD390 are frequently used as structural members. If concrete and longitudinal primary reinforcement reach at the same time the ultimate compressive strength $f'_c$ and the compressive yield strength $f'_{yd}$, respectively, then the upper-bound of the extreme ultimate axial compressive capacity $N'_{ou}$ of the tied column can be given by

$$N'_{ou} = A_cf'_c+A_sf'_{yd}. \quad (6)$$

If putting the compressive strain at $f'_c$ to be 2.0‰(= permillage) [3-10], then $f'_{yd}$ = 400N/mm$^2$. Therefore, Eq. (6) is valid when the yield strength of primary reinforcement is below 400N/mm$^2$ (SD 390) and the deformation of concrete is given by the limit range as follows:

$$0.0020\leq \varepsilon \leq 0.0035 \quad \text{for } f'_{ck}\leq 50\text{N/mm}^2,$$

$$0.0020\leq \varepsilon \leq 0.0025 \quad \text{for } f'_{ck}\geq 60\text{N/mm}^2,$$

where, $f'_{ck}$ shall be assumed to be nearly equal to $f'_c$ practically. Thus, the upper-bound of the extreme ultimate axial compressive load-carrying capacity of the tied column can be transformed into Eq. (7).

$$N'_{ou} \approx A_c(1+mp)f'_{ck}, \quad (7)$$

where, $p$ is the steel ratio $A_s/A_c$, and $m$ is the strength ratio $f'_{yd}/f'_c$. Basically, Eq. (7) means that the upper-bound of the ultimate load carrying capacity increases with increases of the strength ratio, the steel ratio, that is, the quantity of steel and the concrete strength. Especially, it is worth notice that the relative ratio of the compressive yield strength of primary rebar to the concrete strength plays an important role in the ultimate load-carrying capacity.

This theoretical analysis is very useful for the design concept for the RC column.

### 3.2. In Case of High-Strength Structural Reinforcement

When the high-strength reinforcement over the yield strength 400N/mm$^2$ is used as the primary rebar, then the compressive stress of rebar $\sigma_s$ is less than $f'_{yd}$ at the concrete strength $f'_c$, that is,

$$\sigma_s < f'_{yd} \quad \text{at} \quad \sigma_c = f'_c,$$

where, $\sigma_c$ is the compressive stress of concrete. The following relation should be assumed at $\sigma_c = f'_c$ such as Eq. (8) concerning the above-mentioned limit range of deformation:

$$\sigma_s = n\cdot f'_c, \quad (8)$$

where, n is the modular ratio $E_s/E_c$, $E_s$ is the modulus of elasticity equal to 200kN/mm$^2$[10], and $E_c$ is the modulus of elasticity dependent on the characteristic compressive strength of concrete. The upper-bound of the ultimate axial compressive load-carrying capacity of the tied column consisting of the high-strength primary rebar can be obtained as follows:

$$N'_{ou}=A_cf'_c+A_s\sigma_s\approx A_c(1+np)f'_{ck}. \quad (9)$$

Basically, Eq.(9) means that upper-bound of ultimate load-carrying capacity increases with increases of the steel ratio and the concrete strength. Further, the modular ratio $n$ is given by the function of $f'_{ck}$ as follows:

$$n=\phi(f'_{ck}). \quad (10)$$

By virtue of transformation of Eq.(9), the design upper-bound of ultimate load-carrying capacity can be given as follows:

$$N'_{oud}=A\mathcal{f}'_{ck}+A_s\{\mathcal{f}'_{yd}\}, \qquad (11)$$

where, $\mathcal{f}'_{yd}\equiv\phi(f'_{ck})\cdot f'_{ck}$, that is, this term corresponds to the apparent yield strength of rebar.

## 4. FORMURATION OF EXTREME AXIAL COMPRESSIVE LOAD-CARRING CAPASITY CONSIDERING BUCKLING OF PRIMARY REBARS

### 4.1. Buckling Phenomenon of Primary Rebars

In case of the compression test of the RC column model, it is an experienced fact that the effect of primary rebars does not appear remarkably. This reason may depend on the performance that the primary rebars do not show the simple compressive strength perfectly but those result in the elastic failure by virtue of the buckling thoseselves. Figure 1 showing a damaged pier during the Han-Shin Great Earthquake Disaster in Japan,1995,may mean a phenomenal fact that the earthquake load was not only too large but also the load-carrying capacity was too little beyond estimation; further, Fig. 2 expresses the simplified buckling model of the reinforcement cage post the injury of the cover concrete of the real damaged bridge pier as illustrated in Fig. 1.

### 4.2. Upper-Bound of Load-Carrying Capacity Considering Buckling

The load-carrying capacity considering the buckling of primary rebars depends on the buckling load given by the function of the slenderness ratio. The slenderness ratio $\lambda$ is denoted by Eq.(12).

$$\lambda=l/(\phi/4), \qquad (12)$$

where, $l$ and $\phi$ are the length and the diameter of the rebar, respectively. When both ends of rebar are pin-connections, the critical slenderness ratio $\Lambda$ and the buckling stress $\sigma_s$ by Rankine's equation[11] are given by Eqs.(13) and (14), respectively.

$$\Lambda=(\pi^2E_s/f'_{yd})^{1/2}, \qquad (13)$$

$$\sigma_s=f'_{yd}/[1+f'_{yd}\lambda^2/\pi^2E_s], \qquad (14)$$

where, $E_s$ and $f'_{yd}$ are above-mentioned.
The upper-bound of load-carrying capacity $N'_{oub}$ considering the buckling effect, basically, can be expressed by Eq.(15), because the buckling stress when the long column generally is smaller than the simple compresive strength.

$$N'_{oub}=A\mathcal{f}'_c+A_s\sigma_s. \qquad (15)$$

Fig.1. An example of damaged pier during great earthquake.

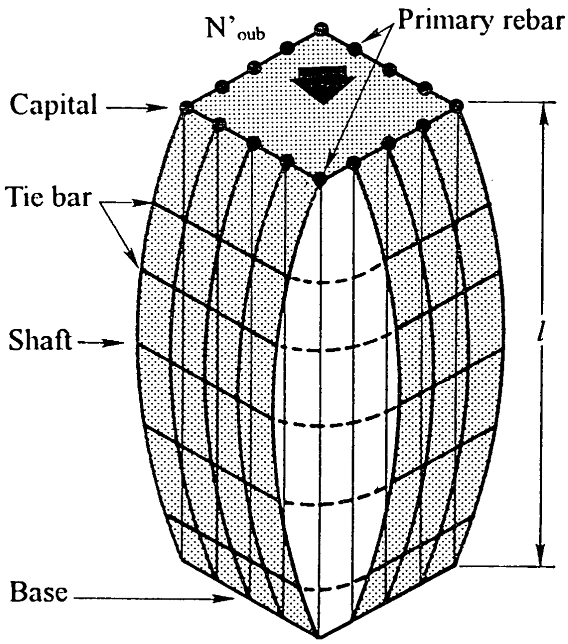



Fig.2. Modeling of buckling of primary rebars in RC column.

### 4.3. Experimental Verification of Upper-Bound of Load-Carrying Capacity Considering Buckling

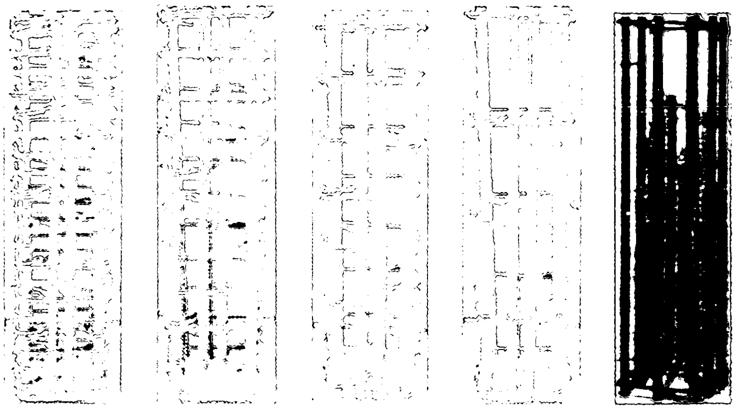
#### 4.3.1. Preparation of RC column model

D13( $\phi$ =12.7mm; SD type $f'_{yd}$=333 N/mm$^2$) and U13( $\phi$ =13.1mm; SBPD type $f'_{yd}$=1424N/mm$^2$) for the primary rebars, and U6.4(SBPD type) for the tie bar were used for preparation of the reinforcement cages. The specimen size of the column model and the core size were 150 × 150 × 530mm and 120 × 120mm, respectively. The pitch nominal spacings were five kinds of 25, 50, 75, 125, and 500mm. Figure 3 illustrates the examples of reinforcement cages when SD type. The average compressive strength of the structural concrete with the maximum size of aggregate of 10mm was 39.4N/mm$^2$ at 28days under-water curing. The procedure placing concrete is first to fill up it into the reinforcement cage, secondly to set down the filled cage into the mould for flexure, thirdly to pour the screening mortar into the part of covering and lastly enough to compact the whole to be in a body by the table type vibrator. The compression test was carried out by use of the 5000kN universal type testing machine.

#### 4.3.2. Experimental result

a) Failure mode

Figures 4 and 5 show the failure modes in cases of the spacings 25mm, 50mm, 75mm, 125mm and 500mm for the SD type primary rebar and for the SBPD type one, respectively. In general, the crackings on the primary rebars and the spall-off of covering concrete are distinguished. The case of the spacing $s$=25mm in both the SD type and the SBPD type is the most ductile and the effective cross-sectional area is never spalled off. The case of the spacing $s$=500mm is the most brittle and that the effective cross-sectional area happens to be deeply spalled off to the extent of about thirty percent as the same as the other paper[12]. The cases of the spacings $s$=50 and 75mm are moderately ductile and have been already observed that the effective cross-section are spalled off only to some extent.



①$s$=25 ②$s$ =50 ③$s$ =75 ④$s$ =125 ⑤$s$ =500

Fig. 3. Reinforcement cages when SD type ($s$ : mm).

①$s$=25 ②$s$ =50 ③$s$ =75 ④$s$ =125 ⑤$s$ =500

Fig. 4. Failure mode of SD type RC column ($s$: mm).

①$s$ =25 ②$s$ =50 ③$s$ =75 ④$s$ =125 ⑤$s$ =500

Fig. 5. Failure modes of SBPD type RC column ($s$:mm).

b) Relation between load-carrying capacity and spacing of tie bars

Figure 6 displays the relationship between the load-carrying capacity and the spacing of tie bars. In any case, the load-carrying capacity increases with decrease of the pitch spacing; furthermore, the use of the higher-strength primary rebar is very advantageous to improve it. Such a general tendency is as similar as previously reported[12]. Especially, a large attention must be paid to the fact that the load-carrying capacity gradually approaches an asymptote, that is, "the upper-bound of ultimate load-carrying capacity" in spite of the difference in quality of primary rebars of RC columns.

### 4.4. Theoretical Verification of Upper-Bound Load-Carrying Capacity Considering Buckling

Table 1 points the terms required for calculating the extreme ultimate load-carrying capacity considering the buckling of primary rebars. It will result in the judgment of a "long column" because of "$\lambda > \Lambda$". The upper-bound of ultimate load-carrying capacities considering the buckling $N'_{oub}$ for the SB type and the SBPD type are plotted together in Fig. 6. The asymptotes of both curves are 623kN for the SD type and 638kN for the SBPD type, respectively; that is, it may safely be said that both curves approaches nearly one point of about 630kN in spite of the difference of strengths of primary rebars. This fact means that the depend on the yield strength of primary rebar diminishes because its buckling strength decreases according as the decrease of the lateral confining effect.

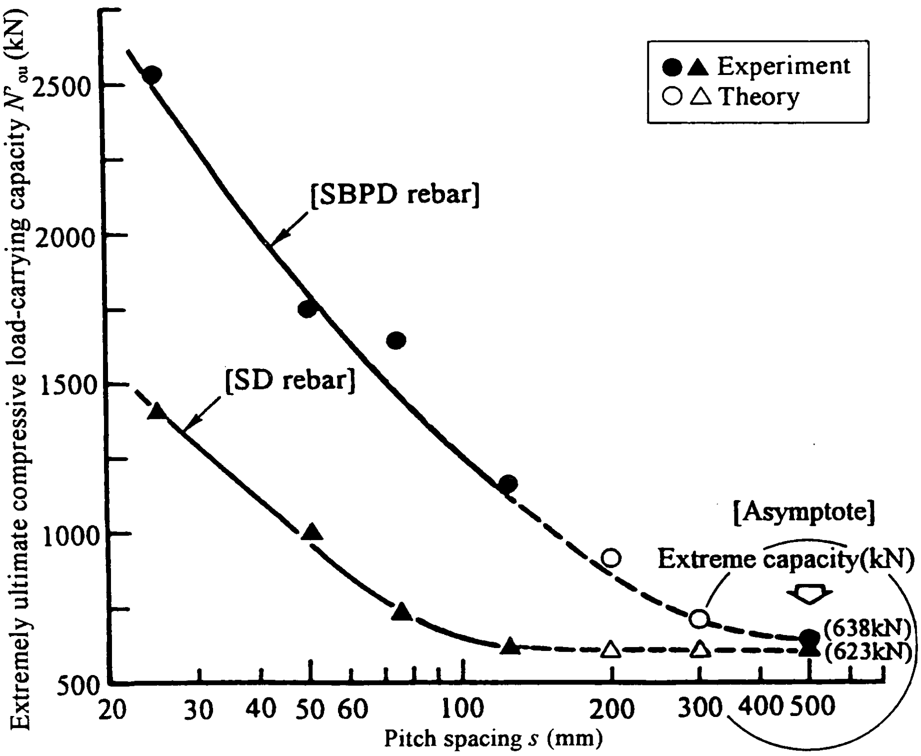



Fig. 6. Relationship between load-carrying capacity and spacing of tie bars.

Table 1. Calculation items for buckling load-carrying capacity.

| Primary rebar | $l$ (mm) | $\phi$ (mm) | $k$ (mm) | $\lambda$ (=$l$ /$k$) | $f_{yd}$(N/mm$^2$) | $E_s$(kN/mm$^2$) |
|---|---|---|---|---|---|---|
| SD type | 530 | 12.7 | 3.18 | 166.7 | 333 | 190 |
| SBPD type | | 12.6 | 3.15 | 168.3 | 1424 | 201 |

| Critical slenderness ratio $\Lambda$ | $\lambda > \Lambda$ | $\sigma_s$(N/mm$^2$) | $A_s$(mm$^2$) | $A_e$(mm$^2$) | $N'_{oub}$(kN) |
|---|---|---|---|---|---|
| 75.0 | Long column | 56.1 | 1520.4 | 12,880 | 623 |
| 37.3 | | 66.8 | 1500.0 | 12,900 | 638 |

The buckling stresses for the yield strengths 333N/mm$^2$ (SD type) and 1424 N/mm$^2$ (SBPD type) are approximately 58 N/mm$^2$ and 67N/mm$^2$, respectively; so, the difference between them is only 9N/mm$^2$. Therefore, it stands to reason that the extreme ultimate load-carrying capacity con- sidering the buckling of primary rebar in the RC column without ties gradually approaches almost a constant value.

Thus, such an extreme ultimate load-carrying capacity, basically should be adopted as the upper-bound for the design compressive load-carrying capacity, taking into the member factor and the structure one. Now, The relative ratios in comparison of the load- carrying capacity by the common equation; Eq.(1) with the one by the practical modified equation ; Eq.(15) are as follows:

① When SD type: 835kN [Eq.(1)]/623kN[Eq.(15)] =1.34

② When SBPD type:2089kN[Eq.(1)]/623kN[Eq.(15)] = 3.27

That is, the common equation for the upper-limit load-carrying capacity gives the 1.34 times excessive larger value for the ordinary steel and the 3.27 times excessive larger value for the high-strength steel. Such a misestimation may be considered to be one of dominant causes for the damage of the structural columns from the earthquake. The failure modes of the experimental RC column display the various patterns as shown in Figs. 4 and 5.

The primary rebars phenomenally buckle in a state of net as shown in Fig. 2, because of the constructional difficulty on the joint of tie bars; therefore, it exists in the safety side to assume that the net fails with the height from the capital to the base.

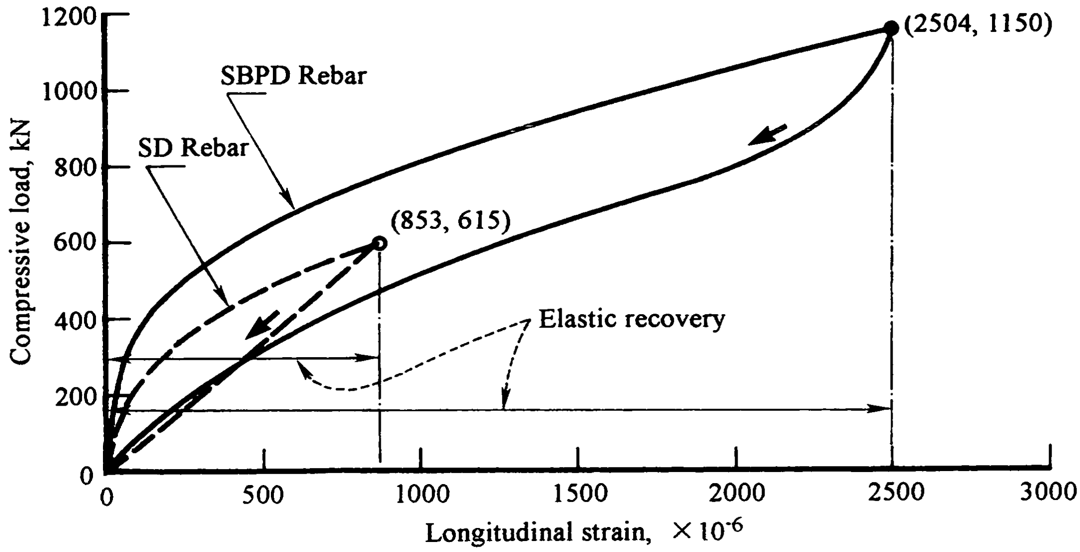



Fig. 7. Load-deformation diagrams of primary rebars in RC columns confined sparsely when $s$=125mm.

### 4.5. Experimental Verification of Buckling Stress

Whether the stress of primary rebar at the onset of plastic deformation concerning the behavior of RC column is in an elastic state or in a plastic one.

Figure 7 plots the load-deformation diagrams when the primary rebars are an ordinary steel and a high-strength one, in either case, with the spacing of tie bars of 125mm.

The comparison of the apparent compressive stress of primary rebar $\sigma_s$ at the extreme loading with its yield strength $f_{yd}$ is as follows:

① When an ordinary rebar,

$\sigma_s = 200 \times 10^3 \times 853 \times 10^{-6} = 170\mathrm{N/mm^2}$

$\ll f_{yd} = 333\mathrm{N/mm^2}$ ; "elastic."

② When a high-strength rebar,

$\sigma_s = 200 \times 10^3 \times 2504 \times 10^{-6} = 500\mathrm{N/mm^2}$

$\ll f_{yd} = 1424\mathrm{N/mm^2}$ ;"elastic."

Such a phenomenal fact means that the "buckling", that is, the "instability failure" must occur certainly even under an ordinary confinement concerning the primary rebars in RC columns in spite of the elastic stress state.

## 5. SYNTHETIC DISCUSSION

In general, when designing the RC column, the judgment whether the primary rebar is the long column or the column must be carried out by Fig. 7, giving the relationship between the buckling stress and the slenderness ratio as the parameter of the yield strength of primary rebar, as the first step; when the "short column" and the ordinary steel concerning the primary rebar, Eq.(6) or Eq.(7) should be adopted. On the other hand, when the "short column" and high-strength steel concerning the primary rebar, Eq.(9) or Eq.(11) should be adopted. If the judgment is the "long column", then Eq.(15) should be adopted, by using Eq.(14) referring

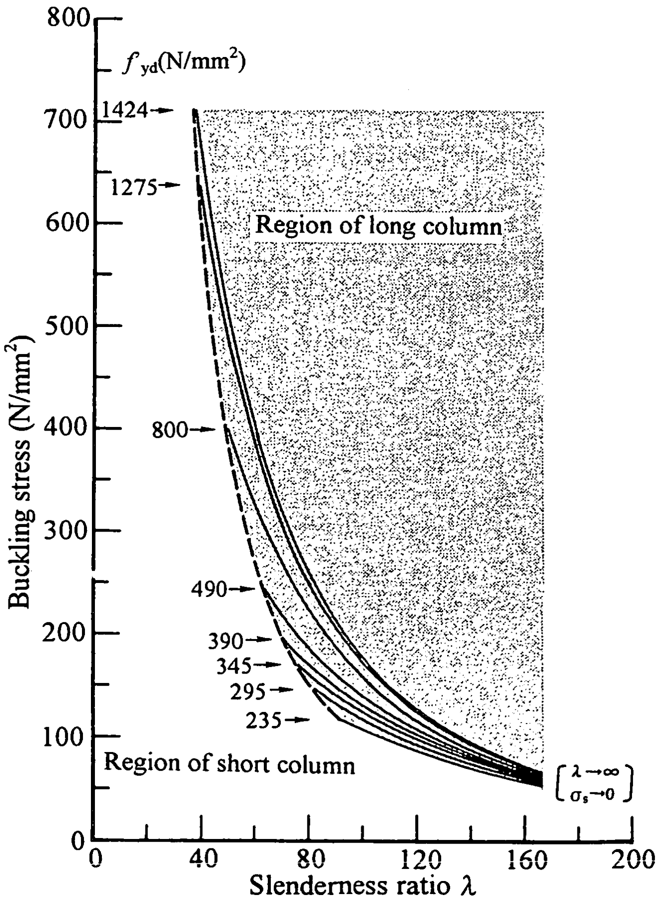



Fig. 8. Relationship between buckling stress and slenderness ratio as parameter of yield strength of primary rebar.

Table 2. Design steps of procedure.

| | | |
|---|---|---|
| Judgment concerning long column or short column [cf.Fig.9] | Short column | Ordinary steel for $f_{yd}$<400N/mm$^2$[Eq.(6)or(7)] |
| | | High-strength steel for $f_{yd} \geqq$400N/mm$^2$[Eq.(9)or(11)] |
| | Long column [cf.Eq.(15) or Eq.(16)] | |

to Fig.8. Note worthily, the primary rebar does not yield, in consideration of the buckling but it behaves only in state of the elastic failure. Especially, when the slender- ness ratio of primary rebar is enough large like the existing column, it can be well understood that its load-carrying capacity results in diminish owing to $\lim_{\lambda \to \infty} \sigma_s$[Eq.(14)]=0. In the final analysis, when estimating the safest load-carrying capacity of the RC column, the upper-bound equation ignoring not only the effect of tie bars but also that of primary rebars must be adopted, as given by

$$N'_{oub}=A_c f'_c \simeq A_c f'_{ck}. \qquad (16)$$

Table 2 summarizes these steps of procedure.

## 6. CONCLUSIONS

(1) The present common equations for the upper-bound of compressive load-carrying capacity concerning both tied and spiral columns are the complex type consisting of the "serviceable limit state design" and the ultimate limit state design; so, such a inconsistent procedure is contrary to the "ultimate limit state design concept ".
(2) The design compressive strength $f'_{cd}$ in the common equations should be substituted for the characteristic compressive strength $f'_{ck}$.
(3) The judgment concerning the long column and the short one of the primary r
ebar in the RC column design must be carried out.
(4) If the primary rebar is a short column and an "ordinary steel", Equations (6) and (7) must be adopted selectively.
(5) When a short column and a "high-strength steel", Equations (9) and (11) must be adopted selectively.
(6) If the primary rebar is a long column, Equation (15) considering the buckling stress must be used, being the frequent cases, practically.
(7) The stress of primary rebars confined ordinarily under the extreme load was in an elastic state experimentally in the present paper, too; therefore, such a fact suggests the occurrence of an "instability failure".
(8) In general the common equation for the upper- bound of load-carrying capacity gives the excessive larger estimation than the existing state; so, it may be attended with danger as a dominant cause for the damage of the structural columns from the earthquake.

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