Natural Disaster Science, Volume 6, Number 1, 1984, pp. 17–30

APPLICATION OF THE EQUI-RISK LINE THEORY TO THE DESIGN OF A DETENTION RESERVOIR

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(Received 4 Jun., 1984 and in revised form 27 Sept., 1984)

ABSTRACT

To drain or to reserve? That is the question discussed in this paper. The capacities of drainage and storage facilities are determined to minimize the total cost under the condition that chances of flooding do not exceed a tolerable level. This paper presents a pilot work to derive an analytical expression of the chance constraint for a flood control system with a storage facility.

1. INTRODUCTION

Storage facilities have assumed a major role in flood control planning. As an example, the design flood discharge of the Neya River system, one of the most intractable urban rivers in Japan, is shown in Fig. 1. On the average, in this system, 25% of the peak discharge is handled by pumps, and another 25% by detention reservoirs (detention parks). These percentages, however, vary from one area to another within the system. For example, upstream on the second Neya River, the peak discharge is released through the river channel at the rate of 100 m^3/sec, and is stored in the Onji River detention park at 240 m^3/sec. Therefore, in this particular area, up to 70% of the peak discharge is stored in the storage facility. The Neya-Ohkawa River system (the old Yodo River system) presents another example of storage. All three of the large gates at the river mouths can be closed against storm surge. With all the gates closed, a large portion of the storm rainfall must be stored in the basin, some of it being pumped out. This trend of giving an increasingly major role for storage facilities is seen in flood control planning for most major rivers as well as for urban river networks in Japan.

If a flood is to be controlled only by a drainage facility such as a channel or a drainage pump, the facility must be designed so that the chance of the peak discharge of a flood exceeding the drainage capacity stays within a given risk level. Thus, the capacity of a drainage facility is based on a frequency analysis of the "peak discharge" of floods. By contrast, if a storage facility alone, such as a dam, or a detention or underground reservoir, is responsible for the control of floods, its capacity must be large enough to store the total volume of the runoff. Therefore, it should be evaluated through a frequency analysis of the "total runoff". Consequently, when both drainage and storage facilities are used, the risk in a flood control system may be reasonably evaluated from the joint frequency distribution of the "peak discharge" and the "total runoff" of a flood. The capacities of the drainage and storage facilities then can be determined taking into account both the risk and the total cost.

Footnote: Most of this work was presented at the 4th IAHR International Symposium on Stochastic Hydraulics [1].
KEY WORDS: Flood, Risk, Reservoir
Note: Discussion open until 1 December, 1985

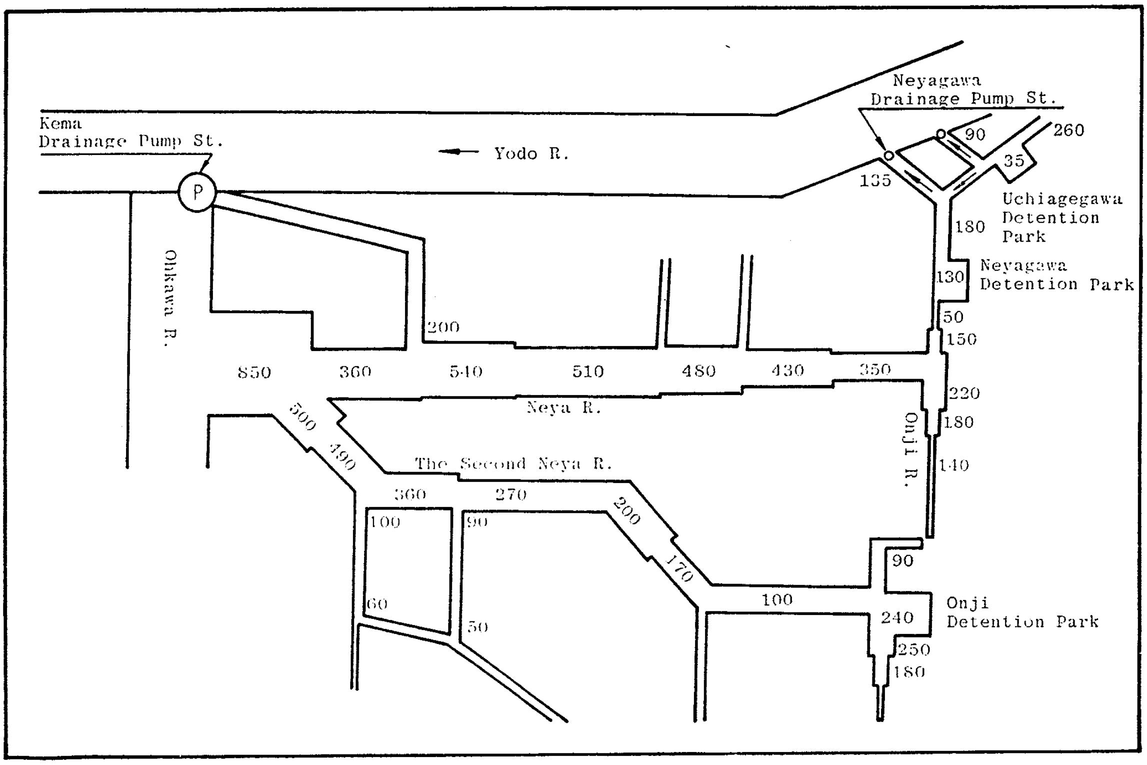

Fig. 1 Design flood discharge of the Neya River system (m³/s)

Using joint frequency distribution, the authors derived an expression for the contour of risk on a plane on which the lateral and vertical axes represent the respective drainage and storage capacities (Fig. 2). Hereafter, this contour will be called the "equi-risk line". On this plane, the chance constraint is defined as the lower area to the left of the equi-risk line (Fi.g 2).

Expressions of the equi-risk line are derived for two types of flood control systems: (1) a single storage facility and a single drainage facility (module 1 in Fig. 3), and (2) a single storage facility and two drainage facilities (module 2). A theoretical analysis of the equi-risk line for module 1 is presented here. Analysis of module 2 has been presented elsewhere in Japanese [2] and is not given in this paper because of its complexity and the limitations of space. Its results, however, are a straightforward extension of the results for module 1, and are rather simple and practical. So, the final result is presented later in "Additional Comments".

The results of the theoretical analysis for module 1 are applied to risk-evaluation in flood control by an actual detention reservoir. The application proves the validity of the presented analysis.

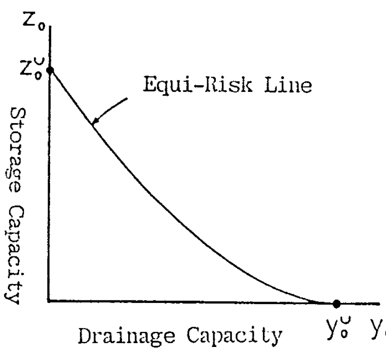

Fig. 2 Schematic explanation of an equi-risk line

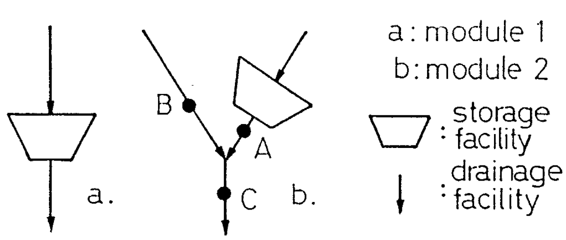

Fig. 3 Modules of flood control systems

2. THEORETICAL ANALYSIS

Fundamental Equations

a. Definition of the Equi-Risk Line

Let y_0 denote the drainage capacity and z_0 the storage capacity. Let λ and ε be the respective average annual frequencies of flood and of failure in flood control. Then, ε or ε/λ is the risk level. If the risk level is constant, when y_0 is sufficient, z_0 may be small or even non-existent. The converse is also true; a small drainage capacity y_0 requires a large storage capacity z_0. From this, we can assume the functional relation between y_0 and z_0,

$$G(y_0, z_0) = \varepsilon \text{ or } G(y_0, z_0) = \varepsilon/\lambda . \tag{1}$$

This equation represents the equi-risk line.

Typical examples of release rules for a storage facility are shown in Fig. 4: They are the constant release rule, the constant-ratio release rule and their combined rule. We shall derive equations of the equi-risk line for the constant and constant-ratio release rules.

b. Constant Release Rule

Let y, z, and z', respectively, denote peak discharge, total volume and the total volume of discharge that exceeds the drainage capacity y_0. The average annual frequency with which the peak discharge y exceeds y_0 is expressed as

$$\{1-F_y(y_0)\}\,\lambda \tag{2}$$

in which F_y is the cumulative distribution function (c.d.f.) of the peak discharge, y. When $y>y_0$, some discharge must be cut or stored in a storage facility. The probability that, subject to $y>y_0$, the storage facility becomes full and fails to control flooding is expressed as

$$\{1-F_{z'}(z_0)\,|_{y>y_0}\} \tag{3}$$

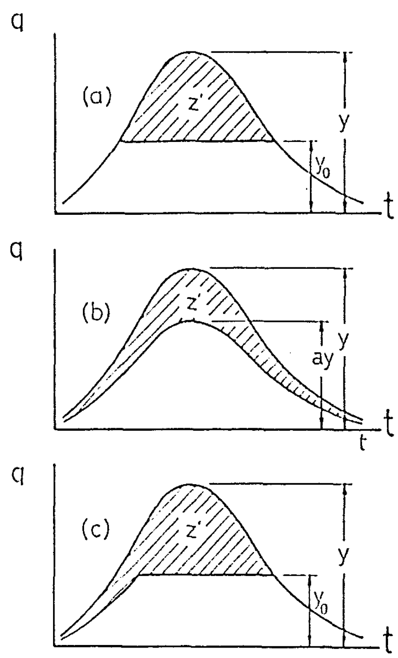

Fig. 4 Typical patterns of the release rule
(a) Constant release rule
(b) Constant-ratio release rule
(c) Combined rule of (a) and (b)

in which $F_{z'}(z')|_{y>y_0}$ is the c.d.f. of z'. Therefore, the equation of the equi-risk line is

$$\{1-F_y(y_0)\}\,\lambda\,\{1-F_{z'}(z_0)|_{y>y_0}\} = \varepsilon\,. \tag{4}$$

In Eq. (4)

$$\begin{aligned} 1-F_y(y_0) &= \mathrm{Prob}\,(y>y_0)\,, \\ 1-F_{z'}(z_0)|_{y>y_0} &= \mathrm{Prob}\,(z'>z_0\,|\,y>y_0) \\ &= \mathrm{Prob}\,(z'>z_0 \cap y>y_0)/\mathrm{Prob}\,(y>y_0)\,. \end{aligned} \tag{5}$$

Substituting (5) in (4),

$$\mathrm{Prob}\,(z'>z_0 \cap y>y_0) = \varepsilon/\lambda\,. \tag{6}$$

When the peak discharge y is less than the drainage capacity y_0, no water is stored in the facility when the constant release rule is applied. Thus,

$$\mathrm{Prob}\,(z'>z_0 \cap y\leqq y_0) = 0\,.$$

Therefore,

$$\begin{aligned} &\mathrm{Prob}\,(z'>z_0) \\ &\quad = \mathrm{Prob}\,(z'>z_0 \cap y>y_0)+\mathrm{Prob}\,(z'>z_0 \cap y\leqq y_0) \\ &\quad = \mathrm{Prob}\,(z'>z_0 \cap y>y_0)\,. \end{aligned} \tag{7}$$

Substituting (7) in (6), the equation for the equi-risk line is reduced to

$$\mathrm{Prob}\,(z'>z_0) = \varepsilon/\lambda\,. \tag{8}$$

Eq. (4) or (8) is the fundamental equation of the equi-risk line for the constant release rule.

We shall discuss some special cases. Let y_0^u represent the capacity of a drainage facility when there is no storage facility and floods are controlled solely by means of the drainage facility. Then, $z_0=0$ and $F_{z'}(z_0)|_{y>y_0}=0$, and eq. (4) is reduced to

$$1-F_y(y_0^u) = \varepsilon/\lambda\,. \tag{9}$$

Eq. (9) shows that y_0^u can be estimated easily using the c.d.f. of the peak discharge without any information about the joint distribution of the peak discharge and the total volume of runoff. Similarly, the capacity of a storage facility z_0^u which is necessary to control a flood solely by storage without drainage can be estimated using only the c.d.f. of the total volume of the runoff. Therefore, the equi-risk line for the constant release rule begins or ends at two points (y_0^u, 0) and (0, z_0^u), which can be easily spotted only if the risk level and the c.d.f.s of the peak discharge and of the total volume are given individually. Then, all we have to do is to derive the expression of the equi-risk line between the two points.

c. Constant-Ratio Release Rule

Let α denote the rate of discharge from a storage facility. We shall assume that α is constant. The peak release discharge and the total volume of water to be stored then are expressed as αy and $(1-\alpha)\,z$. When $\alpha y>y_0$ or $(1-\alpha)z>z_0$, the system fails to control flooding. The failure probability is

$$\begin{aligned} &\mathrm{Prob}\,\{\alpha y>y_0 \cup (1-\alpha)\,z>z_0\} \\ &\quad = 1-\mathrm{Prob}\,\{\alpha y\leqq y_0 \cap (1-\alpha)\,z\leqq z_0\} \\ &\quad = 1-\mathrm{Prob}\,\{y\leqq y_0/\alpha \cap z\leqq z_0/(1-\alpha)\}\,. \end{aligned} \tag{10}$$

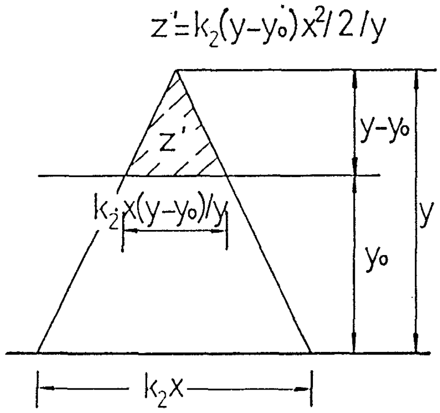

Fig. 5 Triangular hydrograph

Therefore, the equation of the equi-risk line is

$$\{1-F_{yz}(y_0/\alpha, z_0/(1-\alpha))\}\ \lambda = \varepsilon \tag{11}$$

in which $F_{yz}(y, z)$ is the joint c.d.f. of the peak discharge y and the total volume of the runoff z. Eq. (11) is the fundamental equation of the equi-risk line for constant-ratio release rule.

Triangular Hydrograph

a. Expression for Partially-Correlated x and y

Proceeding further with the analysis, we assume that the hydrograph of each flood can be approximated by a triangular shape (see Fig. 5). The duration of some floods may be long and others short, and some floods may have high peaks, others not. Therefore, let both the duration x and the peak discharge y of a flood hydrograph be stochastic variables. The combination of the values of x and y realizes hydrographs of various shapes, such as flat or sharp ones, under a given probability law.

We assume the constant release rule in the following analysis. In Fig. 5,

$$z' = 1/2 \cdot k_2 \cdot x\,(y-y_0)^2/y\,. \tag{12}$$

Therefore, the condition $z'>z_0$ becomes

$$x>2/k_2 \cdot z_0 \cdot y/(y-y_0)^2\,. \tag{13}$$

Then, eq. (8) is represented in terms of x and y with the parameters y_0 and z_0:

$$\text{Prob}\ \{x>2/k_2 \cdot z_0 \cdot y/(y-y_0)^2\} = \varepsilon/\lambda\,. \tag{14}$$

On the y_0-z_0 plane, the domain represented by (13) is the hatched area shown in Fig. 6.

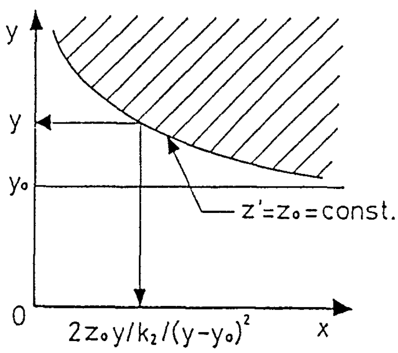

Fig. 6 Domain of integration for Module 1

Therefore, the equation of the equi-risk line is

$$\int_{y_0}^{\infty}\int_{a}^{\infty} \mathrm{g}(x, y)\, \mathrm{d}x\, \mathrm{d}y = \varepsilon/\lambda \tag{15}$$

in which $\mathrm{g}(x, y)$ is the joint probability density function of x and y, and

$$a = 2/k_2 \cdot z_0 \cdot y/(y-y_0)^2\,. \tag{16}$$

Consequently, if the joint probability law of duration and peak discharge of a flood hydrograph is provided, the equation of equi-risk line is specified by the integration of the joint p.d.f.

To outline the basic characteristics of an equi-risk line, we shall derive the equation of equi-risk line for two extreme conditions:

1) duration x and peak y are linearly and perfectly dependent (or proportional), and
2) x and y are independent.

Usually, x and y are partially dependent. Thus, the equi-risk line for a general condition may lie between the lines depicted for the two extreme conditions.

b. Linearly Dependent x and y

Peak y is proportional to duration, x. Thus,

$$y = k_1 x \tag{17}$$

in which k_1 is a constant.
Substituting (17) in (14),

$$\begin{aligned} &\mathrm{Prob}\,\{x > 2/k_2 \cdot z_0 \cdot k_1 x/(k_1 x - y_0)^2\} \\ &\quad = \mathrm{Prob}\,\{x > y_0/k_1 + \sqrt{2z_0/(k_1 k_2)}\,\} \\ &\quad = 1 - \mathrm{F}_x\{y_0/k_1 + \sqrt{2z_0/(k_1 k_2)}\,\} = \varepsilon/\lambda\,. \end{aligned} \tag{18}$$

Therefor,

$$y_0/k_1 + \sqrt{2z_0/(k_1 k_2)} = c \tag{19}$$

in which

$$c = \mathrm{F}_x^{-1}(1-\varepsilon/\lambda) = \mathrm{Const.}$$

After some arithmetical manipulation, eq. (19) is reduced to

$$z_0/z_0^u = \{(y_0^u - y_0)/y_0^u\}^2 \tag{20}$$

in which

$$y_0^u = k_1 c,\ z_0^u = 1/2 \cdot k_1 k_2 c^2\,.$$

Eq. (20) shows that an equi-risk line for linearly dependent x and y is precisely a quadratic parabola as shown in Fig. 7, regardless of the type of distribution of the durations or of the peak discharge of the hydrographs.

c. Independent x and y,

When y is independent of x, no explicit solution of the equation of the equi-risk line can be derived. Thus, some numerical calculation is unavoidable to make clear its characteristics. To reduce the number of cases required for analysis, we shall additionally assume that both the duration x and the peak discharge y are exponentially distributed, an assumption which should be accepted as realistic. Then,

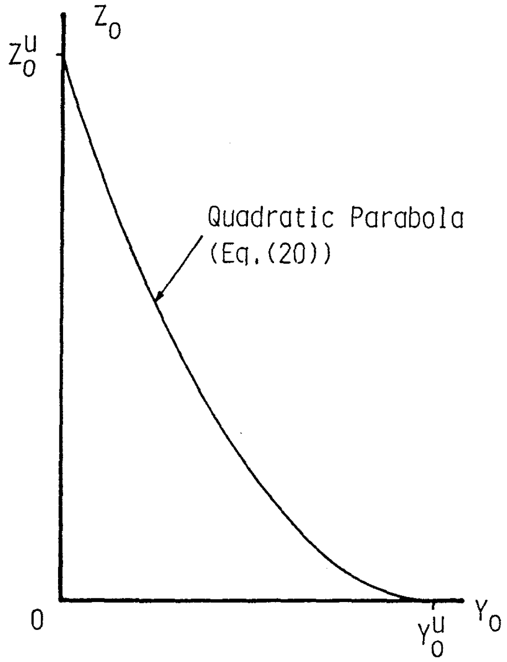

Fig. 7 Equi-risk line for linearly dependent x and y and for the constant release rule

$$g(x, y) = f_x(x)\cdot f_y(y)$$
$$f_x(x) = \beta_x \exp(-\beta_x x)$$
$$f_y(y) = \beta_y \exp(-\beta_y y) \tag{21}$$

in which β_x and β_y are scale parameters of x and y.
Substituting (21) in (15), and nondimensionalizing it,

$$\int_{Y_0}^{\infty} \exp[-\{\eta + Z_0\cdot\eta/(\eta - Y_0)^2\}]\, d\eta = \varepsilon/\lambda \tag{22}$$

in which Y_0 and Z_0 are the respective dimensionless drainage and storage capacities, i.e.,

$$Z_0 = 2\beta_1 z_0/(k_1 k_2),\ Y_0 = \beta_y y_0,\ \eta = \beta_y y\,.$$

Eq. (22) is the dimensionless expression of the equi-risk line for independently and exponentially distributed x and y.

By differentiating (22) with respect to Y_0, the derivative dZ_0/dY_0 is derived. The authors have shown that [3]

(1) $dZ_0/dY_0=0$ at $(Y_0^u, 0)$ and

(2) $dZ_0/dY_0<0$ at $(0, Z_0^u)$.

(1) indicates that an equi-risk line is in contact with the Y_0-axis, and (2) shows that the line intersects with the Z_0-axis at a negative inclination. These results predict that the shape of an equi-risk line for independently and exponentially distributed x and y may be similar to that for linearly dependent x and y which is expressed by eq. (20) or in Fig. 7. Thus, the authors have proposed the following approximate expression for the equation of the equi-risk line for independent x and y;

$$Z_0/Z_0^u = \{(Y_0^u - Y_0)/Y_0^u\}^s \quad \text{or}$$
$$z_0/z_0^u = \{(y_0^u - y_0)/y_0^u\}^s\,. \tag{23}$$

Equi-risk lines can be drawn from numerical calculation using (22). An example of the nu-

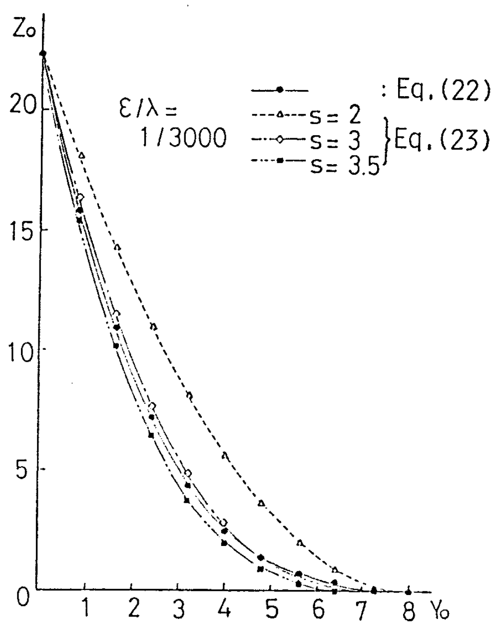

Fig. 8 Comparison of exact and approximate expressions ((22) and (23)) for independent x and y

Table 1. s for various risk levels estimated by the Least Squares Method

ε/λ	s
1/10	2.974
1/100	3.039
1/1000	3.288
1/10000	3.172

merical calculation is shown with a solid line in Fig. 8. Other lines in the figure represent the approximate expression (23) for $s=2$, 3 and 3.5. This indicates that eq. (23) can approximate the exact expression (22) when $s\fallingdotseq 3$. The values of s estimated by the least squares method for various risk-levels ε/λ are shown in Table 1. It is clear from the figure and the table that the equi-risk line for independently and exponentially distributed x and y is approximated by a cubic parabola, which leads us to the conclusion that (1) an equi-risk line can be approximated by a simpel expression (23), and (2) that $s=2$ to 3 for general conditions.

3. ESTIMATION OF 's' FROM OBSERVED DATA

The authors proposed a practical procedure to estimate the value of s from observed hydrographs or hyetgraphs. Logarithmic transformation of (23) leads to

$$\log(z_0/z_0^u) = s\cdot\log\{(y_0^u-y_0)/y_0^u\}\,. \tag{24}$$

In eq. (24), y_0^u and z_0^u can be estimated individually from the probability distributions of peak discharge and total runoff volume. Therefore, if a pair of values, y_0 and z_0, are given, s can be estimated by substituting the values of (y_0^u, z_0^u) and (y_0, z_0) in eq. (24). Practically, the following procedure is useful for estimating s.

a. Selection of Flood Hydrographs

1) Set the threshold discharge y_B.

2) If the observed discharge q' is equal to or less than y_B, discharge q at that time is set to zero. And if $q'>y_B$, $q=q'-y_B>0$.

It is recommended to set y_B so as to keep the number of flood hydrographs at a suitable number, say, about one hundred. A hydrograph in which the peak discharge is greater than y_B is regarded as a flood hydrograph. Then, $y=y''-y_B$, in which y'' is an actually observed peak discharge. If the number of hydrographs selected is N, we have N sets of observed data for y and z'.

Note that equi-risk lines drawn from the transformed discharge data should be moved parallel to the y_0-axis by y_B,

b. Estimation of y_0^u and z_0^u

1) Fix the risk level ε/λ at some level.

2) Estimate y_0^u and z_0^u, where

$$y_0^u = F_y^{-1}(1-\varepsilon/\lambda), \text{ and } z_0^u = F_z^{-1}(1-\varepsilon/\lambda). \tag{25}$$

y_0^u can be estimated either by parametric statistical analysis of the observed peak discharge data, as shown in eq. (25), or by graphic analysis such as Weibull plotting of the data on probability paper.

c. Sets of y_0 and z_0

1) Fix y_0.

2) Select hydrographs of which peaks are greater than y_0.

3) Calculate z', the total volume of discharge that exceeds y_0, for each selected hydrograph.

4) Estimate z_0 for the given y_0 and ε/λ' from

$$z_0 = F_{z'}^{-1}(1-\varepsilon/\lambda') \tag{26}$$

or from Weibull plot of z'. λ' is defined by eq. (27) and (28).
The shift of y_0 in steps and the repetition of procedures 1) to 4) produce sets of values for y_0 and z_0.

d. Estimation of 's'

1) Plot a point at $\{(y_0^u-y_0)/y_0^u, z_0/z_0^u\}$ on full-log paper for each set of y_0 and z_0.

2) Then, the value of s can be estimated as the gradient of a straight line fitted to the points.

The proposed procedure seems complicated when explained in words, but in fact it is not. Readers will find the whole procedure straightforward after actually tracing it once using observed data.

We have applied this procedure to discharge and precipitation sequences. The values of s obtained in the applications range from 1.5 to 4. Taking into account statistical errors, these results support the theoretical prediction that the value of s should be between 2 and 3.

4. APPLICATION

The Basin and the Data

The proposed procedure was applied to a flood control system with a detention reservoir (Fig. 9). The hourly rainfall sequence recorded from Jan. 1974 to July 1983 (9.54 years) was transformed into a discharge sequence through runoff analysis and used as the basic data. The threshold y_B was set at a discharge level corresponding to 5 mm/hr of rainfall intensity. Eighty-four flood hydrographs then were selected.

Analysis

The plot of the peak discharge is shown in Fig. 10. Similar plots had to be made for the total volume z, and for the total volume of discharge exceeding y_0, i.e., z', for some levels of y_0.

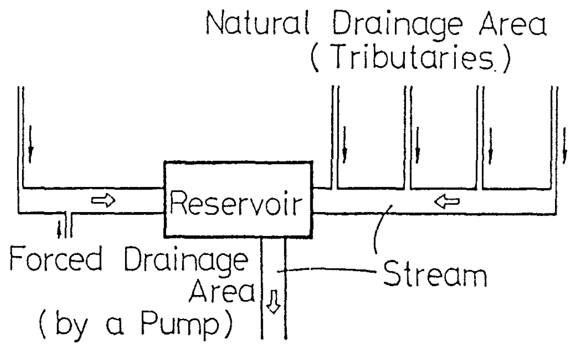

Fig. 9 Model of an existing flood control system with a detention reservoir

Let T denote the average recurrence interval of failure of flood control. When T is sufficient and the drainage capacity is y_0, the frequency of failure is expressed as:

$$\varepsilon = 1/T = \lambda' \cdot p \tag{27}$$

in which λ' is the average frequency of floods exceeding y_0, and p is the frequency at which the storage facility becomes full and fails to control a flood exceeding y_0. Therefore,

$$\lambda' = N'/T_0, \quad p = 1/(\lambda' T) = \varepsilon/\lambda' \tag{28}$$

in which N' is the number of floods exceeding y_0 and T_0 is the peirod of observation of discharge or precipitation.

The values of z' that correspond to the exceedance probability p represent the storage capacity z_0 for the risk-level $1/T$.

A plot such as that in Fig. 10 and a value for z_0 can be obtained for each level of y_0; therefore, the same number of points as the number of y_0 levels are plotted on full-log paper for each risk level $1/T$ (see Fig. 11). The value of s is estimated from Fig. 11.

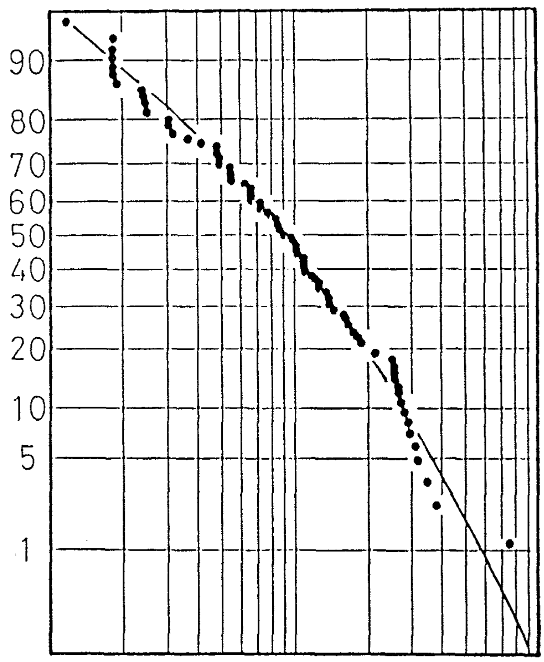

Fig. 10 Example of a frequency analysis using Weibull plot (peak discharge)

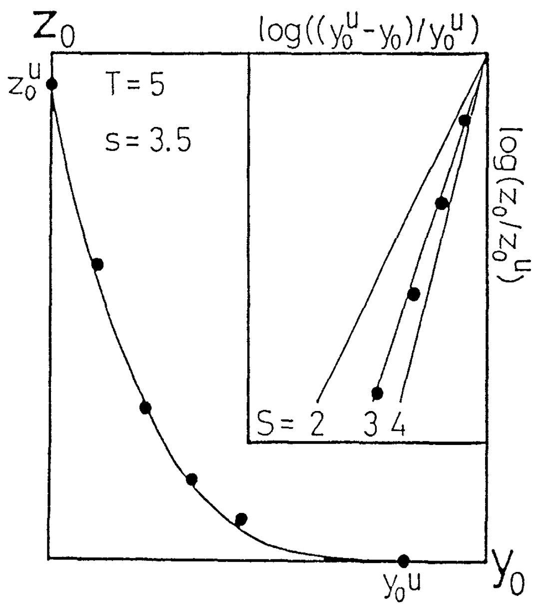

Fig. 11 The estimation of 's'

In the case presented, s ranges from about 3 for $T=5$ to 4 for larger T values. Thus, s is fixed at 3.5 for all T values in the following analysis. Fig. 11 proves the validity of the theory with it's close agreement between the points and eq. (23) (solid line, $s=3.5$).

Equi-Risk Lines

The resulting equi-risk lines are shown in Figs. 12 and 13. Fig. 12 gives the equi-risk lines for the present state of the basin. In the near future, urbanization may encroach on the low-lying paddy fields and hillside forest and cause an increase in the runoff discharge. Therefore, for the future state of the basin, larger facilities should be installed to keep the flood risk within a specified tolerable level, as shown in Fig. 13.

In these figures, the 1st, 2nd and 3rd stages represent the staged capacity expansion of the drainage and storage facilities. For example, point B in Fig. 12 represents the state in which first-stage constructions of both the drainage and storage facilities are complete, and point C the state in which the second-stage capacity expansion of the detention reservoir is complete. A decision-maker can easily ascertain from these figures the shifting of risk-level associated with each capacity expansion, the proper final capacities, and even the preferable sequence of capacity expansions, by jointly considering the economic, environmental and political conditions.

Some Considerations

a. For the Present State of the Basin

1) When the first capacity expansions of the drainage channel and the detention reservoir are completed, the risk level will fall to about 8% per year ($T=13$).
2) To keep the risk level at about 1% ($T=100$), there are two alternatives; expand the drainage channel to its second stage (full capacity) or expand the detention reservoir to its second stage.
3) Expansion of both facilities to their second stages (E) or to their full capacities (D) would be excessive at present.

b. For the Future State of the Basin

1) First-stage expansions are not safe because the values obtained for T are less than 5 years.

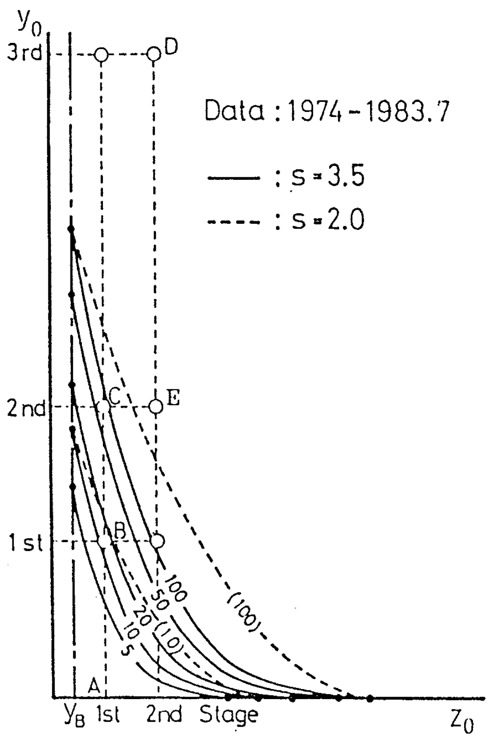

Fig. 12 Equi-risk lines and risk evaluation for the present state of the basin

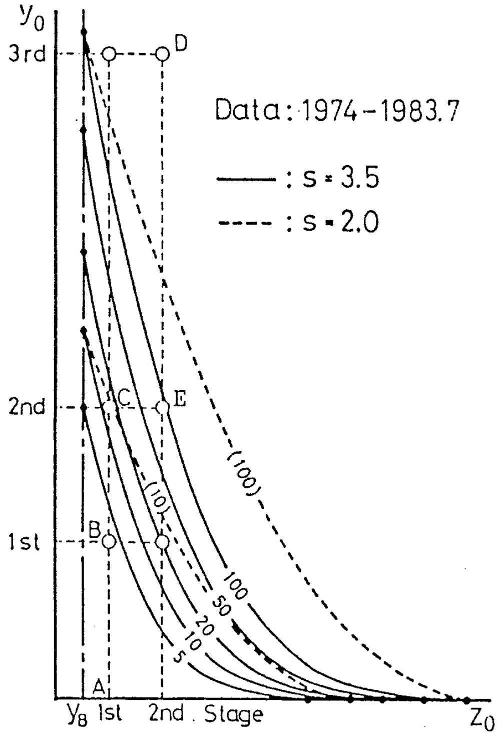

Fig. 13 Equi-risk lines and risk evaluation for the future state of the basin

2) Expansion of both facilities to their second stages (E) must be completed before the basin becomes fully developed.

5. ADDITIONAL COMMENTS

Optimum Capacities

Estimation of construction costs for a drainage facility of capacity y_0 and a storage facility of capacity z_0 is not as difficult as the evaluation of risk. Therefore, equi-cost lines can easily be drawn on the y_0–z_0 plane. Connecting the contact points between the equi-risk and equi-cost lines produces a curve that represents the optimum capacities of the drainage and storage facilities, for the case where flood risk and construction cost are substantial.

Simplest Procedure for Drawing Equi-Risk Lines

The value of s ranges approximately from 2 to 3, and, as in Figs. 8, 12 and 13, $s=2$ gives a safety-side or conservative estimation of an equi-risk line. Therefore, the following simple procedure can be applied to the actual design, for which some allowance is usually required.

1) Estimate y_0^u and z_0^u for a given risk level.
2) Connect $(y_0^u, 0)$ and $(0, z_0^u)$ simply with a quadratic parabola.

Hydrograph Equivalent to an Equi-Risk Line

A hydrograph equivalent to the equi-risk line represented by (23) is shown in Fig. 14. The volume of the area exceeding y_0 in the hydrograph is equal to z_0. In the mathematical expression, the discharge rate y is expressed in terms of time τ, as follows:

$$y = y_0^u - c|\tau|^{1/(s-1)}, \tag{29}$$

in which

$$c = y_0^u(1/\tau_0)^{1/(s-1)},\ |\tau| \leqq \tau_0,\ \tau_0 = s/2 \cdot z_0^u/y_0^u.$$

When $s=2$, the equivalent hydrograph is triangular. When $s=3$, it is represented by two horizontal quadratic parabolas which face each other. Since $s=2$ gives a safety-side approximation, a triangular hydrograph with a peak discharge of y_0^u and a total volume of z_0^u (thus, $\tau_0 = z_0^u/y_0^u$) can be used in practical designs.

Module 2

The authors have derived an approximate expression of an equi-risk line for module 2 in Fig. 3 [2]. The derivation is complicated, but the derived expression is relatively simple. Thus, the final result is presented here. The equi-risk line for point C in Fig. 3 can be approximated by an equirisk line drawn by the following procedure:

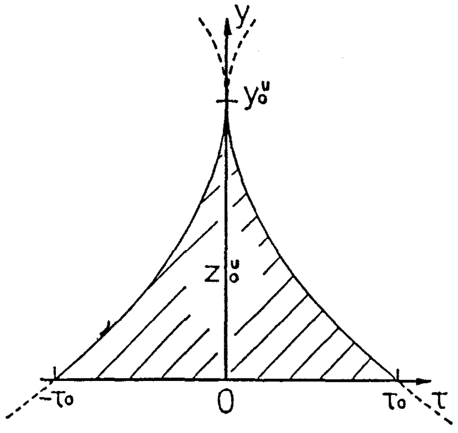

Fig. 14 A hydrograph equivalent to the equi-risk line

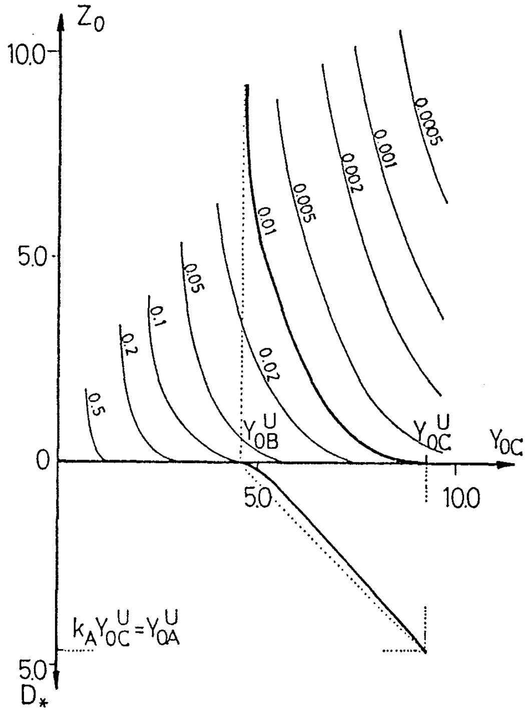

Fig. 15 A typical example of equi-risk lines for Module 2

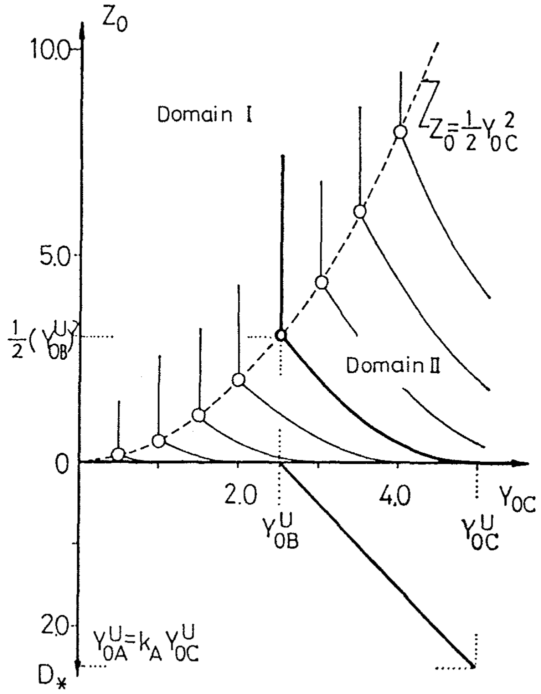

Fig. 16 An example of equi-risk lines (Module 2, linearly dependent x and y): similar to the lines obtained by the simplified practical procedure

1) Estimate y^u_{0A}, y^u_{0B}, y^u_{0C} and z^u_0,

   where $y^u_{0i} = F^{-1}_{yi}(1-\varepsilon/\lambda)$, $z^u_0 = F^{-1}_{zA}(1-\varepsilon/\lambda)$,
   and $i =$ A, B or C.

   F_{yi} and F_{zi} are the c.d.f. of peak discharge and total volume at point i.
2) Calculate $y^{u\prime}_{0B}$ and $z^{u\prime}_0$ by the following equation:

$$(y^{u\prime}_{0B}, z^{u\prime}_0) = S_f(y^u_{0B}, z^u_0) \tag{30}$$

   in which $S_f = 1.0$ to 1.1.
3) Connect $(y^{u\prime}_{0B}, z^{u\prime}_0)$ and $(y^u_{0C}, 0)$ with a quadratic parabola and draw a vertical straight line $y_{0C} = y^{u\prime}_{0B}$ for $z_0 \geqq z^{u\prime}_0$. The resulting equi-risk lines are similar to the ones shown in Fig. 16, whereas equi-risk lines for module 2 generally have the shapes shown in Fig. 15.
4) The optimum release discharge d_* from the storage facility is approximately represented by a straight line connecting the following two points:

$$\begin{cases} d_* = y^u_{0A} \text{ at } y_{0C} = y^u_{0C} \text{ and} \\ d_* = 0 \quad \text{at } y_{0C} = y^{u\prime}_{0B}. \end{cases} \tag{31}$$

Under appropriate conditions, a complex flood control system with many drainage and storage facilities may be lumped into a simplified total system or divided into individual subsystems. Both cases may be considered as module 2. Thus, the extension of the theory to module 2 produces a wider applicability of the theory of the equi-risk line.

6. CONCLUDING REMARKS

The expressions of flood-chance constraint are analytically derived. They are expressed by the equation of the equi-risk line, which represents the functional relation between the capacities of drainage and storage facilities to keep the risk of flooding within certain tolerable levels.

The derived approximate expression of the equation of the equi-risk line is

$$z_0/z_0^u = \{(y_0^u - y_0)/y_0^u\}^s \,. \tag{23}$$

Theoretically, $s=2$ to 3. z_0^u and y_0^u can be estimated easily from the observed peak-discharge and total-volume data.

A practical procedure for estimating s is proposed. The value obtained from the observed data ranges from 1.5 to 4, which is in good agreement with the theoretical prediction. The application of the theory to the design of an actual detention reservoir is presented, which proves the validity of the theory in practical applications.

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